ECE 8540 Analysis of Tracking Systems
Assignment 2

Vivek Koodli Udupa
C12768888

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1 Aim

To derive and develop a nonlinear regression fit for the data set given using 'root finding' method.

2 Objectives

1. Plot the given data.
2. Derive the equation for non-linear regression.
3. Choose an initial value of 'a' and observe its effect on convergence.
4. Find a suitable initial value for 'a' through trial and error method.
5. Plotting the fit

3 Execution

The given task is to fit a function of the form $y = ln(ax)$ where a is the unknown. The equation is non-linear in-terms of the unknown, a. To solve for non-linear terms, we use some variation of gradient decent. All these methods are iterative. They start with an initial approximation and then repeatedly calculate the next guess based on the previous guess. Each of these successive approximations gets closer to the true solution. The iterations are typically stopped when the difference between two successive iterations are below certain threshold. This technique is known as **Non-linear regression.**

3.1 Derivation

we know that,

$$y = ln(ax) \tag{1}$$

$$E = \sum_{i=1}^{N}(y_i - ln(ax_i))^2 \tag{2}$$

$$\frac{\delta E}{\delta a} = \sum_{i=1}^{N}[2(y_i - ln(ax_i))(\frac{1}{a})] = 0 \tag{3}$$

After simplification ,we get

$$\frac{\delta E}{\delta a} = \sum_{i=1}^{N}(\frac{y_i - ln(ax_i)}{a}) \tag{4}$$

$$\therefore f(a) = \sum_{i=1}^{N}(\frac{y_i - ln(ax_i)}{a}) \tag{5}$$

$$\tag{6}$$

The derivative is

$$f'(a) = \sum_{i=1}^{N}[\frac{a\frac{\delta}{\delta a}((ln(ax_i)) - (y_i - ln(ax_i))\frac{\delta}{\delta a}(a)}{a^2}] \tag{7}$$

$$f'(a) = \sum_{i=1}^{N}[\frac{ln(ax_i) - y_i - 1}{a^2}] \tag{8}$$

Now we can find the value of a using the formula

$$a_{n+1} = a_n - \frac{f(a)}{f'(a)} \tag{9}$$

An initial guess of 'a' is taken and the next approximation 'a_{n+1}' is calculated using Equation 9. Using the just obtained value, next approximation is calculated. This process is repeated till we converge to a desirable result.

