Model- Predictive Control of Heat Pump in Smart Buildings

Master's thesis of

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I declare that I have developed and written the enclosed thesis completely by myself, and have not used sources or means without declaration in the text.

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(Vivien Geenen)

Kurzfassung

Abstract

Contents

List of Figures

List of Tables

1. Introduction

1.1. Motivation

Hintergründe... Warum dieses Thema interessant ist

1.2. Objective of this work

Aufgabenstellung

1.3. Related work

Bezug zu bestehenden Arbeiten

1.4. Content structuring

Stukturierung meiner Thesis erläutern

2. Foundations

The main foundations on which this work is based are summarized in this chapter. This includes thermal basics, foundations about thermal modelling, and model predictive control (MPC). ~~Note, that in the following all vectors are denoted as bold letters, e.g. $\mathbf{x}$.~~

2.1. Thermal basics

There are three different mechanisms of heat transfer: Heat conduction, heat convection, and heat radiation[15]. All of these mechanisms are used for the thermal modelling of buildings. For example, conduction is the main part of heat transfer through walls or floors. Convection takes place on the inside and the outside of the building between the walls and the air and radiation is needed for the integration of the impact of the sun, for example.

2.1.1. Conduction

Conduction means that heat energy is directed in a solid or fluid. Molecules within the solid or fluid have higher energy when the temperature is higher. They transfer the energy to neighbouring molecules with smaller energy. Without a heat source, the temperature difference between a hot and a cold location of the molecules decreases.[10]
The equation

$$\dot{\mathbf{q}} = -\lambda \nabla T \tag{2.1}$$

describes the conduction according to Fourier [15]. ~~There is λ the~~ thermal conductivity with the assumption of being constant and $\dot{\mathbf{q}}$ and T represent the specific heat flux and the temperature. The thermal conductivity is dependent on the material, such as concrete, wood or bricks.
To know the heat flux $\dot{Q}$, it is necessary to expand the above equation with the area A,

the thickness of the conductive medium d and a temperature difference ΔT assuming one significant direction of the heat flux $\dot{Q}$ to:

$$\dot{Q} = \frac{A\lambda}{d}\Delta T \tag{2.2}$$

In terms of buildings, the conductive medium could be walls, floors or roofs.

2.1.2. Convection

Macroscopic movements of a fluid lead to a transport of kinetic energy and enthalpy. This mechanism is called convection. These movements are generated by external forces or by internal forces like balancing the pressure or temperature.[15]
Newton's law of cooling describes the convective heat transfer $\dot{Q}$ as

$$\dot{Q} = \alpha A(T_w - T_\infty) \tag{2.3}$$

with the heat transfer coefficient α, especially for building modelling the wall temperature T_w and the environment temperature T_∞ [3] . There are two possibilities to determine the heat transfer coefficient. Both require a temperature difference ΔT and either a temperature gradient $\partial T/\partial x$ or a heat flux $\dot{Q}$. [15]

2.1.3. Radiation

Every body emits heat radiation to the environment with electromagnetic waves. Especially, heat radiation does not need matter for energy transportation. As shown in the following equation, the temperature T of the body influences highly heat radiation.[15]

$$\dot{q} = \sigma T^4 \tag{2.4}$$

This correlation applies to a black body, where $\dot{q}$ is a heat flux and σ represents the Stefan-Boltzmann coefficient. A black body absorbs all heat radiation with all wavelengths from all directions[3]. The consideration of a black body is idealized. For the illustration of a real body (see Equation 2.5), the emissivity ϵ is used. ϵ is material-dependent and lies between 0 and 1.

$$\dot{q} = \epsilon\sigma T^4 \tag{2.5}$$

In general, a body absorbs, transmits, and reflects radiation with the appropriate coefficients a, τ and r. The sum of three coefficients has to be one ($a + \tau + r = 1$) [1]. In particular, the reflection coefficient is needed for describing the influence of radiation from the environment in building modelling.

