Assignment 2: Practice Exercises on Integer Programming

Note: Only IP.1 and IP.2 are part of the assignment. IP.3 and IP.4 are optional practice.

IP.1 [*Knapsack*] Suppose you are taking a trip by plane for spring break and you do not want to check any bags. So you have to bring everything through security, and you need to decide which toiletries (toothpaste, shampoo, etc.) to bring in order to maximize the usefulness of your quart size bag. (Let us say we are ignoring the 3 oz. rule.) Table 1 gives the usefulness estimate of each item together with the volume in the bag that the item would require.

Table 1: Utility and volume for items

Item	Utility	Volume (in quarts)
Shampoo	8	1/3
Conditioner	6	1/3
Toothpaste	10	1/4
Perfume	4	1/5
NyQuil	9	1/3
Lip balm	7	1/6

a) Formulate an IP to solve this problem.

b) If you cannot take more than 3 items in your bag, how should you modify your formulation.

c) If you take conditioner, then you must also take shampoo. How do you model this in your IP?

d) If you take perfume, then you cannot take lip balm. What constraint would you add for this in your IP?

e) How do we model the constraint that if shampoo, NyQuil, or lip balm are brought, then at least two of those must be taken?

IP.2 [*Facility location*] A company is considering opening warehouses in four cities: New York, Los Angeles, Chicago and Atlanta. Each warehouse can ship 100 units per week. The weekly fixed cost of keeping each warehouse open is $400 for New York, $500 for Los Angeles, $300 for Chicago and $150 for Atlanta. Region 1 of the country requires 80 units per week, region 2 requires 70 units per week and region 3 requires 40 units per week. The costs (including production and shipping costs) of sending one unit from a plant to a region are shown in Table 2.

Table 2: Unit shipping cost from warehouse to region

	To		
From	Region 1	Region 2	Region 3
NY	20	40	50
LA	48	15	26
Chicago	26	35	18
Atlanta	24	50	35

a) Formulate an IP to meet weekly demands at minimum cost.

Hint: Create variables to decide on: (i) whether you open a warehouse or not in each city, (ii) the number of units shipped from each warehouse to each region.

b) Write a constraint for: if the New York warehouse is opened, then the Los Angeles warehouse must be opened.

c) Write a constraint for: at most two warehouses can be opened.

d) Write a constraint for: either the Atlanta or the Los Angeles warehouse must be opened.

Hint: Suppose that y_A *and* y_L *are the binary decision variables for Atlanta and Los Angeles warehouses, respectively. What should be the inequality on* $y_A + y_L$*?.*

e) Write a constraint for: for shipping to be economically feasible, at least 20 units must be shipped.

f) Write a constraint for: if New York ships to region 1, then no other warehouses can ship to region 1.

Hint: This is an example of "big M" method. Either the New York warehouse does not ship any unit to region 1 OR all other warehouses do not ship any unit to region 1.

- *The logical constraints are: EITHER* $x_{N,1} \leq 0$ *OR* $x_{L,1} + x_{C,1} + x_{A,1} \leq 0$*, where* $x_{N,1}, x_{L,1}, x_{C,1}, x_{A,1}$ *are the decision variables describing the quantities shipped to region 1 from NY, LA, Chicago and Atlanta, respectively.*
- *Use the "big M" notion covered in Lecture 5 with an extra auxiliary variable to formulate the logical constraints.*

g) Fill the Jupyter notebook named "Problem Set 2 - Jupyter" to implement the resulting integer program.

IP.3 **(Optional problem - Not to be graded)**[*Production planning*] Comquat owns four production plants at which personal computers are produced. Comquat can sell up to 20,000 computers per year at a price of $3,500 per computer. For each plant the production capacity, the production cost per computer, and the fixed cost of operating a plant for a year are given in Table 3.

Table 3: Plant information

Plant	Production capacity	Fixed cost of plant ($ million)	Cost per computer ($)
1	10,000	9	1,000
2	8,000	5	1,700
3	9,000	3	2,300
4	6,000	1	2,900

Determine how Comquat can maximize its yearly profit from computer production.

IP.4 **(Optional problem - Not to be graded)** [*Album design*] You have been assigned to arrange the songs on the cassette version of Madonna's latest album. A cassette tape has two sides (1 and 2). The songs on each side of the cassette must total between 14 and 16 minutes in length. The length and type of each song are given in Table 4.

Table 4: Song information

Song	Type	Length (minutes)
1	Ballad	4
2	Hit	5
3	Ballad	3
4	Hit	2
5	Ballad	4
6	Hit	3
7		5
8	Ballad and hit	4

The assignment of songs to the tape must satisfy the following conditions:

- Each side must have exactly two ballads.
- Side 1 must have at least three hit songs.
- Either song 5 or song 6 must be on side 1.
- If songs 2 and 4 are on side 1, then song 5 must be on side 2.

Explain how you could use an integer programming formulation to determine whether there is an arrangement of songs satisfying these restrictions.