Stock Portfolio Optimization

A
Project Report

submitted for partial fulfillment

Bachelor of Technology degree
in
Computer Science and Engineering

By

Vivek Bhardwaj (1805210063)
Yashraj Singh Bhadauria (1805210066)
Umang Dubey (1805210060)

Under the supervision of
Prof. Maheshwari Tripathi &
Dr. Promila Bahadur

Department of Computer Science and Engineering
Institute of Engineering and Technology, Lucknow
Dr. A.P.J. Abdul Kalam Technical University, Lucknow, Uttar Pradesh.
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Submitted by: Date: 25 May 2022

(1) Name: Vivek Bhardwaj
Roll No : 1805210063
Branch: Computer Science & Engineering

Signature: Vivek Bhardwaj

(2) Name: Yashraj Singh Bhadauria
Roll No : 1805210066
Branch: Computer Science & Engineering
Signature:

(3) Name:Umang Dubey
Roll No : 1805210060
Branch: Computer Science & Engineering
Signature Umang

Certificate

This is to certify that the project report entitled "Stock Portfolio Optimization" presented by Vivek Bhardwaj, Yashraj Singh Bhadauria and Umang Dubey in the partial fulfillment for the award of Bachelor of Technology in Computer Science and Engineering, is a record of work carried out by them under our supervision and guidance at the Department of Computer Science and Engineering at Institute of Engineering and Technology, Lucknow.

It is also certified that this project has not been submitted at any other Institute for the award of any other degrees to the best of my knowledge.

Prof. Maheshwari Tripathi
Department of Computer Science and Engineering
Institute of Engineering and Technology, Lucknow

Dr. Promila Bahadur
Department of Computer Science and Engineering
Institute of Engineering and Technology, Lucknow

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Yashraj Singh Bhadauria
Roll No : 1805210066
Signature : Yashraj

Vivek Bhardwaj
Roll No : 1805210063
Signature : Vivek Bhardwaj

Umang Dubey
Roll No : 1805210060
Signature : Umang

Abstract

The Stock Portfolio Optimisation aimed at solving the problem of a retail investor who wants to invest his/her capital in a way to get expected returns with an acceptable amount of risk. It's always better to keep a diversification in the portfolio then relying on a single peg. The project aims at suggesting alternatives to the previous methods of diversification like 60:40 bond-equity split, equal distribution of funds etc.

We have done a comparative analysis of existing algorithms of Mean Variance Optimisation, Risk Parity, Hierarchical Risk Parity while capturing the shortcomings of all and the time when they shine. The idea is to avoid following a single algorithm for all users and to do personalisation in selecting the best way which matches with the risk profile of the user.

The main focus is on the newly introduced Hierarchical Risk Parity algorithm which distributes the risk among the assets and tries to establish better distribution based on their degree of correlation.

The project also aims to provide users with comparative analysis of the past trends in the portfolio value variation according to different weight allocation methods available which augments the understanding of the investor to make better decisions.

This report consists of experimental data with various combinations of stocks from multiple sectors analyzed on the performance in a way to see the adaptability of the portfolio to random shocks in the market. A graphical representation of the portfolio trends and distribution matrix is provided to the user for choice.

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Introduction

The practice of investment management has been transformed in recent years and in computational investment the most common financial issue is portfolio creation or portfolio optimization. Investment managers must develop portfolios on a daily basis that integrate their risk and return opinions and forecasts. This might be the basic address that a 24 year old Harry Markowitz endeavored to reply more than 6 decades back. His momentous knowledge was to understand that different levels of risk correspond to different optimal portfolios in terms of risk-adjusted returns., thus the idea of “efficient frontier”. One suggestion was that it is once in a while ideal to designate all resources to the ventures with most noteworthy anticipated returns. Instep, we ought to take under consideration the relationship over elective speculations in order to construct a diversified portfolio.

Markowitz left university to work for the RAND Corporation, where he invented the Critical Line Algorithm(CLA), before getting his Ph.D. in 1954.CLA is a quadratic optimization algorithm intended specifically for portfolio optimization problems with inequality constraints. Bailey and López de Prado provide a description and open-source implementation of this approach. Surprisingly, most budgetary experts appear to be unaware of CLA, relying instead on general-purpose quadratic programming strategies that do not guarantee the correct arrangement or a halting time.

Despite Markowitz's brilliant theory, CLA solutions are rather dependable due to a number of practical issues. Small variations in anticipated returns cause CLA to generate quite diverse portfolios, which is a big concern. Even with a tiny input, the deviation is rather substantial. Many authors have chosen to ignore them entirely and concentrate on the covariance matrix. Risk-based asset allocation schemes, such as "risk parity," have resulted as a result of this. However, lowering the return estimates improves but does not eliminate the instability issues. Quadratic programming approaches necessitate the "inversion of a positive-definite covariance matrix (all eigenvalues must be positive)." When the covariance matrix is numerically ill-conditioned, i.e. has a high condition number, this inversion is prone to big mistakes.

The Black-literman model begins from an impartial position utilizing advanced portfolio hypothesis (MPT), and after that takes extra input from investors' sees to decide how the extreme resource allotment ought to veer off from the starting portfolio weights. It at that point experiences a handle of mean-variance optimization (MVO) to maximize anticipated return given one's objective hazard tolerance. The MPT model is said to be limited because it only uses historical market data and then assumes the same returns in the future.
The Black-literman approach allows the investor to apply their own viewpoints before optimizing the asset allocation recommendation.

Optimal capital allotment across resources is seemingly one of the most broadly examined points in quantitative money. Markowitz looked to probability and statistics to further his insights; Markowitz devised a method that allows an investor to trade off risk tolerance and reward expectations analytically, resulting in the optimum portfolio that optimizes return while reducing risk through diversification. Due to its intuitive appeal and theoretical feature as the pareto-optimal in-sample allocation (Kolm, Tütüncü, and Fabozzi 2014), mean-variance optimization (MVO) is the foundation of most applied portfolio optimization algorithms.

Despite its popularity, there are nevertheless a number of criticisms of MVO that are worth highlighting :

A. GIGO (garbage in,garbage out)
B. Allocations to specific asset classes
C. Risk diversification.

Since the global financial crisis in 2008 risk management has become more important, risk parity portfolio design tries to allocate the stocks on the basis of risk associated with them instead of considering them as a single portfolio entity.