The best-known source of heat radiation is the sun, which plays an important role in the thermal modelling of buildings. Objectives in the building, such as radiators, also radiate heat. For example, radiators have equal parts convective and radiative energy transport [5].

2.2. Lumped capacitance model

For modelling the thermal behaviour of buildings, the lumped capacitance model is often used. With this approach, using the ~~electrical~~ analogy, building elements are represented by resistors R and capacitors C. [9]

2.2.1. Electrical analogy

Similar to an electrical network, the potential is represented by the temperature at one node and the heat flux corresponds to the current. We can also use Ohm's law, which is formulated in a thermal way as:

$$\dot{Q} = \frac{\Delta T}{R} \tag{2.6}$$

Combining the above equation with Equation 2.2 or Equation 2.3, the thermal resistance R is determined in conductive cases as [10]:

$$R_\lambda = \frac{d}{A\lambda} \tag{2.7}$$

and in convective cases as[3]:

$$R_\alpha = \frac{1}{\alpha A} \tag{2.8}$$

In sum, the thermal resistances R comply with electrical resistors. Further for modelling thermal networks, the thermal capacitance C is needed. It is calculated from the specific heat capacity c multiplied by the mass m ($C = cm$).

For a better explanation of the structure of a thermal network, a simple example is depicted in Figure 2.1. It represents a heated wall of a building. The heat flux $\dot{Q}$, for example from a radiator, influences the temperature T_{wall}, as well as the capacitance C. And the temperature

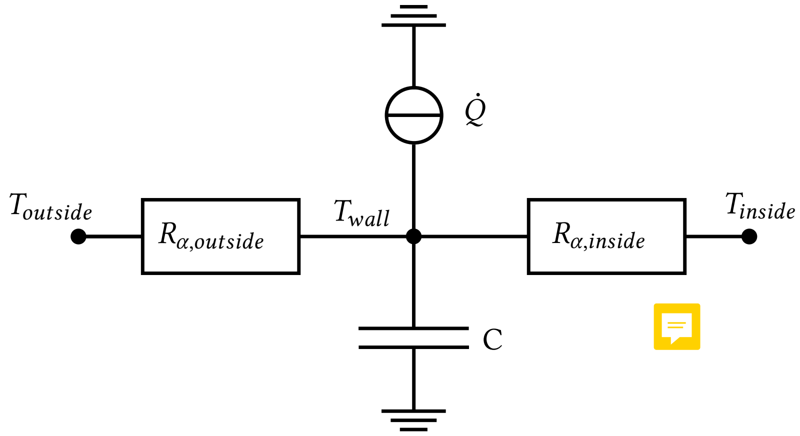


Figure 2.1. Sample RC- network

T_{wall} affects the temperature inside and outside T_{inside} and $T_{outside}$ with their resistances $R_{\alpha,inside}$ and $R_{\alpha,outside}$. The example shows that all connections in the network influence each other. To model the dynamics of the wall in differential equations, Kirchhoff's Current Law is required. It states that the sum of the flowing current to the node is equal to the sum of the flowing current of the node [10]. Because of the thermal analogy of electrical laws, the current is replaced by heat flux. The following differential equation results for the node T_{wall} using Ohm's law ($\dot{Q} = \Delta T/R$) and the relationship $\dot{Q} = C\frac{\partial T}{\partial t}$.

$$C\frac{\partial T_{wall}}{\partial t} = \dot{Q} + \frac{T_{inside} - Twall}{R_{\alpha,inside}} - \frac{T_{wall} - Toutside}{R_{\alpha,outside}} \tag{2.9}$$

In Figure 2.1, the thermal resistances are serially connected. According to the electrical network, resistances in series are equal to their sum.

$$R_{sum} = R_{\alpha,inside} + R_{\alpha,outside} \tag{2.10}$$

A parallel circuitry has windows and walls in buildings, for example. Here the resistances are calculated according to the following schema:

$$\frac{1}{R_{sum}} = \frac{1}{R_{wall}} + \frac{1}{R_{window}} \tag{2.11}$$

In terms of needed more capacitances for describing the thermal model, the summary capacitance is added in a parallel circuitry as:

$$C_{sum} = C_1 + C_2 \tag{2.12}$$

The serial circuitry of capacitances is calculated as follows:

$$\frac{1}{C_{sum}} = \frac{1}{C_1} + \frac{1}{C_2} \tag{2.13}$$

2.3. Model predictive control (MPC)

Wie findest du die Kapitelaufteilung im MPC teil?

