# Vivek Gupta | BSC CS Hons | 20211467 | Practical- 8

---

Plot the integral surface of a given first order PDE with the initial data

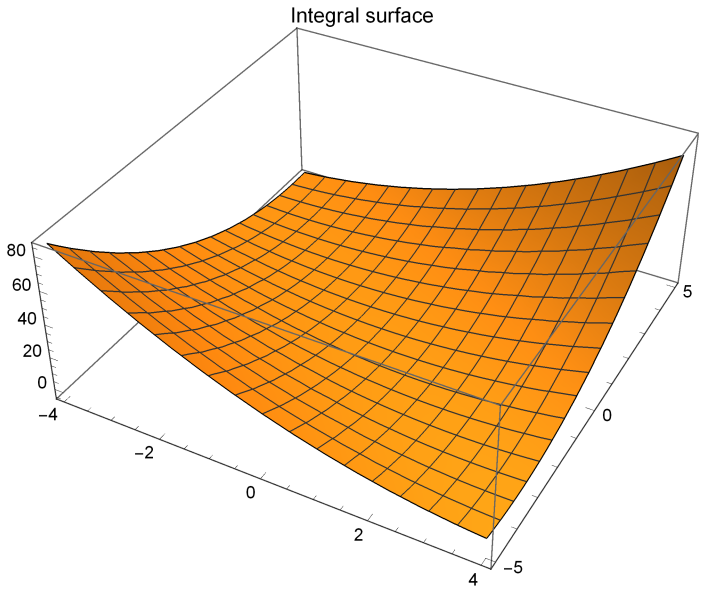
Problem 1: Obtain the solution of the linear equation u[(x,y),x]-u[(x,y),y]=1 with
the Cauchy data u(x,0) = x*x.
Plot the integral surface with in the range {x,-4,4} and {y,-5,5}.
Solution :

In[4]:=
```
pde = D[u[x, y], x] - D[u[x, y], y] == 1
DSolve[{D[u[x, y], x] - D[u[x, y], y] == 1, u[x, 0] == (x * x)}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, -4, 4}, {y, -5, 5}, PlotLabel → "Integral surface "]
```

Out[4]= $-u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 1$

Out[5]= $\{\{u[x, y] \to x^2 - y + 2xy + y^2\}\}$

Out[6]=




DSolve: Equation or list of equations expected instead of $-\partial u^{(0,1)}[x, y] + u^{(1,0)}[x, y]$ in the first argument $\{-\partial u^{(0,1)}[x, y] + u^{(1,0)}[x, y], \partial x^2\, u[x, 0]\}$.

Out[2]=
```
DSolve[{-∂ u^(0,1) [x, y] + u^(1,0) [x, y], ∂ x^2 u[x, 0]}, u[x, y], {x, y}]
```

## Problem 2: Find the solution of the equation y*u[(x,y),x]-2*x*y*u[(x,y),y]=2*x*u[x,y] with the Cauchy data u(0,y) = y*y*y. Plot the integral surface with in the range {x,-7,7} and {y,-5,5}.

## Solution

Plot3D: Options expected (instead of PlotLabel → Integral surface ) beyond position 3 in Plot3D[u[x, y] /. %, {x, −4, 4}, {y, −5, 5}, PlotLabel → Integral surface ]. An option must be a rule or a list of rules.

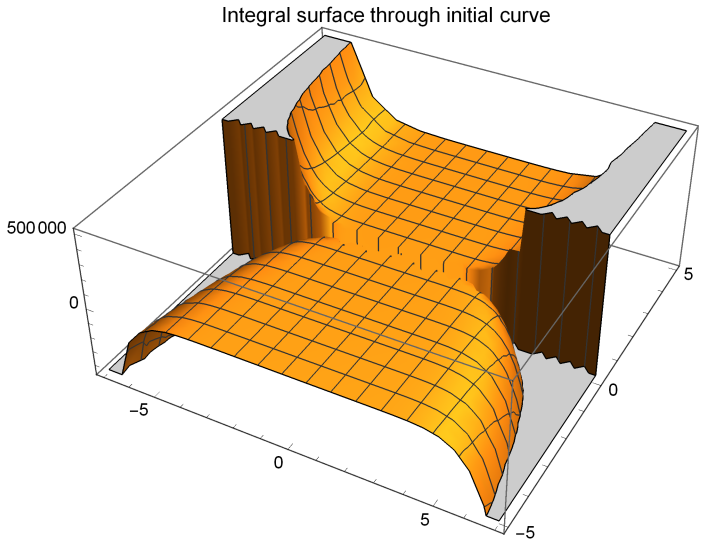
Out[3]=
```
Plot3D[u[x, y] /. %, {x, -4, 4}, {y, -5, 5}, PlotLabel → Integral surface ]
```

