p8106_hw1_qz2266

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Contents

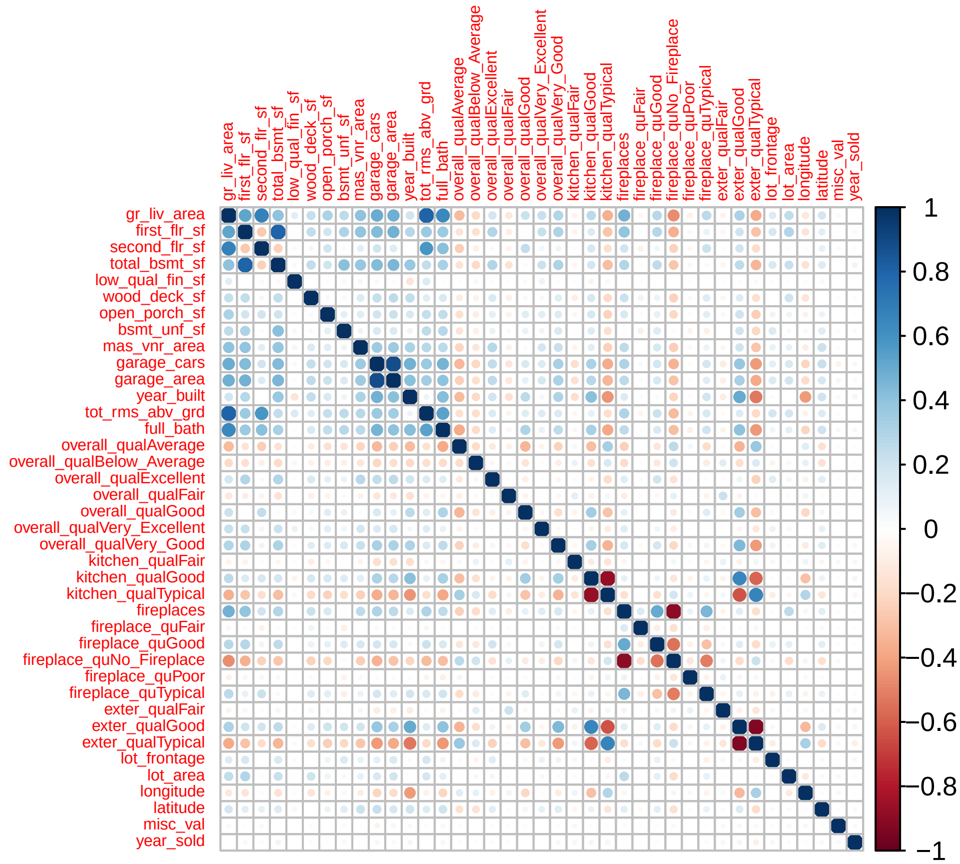
0.0.1 Data Import and cleaning

In this exercise, we predict the sale price of a house using its other characteristics. The training data are in "housing train.csv", and the test data are in "housing test.csv".

```
# read in training data
train = read.csv("data/housing_training.csv") %>%
janitor::clean_names()
train = na.omit(train)

# read in test data
test = read.csv("data/housing_test.csv") %>%
janitor::clean_names()
test = na.omit(test)

# create covariates matrix for training and test
x_train = model.matrix(sale_price ~ ., train)[,-1]
y_train = train$sale_price
x_test <- model.matrix(sale_price ~ ., test)[ ,-1]
y_test <- test$sale_price
```

Check for potential collinearities among predictors in training data

```
# Correlation plot for all predictors
corrplot(cor(x_train), method = "circle", type = "full", tl.cex = 0.5)
```



- From the correlation plot we can see there are high correlations between some of the covariates. This high correlation might cause collinearity problem.
- To fix the potential multicollinearity issue, regularization methods such as lasso, elastic net, or partial least squares could be employed, other than linear model. Please see below for these models.

