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Method of Variation of Parameters

QUESTION 1 : Solve second order differential equation y''[x]+y[x]=tan[x] by method of variation of parameter

Solution:

Step -1: Find complementary function

```
In[4]:= eqn := y''[x] + y[x];
        f[x_] := Tan[x];
        P = DSolve[eqn == 0, y[x], x]

Out[6]= {{y[x] → C[1] Cos[x] + C[2] Sin[x]}}
```

Step -2 Consider fundamental solution function u(x) and v(x)

```
In[9]:= u[x_] := Cos[x];
        v[x_] := Sin[x];
```

Step -3 Find Wronskian W = ({ u[x] , v[x] } , { u'[x] , v'[x] })

```
In[11]:= w = Simplify[Det[{{u[x], v[x]}, {u'[x], v'[x]}}]]

Out[11]= 1
```

Step -4 Find g[x]= (-v[x]f[x])/w and h[x] = (u[x] f[x])/w

In[12]:=
```
g[x_] := (-v[x] × f[x]) / w
h[x] := (u[x] × f[x]) / w
```

Step -5 Find G = Integrate[g[x],x] and H=Integrate[h[x],x]

In[14]:=
```
G = Integrate[g[x], x]
H = Simplify[Integrate[h[x], x]]
```

Out[14]= $\mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right]-\mathrm{Sin}\left[\frac{x}{2}\right]\right]-\mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right]+\mathrm{Sin}\left[\frac{x}{2}\right]\right]+\mathrm{Sin}[x]$

Out[15]= $-\mathrm{Cos}[x]$

Step -6 Find PI=u[x]G+v[x]H

In[16]:=
```
PI = u[x] G + v[x] H
```

Out[16]= $-\mathrm{Cos}[x]\,\mathrm{Sin}[x]+\mathrm{Cos}[x]\left(\mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right]-\mathrm{Sin}\left[\frac{x}{2}\right]\right]-\mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right]+\mathrm{Sin}\left[\frac{x}{2}\right]\right]+\mathrm{Sin}[x]\right)$

QUESTION 2 : Solve second order differential equation y”[x]-2y’[x]=e^x Sin[x] by
method of variation of parameter
Step -1: Find complementary function

In[17]:=
```
eqn := y''[x] - 2 y'[x];
f[x_] := e^x * Sin[x];
P = DSolve[eqn == 0, y[x], x]
```

... DSolve: Equation or list of equations expected instead of 0 in the first argument 0.

Out[19]=
```
DSolve[0, y[x], x]
```

Step -2 Consider fundamental solution function u(x) and v(x)

```
u[x_] := 1/2 Exp[2 x]
v[x_] := 1
```

Step -3 Find Wronskian W = ({ u[x] , v[x] } , { u’[x] , v’[x] })

In[20]:=
```
w = Simplify[Det[{{u[x], v[x]}, {u'[x], v'[x]}}]]
```

Out[20]= 1

Step -4 Find g[x]= (-v[x]f[x])/w and h[x] = (u[x] f[x])/w

```
g[x_] := (-v[x] × f[x]) / w
h[x_] := (u[x] × f[x]) / w
```

Step -5 Find G = Integrate[g[x],x] and H=Integrate[h[x],x]

In[21]:=
```
G = Integrate[g[x], x]
H = Simplify[Integrate[h[x], x]]
```

Out[21]= $-\frac{e^{x}\left(4+\text{Log}[e]^{2}-\text{Cos}[2x]\,\text{Log}[e]^{2}-2\,\text{Log}[e]\,\text{Sin}[2x]\right)}{2\,\text{Log}[e]\left(4+\text{Log}[e]^{2}\right)}$

Out[22]= $\frac{e^{x}\left(-2\,\text{Cos}[2x]+\text{Log}[e]\,\text{Sin}[2x]\right)}{2\left(4+\text{Log}[e]^{2}\right)}$

Step -6 Find PI=u[x]G+v[x]H

In[23]:=
```
PI = u[x] G + v[x] H
```

Out[23]= $-\frac{e^{x}\,\text{Cos}[x]\left(4+\text{Log}[e]^{2}-\text{Cos}[2x]\,\text{Log}[e]^{2}-2\,\text{Log}[e]\,\text{Sin}[2x]\right)}{2\,\text{Log}[e]\left(4+\text{Log}[e]^{2}\right)}+\frac{e^{x}\,\text{Sin}[x]\left(-2\,\text{Cos}[2x]+\text{Log}[e]\,\text{Sin}[2x]\right)}{2\left(4+\text{Log}[e]^{2}\right)}$