In[7]:=
```
pde = y * D[u[x, y], x] - 2 * x * y * D[u[x, y], y] == 2 * x * u[x, y]
sol3 = DSolve[{pde, u[0, y] == y * y * y}, u[x, y], {x, y}]
Plot3D[u[x, y] /. sol3, {x, -7, 7}, {y, -5, 5},
 PlotLabel → "Integral surface through initial curve"]
```

Out[7]= $-2\,x\,y\,u^{(0,1)}[x, y] + y\,u^{(1,0)}[x, y] == 2\,x\,u[x, y]$

Out[8]= $\left\{\left\{u[x, y] \to \frac{\left(x^2 + y\right)^4}{y}\right\}\right\}$

Out[9]=



## Problem 3 : Determine the integral surfaces of the equation u[(x,y),x]+u[(x,y),y]=u[x,y]*u[x,y] , with the data x+y=0,u=1. Plot the integral surface with in the range {x,10,10} and {y,-10,10}.

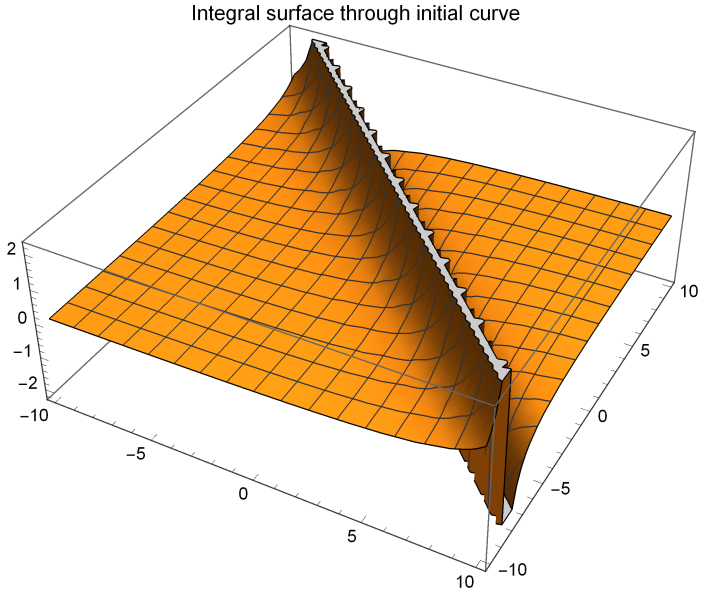
## Solution :

In[10]:=
```
Eqn = D[u[x, y], x] + D[u[x, y], y] == u[x, y] * u[x, y]
DSolve[{D[u[x, y], x] + D[u[x, y], y] == u[x, y] * u[x, y], u[x, -x] == 1}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, -10, 10}, {y, -10, 10},
 PlotLabel → "Integral surface through initial curve "]
```

Out[10]= $u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == u[x, y]^2$

Out[11]= $\left\{\left\{u[x, y] \to -\frac{2}{-2 + x + y}\right\}\right\}$

Out[12]=



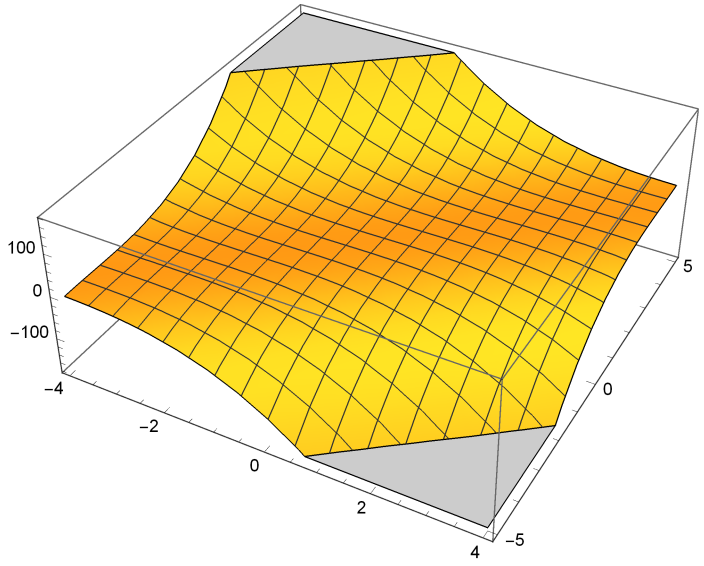
Problem 4 : Obtain the solution of the linear equation u[(x,y),x]+u[(x,y),y]=1 with
the Cauchy data u(x,2x)=x*x*x.
Plot the integral surface with in the range {x,-4,4} and {y,-5,5}.
Solution :

