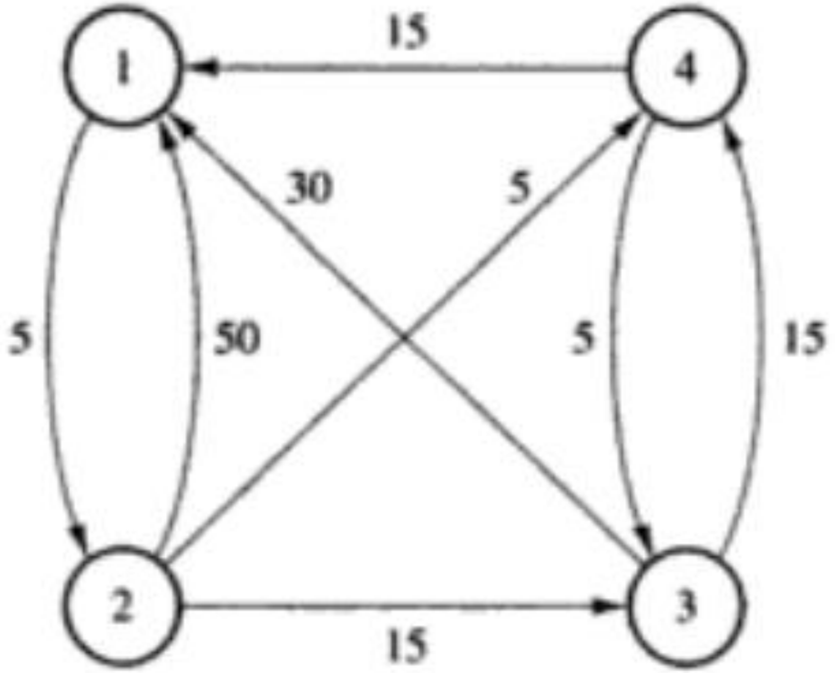
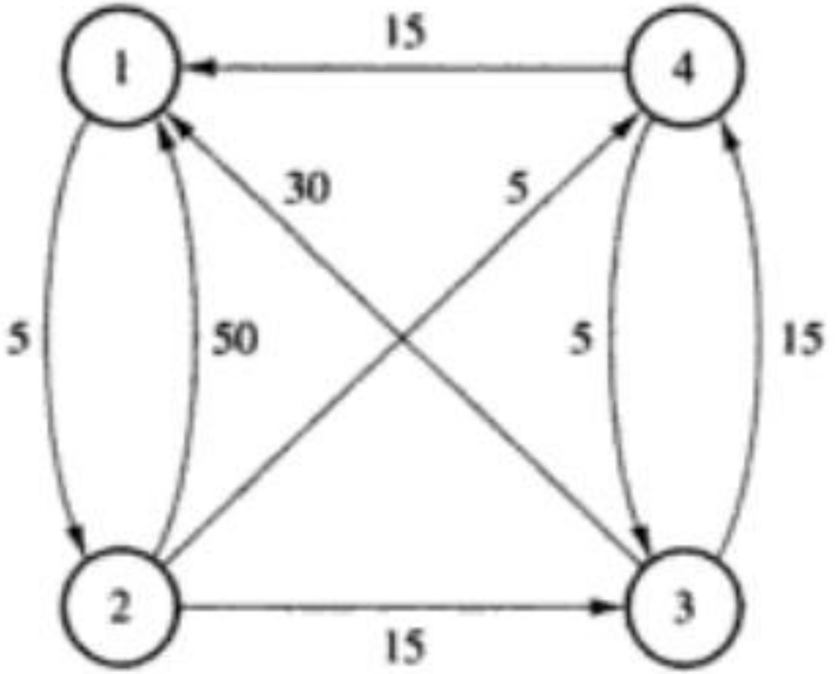

1
2
3
4
15
30
5
5
50
5
15
15

function *Floyd*($L[1..n, 1..n]$): **array** $[1..n, 1..n]$
    array $D[1..n, 1..n]$
    $D \leftarrow L$
    for $k \leftarrow 1$ **to** n **do**
        for $i \leftarrow 1$ **to** n **do**
            for $j \leftarrow 1$ **to** n **do**
                $D[i, j] \leftarrow \min(D[i, j], D[i, k] + D[k, j])$
    return D

$$D_0 = L = \begin{pmatrix} 0 & 5 & \infty & \infty \\ 50 & 0 & 15 & 5 \\ 30 & \infty & 0 & 15 \\ 15 & \infty & 5 & 0 \end{pmatrix}$$