"The idea of model predictive control [...] is [...] to utilize a model of the process in order to predict and optimize the future system behavior" [4]. Applied to a thermal control of a building with the aim of grid-supporting, a model of the thermal behaviour of the building is required to predict the reaction of the system behaviour in the next N time steps, called the prediction horizon. Every time step k, the current state $\mathbf{x_k}$, the output $\mathbf{y_k}$ is measured, and the future system behaviour is obtained computation. The computation of the future system behaviour may include weather forecast, occupancy schedule and the optimization of the control signal $\mathbf{u_k}$ over the optimization horizon $\mathbf{u_{k+N}}$. ~~But,~~ only the first calculated control signal is adopted as input for the plant. Then, the calculations are repeated at every time step. Figure 2.2 visualises the MPC control loop.

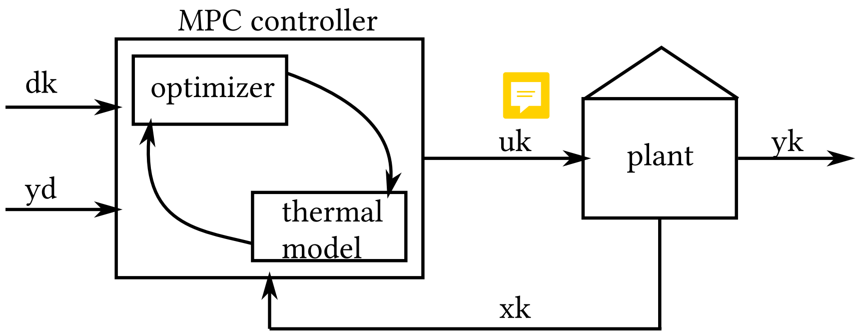


Figure 2.2. MPC structure of the control loop

Concluded, the MPC is "an iterative online optimization over the predictions" [4] compiled by the thermal model of the building. Mathematically explained, the optimizer needs to ~~reduce~~ the following equation according to [16] and [11]:

$$\text{Cost function} \qquad \cancel{minimize} \sum_{k=1}^{N-1} c_k(x_k, u_k, y_k) \tag{2.14}$$

subject to

$$\begin{aligned}
\text{Current state} \quad & x_0 = && x \\
\text{Dynamics} \quad & x_{k+1} = && f(x_k, u_k, d_k) \qquad y_k = g(x_k, u_k, d_k) \\
\text{Constraints} \quad & y_{min} \leq && y_k \leq y_{max} \\
& u_{min} \leq && u_k \leq u_{max}
\end{aligned}$$

c_k represents the cost function, which is explained in detail in subsection 2.3.1 . In terms of building control, y is the internal temperature.

2.3.1. Cost function

The cost function c_k assigns a cost to the control signal u_k and the current state x_k, which is mathematically described in Equation 2.14 , with:

$$c_k = (x_k^T Q x_k + u_k^T R u_k) \tag{2.15}$$

Here Q and R are matrices over which individual elements of the state vector or control signal vector can be weighted differently. [8]

2.3.2. Current state

The current state $\mathbf{x_k}$ is a vector of measured state variables of a building. Every prediction starts from this initial state [11].

