MATH 6904: Mathematic Modelling

MATLAB Assignment

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Question 1:

Consider the linear system of equations:

$\boldsymbol{x_1 + x_2 + 3x_3 + x_4 = 1},$

$\boldsymbol{x_1 - x_2 - x_3 - x_4 = 1},$

$\boldsymbol{3x_1 + x_2 + 5x_3 + 3x_4 = 1},$

$\boldsymbol{x_1 + 5x_2 + 11x_3 + 8x_4 = -2},$

For the unknown vector x = $\boldsymbol{(x_1, x_2, x_3, x_4)^T}$.

(a) In MATLAB, define the matrix A and vector b, such that Ax = b.

In MATLAB, column vectors are already in the transposed form. So, no explicit transpose operation is needed for x = $(x_1, x_2, x_3, x_4)^T$

```
>> A = [1 1 3 1; 1 -1 -1 -1; 3 1 5 3; 1 5 11 8]


A =



     1      1   3     1

     1     -1   -1    -1

     3      1   5     3

     1      5   11    8


>> b = [1; 1; 1; -2]

b =
```

```
    1

    1

    1

   -2
```

(b) Using the backslash command \ determine the solution to the linear system Ax =b

```
 >> x = A\b
Warning: Matrix is singular to working precision.


x =

   NaN

   NaN

  -Inf

   Inf
```

(c) Determine the determinant of A using the det command.

```
>> det(A)

ans =

     0
```

(d) The answer in (b) is only one solution to the linear system. Using an alternative MATLAB command, determine and state the full set of solutions.

```
>> AUG =[A,b]


AUG =


     1      1   3    1      1

     1     -1   -1   -1      1

     3      1   5    3      1

     1      5   11   8     -2

>> AUG = rref(AUG)


AUG =


     1      0   1    0      1
```

0	1	2	0	1
0	0	0	1	-1
0	0	0	0	0

After using the reduced row echelon form of AUG, the equation can be written 'by hand' below:

$$\begin{cases} x_1 = 1 - t \\ x_2 = 1 - 2t \\ x_3 = t \\ x_4 = -1 \end{cases}$$

$\boxed{5}$

Question 2:

Consider the line given by the symmetric equation

$$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z+1}{1}$$

And the plane given by the Cartesian equation 3x – y + 5z = 7

(a) By hand (i.e., the method presented in lectures), determine the point (a, b, c) at which the line intersects the plane.

- The parametric equations of the line are given as:

