Ecuación de Calor

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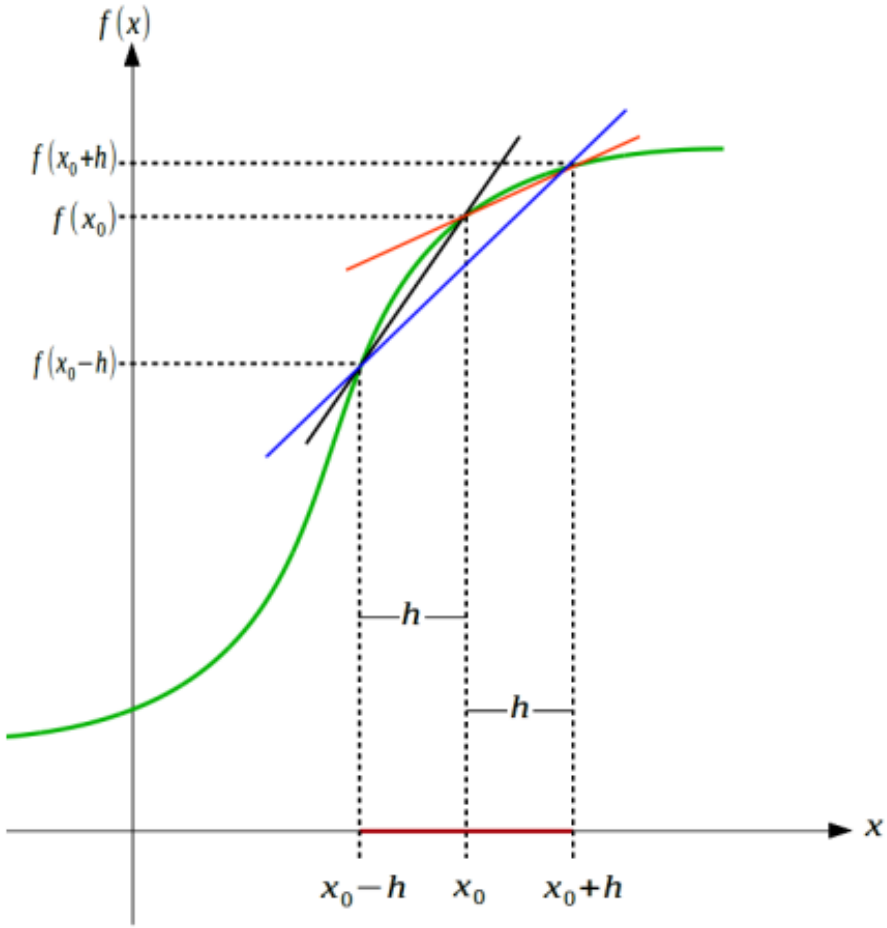
Ecuación de calor

$$u_t - c\Delta u = f$$

El método numérico utilizado para la solución es diferencias finitas.



$$f(x_0) = f(x_0)+f'(x_0)(x-x_0)+\frac{1}{2}f''(x_0)(x-x_0)^2+...+\frac{1}{n!}f^{(n)}(x_0)(x-x_0)^n+... \tag{1}$$

$$f(x_0+h) \approx f(x_0)+f'(x_0)h \tag{2}$$

$$f(x_0-h) \approx f(x_0)-f'(x_0)h \tag{3}$$

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$$f'(x_0) = \frac{f(x_0+h)-f(x_0-h)}{2h} \tag{4}$$

$$u_t = c\Delta u \tag{5}$$

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$$u_{i,j}^{n+1} = u_{i,j}^n + \lambda(u_{i+1,j}^n - 2u_{i,j}^n + u_{i-1,j}^n) + \lambda(u_{i,j+1}^n - 2u_{i,j}^n + u_{i,j-1}^n) \tag{6}$$