3.2 Plots and Observations

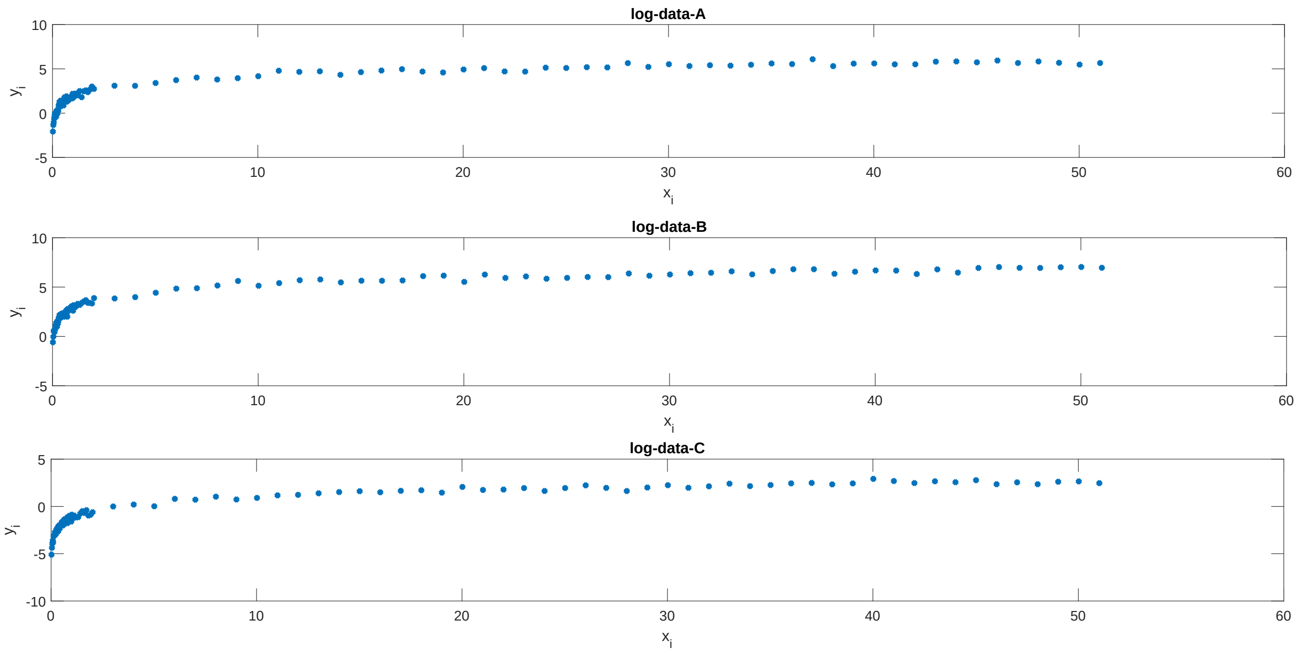

Figure 1: Raw Data plots

Figure 1 is the plot of the raw data that was given. We will use an equation if the form $y = ln(ax)$ to fit the above three data-sets.

Table 1 shows the convergence of the system based on the initial approximation of a. Several trials were conducted with random initial values and Trial 3 looks like a satisfactory solution.

Maximum of 500 iterations were set and the precision (i.e. $abs(a_{n+1} - a)$) set to 0.0001
In Trial 1, the initial approximation was randomly chosen to be 10 for all the three data sets. Data set A converged to 6.7114, Data set B converged to 18.9961. They took 7 and 6 iterations respectively. Whereas Data-set C's calculation was finished by exceeding the MAX iteration and the final value of the unknown was found to be 9.9134e+152, which is very high.

Trials	Data set	Initial 'a' approximation	Final 'a' value	Iterations	MAX iterations
1	A	10	6.7114	7	500
	B	10	18.9961	6	500
	C	10	9.9134e+152	500	500
2	A	5	6.7114	5	500
	B	25	18.9961	6	500
	C	1	3.4673e+152	500	500
3	A	7	6.7114	4	500
	B	20	18.9961	4	500
	C	0.1	0.2899	6	500
4	A	50	8.2663e+153	500	500
	B	70	2.1240e+154	500	500
	C	100	6.6641e+153	500	500

Table 1: Observation of the effects of initial value on convergence

Figure 2 through Figure 5 shows the Non-linear fit drawn using the model $y = ln(ax)$ where a is the unknown , whose value is found is Table 1. Figure 4 seems to be a good fit compared to the others.

