ECE 8540
Analysis of Tracking Systems
Assignment 4

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October - 09, 2018

1 Introduction

In this report we consider the problem of filtering; Kalman filtering to be specific. Filtering is a method of mitigating noise in sensor data. A set of sensor data is used to design a model. The model can be used to smooth out the noise and to make prediction about expected future states. However, in the case of tracking problem, the thing that is to be tracked is not expected to follow a linear predictable behavior all the time. If the object did follow a predictable path, then there would be point in tracking it. In such a situation, the behavior of the object could just be modeled perfectly and the future behavior is completely known.

For example, Consider the problem of tracking a bullet fired through some kind of gun. Initially the bullet travels straight, but as time progresses, the bullet is slowed down due to gravity, air resistance and multiple other factors. Thus the bullet follows certain path for a particular duration and then changes its path for the next set period of time. In order to track such an object, that changes behavior slowly but continuously, it is necessary to change the model fit continuously. In the extreme this leads to the concept of updating the model fit after every new sensor reading is recorded. This is how filtering works.

In the problem of tracking something, we are trying to find the answer for the question " where is the object? ". The answer to this question is rarely if ever certain in real life applications. The answer is mostly a probability distribution of " the object is likely to be in this area ". This is because of the uncertainties in the sensor measurement, object's behavior and other noises that might persist. In this report we will consider data that contains dynamic and measurement noises and we will use Kalman filtering to generate predictions. Kalman filtering is an algorithm that uses a set of measurements collected over time, that contains statistical noises and other inaccuracies and produces estimates of unknown variables that tend to be more accurate than those based on single measurement alone, by estimating a joint probability distribution over the variables for each time frame.

This report deals with the application of kalman filter to track the position of an object in both 1D and 2D space. This report starts with the visual analysis of dataset, building the kalman filter equations and then finally discussing the results of the derived kalman filter.

2 Methods

Kalman filter is a continuous cycle of predict and update. When formulating a problem for the Kalman filter, we consider the following steps:

1. Determine the state variables. Here we are answering the question "what are we tracking?".

2. Write the state transition equations i.e. How things evolve over time.

3. Define the dynamic noise(s). This describes the uncertainties in state transition equation.

4. Determine the observation variables i.e. Sensor readings.

5. Write the observation equations (relating the sensor readings to the state variables).

6. Define the measurement noise(s). These are the uncertainties in observation variables.

7. Characterize the state transition matrix and observation matrix.

8. Check all matrices in the Kalman filter equations to make sure the sizes are appropriate.

2.1 Deriving the equations for 1D Model

We will begin by describing the things that we are tracking. The state X_t can be defined as:

$$X_t = \begin{bmatrix} x_t \\ \dot{x}_t \end{bmatrix} \tag{1}$$

where x_t is the position and $\dot{x}_t$ is the velocity.

The second step is to write the state transition equations that describe the expected behavior of the state variables. Here, we are considering a constant 1D velocity model. The state transition equations for the model are as follows:

$$X_{t+1} = x_t + T\dot{x}_t$$

$$\tag{2}$$

$$\dot{X}_t = \dot{x}_t$$

The third step is to define the dynamic noise(s). Here we will assume that random accelerations can happen between sensor samples.

$$\text{dynamic noise} = \begin{bmatrix} 0 \\ N(0, \sigma_{a1}^2) \end{bmatrix} \tag{3}$$

Notice that the first element of the matrix is zero. The zero indicates that there is no noise in the position of the reading. Noise in position reading would indicate that the object is teleporting.

The forth step is to list the observation variables. Here, we are sensing the x position. Denoting the sensor reading for this as follows:

$$Y_t = \begin{bmatrix} \tilde{x}_t \\ 0 \end{bmatrix} \tag{4}$$

The second element of the matrix is zero because the velocity is not being measured.

The fifth step is to describe the observation equation.

$$\tilde{x}_t = x_t \tag{5}$$

The sixth step is to define measurement noise(s). Here, we will assume that the sensor reading is corrupted by noise.

$$\text{measurement noise} = \begin{bmatrix} N(0, \sigma_{n1}^2) \end{bmatrix} \tag{6}$$

The seventh step is to characterize the co-variance matrices. There are three of them. The co-variance of the state variables is written as:

$$s_t = \begin{bmatrix} \sigma_{x_t}^2 & \sigma_{x_t, \dot{x}_t} \\ \sigma_{x_t, \dot{x}_t} & \sigma_{\dot{x}_t}^2 \end{bmatrix} \tag{7}$$

Co-variance of dynamic noise can be written as:

$$Q = \begin{bmatrix} 0 & 0 \\ 0 & \sigma_{a_1}^2 \end{bmatrix} \tag{8}$$

Co-variance of measurement noise can be written as:

$$R = \begin{bmatrix} \sigma_{n_1}^2 & 0 \\ 0 & 0 \end{bmatrix} \tag{9}$$

The eighth step is to define the state transition and observation matrices. The state transition matrix can be obtained from equation 2.

$$\phi = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \tag{10}$$

The observation matrix can be obtained from equation 4.