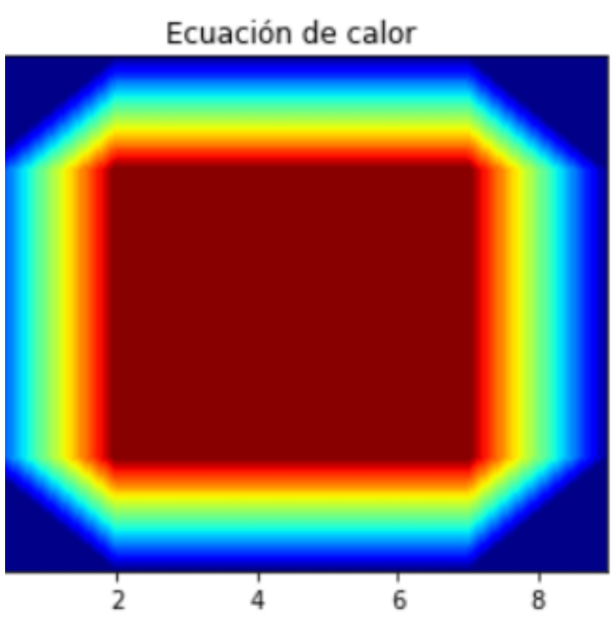
```python
def iteration1(T):
    tmp = np.zeros(T.shape)

# Diferencias finitas
    for i in range(1, T.shape[0]-1):
        for j in range(1, T.shape[1]-1):
            #T[i, j] = T[i,j]+(T[i,j-1]+T[i-1,j]-4*T[i,j]+T[i+1,j]+T[i,j+1])
            tmp[i,j] = T[i,j]+0.25*(T[i,j-1]+T[i-1,j]-4*T[i,j]+T[i+1,j]+T[i,j+1])
    return tmp
```

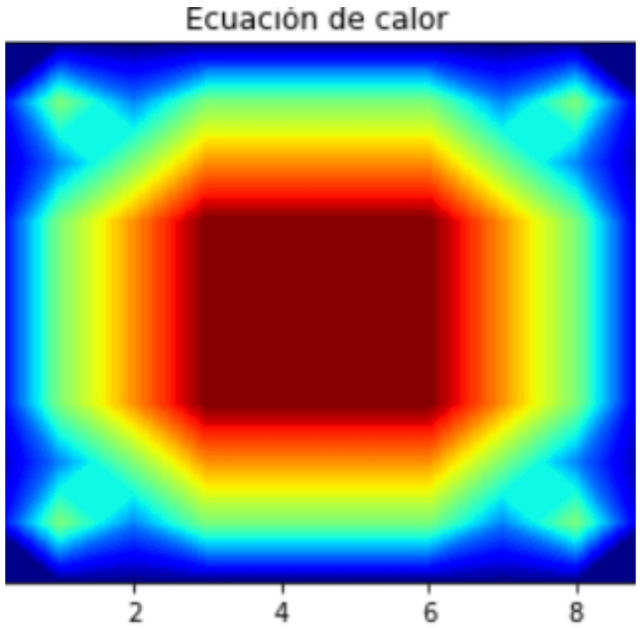
Constante c

Para que la ecuación diferencial y su solución tenga estabilidad y convergencia se tener en cuenta lo siguiente:

Si $\lambda \leq \frac{1}{2}$ se observa que los valores de los errores de la solución no crence, estos tieden a oscila.
si $\lambda \leq \frac{1}{4}$ se observa que los errores de la solución no oscilas.
si $\lambda \leq \frac{1}{6}$ tiende a minimizar los errores por truncamiento.

