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Good morning everyone. Today we are going to present our final presentation on the topic “ Minimax Algorithm”. Our Team members Includes Aadarsh, Priyanshu,Jatin, and Vivek.

In this Presentation, we're going to discuss the Minimax algorithm and its applications in AI. As it's a game theory algorithm, we'll implement a simple game using it. We'll also discuss the advantages of using the algorithm and see how it can be improved.

Let's jump to the Introduction.

Minimax is a decision-making algorithm, typically used in a turn-based, two player game.

The goal of the algorithm is to find the optimal next move.

Tic-tac-toe is a simple game that flexes the basic concepts in programming. For example, a Tic-tac -toe program requires data structures for storing the board and conditional logic for knowing whose turn it is or if someone has won.

In the algorithm, one player is called the maximizer, and the other player is a minimizer. If we assign an evaluation score to the game board, one player tries to choose a game state with the maximum score, while the other chooses a state with the minimum score.

In other words, the maximizer works to get the highest score, while the minimizer tries to get the lowest score by trying to counter moves.

It is based on the **zero-sum game** concept. In a zero-sum game, the total utility score is divided among the players. An increase in one player's score results in the decrease in another player's score. So, the total score is always zero. For one player to win, the other one has to lose. Examples of such games are chess, poker, checkers, tic-tac-toe.

An interesting fact- in 1997, IBM's chess-playing computer Deep Blue (built with Minimax) defeated Garry Kasparov (the world champion in chess).

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Let's Talk about some **Tools** which we had used in this project

First we have HTML is a standard markup language for documents designed to be displayed on a web browser by the use of it we make the structure of the project like grid and its cell

Second, we have CSS. It doesn't have much use but it has a very important role to design and for color to something to make it more beautiful.

Finally, we have JavaScript. In which we write the code and implement the algorithm.

Priyanshu Sisodiya

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Minimax Algorithm

Mini-max algorithm is a recursive or backtracking algorithm which is used in decision-making and game theory. It provides an optimal move for the player assuming that the opponent is also playing optimally.

It is a Backtracking Algorithm in which we move from max to min and then backtrack again to max until we get our relevant results. In this Algorithm the recursive function is used by which it calls it again and again.

There are two players MIN and MAX. Min wants to try to minimize the max move. Max player wants to try to maximize the move itself.

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Terminology

There are four main terminologies.

- **Game Tree:** It is a structure in the form of a tree consisting of all the possible moves which allow you to move from a state of the game to the next state.

A game can be defined as a search problem with the following components:

- **Initial state:** It comprises the position of the board and showing whose move it is.
- **Successor function:** It defines what the legal moves a player can make are.
- **Terminal state:** It is the position of the board when the game gets over.
- **Utility function:** It is a function which assigns a numeric value for the outcome of a game. For instance, in chess or tic-tac-toe, the outcome

is either a win, a loss, or a draw, and these can be represented by the values +1, -1, or 0, respectively. There are games that have a much larger range of possible outcomes; for instance, the utilities in backgammon varies from +192 to -192. A utility function can also be called a payoff function.

Implementation

There are two players involved in a game, called MIN and MAX. The player MAX tries to get the highest possible score and MIN tries to get the lowest possible score, i.e., MIN and MAX try to act opposite of each other.

The general process of the Minimax algorithm is as follows:

Step 1: First, generate the entire game tree starting with the current position of the game all the way upto the terminal states. This is how the game tree looks like for the game tic-tac-toe.

Let us understand the defined terminology in terms of the diagram above.

1. The initial state is the first layer that defines that the board is blank it's MAX's turn to play.
2. Successor function lists all the possible successor moves. It is defined for all the layers in the tree.
3. Terminal State is the last layer of the tree that shows the final state, i.e whether the player MAX wins, loses, or ties with the opponent.
4. Utilities in this case for the terminal states are 1, 0, and -1 as discussed earlier, and they can be used to determine the utilities of the other nodes as well.

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Now Over to Jatin

In Methodology

We Apply the utility function to get the utility values for all the terminal states.

Determine the utilities of the higher nodes with the help of the utilities of the terminal nodes. For instance, in the diagram below, we have the utilities for the terminal states written in the squares.

Let us calculate the utility for the left node(red) of the layer above the terminal. Since it is the move of the player MIN, we will choose the minimum of all the utilities. For this case, we have to evaluate $\text{MIN}\{3, 5, 10\}$, which we know is certainly 3. So the utility for the red node is 3.

Calculate the utility values with the help of leaves considering one layer at a time until the root of the tree.

Eventually, all the backed-up values reach to the root of the tree, i.e., the topmost point. At that point, MAX has to choose the highest value.

In our example, we only have 3 layers so we immediately reach to the root but in actual games, there will be many more layers and nodes. So we have to evaluate $\text{MAX}\{3,2\}$ which is 3.