$$\begin{cases} x = 2t + 1 \\ y = -t + 3 \\ z = t - 1 \end{cases}$$

- On substituting into the plane gives

$$7 = 3x - y + 5z = 3(2t + 1) - (-t + 3) + 5(t - 1)$$

- Thus,

$$t = 1$$

giving the point

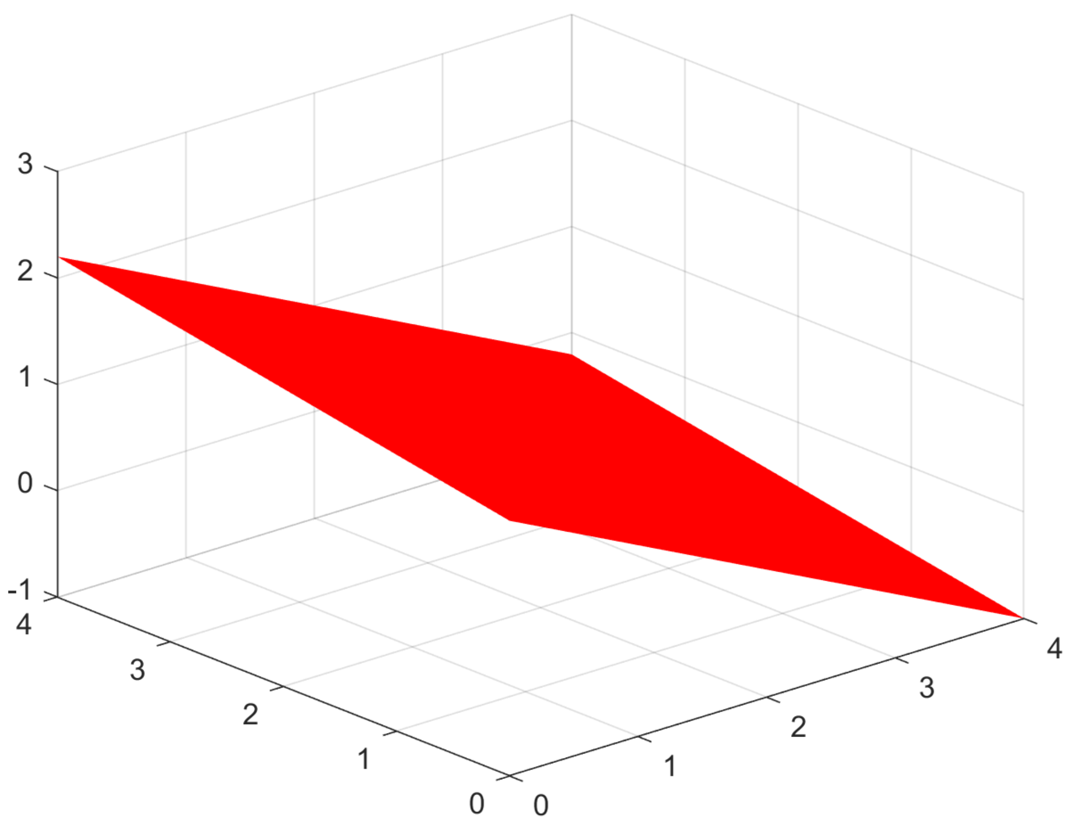
$$(x, y, z) = (3, 2, 0)$$

(b) Define [x,y]=meshgrid(0:4) and using mesh(x,y,z, 'linestyle', 'none', 'facecolor', 'red') plot the plane. Type hold on.

```
>> [x,y] = meshgrid(0:4)
```

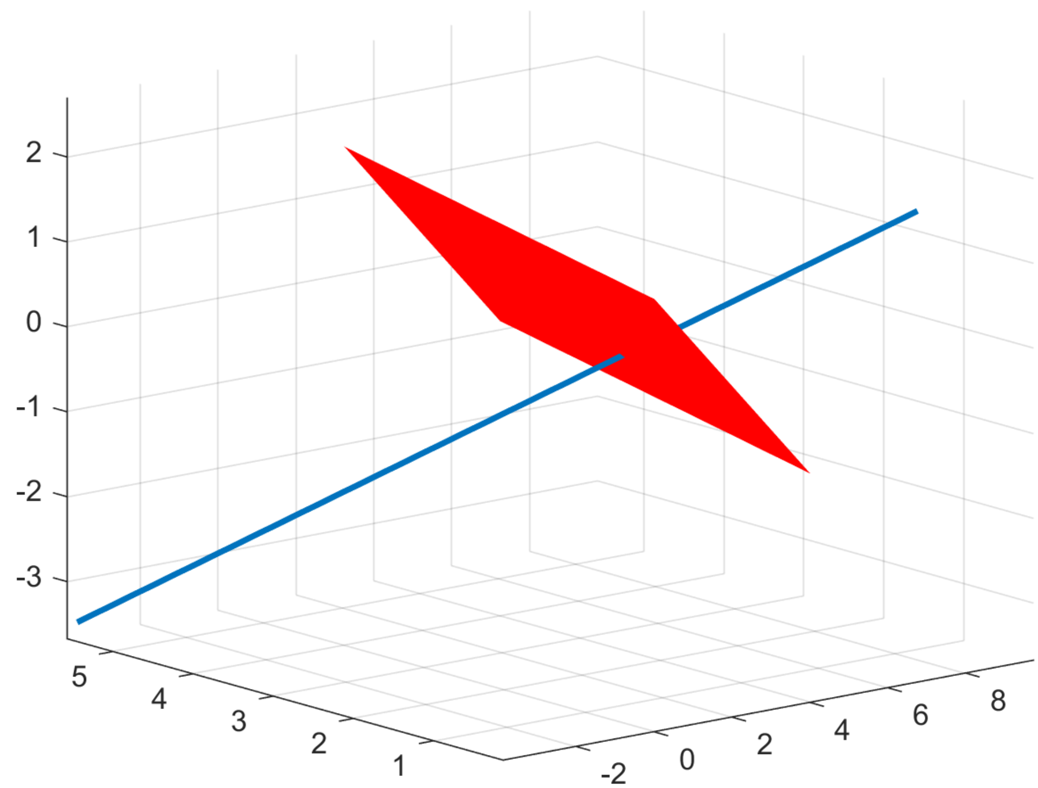
```
x =

     0     1     2     3     4
     0     1     2     3     4
     0     1     2     3     4
     0     1     2     3     4
     0     1     2     3     4

y =

     0     0     0     0     0
     1     1     1     1     1
     2     2     2     2     2
     3     3     3     3     3
     4     4     4     4     4

>> z = (-3*x+y+7)/5

z =

     1.4000    0.8000    0.2000   -0.4000   -1.0000
```

```
    1.6000    1.0000   0.4000   -0.2000   -0.8000

    1.8000    1.2000   0.6000         0   -0.6000

    2.0000    1.4000   0.8000    0.2000   -0.4000

    2.2000    1.6000   1.0000    0.4000   -0.2000


>> mesh(x,y,z,'linestyle', 'none', 'facecolor', 'red')

>> hold on
```