0.0.2 a). Linear model

```
set.seed(1)
lm.fit <- train(x_train, y_train,
                method = "lm",
                trControl = trainControl(method = "repeatedcv", number = 10, repeats = 5))
summary(lm.fit)
```

```
##
## Call:
## lm(formula = .outcome ~ ., data = dat)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -89864 -12424    416  12143 140205
##
## Coefficients: (1 not defined because of singularities)
##                              Estimate Std. Error t value Pr(>|t|)
```

```
## (Intercept)                 -4.985e+06  3.035e+06  -1.642  0.10076
## gr_liv_area                  2.458e+01  1.393e+01   1.765  0.07778 .
## first_flr_sf                 4.252e+01  1.409e+01   3.017  0.00260 **
## second_flr_sf                4.177e+01  1.379e+01   3.029  0.00250 **
## total_bsmt_sf                3.519e+01  2.744e+00  12.827  < 2e-16 ***
## low_qual_fin_sf                     NA         NA      NA       NA
## wood_deck_sf                 1.202e+01  4.861e+00   2.474  0.01350 *
## open_porch_sf                1.618e+01  1.004e+01   1.611  0.10736
## bsmt_unf_sf                 -2.087e+01  1.723e+00 -12.116  < 2e-16 ***
## mas_vnr_area                 1.046e+01  4.229e+00   2.473  0.01353 *
## garage_cars                  4.229e+03  1.893e+03   2.234  0.02563 *
## garage_area                  7.769e+00  6.497e+00   1.196  0.23195
## year_built                   3.251e+02  3.130e+01  10.388  < 2e-16 ***
## tot_rms_abv_grd             -3.838e+03  6.922e+02  -5.545 3.51e-08 ***
## full_bath                   -4.341e+03  1.655e+03  -2.622  0.00883 **
## overall_qualAverage         -5.013e+03  1.735e+03  -2.890  0.00391 **
## overall_qualBelow_Average   -1.280e+04  2.677e+03  -4.782 1.92e-06 ***
## overall_qualExcellent        7.261e+04  5.381e+03  13.494  < 2e-16 ***
## overall_qualFair            -1.115e+04  5.240e+03  -2.127  0.03356 *
## overall_qualGood             1.226e+04  1.950e+03   6.287 4.30e-10 ***
## overall_qualVery_Excellent   1.304e+05  8.803e+03  14.810  < 2e-16 ***
## overall_qualVery_Good        3.798e+04  2.741e+03  13.852  < 2e-16 ***
## kitchen_qualFair            -2.663e+04  6.325e+03  -4.210 2.71e-05 ***
## kitchen_qualGood            -1.879e+04  4.100e+03  -4.582 5.01e-06 ***
## kitchen_qualTypical         -2.677e+04  4.281e+03  -6.252 5.37e-10 ***
## fireplaces                   1.138e+04  2.257e+03   5.043 5.18e-07 ***
## fireplace_quFair            -7.207e+03  6.823e+03  -1.056  0.29106
## fireplace_quGood             6.070e+02  5.833e+03   0.104  0.91713
## fireplace_quNo_Fireplace     3.394e+03  6.298e+03   0.539  0.59002
## fireplace_quPoor            -5.185e+03  7.399e+03  -0.701  0.48362
## fireplace_quTypical         -6.398e+03  5.897e+03  -1.085  0.27814
## exter_qualFair              -3.854e+04  8.383e+03  -4.598 4.66e-06 ***
## exter_qualGood              -1.994e+04  5.585e+03  -3.569  0.00037 ***
## exter_qualTypical           -2.436e+04  5.874e+03  -4.147 3.57e-05 ***
## lot_frontage                 1.024e+02  1.905e+01   5.376 8.90e-08 ***
## lot_area                     6.042e-01  7.864e-02   7.683 2.91e-14 ***
## longitude                   -3.481e+04  2.537e+04  -1.372  0.17016
## latitude                     5.874e+04  3.483e+04   1.686  0.09193 .
## misc_val                     9.171e-01  1.003e+00   0.914  0.36071
## year_sold                   -6.455e+02  4.606e+02  -1.401  0.16132
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 22190 on 1401 degrees of freedom
## Multiple R-squared:  0.9116, Adjusted R-squared:  0.9092
## F-statistic: 380.3 on 38 and 1401 DF,  p-value: < 2.2e-16
```

```r
# prediction
pred.lm = predict(lm.fit, newdata = x_test)
# test error
lm.rmse = RMSE(pred.lm, test$sale_price); lm.rmse
```

```
## [1] 21149.18
```

```
lm.mse = (lm.rmse^2); lm.mse
```

