Vivek Gupta| Bsc(Hons) Computer Science |20211467| Practical- 2

Plotting of second order solution family of differential equation

Question 1: Solve Second order Differential Equation y" + y = 0
Solution :

In[60]:= `DSolve[y''[x] + y[x]== 0, y[x], x]`

Out[60]= $\{\{y[x] \rightarrow c_1 \cos[x] + c_2 \sin[x]\}\}$

Question 2: Solve Second order Differential Equation y" + y' - 6y = 0
Solution :

In[64]:= `DSolve[y''[x] + y'[x] - 6 y[x]== 0, y[x], x]`

Out[64]= "$y[x] \rightarrow e^{-3x} c_1 + e^{2x} c_2$"

Question 3: Solve Second order Differential Equation 4y" + 12y' - 6y = 0
Solution :

In[67]:= `DSolve[4 y''[x] + 12 y'[x] - 6 y[x]== 0, y[x], x]`

Out[67]= $y[x] \rightarrow e^{2\left(-\frac{3}{2}-\frac{\sqrt{15}}{2}\right)x} c_1 + e^{\left(-\frac{3}{2}+\frac{\sqrt{15}}{2}\right)x} c_2$

Question 4: Solve Second order Differential Equation y" - 6y' + 13y = 0
Solution :

In[69]:= `DSolve[y''[x] - 6 y'[x] + 13 y[x]== 0, y[x], x]`

Out[69]= "$y[x] \rightarrow e^{3x} c_2 \cos[2 x] + e^{3x} c_1 \sin[2 x]$"

Question 5: Solve Second order Differential Equation y" - 2y' + y = 0
Solution:

In[71]:= `DSolve[y''[x] - 2 y'[x] + y[x]== 0, y[x], x]`

Out[71]= $\{\{y[x] \rightarrow e^x \cdot_1 + e^x x \cdot_2\}\}$

Plotting Of Solution Of Second order Differential Equations

Question 1: Solve Second order Differential Equation y" + y = 0 and Plot its three Solutions.
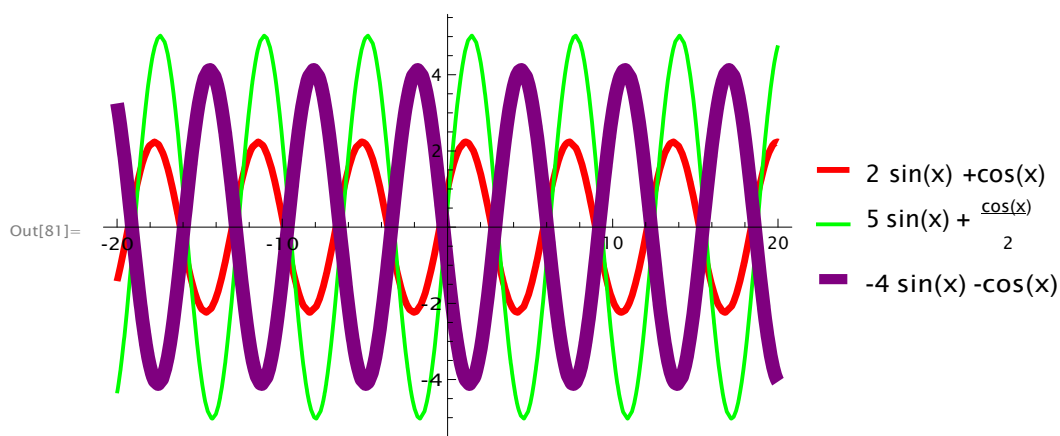### Solution :

In[77]:= `Sol = DSolve[y''[x] + y[x]== 0, y[x], x]`
`Sol1 = y[x] /. Sol [1] / . {C[1]→ 1, C[2]→ 2}`
`Sol2 = y[x] /. Sol [1] / . {C[1]→ 1 / 2, C[2]→ 5}`
`Sol3 = y[x] /. Sol [1] / . {C[1]→ 1, C[2]→ 4}`
`Plot[{Sol1, Sol2, Sol3}, {x, - 20, 20},`
`PlotStyle →{{ Red, Thickness [0.01]}, {Green , Thick}, {Purple , Thickness [0.02]}},`
`PlotLegends →{ Sol1, Sol2, Sol3]`