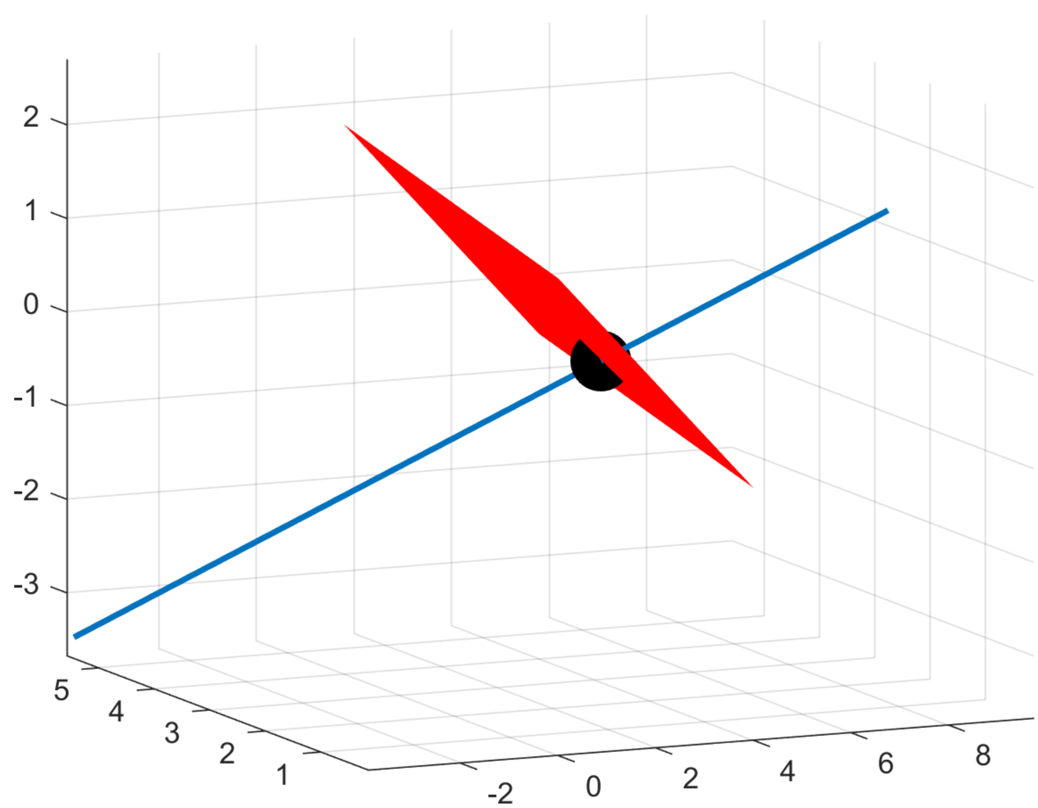
(c) Using the plot3() command, plot the line in the same figure as the plane.

```
>> t = linspace (-3,3,100);

>> x=2*t+1;

>> y=-t+3;

>> z=t-1;

>> plot3(x,y,z,'linewidth',2)
```