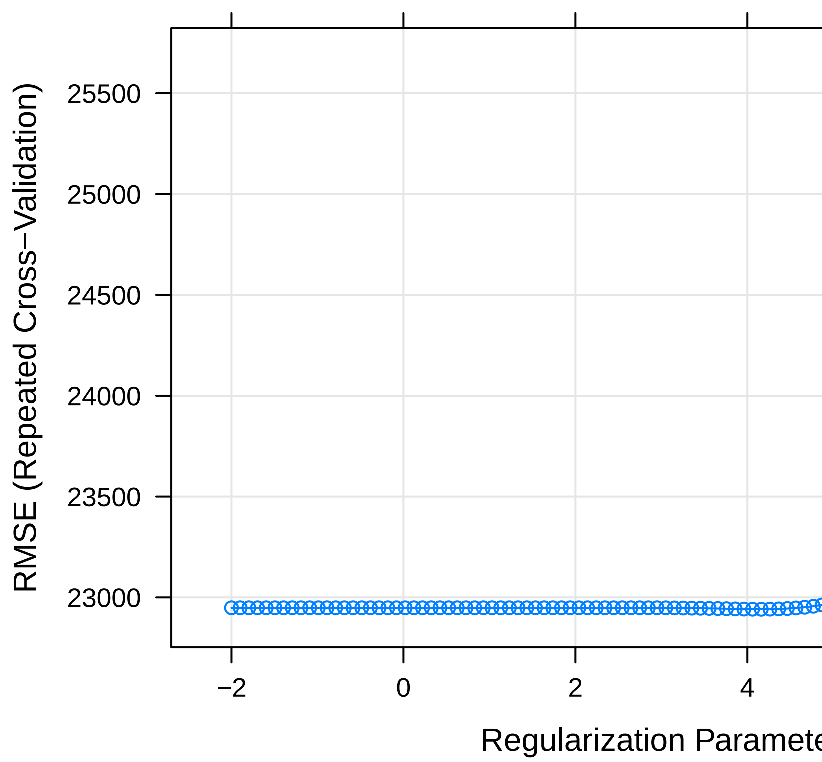
```
## [1] 447287652
```

Pros and cons of the linear model:

- This model is quite straightforward and easy to fit.The estimates are BLUE.The mean-square test error (MSE) for this model is 4.4728765×10^8.
- However, this model is still quite complicated with too many predictors. Moreover, there is multi-collinearity issue and potential overfitting problem.

0.0.3 b). Lasso model

```
set.seed(1)

lasso_fit = train(x_train, y_train,
                  method = "glmnet",
                  tuneGrid = expand.grid(alpha = 1,
                                         lambda = exp(seq(-2, 8, length = 100))),
                  trControl = trainControl(method = "repeatedcv", number = 10, repeats = 5))

plot(lasso_fit, xTrans = log)
```



0.0.3.1 lasso model 1 based on lambda min

```r
# optimal tuning parameters
lasso_fit$bestTune
```

```
##    alpha   lambda
## 62     1 64.17516
```

```r
# show coefficients
coef(lasso_fit$finalModel, lasso_fit$bestTune$lambda)
```

```
## 40 x 1 sparse Matrix of class "dgCMatrix"
##                                       s1
## (Intercept)                -4.830346e+06
## gr_liv_area                 6.540238e+01
## first_flr_sf                8.016798e-01
## second_flr_sf               .
## total_bsmt_sf               3.541418e+01
## low_qual_fin_sf            -4.095881e+01
## wood_deck_sf                1.164153e+01
## open_porch_sf               1.544655e+01
## bsmt_unf_sf                -2.088792e+01
## mas_vnr_area                1.089192e+01
## garage_cars                 4.086270e+03
## garage_area                 8.161397e+00
## year_built                  3.233663e+02
## tot_rms_abv_grd            -3.620617e+03
## full_bath                  -3.849455e+03
## overall_qualAverage        -4.856601e+03
## overall_qualBelow_Average  -1.246350e+04
## overall_qualExcellent       7.545290e+04
## overall_qualFair           -1.075910e+04
## overall_qualGood            1.212418e+04
## overall_qualVery_Excellent  1.356218e+05
## overall_qualVery_Good       3.789366e+04
## kitchen_qualFair           -2.487155e+04
## kitchen_qualGood           -1.723010e+04
## kitchen_qualTypical        -2.533814e+04
## fireplaces                  1.055908e+04
## fireplace_quFair           -7.669876e+03
## fireplace_quGood            .
## fireplace_quNo_Fireplace    1.462970e+03
## fireplace_quPoor           -5.644087e+03
## fireplace_quTypical        -7.011684e+03
## exter_qualFair             -3.340938e+04
## exter_qualGood             -1.515968e+04
## exter_qualTypical          -1.959557e+04
## lot_frontage                9.969303e+01
## lot_area                    6.042705e-01
## longitude                  -3.296544e+04
## latitude                    5.514849e+04
## misc_val                    8.297428e-01
## year_sold                  -5.617527e+02
```

```r
# prediction
pred_lasso = predict(lasso_fit, newdata = x_test)
# test error
lasso_mse = mean((pred_lasso - y_test)^2); lasso_mse
```

```
## [1] 440092572
```

```r
# number of predictors
num_coef = coef(lasso_fit$finalModel, lasso_fit$bestTune$lambda)
sum(num_coef != 0) - 1
```

```
## [1] 37
```

```r
set.seed(1)

lasso_1se = train(x_train, y_train,
                  method = "glmnet",
                  tuneGrid = expand.grid(alpha = 1,
                                         lambda = exp(seq(-2, 8, length = 100))),
                  trControl =  trainControl(method = "repeatedcv", selectionFunction = "oneSE", number =

# optimal tuning parameters based on 1se rule
lasso_1se$bestTune
```