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \tag{11}$$

Note that we have just finished formulating the 1D model for filtering problem. This is just a theoretical model of how we expect the system being tracked to behave. Implementation will be explained in section 2.3

2.2 Deriving the equations for 2D Model

Similar to the derivation shown in section 2.1 this section will show the derivation to a constant 2D velocity model.

We will begin by describing the things that we are tracking. The state X_t can be defined as:

$$X_t = \begin{bmatrix} x_t \\ y_t \\ \dot{x}_t \\ \dot{y}_t \end{bmatrix} \tag{12}$$

where x_t and y_t is the position and $\dot{x}_t$ and $\dot{y}_t$ is the velocity.

The second step is to write the state transition equations that describe the expected behavior

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of the state variables. Here, we are considering a constant 2D velocity model. The state transition equations for the model are as follows:

$$X_{t+1} = x_t + T\dot{x}_t$$

$$Y_{t+1} = y_t + T\dot{y}_t$$

$$\dot{X}_t = \dot{x}_t$$ (13)

$$\dot{Y}_t = \dot{y}_t$$

The third step is to define the dynamic noise(s). Here we will assume that random accelerations can happen between sensor samples.

$$\text{dynamic noise} = \begin{bmatrix} 0 \\ 0 \\ N(0, \sigma_{a1}^2) \\ N(0, \sigma_{a2}^2) \end{bmatrix}$$ (14)

The forth step is to list the observation variables. Here, we are sensing the x and y positions. Denoting the sensor reading for this as follows:

$$Y_t = \begin{bmatrix} \tilde{x}_t \\ \tilde{y}_t \end{bmatrix}$$ (15)

The fifth step is to describe the observation equation.

$$\tilde{x}_t = x_t$$
$$\tilde{y}_t = y_t$$ (16)

The sixth step is to define measurement noise(s). Here, we will assume that the sensor reading is corrupted by noise.

$$\text{measurement noise} = \begin{bmatrix} N(0, \sigma_{n1}^2) \\ N(0, \sigma_{n2}^2) \end{bmatrix}$$ (17)

The seventh step is to characterize the co-variance matrices. There are three of them.
The co-variance of the state variables is written as:

$$s_t = \begin{bmatrix} \sigma_{x_t}^2 & \sigma_{x_t,y_t} & \sigma_{x_t,\dot{x}_t} & \sigma_{x_t,\dot{y}_t} \\ \sigma_{x_t,y_t} & \sigma_{y_t}^2 & \sigma_{y_t,\dot{x}_t} & \sigma_{y_t,\dot{y}_t} \\ \sigma_{x_t,\dot{x}_t} & \sigma_{y_t,\dot{x}_t} & \sigma_{\dot{x}_t}^2 & \sigma_{\dot{x}_t,\dot{y}_t} \\ \sigma_{x_t,\dot{y}_t} & \sigma_{y_t,\dot{y}_t} & \sigma_{\dot{x}_t,\dot{y}_t} & \sigma_{\dot{y}_t}^2 \end{bmatrix}$$ (18)

Co-variance of dynamic noise can be written as:

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{a_1}^2 & \sigma_{a_1,a_2} \\ 0 & 0 & \sigma_{a_1,a_2} & \sigma_{a_2}^2 \end{bmatrix} \tag{19}$$

Co-variance of measurement noise can be written as:

$$R = \begin{bmatrix} \sigma_{n_1}^2 & \sigma_{n_1,n_2} \\ \sigma_{n_1,n_2} & \sigma_{n_2}^2 \end{bmatrix} \tag{20}$$

The eighth step is to define the state transition and observation matrices. The state transition matrix can be obtained from equation 13.

$$\phi = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{21}$$

The observation matrix can be obtained from equation 15.

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{22}$$

Note that we have just finished formulating the 2D model for filtering problem. This is just a theoretical model of how we expect the system being tracked to behave. Implementation will be explained in section 2.3

2.3   Implementation

3   Results

4   Conclusion

5   Appendix