$\lambda = 1/2$


Ecuación de calor
2
4
6
8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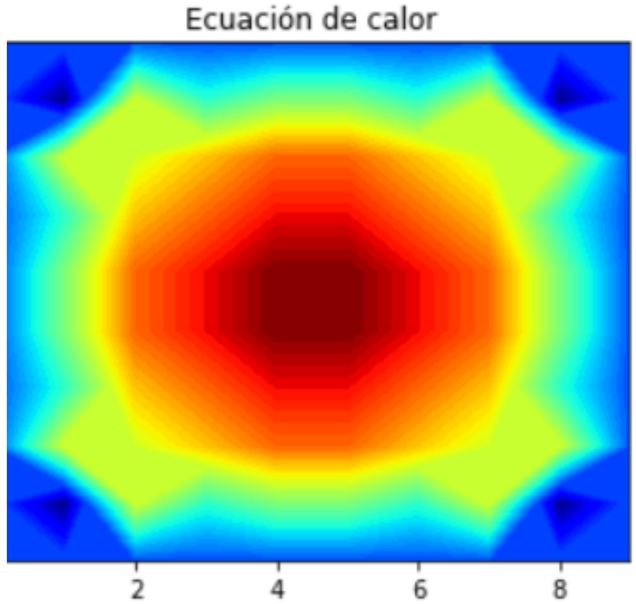
Ecuación de calor
2
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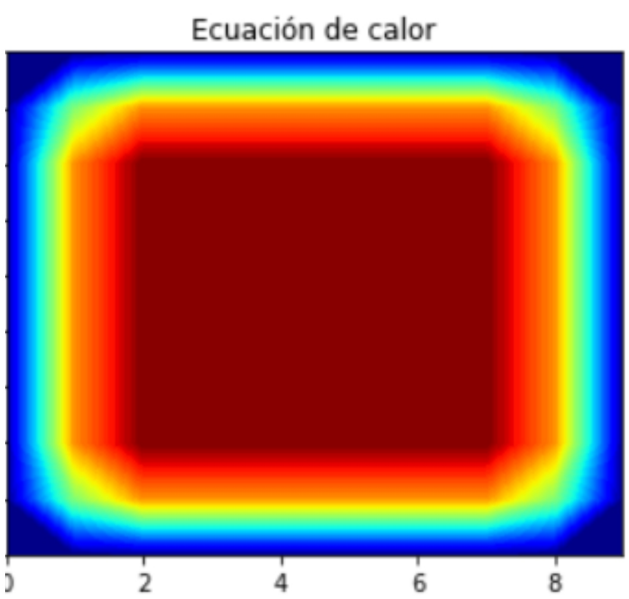
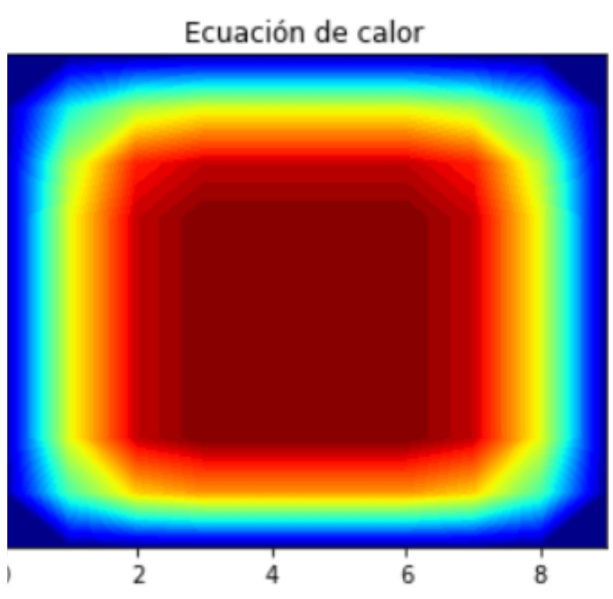
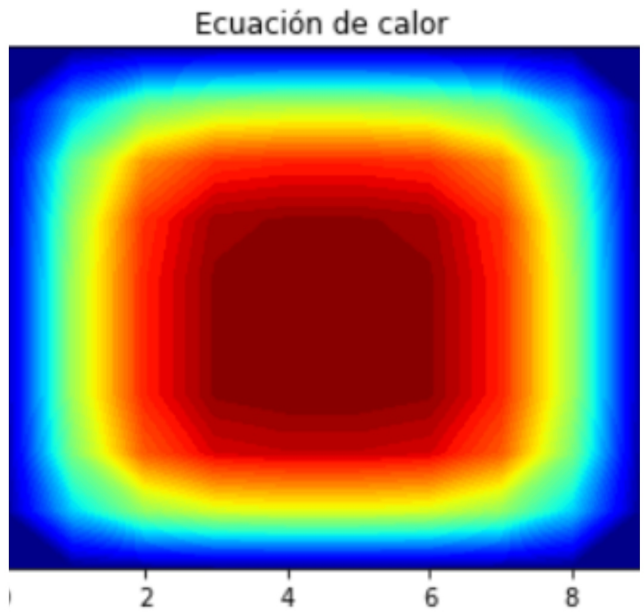
8