0.0.3.2 lasso model 2 based on 1SE

```
##    alpha   lambda
## 80     1 395.3605
```

```r
# show coefficients
coef(lasso_1se$finalModel, lasso_1se$bestTune$lambda)
```

```
## 40 x 1 sparse Matrix of class "dgCMatrix"
##                                        s1
## (Intercept)                 -3.943441e+06
## gr_liv_area                  6.108086e+01
## first_flr_sf                 9.449637e-01
## second_flr_sf                .
## total_bsmt_sf                3.625951e+01
## low_qual_fin_sf             -3.544140e+01
## wood_deck_sf                 1.004751e+01
## open_porch_sf                1.213017e+01
## bsmt_unf_sf                 -2.060701e+01
## mas_vnr_area                 1.293690e+01
## garage_cars                  3.503770e+03
## garage_area                  9.711868e+00
## year_built                   3.152712e+02
## tot_rms_abv_grd             -2.541684e+03
## full_bath                   -1.469004e+03
```

```
## overall_qualAverage        -4.028703e+03
## overall_qualBelow_Average  -1.088208e+04
## overall_qualExcellent       8.701702e+04
## overall_qualFair           -8.812993e+03
## overall_qualGood            1.113742e+04
## overall_qualVery_Excellent  1.557578e+05
## overall_qualVery_Good       3.731931e+04
## kitchen_qualFair           -1.452506e+04
## kitchen_qualGood           -7.943206e+03
## kitchen_qualTypical        -1.672289e+04
## fireplaces                  8.251261e+03
## fireplace_quFair           -3.942182e+03
## fireplace_quGood            2.111321e+03
## fireplace_quNo_Fireplace    .
## fireplace_quPoor           -1.621801e+03
## fireplace_quTypical        -4.236539e+03
## exter_qualFair             -1.699637e+04
## exter_qualGood              .
## exter_qualTypical          -4.789838e+03
## lot_frontage                8.694872e+01
## lot_area                    5.920132e-01
## longitude                  -2.270960e+04
## latitude                    3.807773e+04
## misc_val                    3.236657e-01
## year_sold                  -1.732609e+02
```

```r
# prediction
pred_lasso_1se = predict(lasso_1se, newdata = x_test)
# test error
lasso_1se_mse = mean((pred_lasso_1se - y_test)^2); lasso_1se_mse
```

```
## [1] 420909622
```

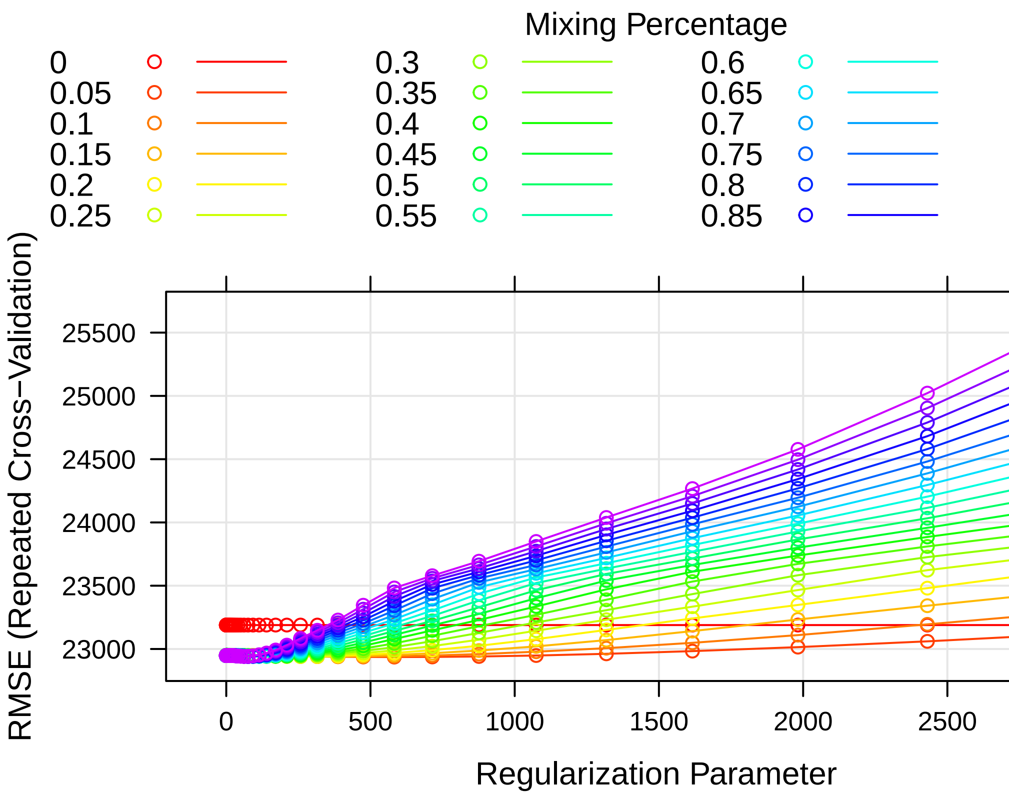
```r
# number of predictors
num_coef_1se = coef(lasso_1se$finalModel, lasso_1se$bestTune$lambda)
sum(num_coef_1se != 0) - 1
```

```
## [1] 36
```