Sensitivity to measurement error, which leads to poor out-of-sample performance, and covariance matrix instability are also key practical concerns. Due to MVO's sample dependence, solely risk-based allocation methods have emerged, which ignore error-prone return estimates. In large N portfolios, risk parity and minimum variance approaches exemplify this strategy, however they are often prone to the instability-critique. CLA has a number of disadvantages (inversion of covariance matrix- the inverse of covariance matrix can change significantly for small changes in portfolio, dependency on the estimation of stock-returns , consider correlations between all the assets and leads to very large dependency graph and not all assets are related to each other).

All of these drawbacks render CLA and other similar allocation algorithms inappropriate for real applications, which is where Hierarchical Risk Parity (HRP) comes in, since it attempts to address and improve on the aforementioned issues. There are three major phases to Hierarchical Risk Parity: -

A. Hierarchical Tree Clustering
B. Matrix Seriation
C. Recursive Bisection

The main idea of HRP is to allocate weights to a portfolio of securities based on

1) the clusters formed by securities (determined on how each security correlates to the portfolio)
2) the volatility of each cluster (more volatile clusters receive lesser weighting, and vice versa)

Hierarchical clustering is used to place our assets into clusters suggested by the data and not by previously defined metrics. This ensures that the assets in a specific cluster maintain similarity. The objective of this step is to build a hierarchical tree in which our assets are all clustered on different levels

A major source of quadratic optimizers' instability: A complete graph with 1/2N(N-1) edges is coupled with a matrix of size N. Because there are so many edges linking the graph's nodes, weights can rebalance with perfect freedom. Because there is no hierarchical structure, modest estimation errors will result in completely different answers. HRP uses a tree structure instead of a covariance structure to achieve three goals: It fully leverages the information included in the covariance matrix, b) weights' stability is recovered, and c) the solution is intuitive by construction, unlike standard risk parity approaches. In deterministic logarithmic time, the method converges.

HRP is robust, visible, and adaptive, allowing the client to present constraints or alter the tree structure without jeopardizing the algorithm's appearance. These characteristics are derived from the fact that HRP does not require invertibility of covariance. HRP can undoubtedly compute a portfolio using an ill-degenerated or even a single covariance framework, which is a remarkable feat for quadratic optimizers. "Monte Carlo tests appear to suggest that HRP has a lower out-of-sample change than CLA or traditional chance equality techniques," says the study.

A single optimizer/approach cannot be utilized for every set of assets, every set is unique in itself and markets are complex , so investors need to cautiously do the stock allocation in order to survive in the market.It is naive to think that one method is best for everything until the end of time. Monte Carlo simulation helps in quickly comparing a variety of optimization methods to find which is most robust in your particular case.

After calculating the expected returns and covariance matrix the data is fed to the simulator which inturn calculates the optimized portfolio using a variety of algorithms. Large number of simulated inputs for this purpose and optimized portfolios are created for all of them.

These optimized portfolios on simulated inputs are error estimated with optimized portfolios using original inputs and various insights are acquired using the same.The insights are presented in a graphical format.

Literature review

2.1 Diversification of portfolio

2.1.1 Introduction

Diversification is a portfolio allocation strategy that aims to minimize idiosyncratic risk by holding assets that are not perfectly positively correlated. Correlation is simply the relationship that two variables share, and it is measured using the correlation coefficient, which lies between $-1 \leq \rho \leq 1$.

<u>The key to a diversified portfolio is holding assets that are not perfectly positively correlated.</u>

i) The risk associated with the portfolio is lower or negligible if it’s diversified. It is because any loss in one asset is likely to be offset by a gain in another asset (which is negatively correlated).

ii) Systematic chance alludes to the chance that's common to the whole showcase, not at all like a peculiar hazard, which is particular to each resource. Enhancement cannot lower efficient chances since all resources carry this risk.

2.2 Markowitz’s Modern Portfolio theory

Harry Markowtiz gave the idea of Portfolio Selection based on diversification in his Paper “Portfolio Selection”. Investors generally consider expected return a desirable thing while the variance is undesirable. Generally high return portfolios are associated with higher risks and low risks are associated with lower return portfolios but Markowitz suggested a way that can help to achieve an optimal stage of diversification.

2.2.1 Acceptable risk:

Investors prefer a less risky portfolio compared to a riskier one. An investor can analyze the risk associated with a portfolio based on statistical measures like variance and standard deviation.

2.2.2 Efficient frontier:

On a coordinate plane, the efficient frontier rates portfolios. The risk is shown on the x-axis, while the return is shown on the y-axis—annualized standard deviation is used to evaluate risk, while compound annual growth rate (CAGR) is used to assess return.

The investor would choose securities that are at the efficient frontier's right end. Securities near the right end of the efficient frontier are predicted to have a high degree of risk with a high potential return, making them suited for risk-averse investors.

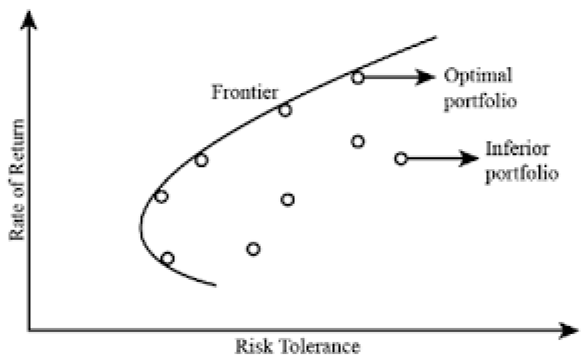

Fig 2.1:- Efficient frontier with Risk tolerance to risk - return

2.2.3 Sharpe Ratio (SR):
(*Expected return per unit of risk*)
It compares the performance of a portfolio with risk free returns. Any value greater than 1 is considered to be good by the investors.

$$S_a = \frac{E[R_a - R_b]}{\sigma_a}$$

2.3 Risk Parity

2.3.1 Post Modern Portfolio Theory

The typical application of Modern Portfolio Theory (MPT) combines asset classes based on their projected returns, risks, and correlations, and then determines the best managers in each asset class once the asset allocation mix is defined. PMPT, on the other hand, is distinct in three ways: first, alpha and beta returns are separated; second, the sizes of alpha and beta are changed to more acceptable values; and third, considerably more diversified portfolios of each are produced. As a result, a PMPT portfolio will not only have returns and risks that are more tailored to the investor's goals, but it will also be far more diversified than a standard portfolio.