In[13]:=
```
D[u[x, y], x] + D[u[x, y], y] == 1
DSolve[{D[u[x, y], x] + D[u[x, y], y] == 1, u[x, 2 x] == x * x * x}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, -4, 4}, {y, -5, 5}]
```

Out[13]= $u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 1$

Out[14]= $\left\{\left\{u[x, y] \to 2 x - x^3 - y + 3 x^2 y - 3 x y^2 + y^3\right\}\right\}$

Out[15]=



## Problem 5 : Obtain the solution of the linear equation u(x+y)*u[(x,y),x]+u(x□y)*u[(x,y),y]=x*x+y*y with the Cauchy data u(x,2x)=0. Plot the integral surface with in the range {x,-4,4} and {y,-5,5}.

Solution :

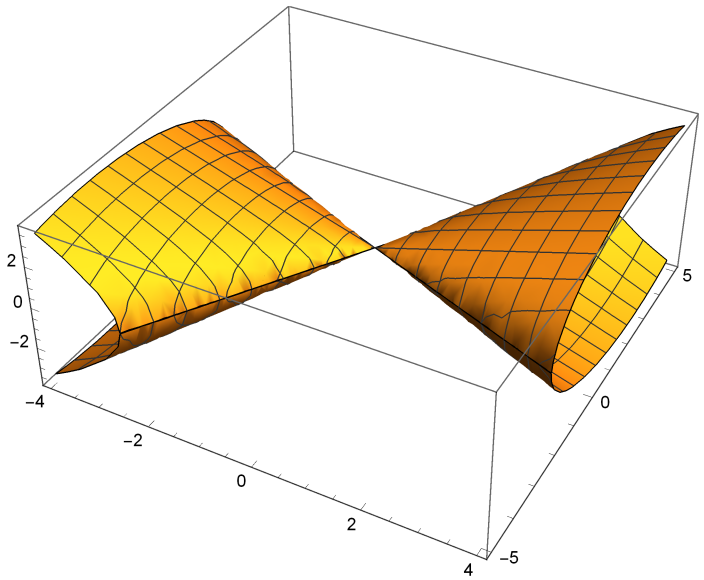
In[16]:=
```
eqn = u[x, y] * (x + y) * D[u[x, y], x] + u[x, y] * (x - y) * D[u[x, y], y] == x * x + y * y
DSolve[{u[x, y] * (x + y) * D[u[x, y], x] + u[x, y] * (x - y) * D[u[x, y], y] == (x * x) + (y * y),
  u[x, 2 x] == 0}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, -4, 4}, {y, -5, 5}]
```

Out[16]= $(x-y)\,u[x,y]\,u^{(0,1)}[x,y]+(x+y)\,u[x,y]\,u^{(1,0)}[x,y]==x^2+y^2$

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[17]= $\left\{\left\{u[x,y]\to-\sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}\right\},\left\{u[x,y]\to\sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}\right\},\right.$
$\left.\left\{u[x,y]\to-\sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}\right\},\left\{u[x,y]\to\sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}\right\}\right\}$

Out[18]=




## Problem 6 : Obtain the solution of the linear equation u[(x,y),x]+u[x,y]*u[(x,y),y]=1 with the Cauchy data u(0,y)=4*y.

Plot the integral surface with in the range {x,-4,4} and {y,-5,5}.

