Model- Predictive Control of Heat Pump in Smart Buildings

Master's thesis of

Vivien Geenen

at the Department of Mechanical Engineering
Institute for Automation and Applied Informatics (IAI)

Reviewer:	Prof. Dr. Veit Hagenmeyer
Second reviewer:	apl. Prof. Dr. Jörg Matthes
Advisor:	Moritz Frahm, M.Sc. and Frederik Zahn, M.Sc

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Karlsruher Institut für Technologie
Fakultät für Maschinenbau
Postfach 6980
76128 Karlsruhe

I declare that I have developed and written the enclosed thesis completely by myself, and have not used sources or means without declaration in the text.

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(Vivien Geenen)

Kurzfassung

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2. Foundations

For the development of this work, some foundations about thermal modeling and model predictive control (MPC) are needed, which are explained in this chapter. Note, that in the following all vectors are in thick print, like **x**.

2.1. Thermal basics

There are three different mechanism, which describe physically the heat transfer: Heat conduction, heat convection and heat irradiation[10]. Every mechanism is used for thermal modelling of buildings. For example, conduction is the main part of heat transfer through walls or floors. Convection takes place inside and outside between the walls and the air and irradiation is needed for the integration of the impact of the sun, for example.

2.1.1. Conduction

Conduction means, that heat energy is directed in a solid or fluid. Molecules within the solid or fluid have higher energy when the temperature is higher. They transfer the energy to neighbour molecules with smaller energy. Without a heat source, the temperature difference between a hot and a cold location of the molecules decrease.[7]
The equation 2.1 describes the conduction according to Fourier [10]. There is λ the thermal conductivity with the assumption of being constant and $\dot{\mathbf{q}}$ and T represent the specific heat flux and the temperature. The thermal conductivity is dependent on the material, such as concrete, wood or bricks.

$$\dot{\mathbf{q}} = -\lambda gradT \tag{2.1}$$

For the further work, it will be relevant to expand the above equation with the area A, the thickness of the conductive medium d and a temperature difference ΔT assuming one significant direction of the heat flux $\dot{Q}$ to:

$$\dot{Q} = \frac{A\lambda}{d}\Delta T \tag{2.2}$$

In terms of buildings, the conductive medium could be walls, floors or roofs.
Further, the Ohm's law of thermodynamics describes the above equation as

$$\dot{Q} = \frac{\Delta T}{R_\lambda} \tag{2.3}$$

and determine the thermal resistance R_λ as [7]

$$R_\lambda = \frac{d}{A\lambda} \tag{2.4}$$

what is required for the RC- modeling of buildings, which is nearly explained in 2.2.

2.1.2. Convection

Macroscopic movements of a fluid lead to transport of kinetic energy and enthalpy, called convection. These movements are generated by external forces or by internal forces like balancing the pressure or temperature.[10]
Newton's law of cooling describes the heat transfer of convection $\dot{Q}$ as

$$\dot{Q} = \alpha A(T_w - T_\infty) \tag{2.5}$$

with the heat transfer coefficient α, especially for building modeling the wall temperature T_w and the environment temperature T_∞ [2] . There are two possibilities to determine the heat transfer coefficient. Both require a temperature difference ΔT and either a temperature gradient $\partial T/\partial x$ or a heat flux $\dot{Q}$. [10]
The conversion of the equation 2.5 leads to the thermal resistance for convection, shown in the next equation.[2]

$$R_\alpha = \frac{\Delta T}{\dot{Q}} = \frac{1}{\alpha A} \tag{2.6}$$

2.1.3. Radiation

Every body emits heat radiation at the environment with electromagnetic waves. Especially, heat radiation ~~only~~ does not need matter for energy transportation. As shown in the following equation, the temperature T of the body influences highly the heat radiation.[10]

$$\dot{q} = \sigma T^4 \tag{2.7}$$

This correlation applies to a black body, where $\dot{q}$ is a heat flux and σ represents the Stefan-Boltzmann coefficient. A black body absorbs all heat radiation with all wave length of all directions[2]. The consideration of a black body is idealized, for the illustration of a real body (see equation 2.8) the emissivity ϵ is used. ϵ is material-dependent and lies between 0 and 1.

$$\dot{q} = \epsilon \sigma T^4 \tag{2.8}$$

In general, a body absorbs, transmits and reflects radiation with the appropriate coefficients a, τ and r. In summary, all coefficients are one $(a + \tau + r = 1)$ [1]. In particular, the reflection coefficient is needed for describing the influence radiation via the environment in building modeling.
The best known source of heat radiation is the sun, which plays an important roll in thermal modeling of buildings. Objectives in the building, such as radiators, also radiate heat. For example, radiators have equal parts convective and radiative energy transport [4].