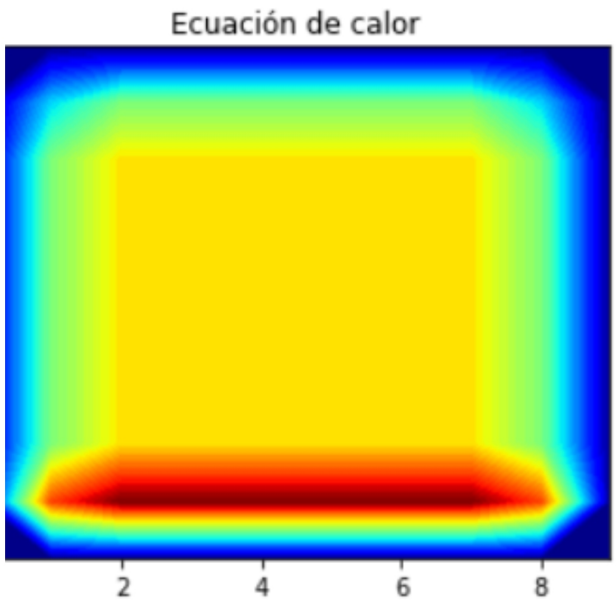
Para condición inicial 100 y valores de frontera (0,0,0,0)