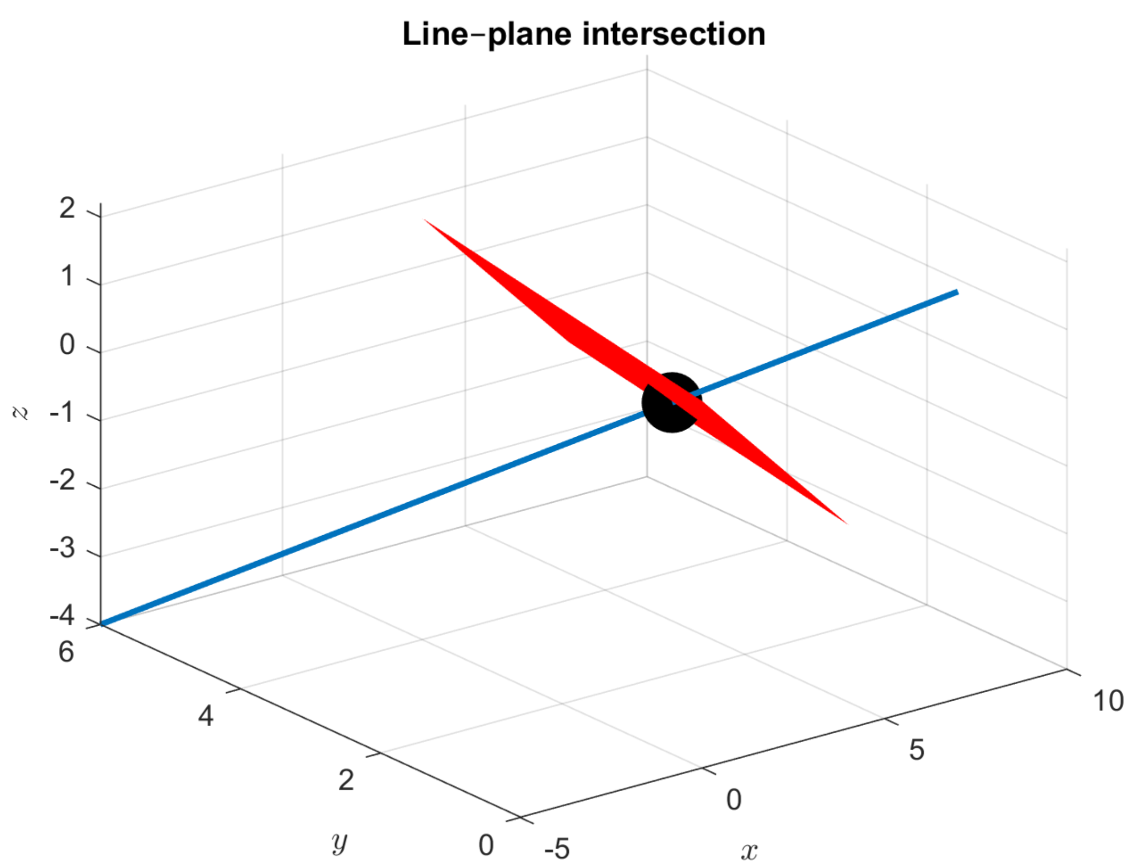
(d) In the same figure, plot the point of intersection (a, b, c) of the line and plane, using the command plot3(a,b,c,'ko','markersize',20, 'MarkerFaceColor','k').

```
>> a = 3;

>> b = 2;

>> c = 0;

>> plot3(a,b,c,'ko', 'markersize', 20, 'MarkerFaceColor',
'k')
```



(e) Add an appropriate title, and x-, y-, z-labels to your figure and save as a PDF.

```
>> title('Line-plane intersection');

>> xlabel('$x$', 'interpreter','latex');

>> ylabel('$y$', 'interpreter','latex');

>> zlabel('$z$', 'interpreter','latex');

>> saveas(gcf, 'Line-plane intersection', 'pdf')
```



Question 3:

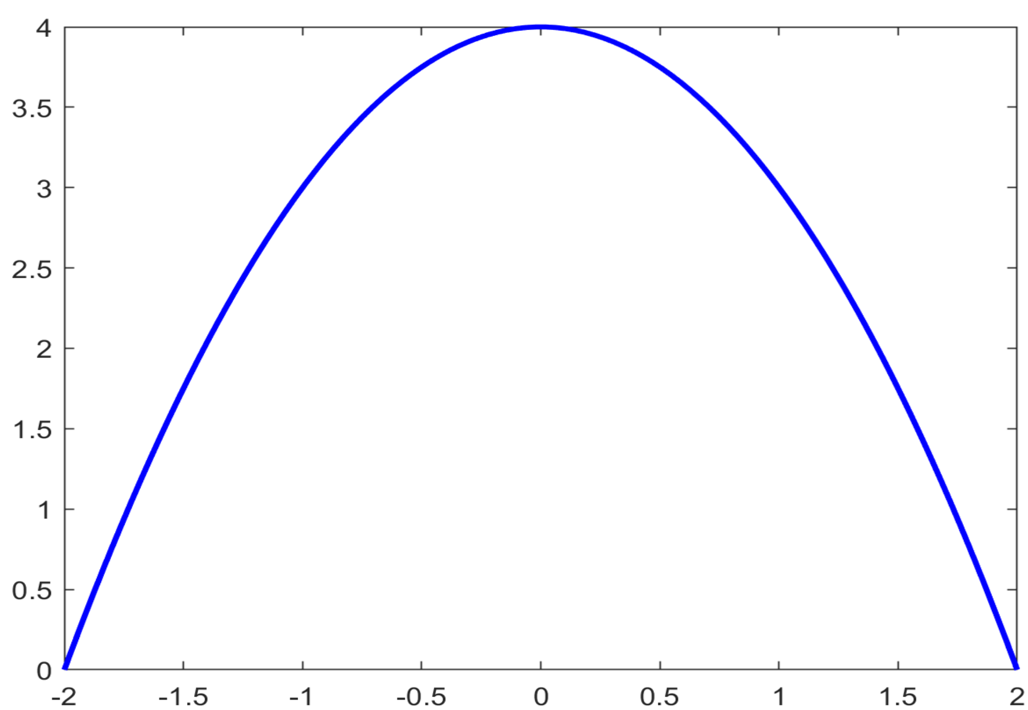
Consider the functions $f(x) = x^2$ and $g(x) = \sqrt{4 - x^2}$

(a) Determine the domain of the composite function (f ∘ g)(x). In Matlab, define the domain of f ∘ g using the linspace command, and define the composite function f ∘ g.

```
>> x = linspace(-2, 2, 100);

>> fog = (sqrt(4-x.^2)).^2;
```