2.3.2 Weighting based on individual risks.

Advocates of the Risk Parity approach suggest that equally weighting asset classes by their risk (volatility) contribution to the portfolio would be a more efficient way to asset allocation. This method effectively provides the same volatility risk budget to each asset class; in other words, under the Risk Parity weighting scheme, each asset class contributes roughly the same predicted fluctuation in the portfolio's dollar value. Under the Markowitz framework, Risk Parity weighting could be seen as ideal if all asset classes have about the same Sharpe Ratios and correlations.

Investors do not need to create expected return assumptions to form portfolios, which is a major advantage of Risk Parity weighing over mean–variance optimization. Only asset class covariances must be specified, which can usually be predicted more precisely than expected returns based on historical data (Merton, 1980). Certainly, covariance estimates can influence portfolio allocation; nevertheless, it's unclear whether low-quality covariance estimates would skew portfolio returns downward.

When asset allocations are modified to the same risk level, the portfolio can achieve a better Sharpe ratio and be more robust to market downturns, according to the risk parity strategy.
The risk parity portfolio aims to confine each asset (or asset class, such as bonds, equities, real estate, etc.) to contribute equally to the portfolio total volatility.
Add formulas for relative risks

2.4 Hierarchical Risk Parity

The risk-based portfolio optimisation method Hierarchical Risk Parity (HRP) has been proven to construct diversified portfolios with robust out-of-sample features without the use of a positive-definite return covariance matrix (Lopez de Prado 2016).

2.4.1 Improvements from the existing algorithms.

Hierarchical risk parity is a random stock market shock by removing the rigorous analytical approach for calculating weights and instead relying on a more approximate machine learning-based approach (hierarchical tree clustering). On the other hand, it produces stable weights. In addition, older algorithms such as CLA perform inversion of the covariance matrix. This is a very unstable operation and tends to have a large impact on performance with small changes in the covariance matrix. By completely removing the dependence on the inversion of the covariance matrix, the hierarchical risk parity algorithm is fast, robust, and flexible.

In fact, HRP can compute a portfolio based on a singular covariance matrix, an impossible feat for quadratic optimizers. The algorithm operates in three stages: Tree clustering, quasi-diagonalization and recursive bisection.

2.4.2 Three steps of Algorithm.

Tree Clustering stage is characterized by breaking the portfolio into various hierarchical clusters.Here we calculate the tree clusters based on a matrix mapping time series data with the number of stocks in the portfolio. The aim is to create a correlation distance matrix using Euclidean Distance.

The Quasi Diagonalisation stage refers to the matrix serialization method. It arranges the stock covariance matrix in a way that higher covariance is present at the diagonal and lower covariance at off diagonal elements.

Recursive Bisection stage is characterized by assigning the weights to the large cluster in a top down manner by recursively distributing the weights between the children.

2.4.3 Conclusion:

We get to know that relying on an approximate machine learning based approach (hierarchical tree-clustering) instead of fully analytical approach Hierarchical Risk Parity produces weights which are stable to random shocks in the stock-market.

2.5 Monte Carlo Simulation

Since portfolio optimization decisions via algorithms are based on calculated inputs which might be unstable and might even underperform compared to a naive equal allocation between assets a few times. Many times the blind allocation of assets using any algorithm without a deep analysis can lead to losses , just for those cases Monte Carlo Simulation is a tool that comes to rescue.

2.5.1 Multiple Simulation to predict better

Monte Carlo Simulation is used to predict the probability of various outcomes of an event. It involves simulating input values using random seed to achieve an average of multiple results being generated.

2.5.2 Five Step process

MCOS consists of five steps, as outlined below. The input variables are the array of expected outcomes (μ), and the covariance matrix of the expected outcomes (V). These variables may incorporate priors, following the Black-Litterman method or similar.

2.5.3 Provides Comparative Analysis

Estimation of error between portfolio created using original inputs and simulated inputs over various algorithms using this approach presents a holistic comparison between various optimizers for a particular set of assets on which optimization is to be performed.

Methodology

3.1 Markowitz - Mean Variance Optimization Method

3.1.1 Overview

The foundation of Modern Portfolio Theory ("MPT") was established in 1952 by Harry Markowitz. Later he was awarded a Nobel Prize for his work. The Main factor of the MPT theory is diversification. Most speculations are either tall hazard and tall return or low chance and low return. Markowitz contended that financial backers could accomplish their best outcomes by picking an ideal blend of the two in view of an assessment of their individual tolerance to risk. The present day portfolio hypothesis (MPT) could be a commonsense strategy for selecting speculations in order to maximize their generally returns inside an satisfactory level of risk

Key Points :

The modern portfolio theory (MPT) is a method that can be used by risk-averse investors to construct diversified portfolios that maximize their returns without unacceptable levels of risk.

I. The advanced portfolio hypothesis can be valuable to speculators attempting to develop productive and expanded portfolios utilizing ETFs.
II. Those Users/Investors who are more concerned with drawback chance might incline toward the post-modern portfolio hypothesis (PMPT) to MPT.

The present day portfolio hypothesis can be utilized to differentiate a portfolio in order to induce a distant better return in general without a greater risk. Another advantage of the cutting edge portfolio hypothesis (and of diversification) is that it can decrease instability. perfect way. The most perfect way to do that's to choose assets that have a negative relationship, such as U.S. treasuries and small-cap stocks. Ultimately, the objective of the present day portfolio hypothesis is to form the foremost effective portfolio possible.

Modern Portfolio Theory implements the main idea of diverseness - owing a portfolio of resources from distinctive classes is less unsafe than holding a portfolio of comparative assets.
There are two types of risk associated with assets:-
i) Unsystematic risk/idiosyncratic risk
ii) Systematic risk

3.1.2 Introduction

A Markowitz model uses the correlation between the stocks to predict an optimal allocation of weight.

3.1.3 Portfolio Frontier

A portfolio frontier, also called an efficient frontier, is a set of portfolios that maximizes expected returns for each degree of standard deviation, according to MPT (risk).