- There are 37 predictors in lasso model 1 based on lambda min, and 36 predictors in lasso model 2 based on 1se rule.
- The selected turing parameters for lowest cv rmse are alpha=1 and lambda=64.18 in lasso model 1. When the 1se rule is applied to lasso model 2, lambda changes to 395.36.
- Lasso model 2 based on 1se rule has smaller test MSE which is 4.2090962×10^8 than lasso model 1 based on lambda min which is 4.4009257×10^8. Therefore, lasso model 2 based on 1se is better.

0.0.4 c). Elastic Net model

```r
set.seed(1)

elnet_fit = train(x_train, y_train,
                  method = "glmnet",
                  tuneGrid = expand.grid(alpha = seq(0, 1, length = 21),
                                         lambda = exp(seq(-2, 8, length = 50))),
                  trControl = trainControl(method = "repeatedcv", number = 10, repeats = 5))

# visualization
myCol <- rainbow(25)
myPar <- list(superpose.symbol = list(col = myCol),
superpose.line = list(col = myCol))
plot(elnet_fit, par.settings = myPar)
```



0.0.4.1 Elastice net model 1

```r
# tuning parameter
elnet_fit$bestTune
```

```
##    alpha   lambda
## 92  0.05 582.5103
```

```r
# show coefficients
coef(elnet_fit$finalModel, elnet_fit$bestTune$lambda)
```

```
## 40 x 1 sparse Matrix of class "dgCMatrix"
##                                      s1
## (Intercept)                -5.112549e+06
## gr_liv_area                 3.877324e+01
## first_flr_sf                2.669846e+01
## second_flr_sf               2.545218e+01
## total_bsmt_sf               3.494244e+01
## low_qual_fin_sf            -1.586458e+01
## wood_deck_sf                1.232562e+01
## open_porch_sf               1.688120e+01
## bsmt_unf_sf                -2.072755e+01
## mas_vnr_area                1.165596e+01
## garage_cars                 4.046453e+03
## garage_area                 8.893308e+00
## year_built                  3.191927e+02
## tot_rms_abv_grd            -3.440136e+03
## full_bath                  -3.692753e+03
## overall_qualAverage        -5.116812e+03
## overall_qualBelow_Average  -1.270813e+04
## overall_qualExcellent       7.582763e+04
## overall_qualFair           -1.147347e+04
## overall_qualGood            1.198272e+04
## overall_qualVery_Excellent  1.363894e+05
## overall_qualVery_Good       3.765814e+04
## kitchen_qualFair           -2.368649e+04
## kitchen_qualGood           -1.610599e+04
## kitchen_qualTypical        -2.415667e+04
## fireplaces                  1.082895e+04
## fireplace_quFair           -7.857833e+03
## fireplace_quGood            1.486635e+02
## fireplace_quNo_Fireplace    1.819327e+03
## fireplace_quPoor           -5.804038e+03
## fireplace_quTypical        -6.962451e+03
## exter_qualFair             -3.296303e+04
## exter_qualGood             -1.455999e+04
## exter_qualTypical          -1.915475e+04
## lot_frontage                1.001690e+02
## lot_area                    6.032320e-01
## longitude                  -3.515266e+04
## latitude                    5.775817e+04
## misc_val                    8.684001e-01
## year_sold                  -5.747735e+02
```

```r
# prediction
pred_elnet = predict(elnet_fit, newdata = x_test)
# test error
elnet_mse = mean(RMSE(pred_elnet, y_test)^2); elnet_mse
```

```
## [1] 438591167
```

```r
set.seed(1)
```

```r
# try to fit elastic net model applying 1se rule to select tuning parameters
elnet_1se = train(x_train, y_train,
                  method = "glmnet",
                  tuneGrid = expand.grid(alpha = seq(0, 1, length = 21),
                                         lambda = exp(seq(-2, 8, length = 50))),
                  trControl =  trainControl(method = "repeatedcv", selectionFunction = "oneSE", numbe

# tuning parameters
elnet_1se$bestTune
```

