Vivian van Oosten **Assignment 2**

IP 1: knapsack

a) Our IP consists of the following:

Decision variables are: x_{shampoo}, $x_{\text{conditioner}}$, x_{shampoo}, $x_{\text{toothpaste}}$, x_{perfume}, x_{nyquil}, $x_{\text{lip balm}}$, where $x \in \{0, 1\}$.

The constraint is:

$$\sum_i x_i v_i \leq 1 \tag{1}$$

where v_i is the volume for item i.

The objective function is

$$\max \sum_i x_i u_i \tag{2}$$

where u_i is the utility for item i.

b) If we do not want to take more than 3 items, the following constraint needs to be added to 1

$$\sum_i x_i \leq 3$$

c) If we take conditioner, we also want to take shampoo. Therefore, we need to add an auxiliary variable to model this relationship. We define an auxiliary variable $C \in \{0, 1\}$ and M which is arbitrarily large, take in this case 100. The additional constraints are:

$$\begin{aligned} x_{\text{conditioner}} + x_{\text{shampoo}} &\geq 2 - M(1 - C) \\ x_{\text{conditioner}} &\leq 0 + MC \end{aligned}$$

d) If we take perfume, we cannot take lip balm. To model this constraint, we add an additional auxiliary variable $P \in \{0, 1\}$. The additional constraints are:

$$\begin{aligned} x_{\text{perfume}} + x_{\text{lip balm}} &\leq 1 + M(1 - P) \\ x_{\text{perfume}} &\leq 0 + MP \end{aligned}$$

e) If shampoo, nyquil or lip balm are brought, then at least two of those must be taken. Again, we add an auxiliary variable $S \in \{0, 1\}$. The additional constraints are:

$$\begin{aligned} x_{\text{perfume}} + x_{\text{lip balm}} + x_{\text{nyquil}} &\geq 2 - M(1 - S) \\ x_{\text{perfume}} + x_{\text{lip balm}} + x_{\text{nyquil}} &\leq 0 + MS \end{aligned}$$

IP 2: Facility location

a) Our IP consists of the following:

The decision variables are:

- location variables x_{NY}, x_{LA}, x_{Chicago}, x_{Atlanta}, where $x \in \{0, 1\}$
- variables for shipping amounts $s_{l,r}$ for $l \in$ {NY, LA, Chicago, Atlanta} and $r \in$ {r1, r2, r3}.

For simplicity, we will call the set of locations *loc* and the set of regions *reg* from now on.

The constraints are:

$$\begin{aligned} \sum_{l \in loc} x_l s_{l,r1} &\geq 80 \\ \sum_{l \in loc} x_l s_{l,r2} &\geq 70 \\ \sum_{l \in loc} x_l s_{l,r3} &\geq 40 \end{aligned} \tag{3}$$

The objective is to minimize cost:

$$\min \sum_{l \in loc} \left(x_l c_l + \sum_{r \in reg} s_{l,r} c_{l,r} \right), \tag{4}$$

where c_l is the cost of having that location and $c_{l,r}$ is the cost of shipping an item from location l to region r.

b) If the New York warehouse is opened, the Los Angeles warehouse must be opened as well. To model this constraint, we need an auxiliary variable $N \in \{0, 1\}$. We add the following constraints to 3

$$\begin{aligned} x_{\mathrm{NY}} + x_{\mathrm{Atlanta}} &\geq 2 - M(1 - N) \\ x_{\mathrm{NY}} &\leq 0 + MN \end{aligned}$$

c) If at most 2 warehouses can be opened, we need the following constraint

$$\sum_{l \in loc} x_l \leq 3$$

d) Either the Atlanta or the Los Angeles warehouses must be opened. We assume that this means both cannot be opened simultaneously. Therefore we add the constraints:

$$\begin{aligned} x_{\mathrm{Atlanta}} + x_{\mathrm{Los\ Angeles}} &\leq 1 \\ x_{\mathrm{Atlanta}} + x_{\mathrm{Los\ Angeles}} &\geq 1 \end{aligned}$$

e) A constraint for all shipping to only be 20 or more units. We need an auxiliary variable $T \in \{0, 1\}$ and the following constraints $\forall r \in reg \forall l \in l$

$$
\begin{aligned}
s_{l,r} &\geq 20 - M(1 - T) \\
s_{l,r} &\leq 0 + MT
\end{aligned}
$$

f) If the New York warehouse ships to region 1, no other warehouses can do so. For this, we need an auxiliary variable $Y \in \{0, 1\}$. The additional constraints are:

$$
\begin{aligned}
x_{N,1} &\leq 0 + M(1 - Y) \\
x_{\text{Atlanta},1} + x_{\text{Chicago},1} + x_{\text{LA},1} &\leq 0 + MY
\end{aligned}
$$

g) For the Jupyter notebook, see the other submitted document.