$$D_1 = \begin{pmatrix} 0 & 5 & \infty & \infty \\ 50 & 0 & 15 & 5 \\ 30 & 35 & 0 & 15 \\ 15 & 20 & 5 & 0 \end{pmatrix} \qquad D_2 = \begin{pmatrix} 0 & 5 & 20 & 10 \\ 50 & 0 & 15 & 5 \\ 30 & 35 & 0 & 15 \\ 15 & 20 & 5 & 0 \end{pmatrix}$$

$$D_3 = \begin{pmatrix} 0 & 5 & 20 & 10 \\ 45 & 0 & 15 & 5 \\ 30 & 35 & 0 & 15 \\ 15 & 20 & 5 & 0 \end{pmatrix} \qquad D_4 = \begin{pmatrix} 0 & 5 & 15 & 10 \\ 20 & 0 & 10 & 5 \\ 30 & 35 & 0 & 15 \\ 15 & 20 & 5 & 0 \end{pmatrix}$$

$$\textbf{if } D[i,k]+D[k,j] < D[i,j] \textbf{ then } D[i,j] \leftarrow D[i,k]+D[k,j]$$
$$P[i,j] \leftarrow k$$

- `

$$P = \begin{pmatrix} 0 & 0 & 4 & 2 \\ 4 & 0 & 4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$

$$c_{ij} = \sum_{k=1}^{q} a_{ik} b_{kj}, \qquad 1 \le i \le p, 1 \le j \le r.$$

for $i \leftarrow 1$ **to** p **do**
    for $j \leftarrow 1$ **to** r **do**
        $C[i, j] \leftarrow 0$
        for $k \leftarrow 1$ **to** q **do**
            $C[i, j] \leftarrow C[i, j] + A[i, k] B[k, j]$

$$M = M_1 M_2 \cdots M_n$$

$$\begin{aligned} M &= (\cdots((M_1 M_2) M_3) \cdots M_n) \\ &= (M_1(M_2(M_3 \cdots (M_{n-1} M_n) \cdots))) \\ &= (\cdots((M_1 M_2)(M_3 M_4)) \cdots), \end{aligned}$$

$((AB)C)D$	10582
$(AB)(CD)$	54201
$(A(BC))D$	2856
$A((BC)D)$	4055
$A(B(CD))$	26418

$$M = (M_1 M_2 \cdots M_i)(M_{i+1} M_{i+2} \cdots M_n).$$

$$T(n) = \sum_{i=1}^{n-1} T(i)T(n-i).$$

$T(n)$ is in $\Omega(4^n/n^2)$

n	1	2	3	4	5	10	15
$T(n)$	1	1	2	5	14	4862	2674440

$$
\begin{array}{rlll}
s = 0: & m_{ii} & = 0 & i = 1, 2, \ldots, n \\
s = 1: & m_{i,i+1} & = d_{i-1} d_i d_{i+1} & i = 1, 2, \ldots, n-1 \\
1 < s < n: & m_{i,i+s} & = \min\limits_{i \le k < i+s} (m_{ik} + m_{k+1,i+s} + d_{i-1} d_k d_{i+s}) & \\
 & & & i = 1, 2, \ldots, n-s
\end{array}
$$

$$(13, 5, 89, 3, 34).$$

$$
\begin{array}{cccccc}
 & j=1 & 2 & 3 & 4 & \\
i=1 & 0 & 5785 & 1530 & 2856 & \\
 & & & & & s=3 \\
2 & & 0 & 1335 & 1845 & \\
 & & & & & s=2 \\
3 & & & 0 & 9078 & \\
 & & & & & s=1 \\
4 & & & & 0 & \\
 & & & & & s=0
\end{array}
$$

$$
\begin{aligned}
m_{13} &= \min(m_{11} + m_{23} + 13 \times 5 \times 3, m_{12} + m_{33} + 13 \times 89 \times 3) \\
&= \min(1530, 9256) = 1530 \\
m_{24} &= \min(m_{22} + m_{34} + 5 \times 89 \times 34, m_{23} + m_{44} + 5 \times 3 \times 34) \\
&= \min(24208, 1845) = 1845.
\end{aligned}
$$

Finally for $s = 3$

$$
\begin{aligned}
m_{14} = \min(&m_{11} + m_{24} + 13 \times 5 \times 34, && \{k = 1\} \\
&m_{12} + m_{34} + 13 \times 89 \times 34, && \{k = 2\} \\
&m_{13} + m_{44} + 13 \times 3 \times 34) && \{k = 3\} \\
= \min&(4055, 54201, 2856) = 2856.
\end{aligned}
$$

$$
\begin{aligned}
\sum_{s=1}^{n-1}(n-s)s &= n\sum_{s=1}^{n-1}s - \sum_{s=1}^{n-1}s^2 \\
&= n^2(n-1)/2 - n(n-1)(2n-1)/6 \\
&= (n^3-n)/6,
\end{aligned}
$$

i	j	0	1	2	3	4	5	6
	y_j		B	D	C	A	B	A
0	x_i	0	0	0	0	0	0	0
1	A	0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B	0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C	0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4	B	0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
5	D	0	↑ 1	↖ 2	↑ 2	↑ 2	↑ 3	↑ 3
6	A	0	↑ 1	↑ 2	↑ 2	↖ 3	↑ 3	↖ 4
7	B	0	↖ 1	↑ 2	↑ 2	↑ 3	↖ 4	↑ 4