0.0.4.2 Elastic net model 2 based on 1se

```
##    alpha   lambda
## 50     0 2980.958
```

```r
# show coefficients
coef(elnet_1se$finalModel, elnet_1se$bestTune$lambda)
```

```
## 40 x 1 sparse Matrix of class "dgCMatrix"
##                                     s1
## (Intercept)                -6.335390e+06
## gr_liv_area                 3.239590e+01
## first_flr_sf                2.625217e+01
## second_flr_sf               2.206668e+01
## total_bsmt_sf               3.197020e+01
## low_qual_fin_sf            -1.786580e+01
## wood_deck_sf                1.581469e+01
## open_porch_sf               2.360529e+01
## bsmt_unf_sf                -1.879961e+01
## mas_vnr_area                1.765883e+01
## garage_cars                 3.835315e+03
## garage_area                 1.380900e+01
## year_built                  2.840685e+02
## tot_rms_abv_grd            -1.777713e+03
## full_bath                  -1.514764e+03
## overall_qualAverage        -5.989414e+03
## overall_qualBelow_Average  -1.282769e+04
## overall_qualExcellent       7.581098e+04
## overall_qualFair           -1.422795e+04
## overall_qualGood            9.779199e+03
## overall_qualVery_Excellent  1.397774e+05
## overall_qualVery_Good       3.386129e+04
## kitchen_qualFair           -1.663421e+04
## kitchen_qualGood           -9.438848e+03
## kitchen_qualTypical        -1.770321e+04
## fireplaces                  9.919400e+03
## fireplace_quFair           -7.656381e+03
## fireplace_quGood            1.644805e+03
## fireplace_quNo_Fireplace    3.759105e+02
## fireplace_quPoor           -5.682051e+03
## fireplace_quTypical        -5.611492e+03
## exter_qualFair             -2.585950e+04
## exter_qualGood             -6.657289e+03
```

```
## exter_qualTypical          -1.321177e+04
## lot_frontage                9.486833e+01
## lot_area                    5.855032e-01
## longitude                  -4.434608e+04
## latitude                    6.263526e+04
## misc_val                    8.342702e-01
## year_sold                  -4.693104e+02
```

```
# prediction
pred_elnet_1se = predict(elnet_1se, newdata = x_test)
# test error
elnet_mse_1se = mean(RMSE(pred_elnet_1se, y_test)^2); elnet_mse_1se
```

```
## [1] 426357707
```

