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EFFECT OF DIMENSION ON DIRECT SEARCH METHODS FOR CONSTRAINED OPTIMIZATION

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Abstract—A simple technique, involving a series of disjointed optimization problems, for finding the effect of the number of variables on direct search methods is described. The computational results obtained with this technique show that the dimension has a significant effect on the performance of direct search methods. Both the accuracy of the solution and the computational effort, which could increase by as much as sixteenfold when the number of variables is doubled, are affected.

INTRODUCTION

Direct search methods are still in use because of their simplicity; they also find application in global optimization because of their ability to find the global optimum. It is generally believed that the computational effort required for solving an optimization problem by any direct search method (and, in general, by any method) increases more than proportionally with the dimension of (or the number of variables in) the problem. But there are very few published results to support this belief. In this article, a simple way to find the effect of dimension on direct search methods is described, and some results are presented.

FORMULATION OF TEST PROBLEMS

Assume that a "basic" optimization problem containing n variables, of the form

$$\min f(x) \qquad \text{wrt } x, \tag{1}$$

subject to a set of inequality constraints $g(x) \geq 0$, is available. Then a larger optimization problem containing mn variables can be formulated as follows:

$$\min \sum_{i=1}^{m} f(x^i) \qquad \text{wrt } x^1, x^2, \ldots, x^m \tag{2}$$

subject to $g(x^i) \geq 0$ for $i = 1,2,\ldots,$m.

Here each x^i is a vector consisting of n components, and hence there are mn variables. The functions f and g in eqn (2) are same as those in eqn (1). The number of constraints, the minimum value and the number of active inequality constraints at the solution for eqn (2) will be exactly m times the respective values for eqn (1). By selecting $m = 1,2,3,\ldots$, one can obtain a series of optimization problems containing n, $2n$, $3n$ ... variables. The basic optimization problem may be considered a building block in the large disjointed optimization problem, like a monomer in a polymer chain. But the interconnections among the various blocks are absent.

In the series of problems, dimension increases by a constant amount n; however, the complexity of the problem is unaffected in the following sense. Since the optimization problem in eqn (2) is readily decomposable or separable, the solution of this problem by a particular method should take no more than m times the computational effort required for solving the basic problem in eqn (1) by the same method. Considering the methodology of gradient search methods, it may be argued that these methods, when applied to the above series of problems, will not show any effect on dimension; that is, the computational effort will not be more than proportional to the dimension of the problem. The numerical results (Table 1) confirm this.

But the behavior of direct search methods on the present series of problems, except that of Powell's [1] conjugate direction method, which is likely to behave like a gradient search method, is unknown. To find this behavior, tests were done using various basic problems. All tests were done on the same computer system. For the tests, the complex method (CM) of Box [2] and the adaptive random search technique (ARST) of Heuckroth *et al* [3] were selected as representative of direct search methods for constrained optimization; the method of multipliers (MM) from the class of gradient methods was also tried.

RESULTS AND DISCUSSION

Computational results for one basic optimization problem and the corresponding series of problems are presented in Table 1; the results for other problems are available in the thesis by Rangaiah [4]. This basic problem, which corresponds to problem T9 in the work of Pierre and Lowe [5], contains seven variables, three inequality constraints and two equality constraints. This problem was used as is for MM. Since direct search methods cannot handle equality constraints, problem T9 was simplified using equality constraints to eliminate two variables. The resulting problem containing five variables and five inequality constraints was used for CM and ARST.

The performance of CM or of ARST, which depends on the sequence of random numbers, is random. So each problem was solved 10 times by CM or ARST, every time with a different random number initiator, in order to obtain the average performance. In effect, the optimization path was altered in every

Table 1. Results of numerical tests on the effect of dimension

Method	N	NI	NE	NF	NC or NG	Execution time (s)	Optimum value	Expected optimum value
Complex method	5	5	0	547	862	2.2	−44.460	−44.4687
	10	10	0	2837	4145	19.1	−87.531	−88.9374
	15	15	0	8229	9615	103.2	−129.516	−133.4061
Adaptive random search technique	5	5	0	773	2448	3.8	−44.430	−44.4687
	10	10	0	6379	27719	60.8	−86.558	−88.9374
	15	15	0	6658	48238	150.4	−127.250	−133.4061
Method of multipliers	7	3	2	73	36	1.6	−44.469	−44.4687
	14	6	4	106	54	3.1	−88.937	−88.9374
	21	9	6	128	66	5.3	−133.406	−133.4061

N = (total) number of variables; NI = (total) number of inequality constraints; NF = number of objective function (or augmented Lagrangian in case of MM) evaluations; NC = number of constraints evaluations; NG = number of gradients of augmented Lagrangian evaluations.

trial. The results for CM and ARST in Table 1 are the average of ten trials.

The results in Table 1 show that the computational effort (measured in terms of either the number of function and constraints evaluations or execution time) required for solution by CM or ARST increases greatly with the dimension of the optimization problem. The increase is as much as sixteenfold when the number of variables is doubled. Also, the discrepancy between the optimum solution obtained by CM or ARST and the exact optimum solution (observe the last two columns in Table 1) increases with the dimension of the problem. On the other hand, the behavior of MM is as expected, and the exact optimum solution is obtained irrespective of the number of variables.

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