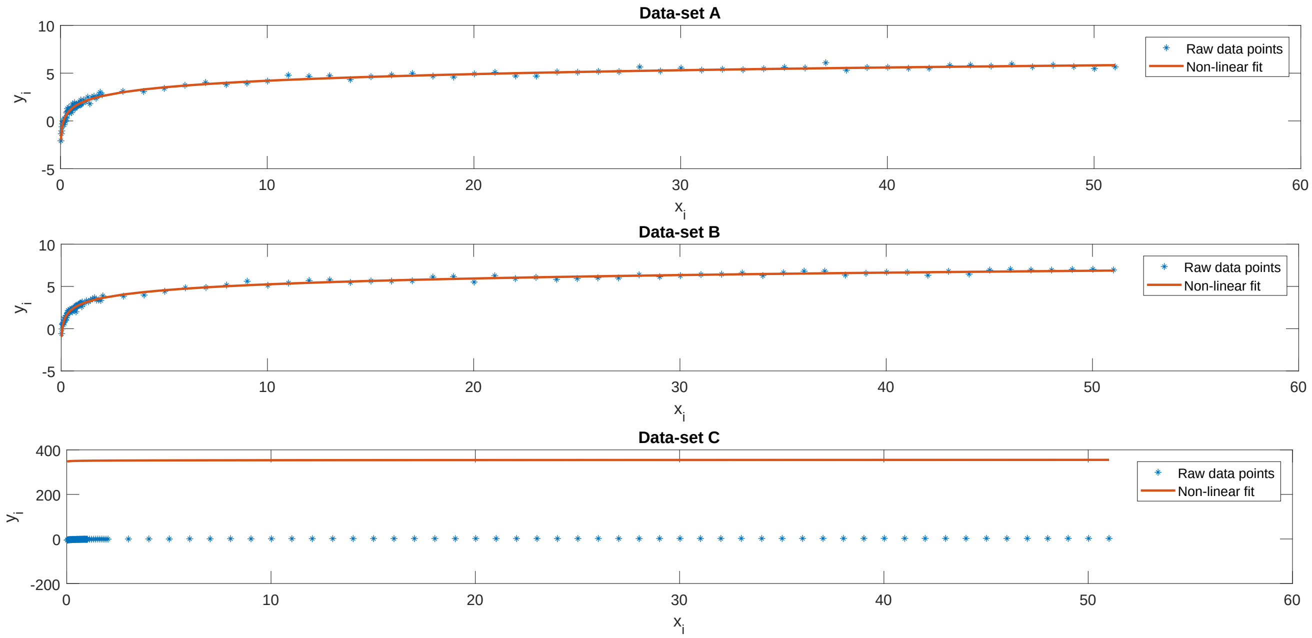

Figure 2: Non-linear fit using the value of unknown obtained in Trial 1

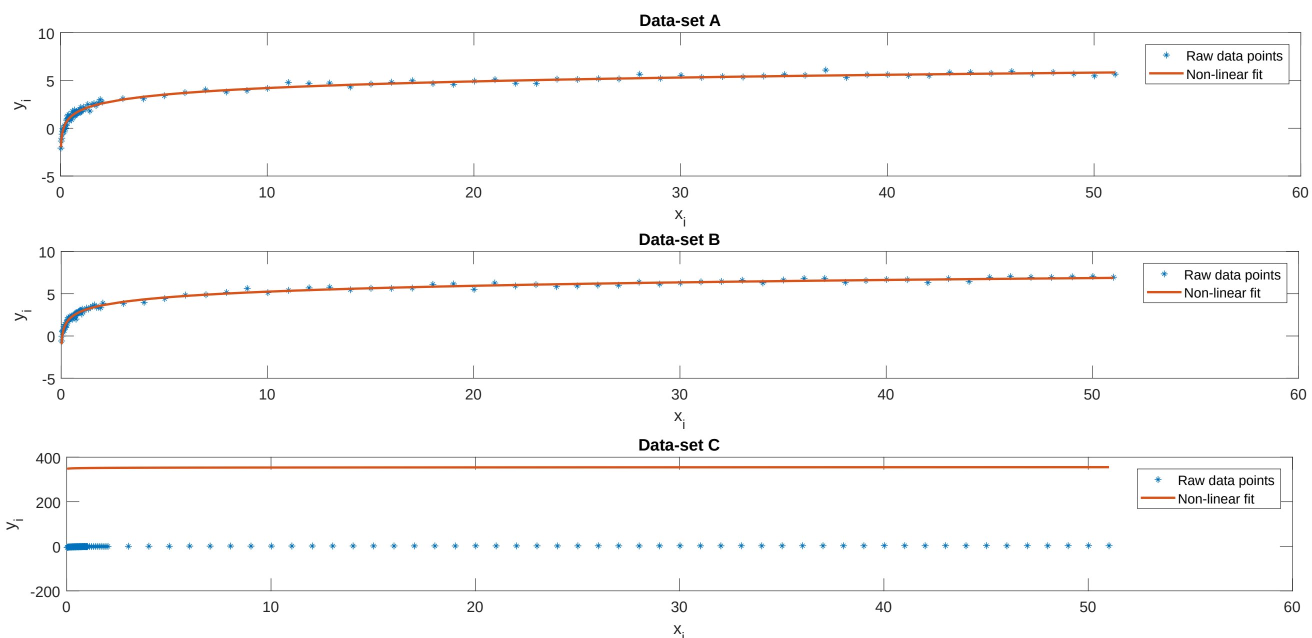

Figure 3: Non-linear fit using the value of unknown obtained in Trial 2

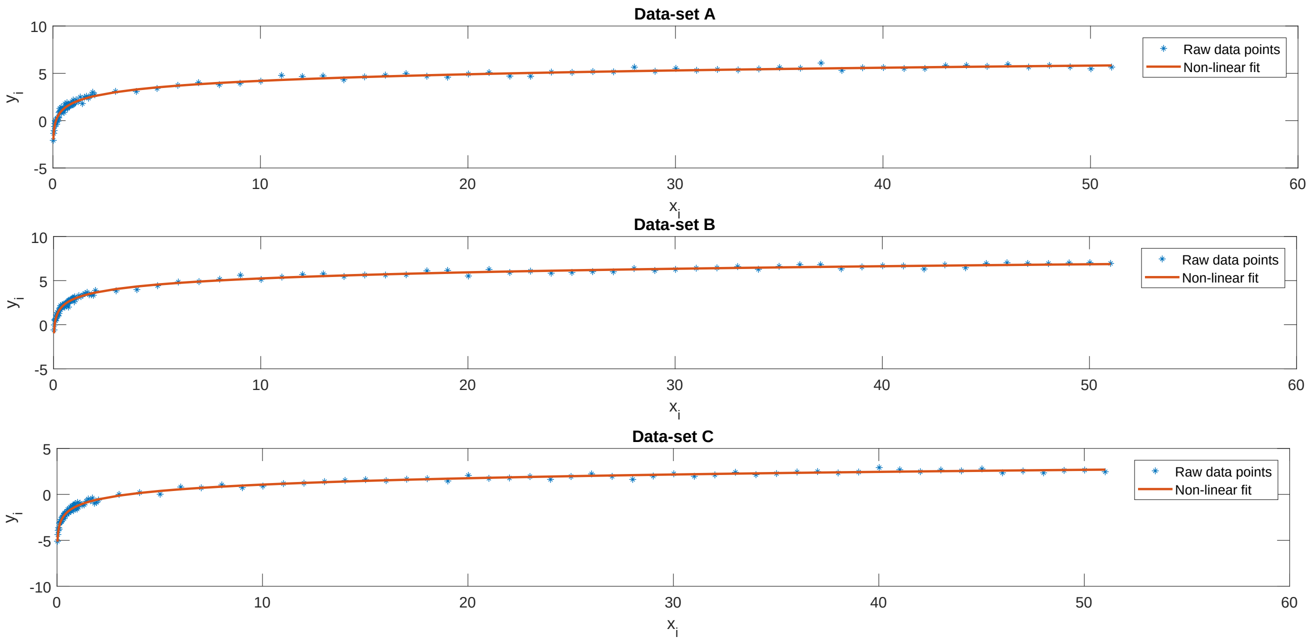

Figure 4: Non-linear fit using the value of unknown obtained in Trial 3

