1 Projection Matrix.

$P = X(X^TX)^{-1}X^T \quad M = I_n - P \quad y = X\beta + e$

$\hat{\beta} = (X^TX)^{-1} X'y$

a) P is idempotent

$P.P = X(X^TX)^{-1}(X^T.X)(X^TX)^{-1}X^T$

$= X(X^TX)^{-1} I_n X^T$

$= X(X^TX)^{-1}X^T$

$= P$

b) M is idempotent

$M.M = (I_n - P)(I_n - P)$

$= I_nI_n - I_nP - PI_n + P.P$

$= I_n - I_nP - PI_n + P$ ---(from a)

$= I_n - \cancel{I_nP} - \cancel{P} + \cancel{P}$

$= I_n - P = \underline{M}$

c) $PY = \hat{Y}$

$\hat{Y} = X\hat{\beta} \longrightarrow P$

$= X(X^TX)^{-1}X^TY$

$= PY$

d) $MY = \hat{e}$

$\hat{e} = y - \hat{y}$

$= Y - PY$

$= Y(I_n - P)$

$= MY$

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(iv) $PY + MY = Y$

$$PY + MY = PY + (I_n - P)Y$$
$$= PY + I_nY - PY$$
$$= I_nY = \underline{\underline{Y}}$$

(v) $\hat{y} \perp \vec{e}$

$$= \hat{y} \cdot \vec{e}$$
$$= PY \cdot MY$$
$$= PY(1-P)Y$$
$$= PY(P - P.P)Y$$
$$= Y(P-P)Y$$
$$= Y(0)Y$$
$$= \underline{\underline{0}}$$

Hence proved its orthogonal.