2.3.3. Dynamics

The state-space formulation (SSF) is an alternative representation of a linear differential equation, which models a physical system. In this work, it is used for the formulation of the thermal model, which is required for the MPC. The SSF consists of the state $\mathbf{x}$, the control signal $\mathbf{u}$, the disturbances $\mathbf{d}$ and the output of the system $\mathbf{y}$ represented in Equation 2.16. The system matrix is A, B_1 and B_2 are called the input matrices, C is the output matrix, D_1 and D_2 are the pass-through matrices.

$$\begin{aligned} \dot{\mathbf{x}} &= A\mathbf{x} + B_1\mathbf{u} + B_2\mathbf{d} \\ \mathbf{y} &= C\mathbf{x} + D_1\mathbf{u} + D_2\mathbf{d} \end{aligned} \tag{2.16}$$

In a thermal model of a building, some authors ([5], [12],...) use the state as a vector of some temperatures, the control signal as a signal for the heating system, the disturbances can describe the influence by the weather or occupants and the output of the system contains frequently the temperature inside of the building.

2.3.4. Constraints

Dealing with constraints is one of the most important advantages of MPC. Thereby, constraints can be used for the state, the output, and the input. In terms of building control, output constraints and input constraints are reasonable, as mathematically described in the Equation 2.14. That means, the output constraints could be a temperature range, which feels comfortable for occupants. And the constraints for the input are given as minimal (= 0) and maximal values of the possible performances. General, logical and physical ranges are constrained. There are different forms of constraints, but linear constraints are frequently used for MPC because they simplify the optimization problem. Constraints can also be time dependant. This is beneficial for embedding diverse temperature ranges during the night and the day or during the working time of occupants when they are not at home. [12]

3. Modelling

After explaining thermal basics and the electrical analogy, the foundations are used in this chapter. How I obtain a thermal model and the resulting thermal model is introduced. Later the model is required for the MPC to predict the thermal reactions of the building.

3.1. The building

Since the thermal model and the MPC are based on a real building, the building is described in more detail below.
The building ~~stands~~ at the "Campus Nord" ~~from~~ the "Karlsruher Institut für Technologie (KIT)" and is part of the "Energy Lab 2.0", "a research infrastructure for renewable energy" [7]. It is equipped with a kitchen, a bathroom, five rooms and a technical room. For a better orientation, you can see in Figure 3.1 a part of the construction plan of the building. ~~It can be noted that~~ the building is designed as a single-family house, but for practical reasons, it is used as an office. The living area is around 100 m^2. In ~~the building are two options to heat~~ or cool with a ground-source heat pump or an air heat pump. The focus is placed on the air heat pump because the most commonly used heat pumps in Germany are air heat pumps [2]. Further information about the air heat pump you can read in the "Technische Unterlagen Montageanleiung" [6]. In cooperation with the heat pump, there is a water reservoir for saving energy. The volume is 1000 litres (further information in [17]). However, the heating system is not completely installed so heating or cooling is not possible actual.
One of the main features of the building is the number of sensors. The air temperature is measured in every room, as well as the temperature in the middle of the exterior wall, the screed temperature, the floor plate temperature, and the temperature of the inner wall between room three and room four (see Figure 3.1). Furthermore, the consumption of the ~~actual~~ electrical power is also detected. There are much more sensors, but in this case, are only needed the mentioned sensors.

