Real Business cycles.

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

$$u(c_t, l_t) = \frac{(c^{\phi} l^{1-\phi})^{1-\rho}}{1-\rho}$$

s/t : $\quad y_t = z_t k_t^{\alpha} n_t^{1-\alpha}$

$$1 = n_t + l_t$$

$$y_t = c_t + i_t$$

$$k_t = i_t + (1-\delta) k_t$$

Ans:

$$k_t = i_t + (1-\delta) k_t$$

$$\cancel{k_t} = i_t + \cancel{k_t} - \delta k_t$$

$$i_t = \delta k_t$$

$$y_t = c_t + \delta k_t$$

$$c_t + \delta k_t = z_t k_t^{\alpha} n_t^{(1-\alpha)}$$

$$n_t = 1 - l_t$$

$$c_t + \delta k_t = z_t k_t^{\alpha} (1-l_t)^{(1-\alpha)}$$

$$c_t + \delta k_t - z_t k_t^{\alpha} (1-l_t)^{(1-\alpha)} = 0$$

$$\underset{\substack{c_t, l_t, k_t \\ \lambda}}{\text{Max}} = \text{Max}(U) = E_0 \sum_{t=0}^{\infty} \beta^t \frac{(c_t^{\phi} l_t^{1-\phi})^{1-\rho}}{1-\rho} + \lambda \left[c_t + \delta k_t - z_t k_t^{\alpha} (1-l_t)^{(1-\alpha)} \right]$$

Scanned with CamScanner

$$\frac{du}{dC_t} = \frac{\beta^t \cancel{(1-\rho)}}{\cancel{1-\rho}} \left(C_t^{\phi} L_t^{1-\phi}\right)^{-\rho} L_t^{1-\phi} \phi C_t^{\phi-1} + \lambda C_t = 0$$

$$= \beta^t \left(C_t^{\phi} L_t^{1-\phi}\right)^{-\rho} L_t^{1-\phi} \phi C_t^{\phi-1} + \lambda C_t = 0 \quad \text{—— ①}$$

$$\frac{du}{dL_t} = \frac{\beta^t}{\cancel{1-\rho}} \cancel{(1-\rho)} \left(C_t^{\phi} L_t^{1-\phi}\right)^{-\rho} C_t^{\phi}(1-\phi) L_t^{-\phi} + \lambda\left[0+0-Z_t K_t^{\alpha}(1-\alpha)(1-L_t)^{-\alpha}(-1)\right]$$

$$= \beta^t \left(C_t^{\phi} L_t^{1-\phi}\right)^{-\rho} C_t^{\phi} (1-\phi) L_t^{-\phi} + \lambda Z_t K_t^{\alpha}(1-\alpha)(1-L_t)^{-\alpha} = 0$$

$$\text{—— ②}$$

$$\frac{du}{dK_t} = 0 + \lambda\left[0 + \delta - Z_t (1-L_t)^{1-\alpha} \alpha K_t^{\alpha-1}\right] = 0$$

$$\delta = Z_t (1-L_T)^{1-\alpha} \alpha K_t^{\alpha-1} \quad \text{—— ③}$$

$$\frac{du}{d\lambda} = 0 + \left[C_t + \delta K_t - Z_t K_t^{\alpha}(1-L_t)^{1-\alpha}\right] = 0$$

$$C_t + \delta K_t = Z_t K_t^{\alpha}(1-L_t)^{1-\alpha} \quad \text{—— ④}$$

$$\frac{①}{②} \quad \frac{\cancel{\beta^t \left(C_t^{\phi} L_t^{1-\phi}\right)^{-\rho}} L_t^{1-\phi} \phi C_t^{\phi-1}}{\cancel{\beta^t \left(C_t^{\phi} L_t^{1-\phi}\right)^{-\rho}} C_t^{\phi}(1-\phi) L_t^{-\phi}} = \frac{-\cancel{\lambda} \cdot C_t}{-\cancel{\lambda} Z_t K_t^{\alpha}(1-\alpha)(1-L_t)^{-\alpha}}$$

$$\frac{\phi L_t C_t^{-1}}{(1-\phi)} = \frac{C_t}{Z_t K_t^{\alpha}(1-\alpha)(1-L_t)^{-\alpha}}$$

$$\phi L_t \left[Z_t K_t^{\alpha}(1-\alpha)(1-L_T)^{-\alpha}\right] = (1-\phi) C_t^2 \quad \text{—— ⑤}$$

