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Find the Characteristics for the first order PDE and Plotting them

Example 1 :

Find the Characteristics of the equation (u – y) u[(x, y), x] + y * u[(x, y), y] = x + y and plot them.

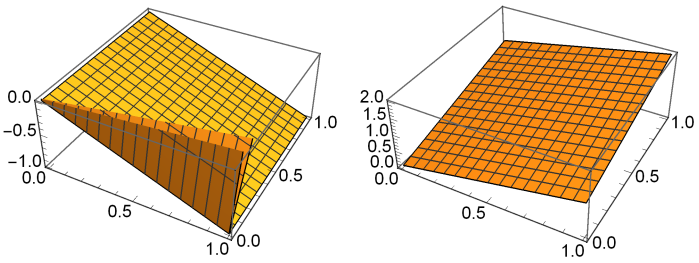
Solution :

The characteristics system is dx / (u – y) = dy / y = du / (x + y) using (i) + (ii) + (iii), we have v = (u + x) / y = c1, is a first integral.using (i) + (ii) = (iii), we have w = (x + y)^2 – u * u = c2 is a second first integral.

In[37]:=
```
f0 = Plot3D[-x, {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
f1 = Plot3D[5 y - x, {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
f2 = Plot3D[10 y - x, {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
g1 = Show[f0, f1, f2];
h0 = Plot3D[x + y, {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
h1 = Plot3D[Sqrt[(x + y)^2 + 5], {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
h2 = Plot3D[Sqrt[(x + y)^2 + 10], {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
g2 = Show[h0, h1, h2];
Show[GraphicsArray[{g1, g2}]]
```

GraphicsArray: GraphicsArray is obsolete. Switching to GraphicsGrid.

Out[45]=



Example 2 :

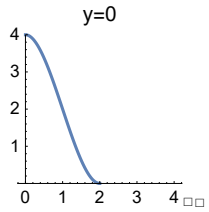
The solution of the equation u[(x, y), y] + u[x, y] * u[(x, y), x] = 0 can be interpreted as a vector field on the x – axis varying with time y.Find the integral satisfying the initial condition u (s, 0) = h (s), where h is a given function.

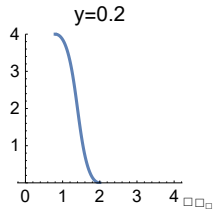
Solution :

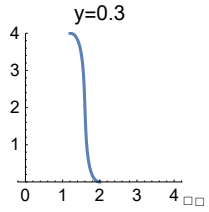
We plot the curves

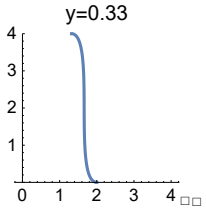
{Ct : x = s + t (s^3 – 3 s^2 + 4), u = s^3 – 3 s^2 + 4}

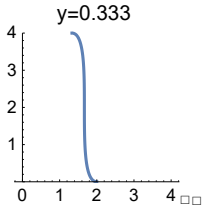
In[61]:=
```
u[s_] := s^3 - 3 s^2 + 4;
x[s_, t_] := s + t * u[s];
h0 = ParametricPlot[{x[s, 0], u[s]}, {s, 0, 2}, PlotRange → {0, 4}, PlotLabel → "y=0"]
h1 = ParametricPlot[{x[s, 0.2], u[s]}, {s, 0, 2}, PlotRange → {0, 4}, PlotLabel → "y=0.2"]
h2 = ParametricPlot[{x[s, 0.3], u[s]}, {s, 0, 2}, PlotRange → {0, 4}, PlotLabel → "y=0.3"]
h3 = ParametricPlot[{x[s, 0.33], u[s]}, {s, 0, 2}, PlotRange → {0, 4}, PlotLabel → "y=0.33"]
h4 = ParametricPlot[{x[s, 0.333], u[s]},
  {s, 0, 2}, PlotRange → {0, 4}, PlotLabel → "y=0.333 "]
h5 = ParametricPlot[{x[s, 0.4], u[s]}, {s, 0, 2}, PlotRange → {0, 4}, PlotLabel → "y=0.4"]
Show[GraphicsArray[{{h0, h1, h2}, {h3, h4, h5}}].FrameTicks → None, Frame → False]
```

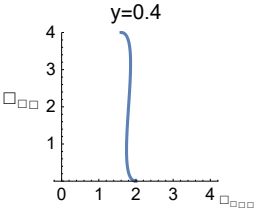












GraphicsArray: GraphicsArray is obsolete. Switching to GraphicsGrid.

Show: No graphical objects to show.

Out[69]= Show[y=0 y=0.2 y=0.3 y=0.33 y=0.333 y=0.4 .FrameTicks → None, Frame → False]