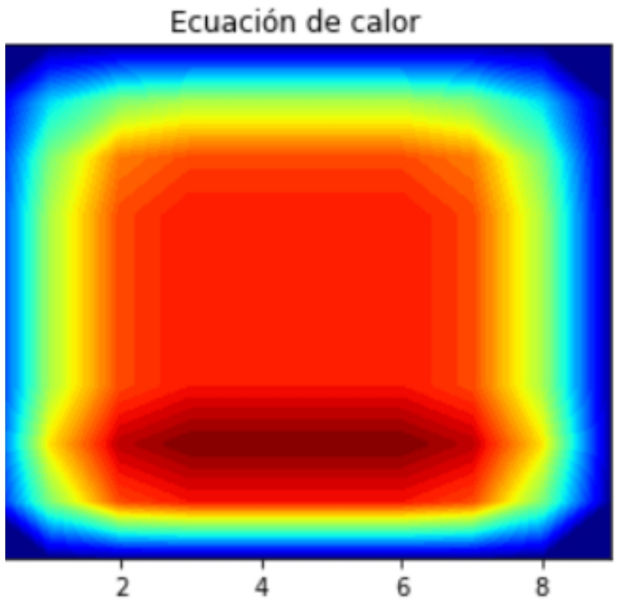




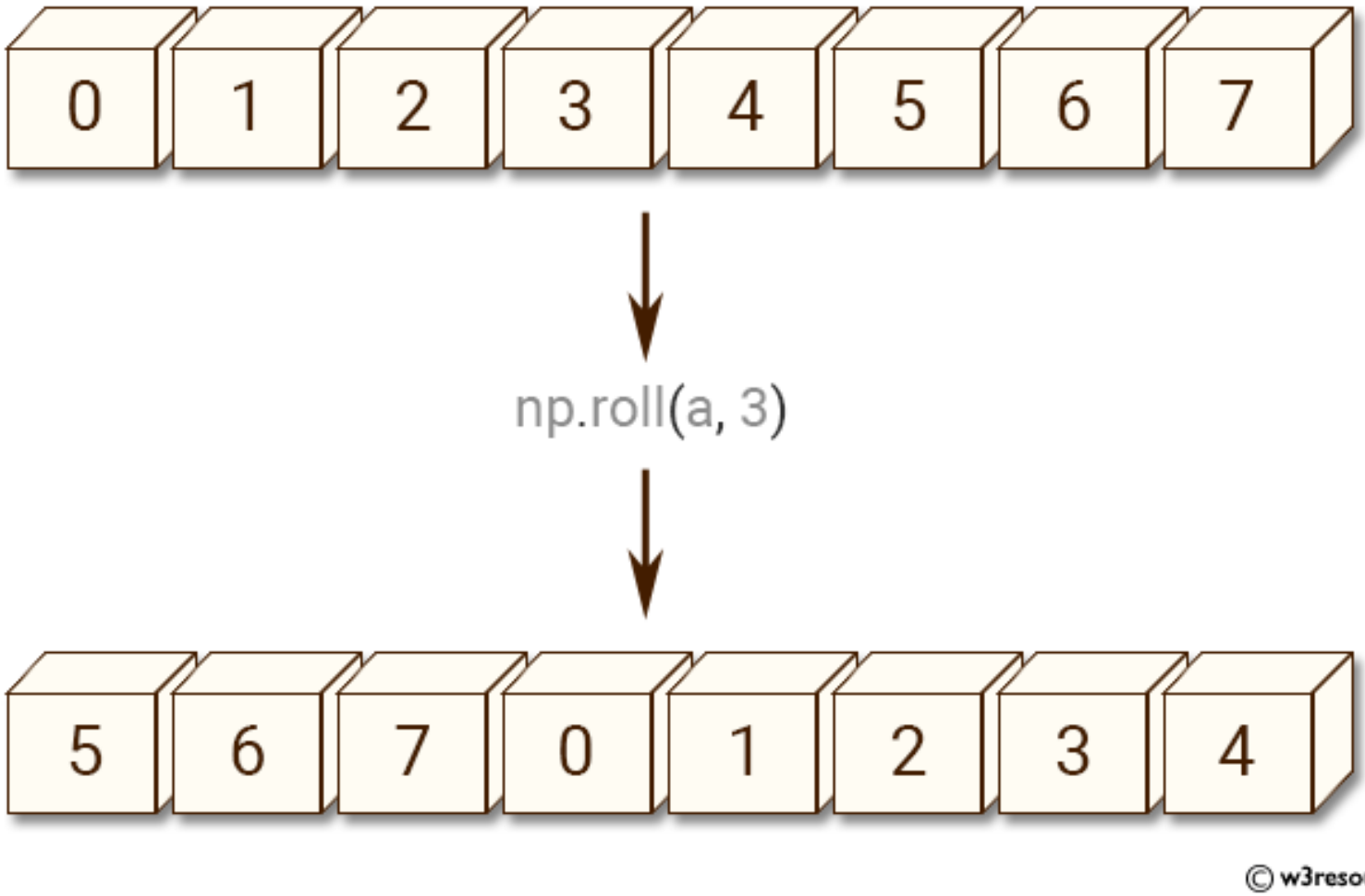


Para condición inicial 100 y valores de frontera (300,0,0,0)





np.roll



Código con np.roll

```
def iteration(T):
    tmp = np.zeros(T.shape)
    #T[i, j] = T[i,j]+(T[i,j-1]+T[i-1,j]-4*T[i,j]+T[i+1,j]+T[i,j+1])
    tmp=T+0.25*(np.roll(T,1,axis=0)+np.roll(T,-1,axis=0)+np.roll(T,1,axis=1)+np.roll(T,-1,axis=1)
    return tmp
```

Tiempo

ITERACIÓN	CON ROLL	SIN ROLL
3	0.946	0.765
10	1.452	1.498
20	1.982	2.22
30	2.50	2.74
40	2.79	3.00

Utilizando bash

```
def guardaT(j,T0):
    P=T0
    for i in range(j):
        if i>=0:
            P=iteration(P)
    return P

for i in range(4):
    Te = condiciones(5,5, rand.randrange(100,500),rand.randrange(10,100),rand.randrange(100,500),rand
    Tem= guardaT(3,Te)
```