Out[77]= $\{\{y[x] \rightarrow \cdot_1 \text{Cos}[x] + \cdot_2 \text{Sin}[x]\}\}$

Out[78]= $\text{Cos}[x] + 2 \text{Sin}[x]$

Out[79]= $\dfrac{\text{Cos}[x]}{2} + 5 \text{Sin}[x]$

Out[80]= $-\text{Cos}[x] - 4 \text{Sin}[x]$

Out[81]=

Legend:
- 2 sin(x) +cos(x)
- 5 sin(x) + $\dfrac{\cos(x)}{2}$
- -4 sin(x) -cos(x)

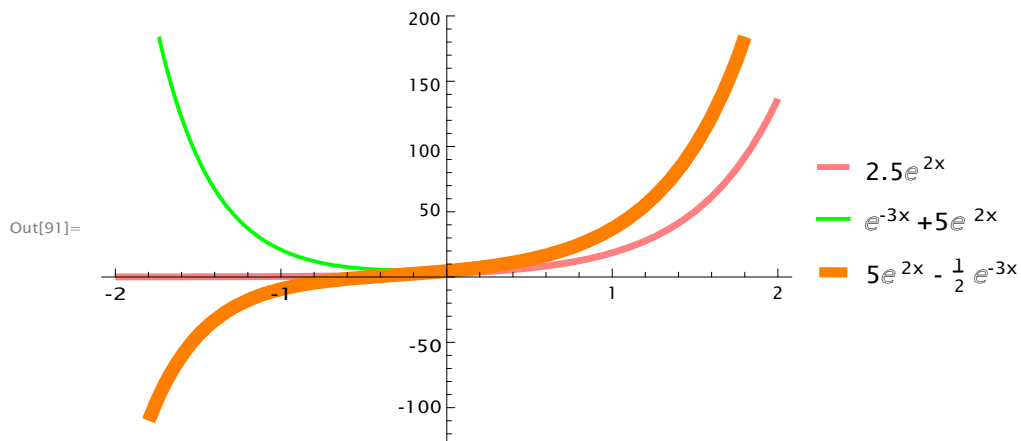### Question 2: Solve Second order Differential Equation y" + y' - 6y = 0 and Plot its three Solutions.

Solution:

In[87]:= Sol = DSolve[y''[x] + y'[x] - 6 y[x]==0, y[x], x]Sol1
= y[x] /. Sol [[1]] /. {C[1]→ 0, C[2]→ 2.5}
Sol2 = y[x] /. Sol [[1]] /. {C[1]→ 1, C[2]→ 5}
Sol3 = y[x] /. Sol [[1]] /. {C[1]→ 1/2, C[2]→ 5}
Plot[{Sol1, Sol2, Sol3}, {x, -2, 2},
PlotStyle →{{ Pink, Thickness [0.01]}, {Green , Thick}, {Orange , Thickness [0.02]}},
PlotLegends →{Sol1, Sol2, Sol3}]

Out[87]= "y[x]→ $e^{-3x} \cdot {}_1 + e^{2x} \cdot {}_2$"