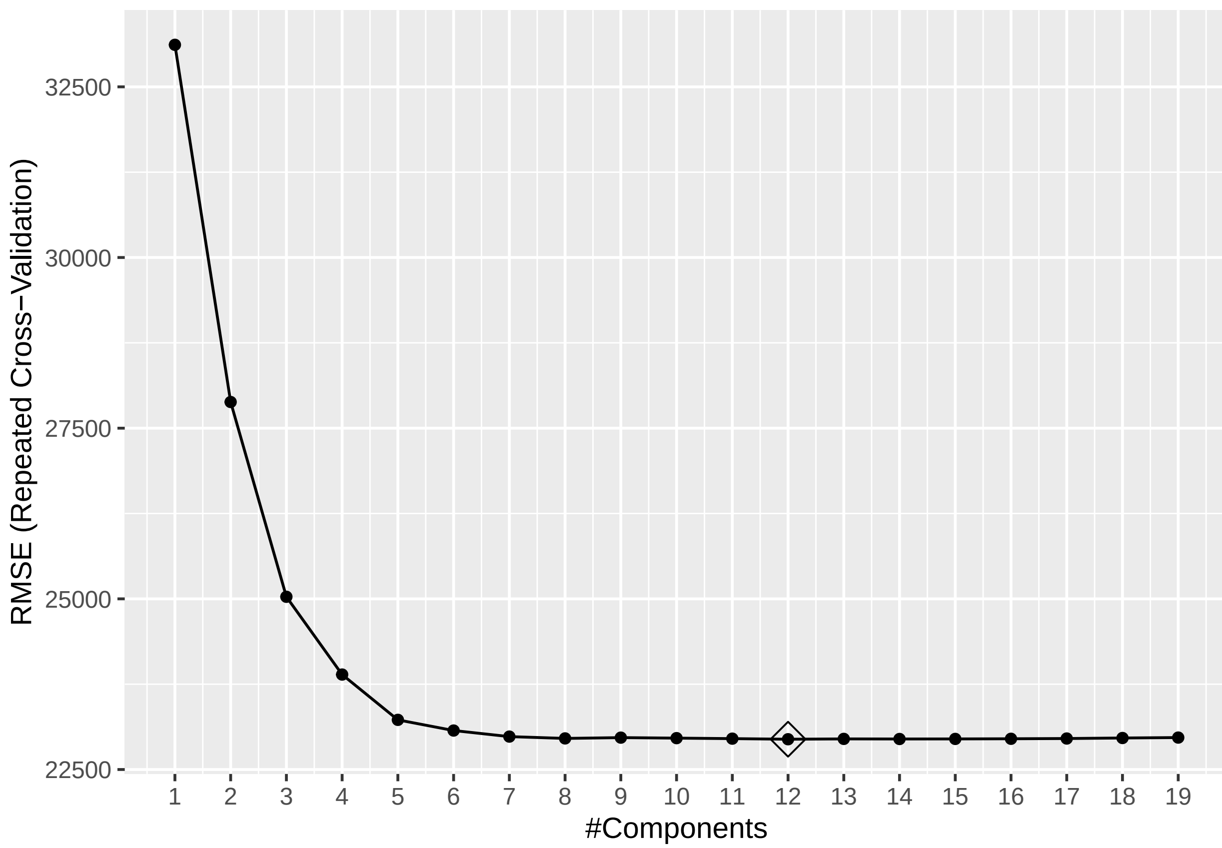
- The selected tuning parameters of elastic net model 1 is alpha = 0.05 and lambda = 582.5 and test error is 4.3859117×10^8. If 1se rule is applied to elastic net model, the tuning parameters is alpha = 0 and lambda is 2980.96, and test error is 4.2635771×10^8 .
- As we know, elastic net allows us to tune the alpha parameter where alpha = 0 corresponds to ridge and alpha = 1 to lasso. That means we can choose an alpha value between 0 and 1 to optimize the elastic net. In elastic net model 2, we found alpha = 0. Therefore, the penalty function reduces to the ridge term. Thus, we don't need to apply 1se rule to select tuning parameters in this elastic net model. It doesn't help to optimize this model.

0.0.5 d). Partial least squares model

```
set.seed(1)

pls_fit <- train(x_train, y_train,
                 method = "pls",
                 tuneGrid = data.frame(ncomp = 1:19),
                 trControl =  trainControl(method = "repeatedcv", number = 10, repeats = 5),
                 preProcess = c("center", "scale"))
summary(pls_fit)
```

```
## Data:    X dimension: 1440 39
##  Y dimension: 1440 1
## Fit method: oscorespls
## Number of components considered: 12
## TRAINING: % variance explained
##           1 comps  2 comps  3 comps  4 comps  5 comps  6 comps  7 comps
## X           20.02    25.93    29.67    33.59    37.01    40.03    42.49
## .outcome    79.73    86.35    89.36    90.37    90.87    90.99    91.06
##           8 comps  9 comps  10 comps  11 comps  12 comps
## X           45.53    47.97     50.15     52.01     53.69
## .outcome    91.08    91.10     91.13     91.15     91.15
```

```
ggplot(pls_fit, highlight = TRUE) +
scale_x_continuous(breaks = seq(0,20,by = 1))
```




```
# prediction
pls_pred = predict(pls_fit, newdata = x_test)
# test error
pls_mse = mean((pls_pred - y_test)^2); pls_mse
```

```
## [1] 449622718
```

From the summary of this partial least squares model we found the number of components is 12. The test error is 4.4962272×10^8.