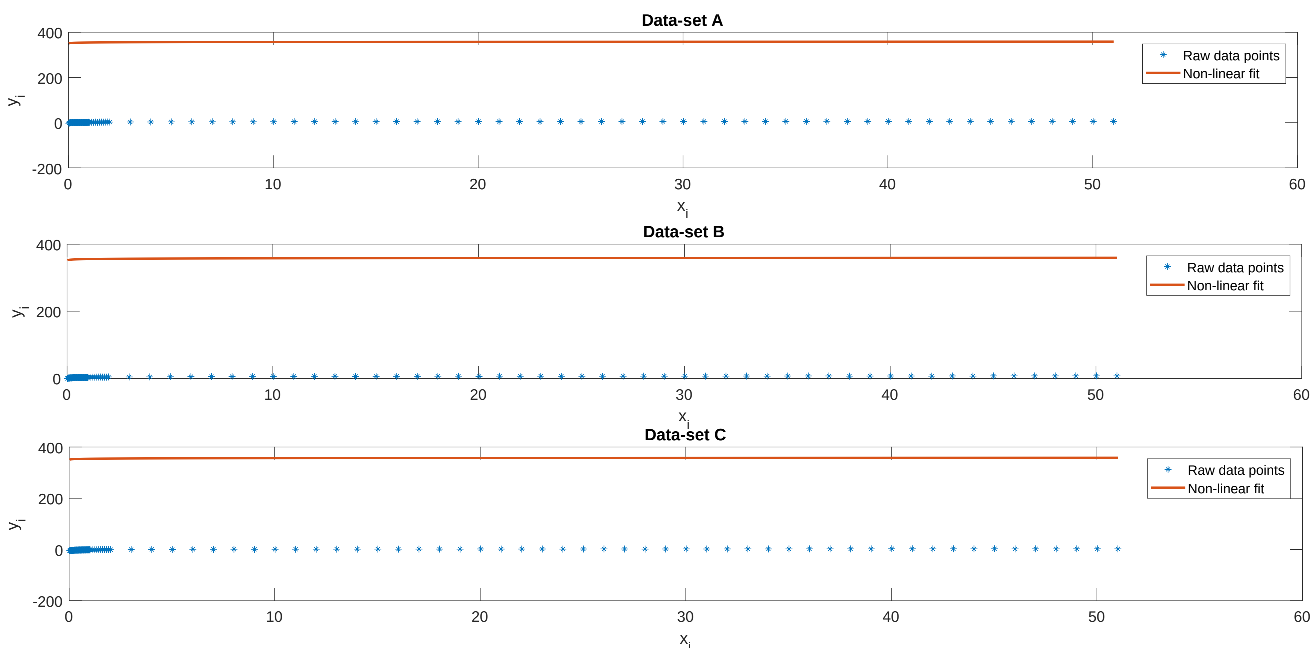

Figure 5: Non-linear fit using the value of unknown obtained in Trial 4

4 Conclusion

The Raw data points were plotted and a Non-linear model $y = ln(ax)$ was chosen for the fit. The values of the unknowns were calculated using the 'root finding' successive iteration method and the results are shown in Table 1.