2.2. Lumped capacitance model

For modeling the thermal behavior of buildings, the lumped capacitance model is often used. With this approach, using the electrical analogy, building elements can be easily represented by resistors R and capacitors C. [6]

2.2.1. Electrical analogy

Similar to an electrical network, the potential is represented by the temperature at one node, the heat flux corresponds to the current and thermal resistances R comply with electrical resistors and thermal capacitance C with electrical capacitors. The thermal capacitance is the specific heat capacity c multiplied by the mass m $(C = cm)$. All connections in a network have impact on each other. For a better explanation, a simple exampling thermal network is mapped in the next figure, which represents a heated wall of a building. The heat flux $\dot{Q}$,

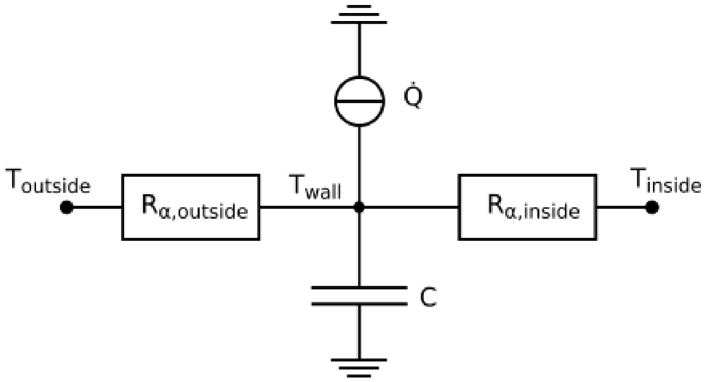

Figure 2.1. Sample RC- network

for example from an radiator, influence the temperature T_{wall}, as well as the capacity C, the temperature inside and outside T_{inside} and $T_{outside}$ with their resistances $R_{\alpha,inside}$ and $R_{\alpha,outside}$. For creating the differential equation, the Kirchhoff's Current Law is required. It states that the sum of the flowing current to the node is equal to the sum of the flowing current off the node [7]. Since there is a thermal modeling case, the current is replaced by the heat flux. The following differential equation results for the node T_{wall} using the Ohm's law ($\dot{Q} = \Delta T/R$) and the relationship $\dot{Q} = C\frac{\partial T}{\partial t}$.

$$C\frac{\partial T_{wall}}{\partial t} = \dot{Q} + \frac{T_{inside} - Twall}{R_{\alpha,inside}} - \frac{T_{wall} - Toutside}{R_{\alpha,outside}} \tag{2.9}$$

In the shown figure 2.1, the thermal resistances are serial connected. According to the electrical network, the summary resistance is the sum of these two resistance.

$$R_{sum} = R_{\alpha,inside} + R_{\alpha,outside} \tag{2.10}$$

A parallel circuitry has windows and walls in buildings, for example. Here the resistances are calculated according to the following schema:

$$\frac{1}{R_{sum}} = \frac{1}{R_{wall}} + \frac{1}{R_{window}} \tag{2.11}$$

In terms of needed more capacities for describing the thermal model, the summary capacity are added in a parallel circuitry as:

$$C_{sum} = C_1 + C_2 \tag{2.12}$$

The serial circuitry of capacities will not be used in this work, ~~thus as it is not nearly explained.~~

2.3. Model predictive control (MPC)

"The idea of model predictive control [...] is [...] to utilize a model of the process in order to predict and optimize the future system behavior." [3] Applied to a thermal control of a building with the aim of grid- supporting, a model of the thermal behavior of the building is required to predict the reaction of the system behavior in the next N time steps, called the prediction horizon. Every time step k, the current state x_k, the output y_k and the future system behavior is obtained via measurements and computation. The computation of the future system behavior includes weather forecast, occupant schedule and the optimization of the control signal u_k over the optimization horizon u_{k+N}. But, only the first calculated control signal is adopt as input for the plant. Then, the proceeding repeat every time step the calculations. The following figure visualise the MPC control loop.

