1) General least square estimator

$Var(e) = \sigma^2 I_n$
$E(ee'/x) = Var(e)$
$E(e_i^2/x) = \sigma^2$

$Var(e) = \Sigma$ .... (given)
$e'e = \Sigma$
$e'\Sigma^{-1}e = 1$

Minimizing sum of squared errors to find optimal value of β (co-efficients of x)

$$Y = \beta x + e$$
$$e = Y - \beta x$$

$$e'\Sigma^{-1}e = (Y-\beta x)'\Sigma^{-1}(Y-\beta x)$$
$$= (Y' - x'\beta')\Sigma^{-1}(Y - \beta x)$$
$$= \cancel{(Y'\Sigma^{-1}Y - x'\beta'\Sigma^{-1}Y - Y'\Sigma^{-1}\beta x +}$$
$$e'\Sigma^{-1}e = Y'\Sigma^{-1}Y - x'\beta'\Sigma^{-1}Y - Y'\Sigma^{-1}\beta x + x'\beta'\Sigma^{-1}\beta x$$
$$= Y'\Sigma^{-1}Y - 2x'\beta'\Sigma^{-1}Y + x'\beta'\Sigma^{-1}\beta x$$

$$\frac{d\, e'\Sigma^{-1}e}{d\beta} = 0 - 2x'\Sigma^{-1}Y + 2x'\Sigma^{-1}x\beta$$

$$-2x'\Sigma^{-1}Y + 2x'\Sigma^{-1}x\hat{\beta} = 0$$
$$\cancel{2}x'\Sigma^{-1}x\hat{\beta} = \cancel{2}x'\Sigma^{-1}Y$$
$$(x'\Sigma^{-1}x)^{-1}(x'\Sigma^{-1}x)\hat{\beta} = (x'\Sigma^{-1}x)^{-1}x'\Sigma^{-1}Y$$
$$\hat{\beta} =$$

$$\boxed{\hat{\beta} = (x'\Sigma^{-1}x)^{-1}x'\Sigma^{-1}Y}$$

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