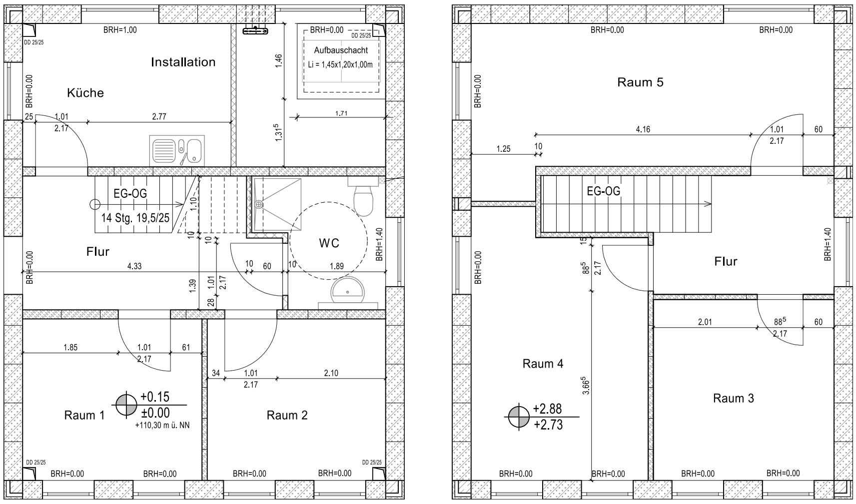
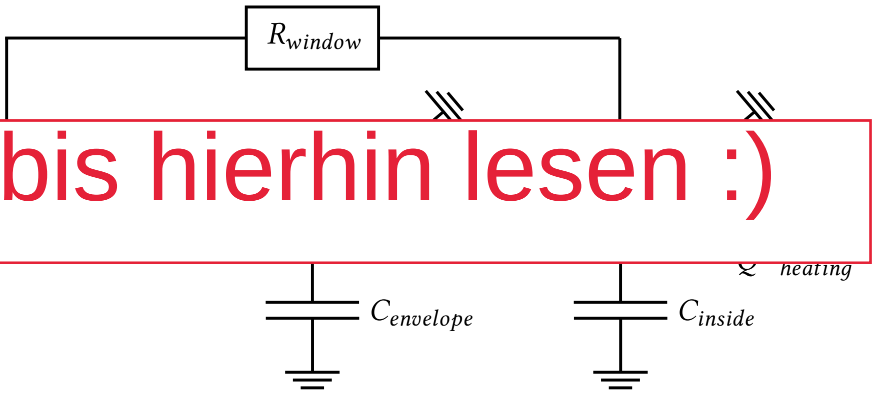


Figure 3.1. construction plan of the building [14]



nur bis hierhin lesen :)

Figure 3.2. structure of the thermal model in RC- analogy

3.2. The thermal model

The focus of this work is on the MPC part, so a simple thermal model is required. Nevertheless, no necessary information must be missing. In this case, the thermal storage possibilities, the temperature inside the building, and the heating system's influence have to be represented in the model. The storage is needed because the objective of the MPC is to heat during the grid has too much power/a too high frequency and to save the energy in the building during grid requires energy. The output of the model needs to be the temperature inside since the MPC aims to be in a pleasant temperature range to ensure customer comfort. Last, the influence of the heating system must be recognisable in the model, as it is the input of the plant.

$$
\begin{aligned}
C_{inside} * \frac{\partial T_{inside}}{\partial t} &= \dot{Q}_{heating} + \dot{Q}_{sun,inside} - \frac{T_{inside} - T_{envelope}}{R_{inside}} - \frac{T_{inside} - T_{outside}}{R_{window}} \\
C_{envelope} * \frac{\partial T_{envelope}}{\partial t} &= \dot{Q}_{sun,envelope} - \frac{T_{envelope} - T_{outside}}{R_{envelope}} + \frac{T_{inside} - T_{envelope}}{R_{inside}} \\
\dot{Q}_{WR} &= \dot{Q}_{heating} + \dot{Q}_{HP} - \dot{Q}_{loss} - \dot{Q}_{SW}
\end{aligned}
\tag{3.1}
$$

3.3. Grey-box modelling

Creating a model can be made with three types of models, the so-called white-box models, grey-box models or black-box models. White-box models describe the real system only physically. Black-box models, on the other hand, have no physical description. They are created with data. And grey-box models are in between these two options [13]. All possibilities are used in the thermal modelling of buildings [9].

The chosen approach for the MPC is the grey-box model because of its advantages.

3.4. Data collection

3.5. Validation of the thermal Model

4. Model predictive control

4.1. Optimization

4.2. Constrains

4.3. Cost function

5. Results

6. Conclusion

7. Outlook

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A. Appendix

A.1. First Section

Figure A.1. A figure

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