The value of the unknowns for data-set A, B and C were found to be **a1 = 6.7114**, **a2 = 18.9961** and **a3 = 0.2899** respectively.

All the calculations were performed using MATLAB and the code is listed in the Appendix.

5 Appendix

5.1 MATLAB Code

```
close all
clear
clc

A = dlmread('log-data-A.txt'); %Read table A
B = dlmread('log-data-B.txt'); %Read table B
C = dlmread('log-data-C.txt'); %Read table C


N = length(A(:,1)); %Total number of data points

%initial guesses
% a_init = 3;
% b_init = 25;
% c_init = 0.4;
 a_init = 50;
 b_init = 70;
 c_init = 100;

a1 = find_a(A,a_init,'Data-A')
a2 = find_a(B,b_init,'Data-B')
a3 = find_a(C,c_init,'Data-C')

y1 = log( a1 .* A(:,1) );
y2 = log( a2 .* B(:,1) );
y3 = log( a3 .* C(:,1) );

%Plots

%Raw data plot
% figure(1)
% subplot(3,1,1)
% plot(A(:,1),A(:,2),'x','markersize',5,'linewidth',3);
% xlabel('x_i');
% ylabel('y_i');
% title("log-data-A");
% set(gca,'FontSize',12);
%
% subplot(3,1,2)
% plot(B(:,1),B(:,2),'x','markersize',5,'linewidth',3);
% xlabel('x_i');
% ylabel('y_i');
% title("log-data-B");
% set(gca,'FontSize',12);
%
% subplot(3,1,3)
% plot(C(:,1),C(:,2),'x','markersize',5,'linewidth',3);
% xlabel('x_i');
% ylabel('y_i');
```

```
% title("log-data-C");
% set(gca,'FontSize',12);

%Plot A
figure(2)

subplot(3,1,1)
plot(A(:,1),A(:,2),'*','markersize',5);
hold on;
plot( A(:,1),y1, 'LineWidth',2 );
hold off;
title("Data-set A");
xlabel('x_i');
ylabel('y_i');
legend('Raw data points','Non-linear fit');
set(gca,'FontSize',13)

%Plot B
subplot(3,1,2)
plot(B(:,1),B(:,2),'*','markersize',5);
hold on;
plot( B(:,1),y2,'LineWidth',2 );
hold off;
title("Data-set B");
xlabel('x_i');
ylabel('y_i');
legend('Raw data points','Non-linear fit');
set(gca,'FontSize',13)

%Plot C

subplot(3,1,3)
plot(C(:,1),C(:,2),'*','markersize',5);
hold on;
plot( C(:,1),y3,'LineWidth',2 );
hold off;
title("Data-set C");
xlabel('x_i');
ylabel('y_i');
legend('Raw data points','Non-linear fit');
set(gca,'FontSize',13)
```

```
function a = find_a(Data, initial_guess, Name)

    disp("Executing for ");
    disp(Name);

    %Find the unknown using iritative root finding method
    iti = 500; %Total number of itirations
    a = initial_guess; %initial guess

    x = Data(:,1);  %X co-ordiante values
    y = Data(:,2);  %Y co-ordinate points

```

```
    N = length(x); %Total number of data points

    for j = 1:iti
        F = 0;
        F_dash = 0;
        for i = 1:N
%               F = F + ( ( y(i) - log(a*x(i) ) ) / a ) ;
%               F_dash = F_dash + log( a*x(i) ) - ( y(i) + 1 );
                F = F + ( ( y(i) - log( a * x(i) ) ) / a  );
                F_dash = F_dash + ( ( log( a * x(i) ) - y(i) - 1 ) / ( a*a) );
        end
        an = a - (F / F_dash);
        if( abs(an - a) < 0.0001 )
            break;
        end
        a = an;
    end
    disp("Itirations taken");
    disp(j);


end
```