Out[88]= $2.5 \, e^{2x}$

Out[89]= $e^{-3x} + 5 \, e^{2x}$

Out[90]= $-\dfrac{1}{2} \, e^{-3x} + 5 \, e^{2x}$

Out[91]=

Question 3: Solve Second order Differential Equation 4y" + 12y' + 9y = 0 and Plot its four Solutions for
(i) C[1] = -1 , C[2] = 4
(ii) C[1] = -3 , C[2] = 6
(iii) C[1] = -10 , C[2] = 7
(iv) C[1] = -1.5 , C[2] = -5
Solution :

```
In[132]:= Sol = DSolve[4 y ''[x] + 12 y '[x] + 9 y[x]==  0, y[x], x]
          Sol1 = y[x] /. Sol[1]/ . {C[1]→   1, C[2]→   4}
          Sol2 = y[x] /. Sol[1]/ . {C[1]→   3, C[2]→   6}
          Sol3 = y[x] /. Sol[1]/ . {C[1]→   10, C[2]→   7}
          Sol4 = y[x] /. Sol[1]/ . {C[1]→   1.5, C[2]→   5}
          Plot[{Sol1, Sol2, Sol3, Sol4}, {x, - 2, 2},
          PlotStyle →{{ Red, Thickness[0.01]}, {Green , Thick},
              {Purple , Thickness[0.02]}, {Yellow , Thickness[0.03]}},
          PlotLegends →{Sol1, Sol2, Sol3, Sol4}]
```

Out[132]=
"y[x]→ $e^{-3x\cdot2}$ ·₁ + $e^{-3x\cdot2}$ x ·₂"
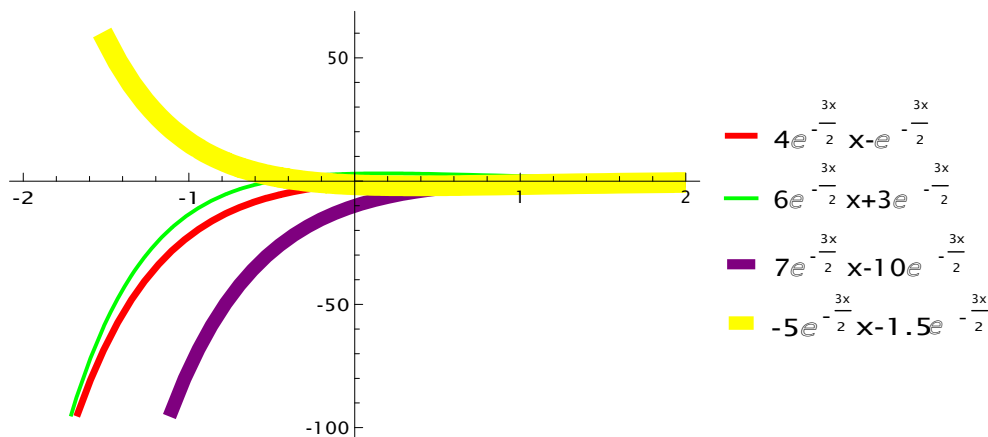
Out[133]=
$-e^{-3x\cdot2} + 4\,e^{-3x\cdot2}\,x$

Out[134]=
$3\,e^{-3x\cdot2} + 6\,e^{-3x\cdot2}\,x$

Out[135]=
$-10\,e^{-3x\cdot2} + 7\,e^{-3x\cdot2}\,x$

Out[136]=
$-1.5\,e^{-3x\cdot2} - 5\,e^{-3x\cdot2}\,x$

Out[137]=

Question 4: Solve Second order Differential Equation 4y" - 6y' +13y = 0 and Plot

its any three Solutions.
 Solution :