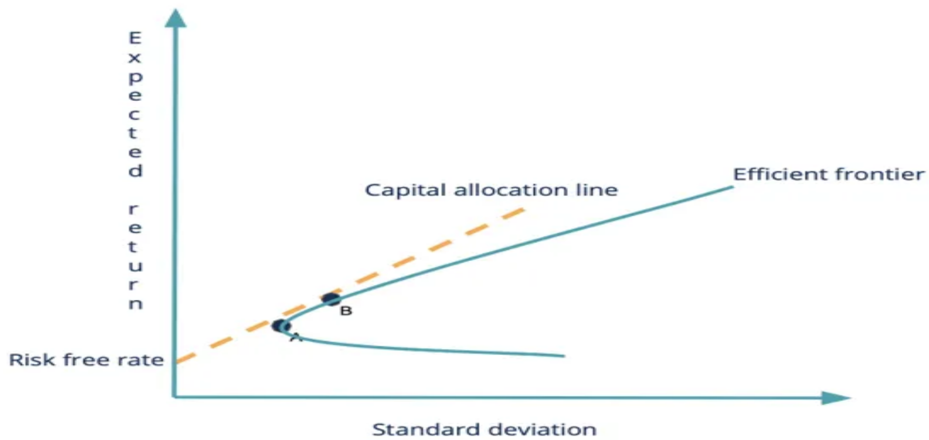

Fig 3.1: - Capital Assignment Line for asset allocation

The efficient frontier graphically demonstrates the value of diversification. The efficient frontier's curvature demonstrates how diversification can improve a portfolio's risk vs return profile.

The "efficient frontier" – the mix of risky assets that maximizes expected return for a given degree of standard deviation – is found in the upper portion of the bend (from point A onwards). As a result, any portfolio on this side of the curve gives the best possible expected returns for a given degree of risk.

I. Point “A” on the productive wilderness is the least fluctuation portfolio – the combination of risky-assets that minimizes standard deviation/risk.
II. Point “B” is the ideal advertised portfolio, which comprises at slightest one risk-free asset. It is delineated by the line that's digression to the effective wilderness, which is additionally called the Capital Assignment Line (CAL).

3.1.4 Expected Returns

The anticipated return of a portfolio is the anticipated esteem of the likelihood conveyance of the conceivable returns it can give to investors.

Consider a speculator holding a portfolio with $4,000 contributed in Resource Z and $1,000 contributed in Resource Y. The anticipated return on Z is 10% ,and the anticipated return on Y is 3%. The anticipated return of the portfolio is:

E(R) = [(τ Σ)-1 + PT ΩP]-1 [(τ Σ)-1 Π + PT ΩQ] , t = scalar number denoting the uncertainty of CAPM distribution.

Expected Return = [($4,000/$5,000) * 10%] + [($1,000/$5,000) * 3%] = [0.8 * 10%] + [0.2 * 3%] = 8.6%

3.1.5 Extensions

Since MPT's introduction in 1952, many attempts have been made to improve the model, especially by using more realistic assumptions.

Post-modern portfolio theory extends MPT by adopting non-normally distributed, asymmetric, and fat-tailed measures of risk. This helps with some of these problems, but not others.

Black–Litterman model optimization is an extension of unconstrained Markowitz optimization that incorporates relative and absolute 'views' on inputs of risk and returns from.

3.1.5.1 Black-Litterman Model

The Black-Litterman (BL) Show is an expository instrument utilized by portfolio supervisors to optimize resource assignment inside an investor's hazard resilience and advertise sees. Worldwide financial specialists, such as annuity stores and protection companies, have to choose how to designate their ventures over distinctive resource classes and nations.

The Black-Litterman Demonstrate is utilized to decide ideal resource allotment in a portfolio Black-Litterman Show takes the Markowitz Show one step further Incorporates an investor's possess sees in deciding resource assignments.

Key Points :

- Resource returns are ordinarily distributed Different disseminations may well be used, but using typical is the simplest
- Variance of the earlier and the conditional dispersions approximately the genuine cruel are known Actual genuine cruel returns are not known

Advantages & Disadvantages :

Advantages :
- Investor's can embed their view
- Control over the certainty level of views
- More natural translation, less extraordinary shifts in portfolio weights

Disadvantages :
- Black-Litterman demonstrate does not provide the leading conceivable portfolio, just the leading portfolio given the sees stated
- As with any show, delicate to assumptions Model accept that sees are autonomous of each other

3.2 Equal Risk Parity Based Stock Allocation

3.2.1 Overview

Markowitz's portfolio has been intensely criticized for over half a century and has never been completely grasped by professionals, among numerous reasons because:
- it as it were considers the chance of the portfolio as a entire and overlooks the chance broadening (i.e., concentrates as well much chance in few resources, which was watched within the 2008 money related crisis)
- it is exceedingly delicate to the estimation blunders within the parameters (i.e., little estimation blunders within the parameters may alter the planned portfolio drastically).

Although portfolio administration did not alter much amid the 40 a long time after the seminal works of Markowitz and Sharpe, the advancement of hazard budgeting procedures checked an vital turning point in developing the relationship between chance and resource administration.

Risk management has been more important than execution administration in portfolio optimization since the global financial crisis of 2008. Without a doubt, risk equality got to be a well known budgetary show after the worldwide money related emergency in 2008.

The alternative risk parity portfolio design has gotten a lot of interest from both the theoretical and practical sides because it: (1) diversifies risk among assets rather than capital, and (2) is less susceptible to parameter estimate mistakes.

This approach is now being used by pension funds and institutional investors to construct intelligent indexing and redefine long-term investment objectives.

3.2.2 Introduction

In this approach of optimization of portfolio the centered part is Allocation of Risk rather tha Allocation of Assets. When asset allocations are modified to the same risk level, the portfolio can achieve a better Sharpe ratio and be more robust to market downturns, according to the risk parity strategy.

While the minimum variance portfolio aims to reduce variance (with the drawback that a few assets may contribute the most to risk), the risk parity portfolio aims to ensure that each asset (or asset class, such as bonds, stocks, real estate, and so on) contributes equally to the overall volatility of the portfolio.