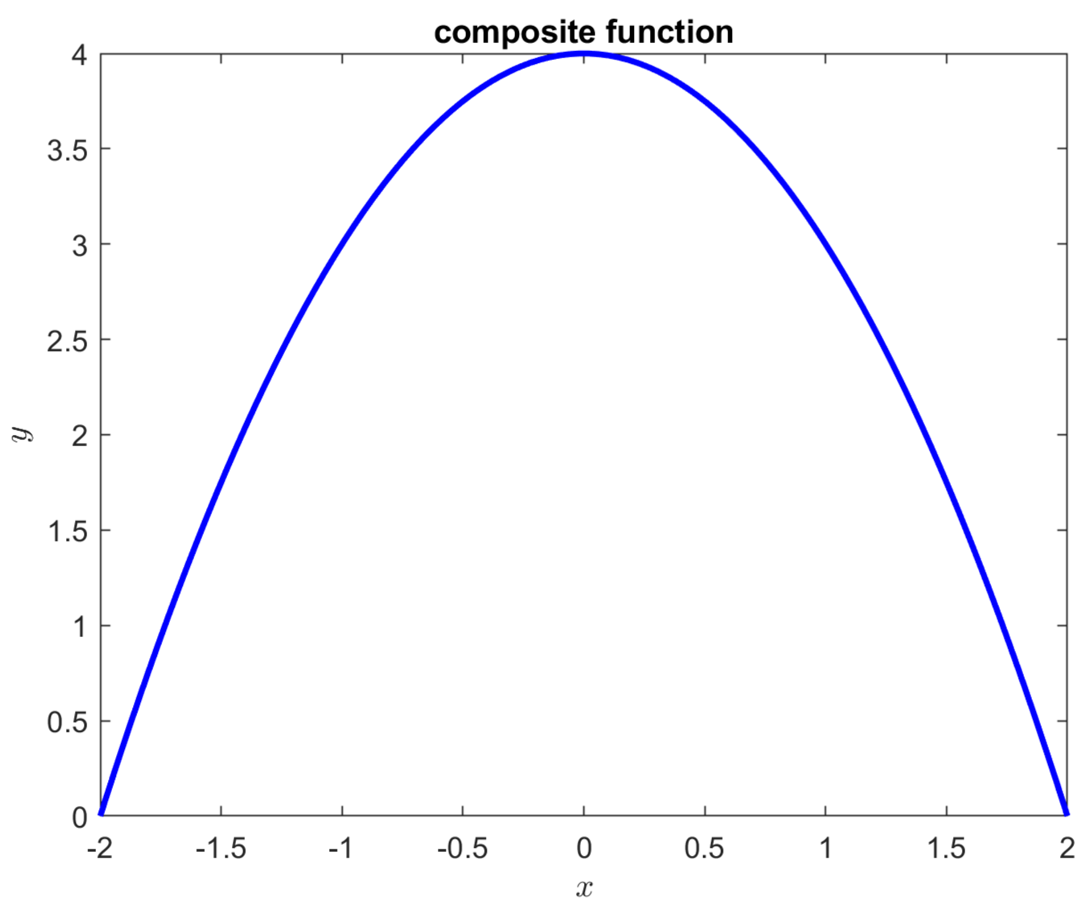
(b) Plot the composite function using the plot() command.

```
>> plot (x, fog, '-b', 'linewidth',2);
```



(c) Add an appropriate title, and x-, y-labels to your figure and save as a PDF

```
>> title('composite function');

>> xlabel('$x$','interpreter','latex');

>> ylabel('$y$','interpreter','latex');

>> saveas(gcf, 'composite function', 'pdf')
```



Question 4:

Consider the function

$$\boldsymbol{A(t) = 2b^2t^4 - 2t^2}$$

Where $\boldsymbol{b \in R}$

(a) Use the syms command to create the symbolic variables t and b, and then create the symbolic function A.

```
>> syms b t

>> A = 2*b^2*t^4-2*t^2

 A =

 2*b^2*t^4 - 2*t^2
```

(b) Use the diff command to find dA/dt. (See section 1.9.1 of the Matlab Manual.)

```
>> syms t

>> diff(2*b^2*t^4-2*t^2)



ans =



8*b^2*t^3 - 4*t
```

(c) Use the solve command to solve dA/dt = 0. (See section 1.9.3 of the Matlab Manual.)

```
>> solve(8*b^2*t^3 - 4*t==0)

ans =

         0

-2^(1/2)/(2*b)

 2^(1/2)/(2*b)
```

(d) Use the subs(A,t) command to compute A(5b).

```
>> syms b t

 >> subs(2*b^2*t^4-2*t^2, t, 5*b)

ans =

1250*b^6 - 50*b^2
```

Question 5:

Use MATLAB to determine the following integrals.

(a) Determine

$$\int \boldsymbol{tan(x)\,dx}.$$

(**Use the syms and int commands. See section 1.9.1 of the MATLAB Manual**).

```
>> syms x

>> int(tan(x),x)


ans =


-log(cos(x))
```

(b) Determine

$$\int \boldsymbol{log\,(x^2)\,dx}.$$

Where log is the natural logarithm.

```
>> syms x

>> int(log(x^2),x)
```

```
ans =


x*(log(x^2) - 2)
```

(c) Determine

$$\int x^2 \sin(2x)\ dx.$$

```
>> syms x
>> int(x^2*sin(2*x),x)


ans =


(x*sin(2*x))/2 + (2*sin(x)^2 - 1)*(x^2/2 - 1/4)
```