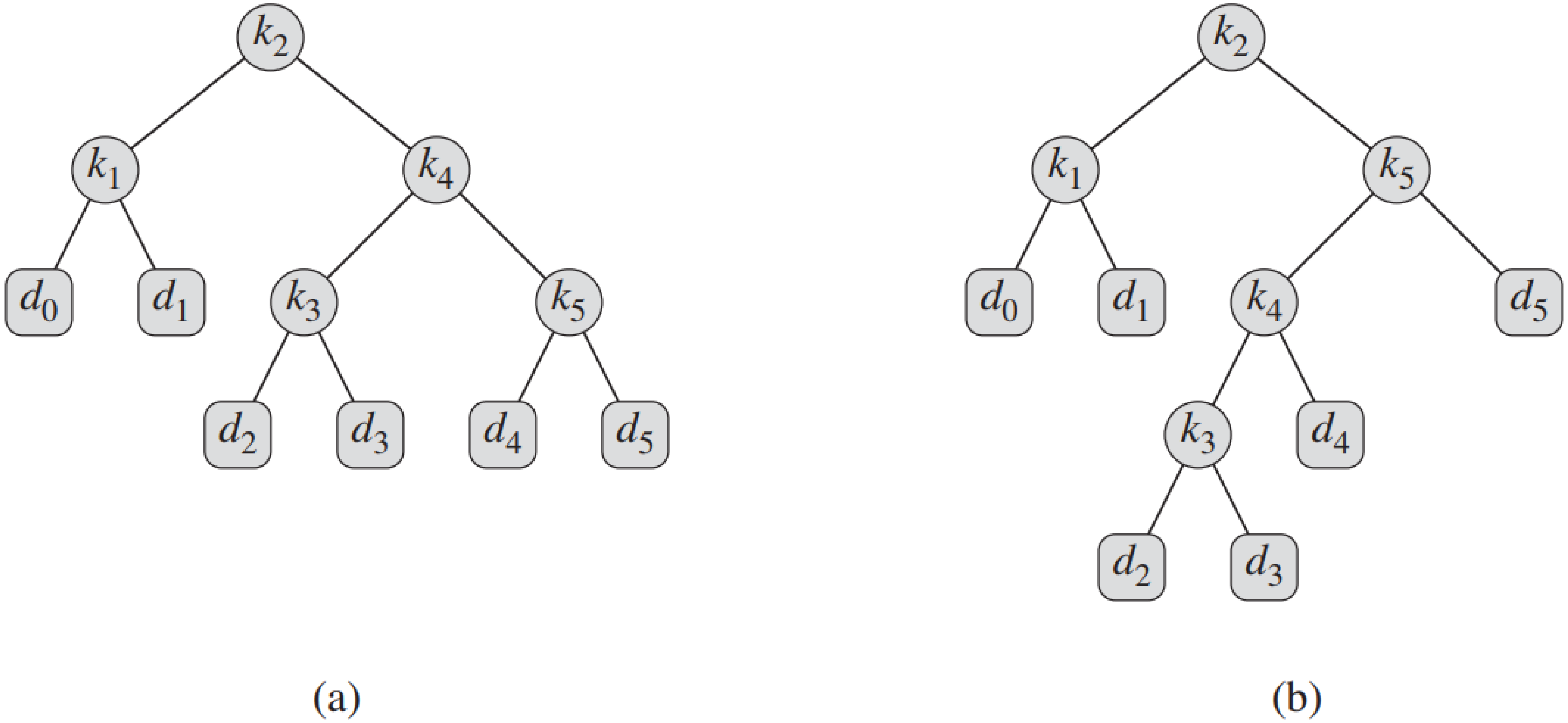
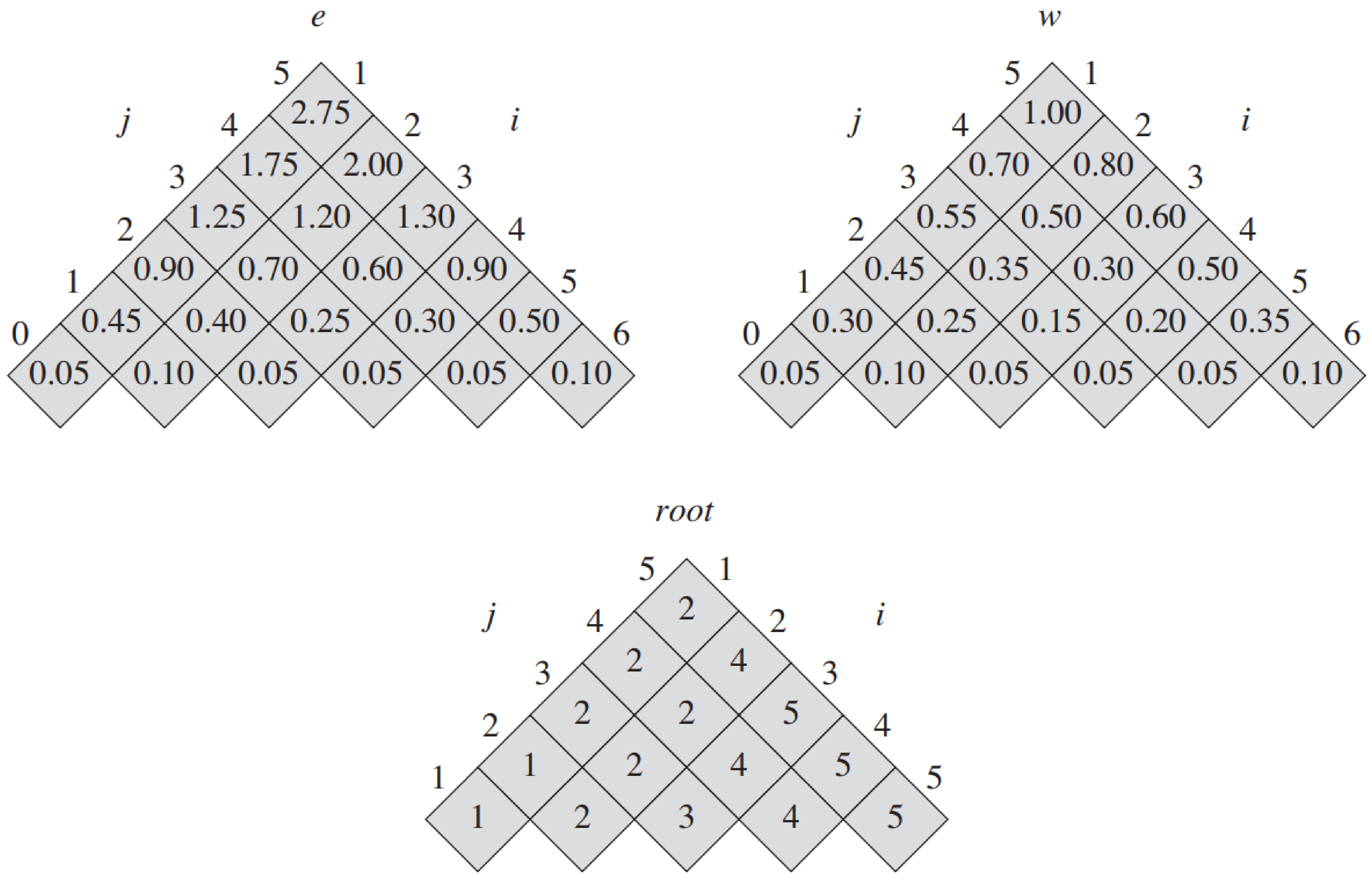


Figure 15.9 Two binary search trees for a set of $n = 5$ keys with the following probabilities:

i	0	1	2	3	4	5
p_i		0.15	0.10	0.05	0.10	0.20
q_i	0.05	0.10	0.05	0.05	0.05	0.10

(a) A binary search tree with expected search cost 2.80. **(b)** A binary search tree with expected search cost 2.75. This tree is optimal.


e
j
i
5 1
2.75
4 2
1.75 2.00
3 3
1.25 1.20 1.30
2 4
0.90 0.70 0.60 0.90
1 5
0.45 0.40 0.25 0.30 0.50
0 6
0.05 0.10 0.05 0.05 0.05 0.10
w
j
i
5 1
1.00
4 2
0.70 0.80
3 3
0.55 0.50 0.60
2 4
0.45 0.35 0.30 0.50
1 5
0.30 0.25 0.15 0.20 0.35
0 6
0.05 0.10 0.05 0.05 0.05 0.10
root
j
i
5 1
2
4 2
2 4
3 3
2 2 5
2 4
1 2 4 5
1 5
1 2 3 4 5