The risk parity portfolio (RPP), also known as the equal risk portfolio (ERP), aims to "equalize" risk by ensuring that each asset's risk contribution is equal, rather than just having an equal capital allocation like the equally weighted portfolio (EWP):

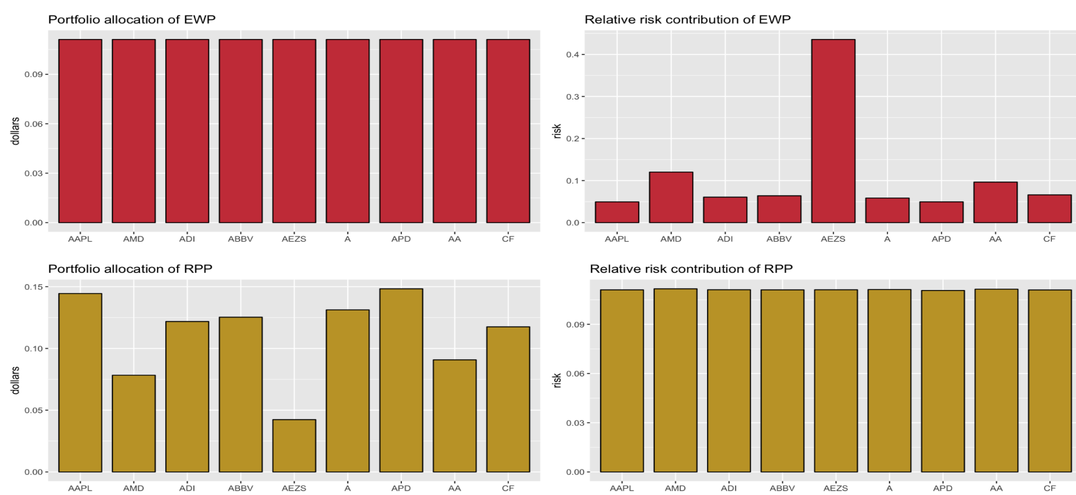

Fig 3.2 :- Comparison of EWP and RPP for portfolio Asset Allocation

3.2.3 Risk Parity Portfolio

A risk parity portfolio aims to achieve a balanced risk profile across all asset classes and portfolio components. Lower risk asset classes will have higher notional allocations than higher risk asset classes as a result.

Risk parity is all about finding the right balance. Designing a portfolio based on a fundamental understanding of the environmental sensitivities inherent in the price structure of asset classes is the most dependable strategy to attain stable balance. Bridgewater is the core of the All Weather method.

3.2.3.1 The Ray Dalio Approach

A chance equality portfolio indicates a lesson of portfolios whose resources confirm the taking after equalities

$$w_i \frac{\partial f(\mathbf{w})}{\partial w_i} = w_j \frac{\partial f(\mathbf{w})}{\partial w_j}, \forall i, j$$

where f is a positively homogeneous function of The portfolio's overall risk is calculated by degree one, and the portfolio weight vector is w. In other words, every asset in a risk parity portfolio has the same marginal risk contribution. The portfolio's standard deviation, a frequent proxy for volatility, is a popular candidate for f.

$f(\mathbf{w}) = \sqrt{\mathbf{w}^T \mathbf{\Sigma} \mathbf{w}}$, where Σ is the covariance matrix of assets

$$w_i(\Sigma \mathbf{w})_i = b_i \mathbf{w}^T \Sigma \mathbf{w}, \forall i,$$

where $\mathbf{b} \triangleq (b_1, b_2, \ldots, b_N)$ $\left(\text{with } \mathbf{1}^T\mathbf{b} = 1 \text{ and } \mathbf{b} \geq \mathbf{0}\right)$ is the vector of desired marginal risk contributions.

3.2.4 Discussion and Conclusion

On the off chance that the source of short-term chance may be an overwhelming concentration in a single sort of resource, this approach brings with it a critical hazard of destitute long-term returns that threatens the ability to meet future commitments. This is often since each resource is vulnerable to destitute performance that can last for a decade or more, caused by a supported move within the financial environment — Bridgewater.

3.3 Hierarchical Risk Parity Based Stock Allocation

3.3.1 Overview

Hierarchical Risk Parity (abbreviated HRP) is an optimization technique for portfolios that is based on risk , which demonstrates capability of generation of diversified portfolios. This algorithm relies on machine learning techniques to identify the underlying structure of hierarchy within the portfolio , this helps in division of capital between mutually exclusive clusters of assets and makes the cluster of similar assets compete for capital. Thus the resulting capital allocation is varied and much safer comparatively.

Here via this algorithm , this methodology proposes a method of utilizing the information created by clustering process which helps in achieving out-of-sample risk and return characteristics.HRP delivers highly diversified allocations with low volatility, low portfolio turnover and competitive performance metrics.

3.3.2 Introduction

Hierarchical risk parity (HRP) is a risk parity allocation algorithm which was introduced by Lopez de Prado in 2016 aimed for improving upon Mean-variance optimisation (presented in the previous approach) by working on its shortcomings.The idea is straightforward: if some assets have greater correlation between themselves than with the others, first diversify weights between them, and after between the group of this correlated assets and others.The method benefits from the out-of-sample robustness of risk parity portfolios, and can be applied to singular covariance matrices, hence solving the stability problem. Apart from desirable quantitative properties, the method has the intuitive appeal of allowing groups of related assets (as opposed to individual assets) to compete for capital in the portfolio, improving diversification over risk sources i.e portfolio owners can put in inputs to divide capital among assets within a particular cluster. Alongwith Lopez de Prado (2016) , multiple other researchers have also presented their variations of HRP like Alipour et al. (2016) and Raffinot (2016).

Here this methodology provides implementation of the HRP algorithm for portfolio optimization along with a detailed explanation of its reasoning, relevance and utility. The HRP portfolio has been compared with classic Mean Variance Optimizer and Classical Risk parity approach.

3.3.3 The Hierarchical Risk Parity Algorithm

This algorithm exploits the intrinsic hierarchy in the correlation matrix.In its essence it calculates inverse-variance weights for groups of coherent assets, and with each layer moves down and applies to ever smaller sub-groups until leaf nodes (individual assets) are encountered.
This algorithm was proposed by Lopez de Prado in 2016.

I. Stage 1: Tree Clustering

   A. Build a distance matrix from the correlation Matrix
      where, $d(i,j) = \sqrt{1/2\,(1 - \varrho\,(i,j))}$
   B. Followed by clustering using some heuristic over Euclidean distance that is calculated from dist[][].