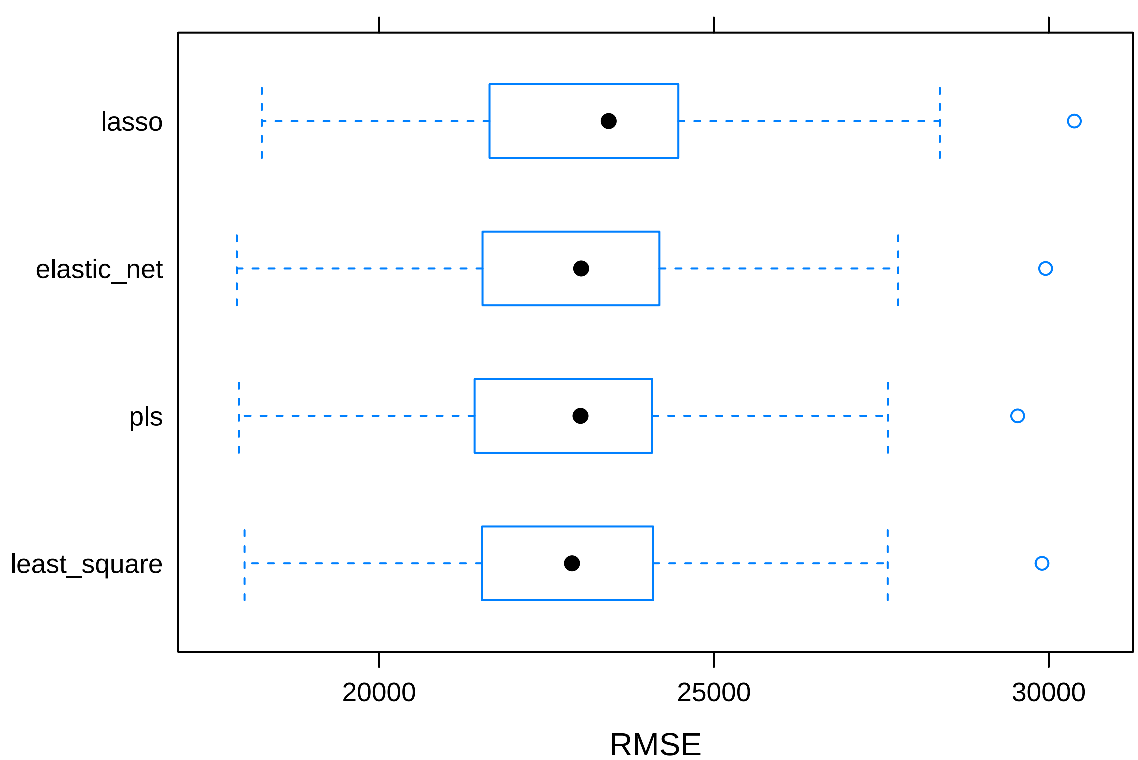
```
set.seed(1)

# compare four models
resamp <- resamples(list(least_square = lm.fit, lasso = lasso_1se, elastic_net = elnet_fit, pls = pls_fi
summary(resamp)
```

0.0.5.1 Model comparison

```
##
## Call:
## summary.resamples(object = resamp)
##
```

```
## Models: least_square, lasso, elastic_net, pls
## Number of resamples: 50
##
## MAE
##                  Min.  1st Qu.   Median     Mean  3rd Qu.     Max. NA's
## least_square 13590.23 16046.61 16694.90 16712.84 17491.07 19148.35    0
## lasso        13612.12 16032.47 16832.86 16653.34 17423.12 19210.70    0
## elastic_net  13491.87 15930.93 16567.62 16626.55 17392.02 19166.44    0
## pls          13541.68 16090.73 16727.26 16716.20 17492.65 19113.09    0
##
## RMSE
##                  Min.  1st Qu.   Median     Mean  3rd Qu.     Max. NA's
## least_square 17991.36 21596.77 22880.98 22978.67 24085.83 29899.57    0
## lasso        18249.56 21725.37 23428.71 23238.78 24452.90 30382.60    0
## elastic_net  17875.42 21562.96 23017.69 22936.08 24166.08 29953.16    0
## pls          17907.12 21463.52 23008.08 22943.72 24070.28 29535.42    0
##
## Rsquared
##                   Min.   1st Qu.    Median      Mean   3rd Qu.      Max. NA's
## least_square 0.8600209 0.8924164 0.9059332 0.9028661 0.9149852 0.9387696    0
## lasso        0.8593599 0.8916075 0.9038168 0.9009386 0.9118428 0.9364788    0
## elastic_net  0.8603607 0.8931148 0.9069545 0.9032441 0.9146375 0.9393932    0
## pls          0.8603467 0.8921784 0.9071393 0.9030770 0.9155912 0.9392286    0
```

```r
# make a boxplot to show RMSE of 4 models
bwplot(resamp, metric = "RMSE")
```



- As we discussed above, linear model has multiple downsides such as violation of the principle of parsimony, multicollinearity, etc.
- As for the rest 3 models, from the summary and boxplot we found elastic net model has the lowest RMSE, lowest MAE, as well as highest R_squred. In addition, it's more difficult to interpret the results of partial least squares model.
- Therefore, I will choose elastic net model as the final model for predicting the response.