Utilizando Bash

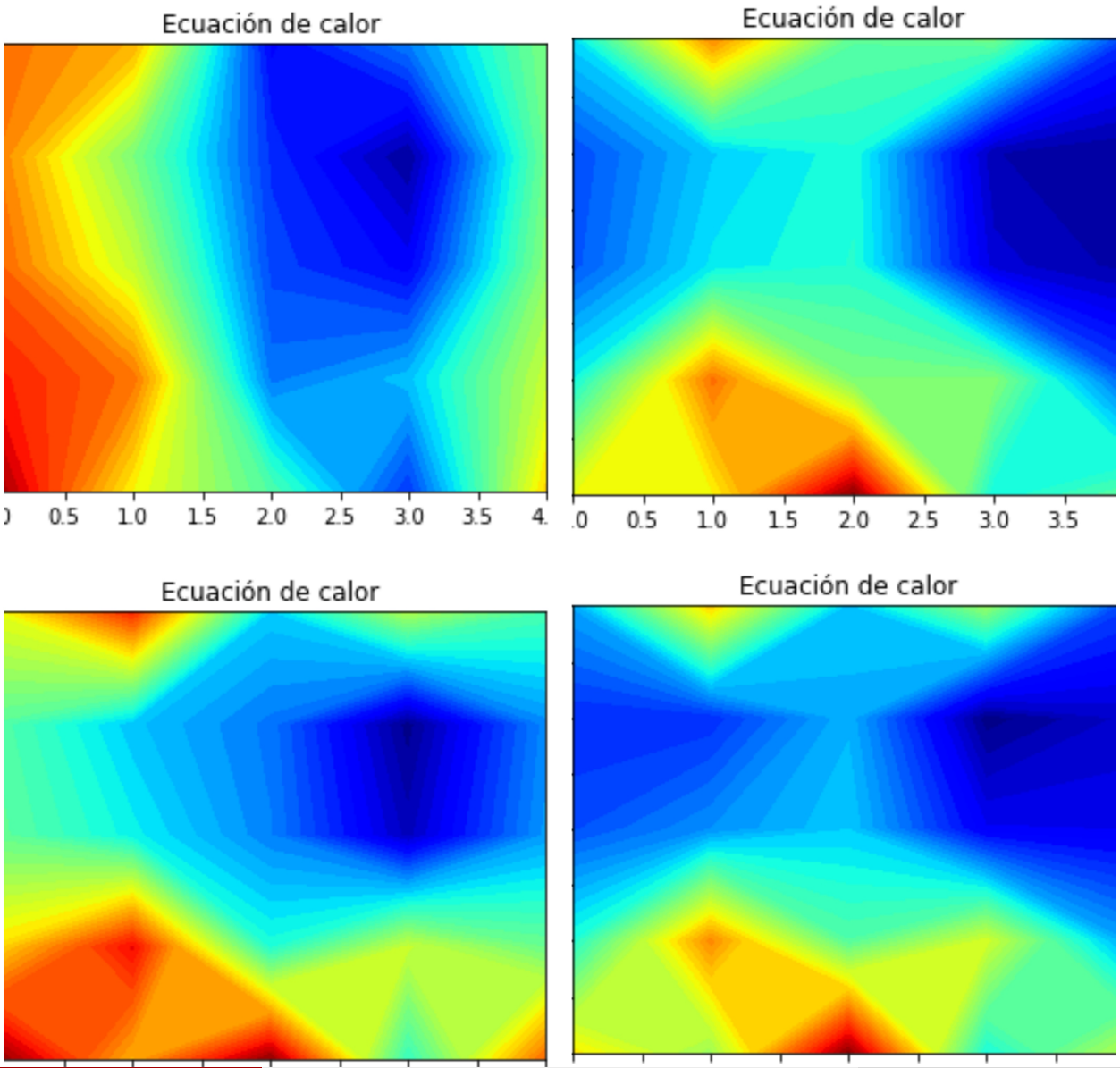
```
174.1   154.3   142.1   125.9   156.4
168.6   162.7   129.9   134.3   150.9
164.1   150.7   126.9   122.3   146.4
162.1   145.3   125.6   116.9   144.4
163.2   158.6   122.5   130.3   145.5


120.0   120.7   133.0   110.4   113.6
110.4   125.8   115.3   115.6   104.0
104.4   109.8   111.3   99.56   98.0
104.0   108.7   111.0   98.44   97.58
109.3   125.0   113.7   114.7   102.9


201.8   182.5   201.0   160.1   187.8
178.8   195.2   158.3   172.9   164.8
164.3   156.6   148.7   134.2   150.3
163.2   153.5   147.9   131.1   149.2
175.7   192.9   154.0   170.5   161.7


168.8   156.9   200.5   141.3   159.0
141.9   178.7   148.1   163.1   132.2
123.7   130.0   135.9   114.3   113.9
120.0   120.3   133.5   104.7   110.3
132.3   171.5   134.8   155.9   122.5
```

Utilizando Bash



Utilizando Clases

```
class Mapa(object):

    def __init__(self,nx,ny,Ttop,Tbottom,Tleft,Tright,Tini):
        self.x = nx
        self.y = ny
        self.Tt = Ttop
        self.Tb = Tbottom
        self.Tl = Tleft
        self.Tr = Tright
        self.Ti = Tini


    #Para llenar mi matriz de ceros inicialmente
        self.T = np.zeros((nx,ny))
#El fill rellena la matriz con un valor, en este caso será la temperatura inicial llamada (Tini)
        self.T.fill(Tini)
#print(T)
# Condiciones de frontera
        self.T[0, :] = Ttop
        self.T[-1, :] = Tbottom
        self.T[:, -1] = Tright
        self.T[:, 0] = Tleft

    def iteration(self,x):
        for i in range(x):
            tmp = np.zeros(self.T.shape)
            tmp=self.T+0.25*(np.roll(self.T,1,axis=0)+np.roll(self.T,-1,axis=0)+np.roll(self.T,1,axis=1)+np.ro
            return tmp
```