II. Stage 2: Quasi Diagonalisation

   A. Reorganizes rows & cols of covariance matrix in a fashion such that largest values lie along the diagonal.
   B. Renders property: Similar investments come together while dissimilar investments fall far apart.

III. Stage 3: Recursive Bisection (Bottom Up and Top Down)

   A. Distribute the allocation through recursive bisection based on cluster covariance.

3.3.3.1 Clustering

This step breaks down the assets in our portfolio into different hierarchical clusters using the famous Hierarchical Tree Clustering algorithm. The tree cluster is calculated based on the *TxN* matrix of stock returns where *T* represents the timeseries of the data and *N* represents the number of stocks in our portfolio. This method brings the items together into forming a cluster rather than breaking it down into singular entities.

Here is how clustering is done in a step by step manner :

1. Given a *T XN* matrix of stock returns, calculate the correlation of each stock's returns with the other stocks which gives us an *N X N* matrix of these correlations, ρ
2. The correlation matrix is converted to a correlation-distance matrix D, where,

$$D(i, j) = \sqrt{0.5 * (1 - \rho(i, j))}$$

3. Now, we calculate another distance matrix *D(bar)* where

$$\overline{D}(i, j) = \sqrt{\sum_{k=1}^{N} (D(k, i) - D(k, j))^2}$$

It is formed by taking the Euclidean distance between all the columns in a pair-wise manner.

4. Clusters of assets are made using these distances in a recursive manner. Set of clusters, *U*. The first cluster *(i*,j*)* is,

$$U[1] = argmin_{(i,j)} \overline{D}(i, j)$$

Instance :

	a	b	c	d	e
a	0	17	21	31	23
b	17	0	30	34	21
c	21	30	0	28	39
d	31	34	28	0	43
e	23	21	39	43	0

Table:3.1:- Distance matrix for HRP

stocks *a* and *b* have the minimum distance value and thus combine to form one cluster.

5. The columns & rows w.r.t to the stocks(clusters) taken together are removed and replaced by another node whose distance from other entries is calculated by the formula

$$\overline{D}(i, U[1]) = min(\overline{D}(i, a), \overline{D}(i, b))$$

Distance matrix *D* is re-evaluated using the distance of other assets from the newly formed cluster. And the steps 4 & 5 are repeated till a single cluster is formed i.e the Distance matrix remains *1X1*.

6. The above procedure recursively combines 2 nodes in each iteration forming a hierarchical cluster that can be viewed by a dendogram, as below :

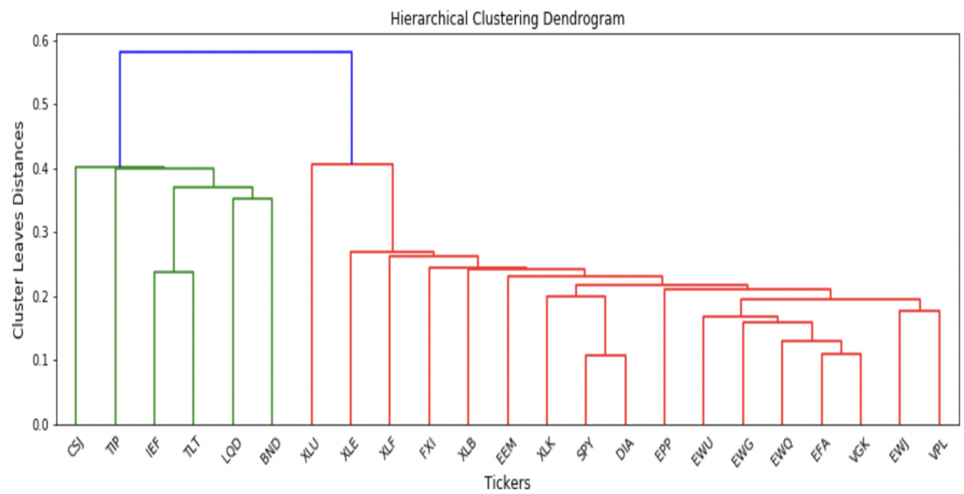

Fig:3.3 :- Hierarchical Tree Structure for Clustering

3.3.3.2 Quasi-Diagonalization

This step uses the clustering acquired from the previous step to rearrange the covariance matrix into a more diagonal representation. Greater the correlation between the assets , more the proximity to the diagonal this means similar investments are placed together and dissimilar investments are placed far apart.

In technical terms greater covariances are placed along the diagonal and smaller covariances are present around this. Also due to the fact that the off-diagonal elements are not perfectly zeros, this matrix is referred to as a *quasi-diagonal covariance matrix*

3.3.3.3 Recursive Bisection

This step deals with the actual allocation of the stocks i.e what weights are to be attached with which stock.

1. Initialize weights Wi = X | i ∈ [1,N]
2. The Hierarchical Tree Structure (i.e a binary tree) is worked on in top-down fashion where each cluster has a left and a right child , say *V1 & V2.*
   Variance is calculated for each cluster using

$$V_{adj} = w^T V w$$

where

$$w = \frac{diag[V]^{-1}}{trace(diag[V]^{-1})}$$

3. Weighting factor is calculated based on the new covariance matrix

$$\alpha_1 = 1 - \frac{V_1}{V_1 + V_2}; \alpha_2 = 1 - \alpha_1$$

4. The weight of left and right subtrees are then calculated and in recursive fashion.

$$W_1 = \alpha_1 * W_1$$

$$W_2 = \alpha_2 * W_2$$

3.4 Monte Carlo Simulation on input to pick better algorithm

3.4.1 Overview

Most of the time portfolio optimization decisions are taken by blindly picking up any of the above algorithms , in the hope that the results will always align with the requirements and lead to better results. But the results sometimes are unstable and even after optimization of the portfolio there are not any good gains which totally remove the point of optimization.

Different optimization approaches might cater to different sets of inputs and this methodology tries to exploit on that front trying out various algorithms for a given set of input and identifying which algorithm has what to offer , as , and it is unrealistic to expect that one method will dominate all under varied circumstances

3.4.2 Introduction

The problem monte carlo is trying to solve here is formulated as a

> System with N random variables, where the expected value of draws is μ, and the variance of these draws is V ,the covariance matrix and we need to calculate ω.

