Concept

Jingwen Xu

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1. Simulate original nonnegative matrices X_1 ($p_1 \times n_1$) and X_2 ($p_2 \times n_2$). Choose the factorization ranks r_1 and r_2.
2. Initialize W_1 and H_1 for X_1, W_2 and H_2 for X_2 by randomly assignning each entry of them from the lognormal distribution.
3. Set up the objective function:

$$\frac{1}{2} \parallel X_1 - W_1 H_1 \parallel_F^2 + \frac{\lambda_1}{2} \parallel X_2 - W_2 H_2 \parallel_F^2 - \lambda_2 tr(W_2^T A W_1) + \mu(\parallel W_1 \parallel_F^2 + \parallel W_1 \parallel_F^2)$$

Where,

$$\mu = \parallel X_1 - W_1^{(N)} H_1^{(N)} \parallel_F^2 / (\parallel W_1^{(N)} \parallel_F^2 + \parallel W_2^{(N)} \parallel_F^2)$$

If we want to extend the above function to muti-model(join more than 2 models), the difficulty is to join more matrices in $tr(W_2^T A W_1)$. It should be a question about tensor analysis.

4. For the input sparse matrices, iterate for N times with ALS firstly and then use MU to iterate until converging to a stationary point.

for t = 1, 2, 3, . . . , N, do

$$W_1^{(t)} \leftarrow max(argmin_{Z_1 \in R^{p_1 \times r_1}} \parallel X_1 - Z_1 H_1^{(t-1)} \parallel_F, 0)$$
$$W_2^{(t)} \leftarrow max(argmin_{Z_2 \in R^{p_2 \times r_2}} \parallel X_2 - Z_2 H_2^{(t-1)} \parallel_F, 0)$$
$$H_1^{(t)} \leftarrow max(argmin_{Y_1 \in R^{r_1 \times n_1}} \parallel X_1 - W_1^{(t-1)} Y_1 \parallel_F, 0)$$
$$H_2^{(t)} \leftarrow max(argmin_{Y_2 \in R^{r_2 \times n_2}} \parallel X_2 - W_2^{(t-1)} Y_2 \parallel_F, 0)$$

end

Then,

for t = 1, 2, 3, . . . , ?, do

$$W_1^{(t)} \leftarrow W_1^{(t-1)} \frac{X_1 H_1^{T(t-1)} + \frac{\lambda_2}{2} A^T W_2^{(t-1)}}{W_1^{(t-1)} H_1^{t-1} H_1^{T(t-1)} + 2\mu W_1^{(t-1)}}$$

$$W_2^{(t)} \leftarrow W_2^{(t-1)} \frac{X_2 H_2^{T(t-1)} + \frac{\lambda_2}{2\lambda_1} A^T W_1^{(t-1)}}{W_2^{(t-1)} H_2^{t-1} H_2^{T(t-1)} + 2\mu W_2^{(t-1)}}$$

$$H_1^{(t)} \leftarrow H_1^{(t-1)} \frac{W_1^{T(t-1)} X_1}{W_1^{T(t-1)} W_1^{(t-1)} H_1^{(t-1)}}$$

$$H_2^{(t)} \leftarrow H_2^{(t-1)} \frac{W_2^{T(t-1)} X_2}{W_2^{T(t-1)} W_2^{(t-1)} H_2^{(t-1)}}$$

end for (how to add a time limit and find the stationary point)