$$\frac{③}{④} \quad \frac{\delta}{C_t + \delta K_t} = \frac{\cancel{Z_t} \cancel{(1-L_t)^{1-\alpha}} \alpha K_t^{\alpha-1}}{\cancel{Z_t} K_t^{\alpha} \cancel{(1-L_t)^{1-\alpha}}} = \frac{\alpha}{K_t}$$

$$\frac{\delta}{C_t + \delta k_t} = \frac{\alpha}{k_t}$$

$$\delta k_t = \alpha C_t + \alpha \delta k_t$$

$$\alpha C_t = \delta k_t - \alpha \delta k_t$$

$$\alpha C_t = \delta k_t (1-\alpha)$$

$$C_t = \frac{\delta(1-\alpha)}{\alpha} k_t$$

$$\delta(1-\alpha) k_t = \alpha C_t \qquad \text{—— (6)}$$

$$\frac{(5)}{(6)} \qquad \frac{\phi l_t [z_t k_t^{\alpha} \cancel{(1-\alpha)} (1-l_t)^{-\alpha}]}{\delta \cancel{(1-\alpha)} k_t} = \frac{(1-\phi) \cancel{C_t}}{\alpha \cancel{C_t}}$$

$$\boxed{C_t = \frac{\phi l_t [z_t k_t^{\alpha-1} (1-l_t)^{-\alpha}] \alpha}{\delta(1-\phi)}}$$

Rearranging

$$\frac{(1-\phi)}{\phi} \frac{C_t}{l_t} = \frac{z_t k_t^{\alpha} \cancel{k_t^{-1}} (\cancel{1} - \cancel{1} + n_t)^{-\alpha} \, l}{\cancel{\delta} \, l_t \cancel{k_t^{-1}}}$$

$$= z_t k_t^{\alpha} \cdot n_t^{-\alpha} (1-\alpha)$$

$$\boxed{\frac{(1-\phi) C_t}{\phi l_t} = z_t (1-\alpha) \left(\frac{k_t}{n_t}\right)^{\alpha}}$$

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \frac{(c^{\phi} l^{1-\phi})^{1-\rho}}{1-\rho}$$

$$s.t: \quad y_t = z_t k_t^{\lambda} n_t^{1-\lambda}$$

$$1 = n_t + l_t$$

$$y_t = c_t + i_t$$

$$k_{t+1} = i_t + (1-\delta) k_t$$

$$c_t + i_t = z_t k_t^{\lambda} n_t^{1-\lambda}$$

$$c_t + \cancel{k_{t+1}} i_t = z_t k_t^{\lambda} (1-l_t)^{1-\lambda}$$

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \frac{(c_t^{\phi} l_t^{1-\phi})^{1-\rho}}{1-\rho} + \lambda [c_t + i_t - z_t k_t^{\lambda} (1-l_t)^{1-\lambda}] + \eta_t [k_{t+1} - i_t \mp (1-\delta) k_t]$$

$$\frac{dU}{dc_t} = \beta^t \frac{\cancel{1-\rho}}{\cancel{1-\rho}} \phi c_t^{\phi-1} l_t^{1-\phi} + \lambda = \beta^t \phi c_t^{\phi-1} l_t^{1-\phi} + \lambda = 0$$

$$\frac{dU}{di_t} = 0 + \lambda - \eta = 0$$

$$\frac{dU}{d\lambda_t} = c_t + i_i - z_t k_t^{\lambda} (1-l_t)^{1-\lambda} = 0$$

$$\frac{dU}{d\eta_t} = k_{t+1} - i_t - (1-\delta) k_t = 0$$

$$\frac{du}{dk_t} = 0 + \lambda [-z_t (1-l_t)^{1-\lambda} \lambda k_t^{\lambda-1}] + \eta_{t-1} - (1-\delta) \eta_t = 0$$

$$\beta^t \phi c_t^{\phi-1} l_t^{1-\phi} + \eta = 0$$

$$\cancel{\lambda[-z_t(1-l_t)^{1-\alpha} \alpha k_t^{\alpha-1} + \eta_{t-1} = (1-\delta)\eta_t}$$

$$\lambda[-z_t(1-l_t)^{1-\alpha} \alpha k_t^{\alpha-1}] + \eta_{t-1} = (1-\delta)\eta_t$$

$$\boxed{c_t^{\phi(1-\phi)-1} l_t^{(1-\phi)(1-\phi)} = \beta E_t \left\{ c_{t+1}^{\phi(1-\phi)-1} l_{t+1}^{(1-\phi)(1-\phi)} \cdot \left[\alpha z_{t+1} \left(\frac{n_{t+1}}{k_{t+1}}\right)^{1-\alpha} + (1-\delta) \right] \right\}}$$