These variables μ and V are typically unknown and are calculated using the historical stock price data which is acquired from yahoo finance.
In this method multiple simulated values of μ and V are calculated using a random seed and then Ledoit-Wolf shrinkage and then denoising to get $\hat{\mu}$ and $\hat{V}$ respectively.
Now $\hat{\omega}$ is calculated for the simulated values multiple times.
The mean of the calculated values gives the glimpse on how sensitive is portfolio w.r.t to any particular algorithm.

Thus MCOS method is used for estimating $\hat{\omega}$ while controlling for noise-induced and signal-induced instabilities.

3.4.3 The Monte Carlo Estimation method

There are basically five steps in this method , with input being the list of expected price of the set of stocks in the portfolio and the covariance matrix.

3.4.3.1 Calculating simulated predictions and covariance

The algorithm uses the original matrix X of size TxN i.e the matrix of time series data of N stocks for T days in order to obtain the simulated pair $\{\hat{\mu}, \hat{V}\}$ by multivariate using a random seed.

Ledoit-Wolf shrinkage may be applied to X if required.
Now this simulated pair is used instead of the original $\{\mu, V\}$ corresponds to the true values.

3.4.3.2 Removing the noise

In this step noise reduction is done on the covariance matrix , this helps in preventing the instability of input data to the algorithms. This project uses a Kernel Density Estimate (KDE) algorithm to fit the Marcenko-Pastur distribution to the empirical distribution of eigenvalues. This helps in the separation of noisy eigenvalues from signal related eigenvalues.

3.4.3.3 Applying allocation algorithms

In this step a variety of portfolio allocation algorithms namely , Mean-variance optimizer , Risk parity optimizer , and Hierarchical risk parity optimizer are applied to the simulated values and $\hat{\omega}*$ is estimated.

3.4.3.4 Applying the Simulation

This step is the combination of all the 3 steps mentioned previously, now optimal allocation is done for simulated pairs $\{\hat{\mu}, \hat{V}\}$ via each of the optimizers $\hat{\omega}^*(i)$, where $\hat{\omega}^*$ is the optimal allocation via i-th optimizer.

This calculation is done multiple times , the no. of times is defined by the user , greater the number more accurate will be the predictions made.

3.4.3.5 Error Estimations

In this step the optimal allocation is done for original pair $\{\mu, V\}$ for each optimizer ,i.e. $\omega^*(i)$, where ω^* is the optimal allocation via i-th optimizer and the results are compared with estimated $\hat{\omega}*$.

The estimation error may be evaluated in the following terms for each i :

- the mean difference in expected outcomes, $(\omega*(i) - \hat{\omega}*(i))'\mu$
- the mean difference in variance, $(\omega*(i) - \hat{\omega}*(i))'V(\omega*(i) - \hat{\omega}*(i))$
- the mean difference in Sharpe ratio, $(\omega*(i)-\hat{\omega}*(i))'\mu \;/\; \sqrt{(\omega*(i)-\hat{\omega}*(i))'V(\omega*(i)-\hat{\omega}*(i))}$

3.4.3.6 Plotting the error for Optimizers

In this step the errors calculated in the previous step are plotted on a bar chart using matplotlib for comparison between multiple optimizers and which is better.

4. Experimental Results

Objective: To analyze the performance of a stock portfolio on various optimisation techniques of portfolio allocation.

Methodology: A portfolio of sixteen stocks belonging to different sectors is chosen; a dataset containing the price history of these stocks is being taken starting from 1984 till 2017.
Monte Carlo simulation is being performed for 50 iterations and the bar chart is plotted for the error estimation.

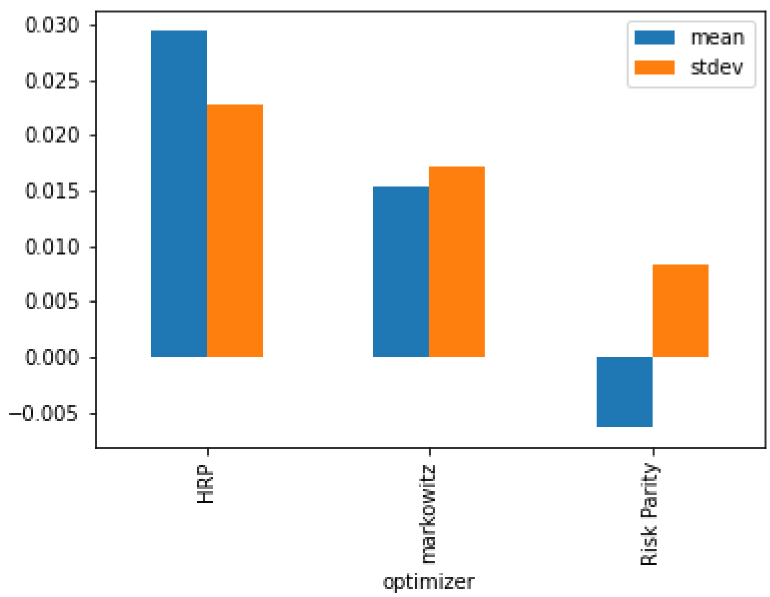

Fig 4.1 Expected Outcome error estimator for mean and standard deviation

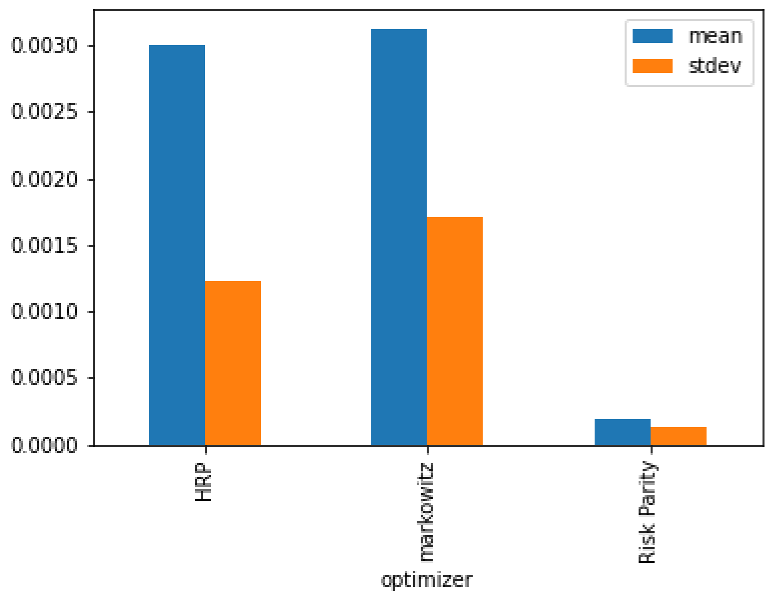

Fig 4.2 Variance error estimator for mean and standard deviation

Allocation of stocks is being performed for different stocks based on all three optimisers below is an example pie chart of suggestion of optimized portfolio as per HRP.