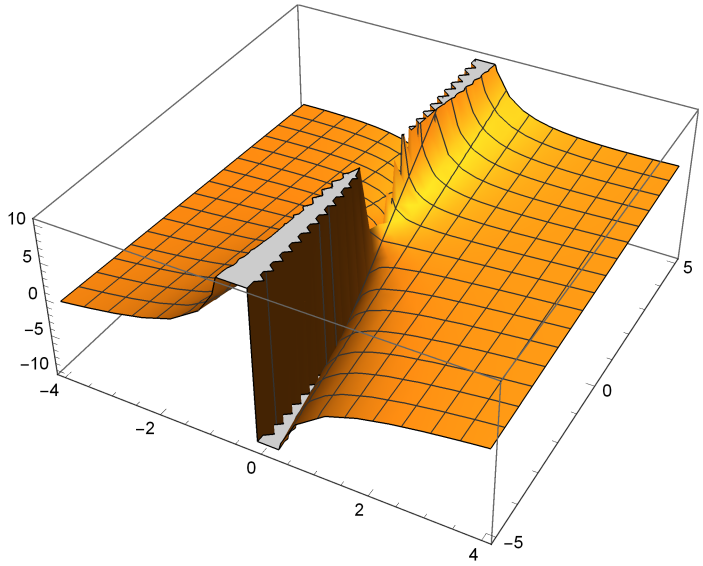
Solution

In[19]:=
```
D[u[x, y], x] + u[x, y] * D[u[x, y], y] == 1
DSolve[{D[u[x, y], x] + u[x, y] * D[u[x, y], y] == 1, u[0, y] == 4 * y}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, -4, 4}, {y, -5, 5}]
```

Out[19]= $u[x, y]\, u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 1$

Out[20]= $\left\{\left\{u[x, y] \to \frac{x + 2\,x^2 + 4\,y}{1 + 4\,x}\right\}\right\}$

Out[21]=



Problem 7 : Obtain the solution of the linear equation u[(x,y),x]+y*u[(x,y),y]=0 with the Cauchy data u(0,y)=4*y.

Plot the integral surface with in the range {x,-4,4} and {y,-5,5}.

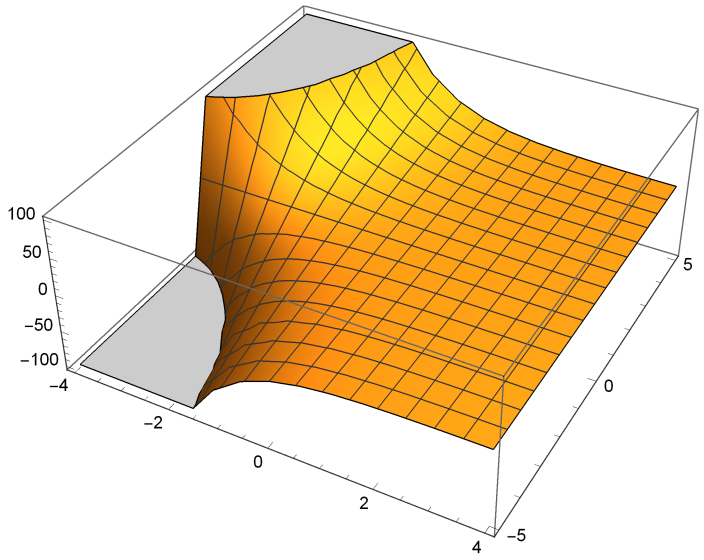
Solution :

In[22]:=
```
D[u[x, y], x] + y * D[u[x, y], y] == 0
DSolve[{D[u[x, y], x] + y * D[u[x, y], y] == 0, u[0, y] == 4 * y}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, -4, 4}, {y, -5, 5}]
```

Out[22]= $y\,u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 0$

Out[23]= $\{\{u[x, y] \to 4\,e^{-x}\,y\}\}$

Out[24]=



## Problem 8 : Obtain the solution of the linear equation u[(x,y),x]+2*u[(x,y),y]=0 with the Cauchy data u(0,y)=Exp[-y*y].

## Plot the integral surface with in the range {x,-4,4} and {y,-5,5}.

## Solution :

In[25]:=
```
D[u[x, y], x] + 2 * D[u[x, y], y] == 0
DSolve[{D[u[x, y], x] + 2 * D[u[x, y], y] == 0, u[0, y] == Exp[-y * y]}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, -4, 4}, {y, -5, 5}]
```

Out[25]= $2\,u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 0$

Out[26]= $\left\{\left\{u[x, y] \to e^{-(-2x+y)^2}\right\}\right\}$

Out[27]=