In[138]:=
```
Sol = DSolve[4 y ''[x] - 6 y '[x] + 13 y[x]==   0,  y[x],  x]
Sol1 = y[x] /. Sol[1]/ . {C[1]→    1, C[2]→   4}
Sol2 = y[x] /. Sol[1]/ . {C[1]→    3, C[2]→   6}
Sol3 = y[x] /. Sol[1]/ . {C[1]→     10, C[2]→  7}
Plot[{Sol1, Sol2, Sol3}, {x, - 5, 5},
PlotStyle →{{ Red, Thickness[0.01]}, {Green , Thick}, {Purple , Thickness[0.02]}},
PlotLegends →{ Sol1, Sol2, Sol3}]
```
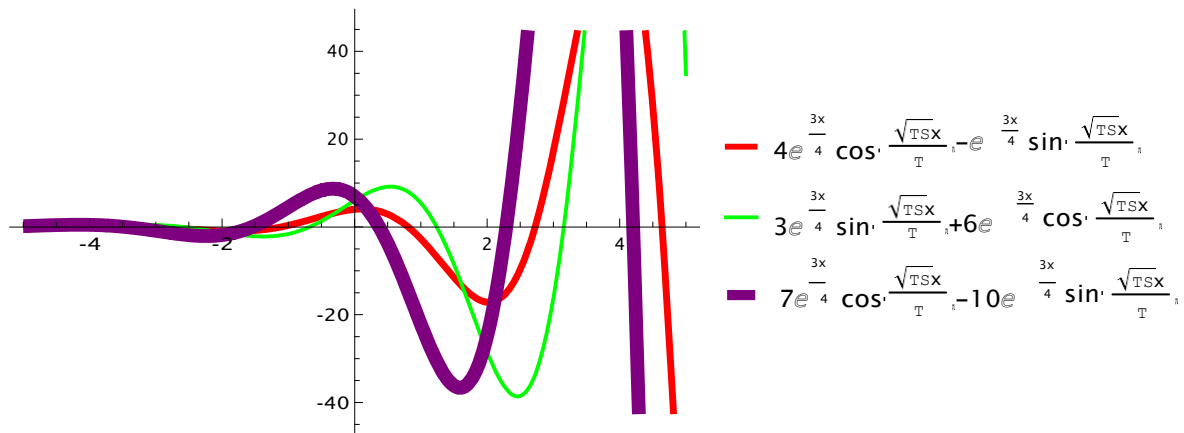
Out[138]=

$$\text{"y[x]} \rightarrow e^{3x\cdot4}\cdot_2 Cos\left[\frac{\sqrt{43}\ x}{4}\right] + e^{3x\cdot4}\cdot_1 Sin\left[\frac{\sqrt{43}\ x}{4}\right]\cdots$$

Out[139]=

$$4\,e^{3x\cdot4} Cos\left[\frac{\sqrt{43}\ x}{4}\right] - e^{3x\cdot4} Sin\left[\frac{\sqrt{43}\ x}{4}\right]$$

Out[140]=

$$6\,e^{3x\cdot4} Cos\left[\frac{\sqrt{43}\ x}{4}\right] + 3\,e^{3x\cdot4} Sin\left[\frac{\sqrt{43}\ x}{4}\right]$$

Out[141]=

$$7\,e^{3x\cdot4} Cos\left[\frac{\sqrt{43}\ x}{4}\right] - 10\,e^{3x\cdot4} Sin\left[\frac{\sqrt{43}\ x}{4}\right]$$

Out[142]=

Question 5: Solve Second order Differential Equation y" - 2y' + y = 0 and Plot its five Solutions.
Solution :

```
In[143]:= Sol = DSolve[y ''[x] - 2 y '[x] + y[x]== 0, y[x], x]
        Sol1 = y[x] /. Sol [1]/ . {C[1]→  0.5, C[2]→  3}
        Sol2 = y[x] /. Sol [1]/ . {C[1]→    3, C[2]→   2}
        Sol3 = y[x] /. Sol [1]/ . {C[1]→    1, C[2]→  7}
        Sol4 = y[x] /. Sol [1]/ . {C[1]→    6, C[2]→  1}
        Sol5 = y[x] /. Sol [1]/ . {C[1]→   1, C[2]→  2 / 3}
        Plot[{Sol1, Sol2, Sol3, Sol4, Sol5}, {x, - 2, 2},
        PlotStyle →{{ Red, Thickness[0.01]}, {Green , Thick}, {Purple , Thickness[0.02]},
            {Yellow , Thickness[0.03]}, {Orange , Thickness[0.04]}},
        PlotLegends →{ Sol1, Sol2, Sol3, Sol4, Sol5}]
```

Out[143]=

$\{\{y[x]\to e^x \cdot_1 + e^x x \cdot_2\}\}$

Out[144]=

$0.5 e^x + 3 e^x x$

Out[145]=

$- 3 e^x - 2 e^x x$

Out[146]=

$-e^x + 7 e^x x$

Out[147]=

$- 6 e^x + e^x x$

Out[148]=

$e^x + \dfrac{2 e^x x}{3}$

Out[149]=