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2019 Mathematical Contest in Modeling(MCM) Summary Sheet

Fictional Dragons in Real World

Abstract

The dragon is a mysterious and formidable creature widely known by us. Three dragons appearing in *Game of Thrones* and *A Song of Ice and Fire* are particularly popular. Through our model, we can analyze the requirements for the dragon's living in details.

To determine the growth of the dragon, we suppose the dragon is geometrically similar as time goes by. We use the Richards equation to simulate, which presents a clear 'S' curve varying with time. This curve could be divided into three parts, including the initial growth stage, the exponential growth stage, and the steady growth stage. The turning points to divide these three stages are at the age of **19** and **131**. The fastest growing speed occurs when the dragon is **52** years old. **It takes 280 years for a dragon to reach the upper limit of weight, which is 281.6 tons.**

To calculate the energy expenditure for the dragon, we suppose that the dragon will not fly and breath fire unless predating. Our model divides the energy required into five parts, which are: the energy for basic metabolism, the energy for growth, the energy for flight, the energy for breathing fire and the energy for recovery of trauma. These five parts are derived by the chemical formula of dragon's respiration, the derivative of 'S' trend curve, the empirical formula between speed consumed minimum power and the dragon's weight, the SFM model of the heating transformation and the coefficient describing the possibility of trauma. **Since it is a coupled model, we create Dragon Hunting Algorithm to calculate the total energy expenditure.** Finally, we estimate that how much area is required for the three dragons, and we use the method solving 'Cattle Grazing Problem' for reference.. The results show that a mature dragon requires about **140,000 million calories** each two days, which includes of **a community of 178 sheep and 470 cattle**, and it needs **at least about 14,000 km^2 area** to support three mature dragons to live.

After that, we build a model analyzing the manifestation of the dragon under different climate conditions. The total energy expenditure and the preys' distribution will both be affected by this factor. We use the density data of sheep and cattle in Australia to simulate. Our results show that **the higher the temperature, the less energy the dragon consumed. The higher the humidity, the lower the energy the dragon needs.** And the influence of temperature on the dragon is greater than humidity.

At last, our model analyzes the interaction between the dragon and the environment. We simulate an ecosystem with three types of preys that differ in distribution density and meat energy. We draw a conclusion that **the dragon has a greater impact on the prey with high energy and dense distribution.**

Sensitivity analysis shows that the trauma coefficient, the ratio of the dragon's bone and muscle and the sheep's density are not sensitive to the total energy expenditure, which implies the robustness of our model.

Keywords: growth curve, the SFM model, Dragon Hunting Algorithm, Maxent method

Contents

1 Introduction

1.1 Background

In the famous fantasy drama television series *Game of Thrones* and its original novel series *A Song of Ice and Fire, Daenerys Targaryen* is one of the most popular characters. *The New York Times* cites her as one of the author's finest creatures, named as "Mother of Dragons", for she has three dragons: *Drogon, Rhaegal,* and *Viserion.* We appreciate *Daenerys Stormborn* and are willing to see her conquering the Westeros with her dragons. But there is another question catching our eyesight now. What if these three fictional dragons are alive today?

Dragons are the creatures standing on the top of the food chain. Their existence will have a great impact on the ecological environment. It will consume quantities of resources to secure their survival. In order to make the existence of the dragon more possible, we need to figure out these circumstances around the dragons.

1.2 Restatement of the Problem

To find out how the dragon will live in in a real world, we need to analyze the following questions:

- How will the dragon grow?
- How will the dragon interact with the ecological environment?
- Will the dragon perform differently in different environments? If true, what is the discrepancy?

In order to answer these questions, we take some of the scientific researches in the field of biology today into consideration.

1.3 Literature Review

The growth curve model is a model for studying the laws of a certain biological indicator's changes varying with time. So far, scientists have used nonlinear curves to fit the growth trajectory related to animals. In 1838, *Verhulst* proposed the Logistic growth curve equation[1]. In 1938, *Von Bertalanffy* corrected the Logistic growth curve to accommodate the metabolic laws of the organism[2], and he applied this in the field of fish weight studies successfully. In 1959, *F.J. Richards* extended the Bertalanffy growth curve and proposed the Richards growth curve[3].

Two general approaches have been in use to determine the distance to a specified level of heat flux hazard from fires [4, 5, 6, 7, 8]. These are the so-called Solid Flame Model and the Point Source Model[9].

Species Distribution Models (SDMs) mainly use species distribution data and environmental data to estimate the niche of a species according to a specific algorithm. And project it into the landscape to indicate the species' preference for habitat in probabilistic. The results can be interpreted as the probability of occurrence of species, habitat suitability or species richness, etc.. In the 1970s, Nix et al first used SDMs to predict the spatial distribution of species[10]. After the 1990s, as the rapid developments of GIS technology and easy refining of remote sensing data related to climate, sea surface, land surface take place, it greatly enhanced the application of SDMs, together with a large number of species distribution models and software emerged.