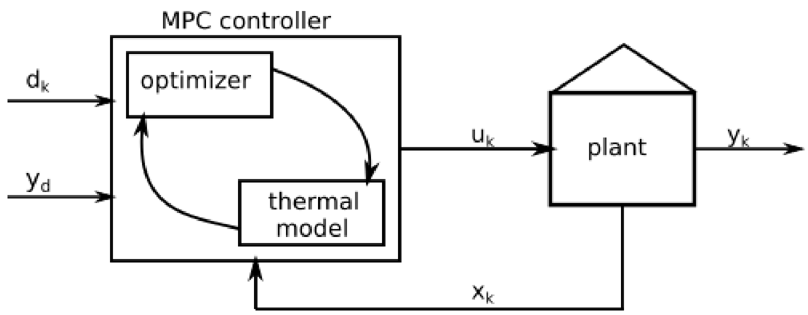

Figure 2.2. MPC structure of the control loop

Concluded, the MPC is "an iterative online optimization over the predictions" [3] compiled by the thermal model of the building. Mathematically explained, the optimizer needs to reduce the following equation according to [11] and [8]:

$$\text{Cost function} \qquad minimize \sum_{k=1}^{N-1} c_k(x_k, u_k, y_k) \tag{2.13}$$

subject to

$$
\begin{array}{rrcll}
\text{Current state} & x_0 = & & x & \\
\text{Dynamics} & x_{k+1} = & & f(x_k, u_k, d_k) & y_k = g(x_k, u_k, d_k) \\
\text{Constraints} & y_{min} \leq & & y_k \leq y_{max} & \\
 & u_{min} \leq & & u_k \leq u_{max} &
\end{array}
$$

c_k represents the cost function, which is nearly explained in the next subsection 2.3.1 . In terms of building control, y is the internal temperature.

2.3.1. Cost function

The cost function c_k optimize the control signal u_k and the current state x_k, which is mathematically described in equation 2.13 , with:

$$c_k = (x_k^T Q x_k + u_k^T R u_k) \tag{2.14}$$

Here Q and R are matrices over which individual elements of the state vector or control signal vector can be weighted differently. [5]

2.3.2. Current state

The current state x_k is a vector of measured state variables of a building. Every prediction starts form this initial state[8].

2.3.3. Dynamics

The state space formulation (SSF) contains the differential equations, which describe a physically system. In this work, it is used for the formulation of the thermal model, which is required for the MPC. The SSF consists of the state $\mathbf{x}$, the control signal $\mathbf{u}$, the disturbances $\mathbf{d}$ and the output of the system $\mathbf{y}$ represented in equation 2.15. The system matrix is A, B_1

and B_2 are called the input matrices, C is the output matrix, D_1 and D_2 are the pass- through matrices.

$$\begin{aligned} \dot{\mathbf{x}} &= A\mathbf{x} + B_1\mathbf{u} + B_2\mathbf{d} \\ \mathbf{y} &= C\mathbf{x} + D_1\mathbf{u} + D_2\mathbf{d} \end{aligned} \tag{2.15}$$

In a thermal model of a building, some authors ([4], [9],...) use the state as a vector of some temperatures, the control signal as a signal for the heating system, the disturbances can describe the influence by the weather or occupants and the output of the system contains frequently the temperature inside of the building.

2.3.4. Constraints

Dealing with constraints is one of the most important advantages of MPC. Thereby, constraints can be used for the state, the output, and the input. In terms of building control, output constraints and input constraints are reasonable, as mathematically described in the equation 2.13. That means, the output constraints could be a temperature range, which feels comfortable for occupants. And the constraints for the input orients at minimal (= 0) and maximal values of the possible performances. General, logical and physical ranges are constrained. There are different forms of constraints, but linear constraints are mostly used for MPC, because they simplify the optimization problem. Constraints can also be time-invariant. This is beneficial for embedding diverse temperature range during the night and the day or during the working time of occupants, when they are not at home. [9]

3. Modeling

3.1. The thermal model

internal mass, extern walls, considering/regarding as single zone building solar irradiation [4]

3.2. Validation of the thermal Model

4. Model predictive control

4.1. Optimization

4.2. Constrains

4.3. Cost function

5. Results

6. Conclusion

7. Outlook

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A. Appendix

A.1. First Section

Figure A.1. A figure

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