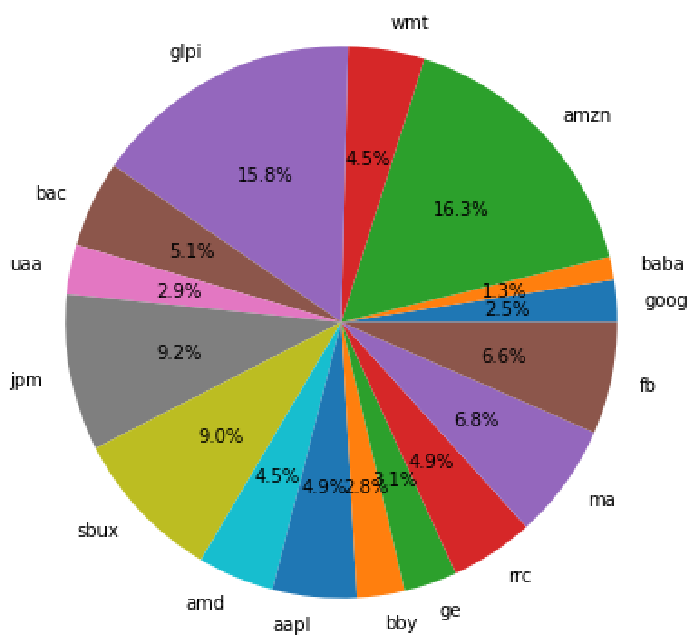

Fig 4.3 Distribution of stocks by amount to be allocated predicted by HRP

The amount is allocated to various stocks and the the performance of portfolios is compared over a period of 5 years and following observations were made:

1. Initially Markowitz performed better than both HRP and Risk Parity but eventually showed a decline with time.
2. HRP and Markowitz suffered major decline in performance with fluctuations in market while risk parity maintained acceptable performance.
3. Hence a user could decide on the basis of the amount of risk acceptable to select an optimal portfolio allocation strategy. If more returns are expected and risks can be ignored than HRP or Markowitz would be better options otherwise Risk Parity showed better stability of returns.

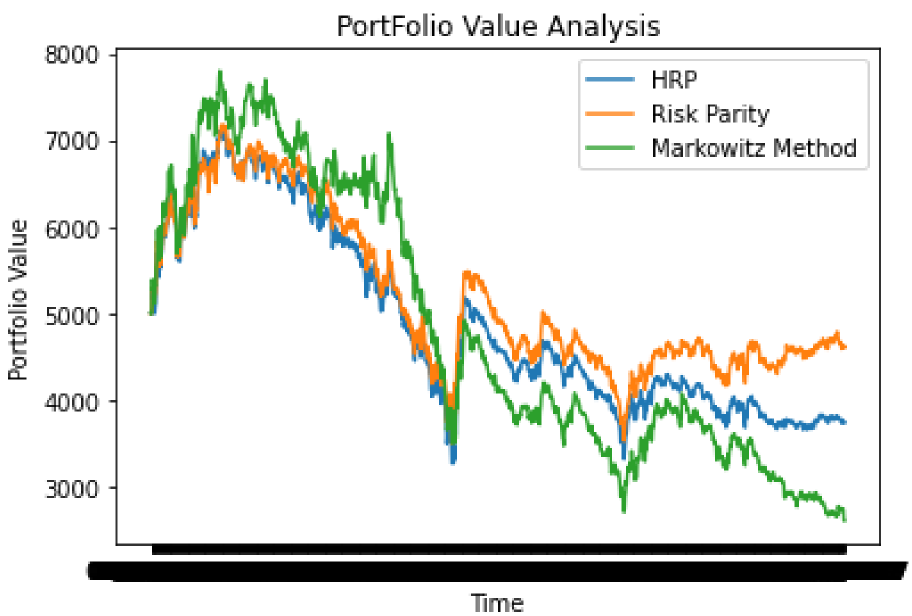

Fig 4.4 Performance of various portfolio allocation strategies over a period of five years.

Conclusions

5.1 Conclusions

Portfolio optimization is a excellent tool for investors looking to maximize their risk-to-reward Ratio. They can do so with the assistance of this project, which can help them choose the best algorithm as per their need and risk appetite. This can help them choose the best strategy knowing the tradeoffs

The decision would always be based on the investors' risk appetite and expected rate of return. Finally while any model or theory has advantages and disadvantages, portfolio managers can maximize the benefits of the portfolio maximizing technique if they employ it diligently.

This model can also help developers to build upon it , create UI and extensions to distribute the capability of this project just by simple plugins.

This is a simple yet powerful tool which can help retail investors ramp up their investment journey after going through the basic strategies / algorithms available for diversification.

Investors simply have to provide the set of stocks they need to invest in , the amount to be invested and choose their strategy according to the analysis we present. The tool will help them with what amount of stock they need to buy.

Right now the tool supports HRP, MPT and Risk Parity as algorithms but the list can be ever growing and can be built upon by developers.

5.2 Future Work

Many different adaptations, tests and experiments have been left for the future due lack of user Traffic and data sets. Future work concerns deeper analysis of some methods, new curiosity and new proposals as per time.

Here are a few things that can be onboarded as an extension to the project :

1. Optimize portfolio with respect to the volatility provided by the investor.
2. Inclusion of multiple categories of assets like gold, bonds etc in the portfolio.
3. A web app with operational UI which makes it easy for non technical users to utilize the capabilities of this project.
4. Add a feature of Trending Assets on the home page which allows naive users to pick in their favorites from trending assets in order to try this tool.

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