Therefore, the best opening move for MAX is the left node(or the red one). This move is called the minimax decision as it maximizes the utility following the assumption that the opponent is also playing optimally to minimize it.

To summarize,

Minimax Decision = $\text{MAX}\{\text{MIN}\{3,5,10\}, \text{MIN}\{2,2\}\}$

= $\text{MAX}\{3,2\}$

= 3

Similarly, for the green node in the same layer, we will have to evaluate $\text{MIN}\{2,2\}$ which is 2.

Application

- The Mini-Max algorithm uses recursion to search through the game-tree.
- The Min-Max algorithm is mostly used for game playing in AI. Such as Chess, Checkers, tic-tac-toe, go, and various two-players games. This Algorithm computes the minimax decision for the current state. Find out the best move against the Opponent.

Now Vivek will continue

Optimization

Game trees are, in general, very time consuming to build, and it's only for simple games that it can be generated in a short time.

If there are b legal moves i.e., b nodes at each point and the maximum depth of the tree is m , the time complexity of the minimax algorithm is of the order $O(b^m)$

To curb this situation, there are a few optimizations that can be added to the algorithm.

Fortunately, it is viable to find the actual minimax decision without even looking at every node of the game tree. Hence, we eliminate nodes from the tree without analyzing, and this process is called pruning.

Alpha-beta pruning

The method that we are going to look at in this Presentation is called alpha-beta pruning.

If we apply alpha-beta pruning to a standard minimax algorithm, it returns the same move as the standard one, but it removes (prunes) all the nodes that are possibly not affecting the

final decision.

Let us understand the intuition behind this first and then we will formalize the algorithm.

Suppose, we have the following game tree:

In this case,

Minimax Decision = $\text{MAX}\{\text{MIN}\{3,5,10\}, \text{MIN}\{2,a,b\}, \text{MIN}\{2,7,3\}\}$

= $\text{MAX}\{3,c,2\}$

= 3

You would be surprised!

How could we calculate the maximum with a missing value? Here is the trick. $\text{MIN}\{2,a,b\}$ would certainly be less than or equal to 2, i.e., $c \leq 2$ and hence $\text{MAX}\{3,c,2\}$ has to be 3.

The question now is do we really need to calculate c? Of course not.

We could have reached a conclusion without looking at those nodes. And this is where alpha-beta pruning comes into the picture.

Definitions:

Alpha: It is the best choice so far for the player MAX. We want to get the highest possible value here.

Beta: It is the best choice so far for MIN, and it has to be the lowest possible value.

Note: Each node has to keep track of its alpha and beta values. Alpha can be updated only when it's MAX's turn and, similarly, beta can be updated only when it's MIN's chance.

work?

1. Initialize $\alpha = -\infty$ and $\beta = \infty$ as the worst possible cases. The condition to prune a node is when α becomes greater than or equal to β .
2. Start with assigning the initial values of α and β to root and since α is less than β we don't prune it.
3. Carry these values of α and β to the child node on the left. And now from the utility value of the terminal state, we will update the values of α and β , so we don't have to update the value of β . Again, we don't prune because the condition remains the same. Similarly, the third child node also. And then backtracking to the root we set $\alpha=3$ because that is the minimum value that α can have.
4. Now, $\alpha=3$ and $\beta=\infty$ at the root. So, we don't prune. Carrying this to the center node, and calculating $\min\{2, \infty\}$, we get $\alpha=3$ and $\beta=2$.
5. Prune the second and third child nodes because α is now greater than β .
6. α at the root remains 3 because it is greater than 2. Carrying this to the rightmost child node, evaluate $\min\{\infty, 2\}=2$. Update β to 2 and α remains 3.
7. Prune the second and third child nodes because α is now greater than β .
8. Hence, we get 3, 2, 2 at the left, center, and right MIN nodes, respectively. And calculating $\max\{3, 2, 2\}$, we get 3. Therefore, without

even looking at four leaves we could correctly find the minimax decision.

Conclusion

Games are very appealing and writing game-playing programs is perhaps even more exciting. What Grand Prix racing is to the car industry, game playing is to AI.

Just as we would not expect a racing car to run perfectly on a bumpy road, we should not expect game playing algorithms to be perfect for every situation.

So is the minimax algorithm. It may not be the best solution to all kinds of computer games that need to have AI.

But given a good implementation, it can create a tough competitor.

We come to Future Ideas

In the future we have some ideas related to the Project in which we want to work on to make it as a generalized grid like $m \times n$ and $m \times m$ where m and n are both greater than 3.

Another One we will also increase the players but till now we have worked on 3 player tic tac toe game and added some more levels like Easy Medium and Hard by increasing the level AI Minimax Algorithm.