2 Assumptions and Notations

2.1 Assumptions and Justifications

- **There is no human intervention in the ecosystem we discuss.** We only discuss the dragons' livelihood without humans, because human's behavior can totally affect the whole objective laws of nature.

- **The dragons will not fly except for predation.** The dragons' flight may consume a lot of energy, and the purposeless flight is not propitious to survival. Thus we assume that the dragon will not fly in addition to predating.
- **The dragon has to hunt for a living.** In the real world, we do not consider the elements of magic. According to the law of conservation of energy, it is impossible for a dragon to create the energy for fire, etc..
- **The dragon will not breath fire unless for predation.** The dragon will try its best to save its energy.
- **The dragons are carnivores.** The dragons are ferocious, and only eat meat containing sufficient energy.
- **The three dragons are far away from each other.** Competition effect is complex. To simplify our model, we set they will not influence each other's predation by the competition effect.

2.2 Notations

Some notations used in this paper are listed in Tab. 2.1 and Tab. 2.2.

Table 2.1: Variable notations

Notation	Definition	Unit
	Variable	
E_n	total energy for expenditure	kJ
E_m	the energy for basic metabolism	kJ
E_g	the energy for growing in t_d	kJ
E_f	the energy for flying	kJ
E_b	the energy for breathing fire	kJ
E_t	the energy of recovery of trauma	kJ
E_v	the cost energy for flying	J/(kg·hour)
E_a	the energy for absorbing all preys	J
m_d	the weight of the dragon	kg
dm_d	the gain of dragon's weight in t_d	kg
dm_b	the gain of dragon's bone weight in t_d	kg
dm_m	the gain of dragon's muscle weight in t_d	kg
ϕ_t	the heat flux producing by the dragon	kg
ϕ_l	the heat flux transfer to the side lateral area	kg
ϕ_b	the heat flux transfer to prey	kg
V_E	the consuming oxygen's volume	ml/(kg·min)
dV_b	the gain of the bone volume	m^3
dV_m	the gain of the muscle volume	m^3
T_e	the temperature of the environment	℃
v_d	the flying speed referring to minimum power	m/s
n	the number of preys eaten by the dragon	none
t_{bi}	the time period for the breeding of prey i	day
t_d	the time period for the predation of the dragon	day
B_i	the breeding amount of prey i during t_{bi}	none

Table 2.2: Constant notations

Notation	Definition	Unit
	Constant	
E_p	the energy produced by 1kg protein	kJ/kg
E_{bone}	the energy for growing 1kg bone	kJ/kg
E_o	the energy produced by 1mol O_2	kJ/mol
E_{pi}	the energy of 1kg prey i	kcal/kg
ρ_o	the air density of oxygen at 1atm	kg/m^3
ρ_b	the density of dragon's bone	kg/m^3
ρ_m	the density of dragon's muscle	kg/m^3
ρ_{pi}	the distribution density of prey i	/km^2
m_{pi}	the weight of the prey i	kg
η	the absorbing coefficient for prey	none
ξ	the trauma coefficient	none
c_{pi}	the heat capacity of the prey i	kJ/(kg·°C)

3 Model Establishment

3.1 The Model of Dragon's Growth

The organism has a common feature on it's growing speed, which can be described as Slow-Fast-Slow. Fig. 3.1 clarifies the growth process of the organism.

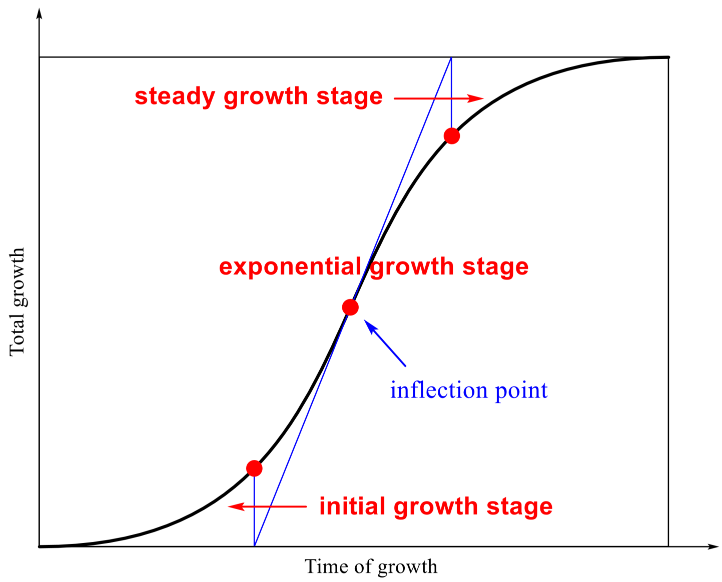


Figure 3.1: Sigmoid Form Curve of Growth Process

As we can see in Fig. 3.1, the total growth of the organism shows an 'S' trend that includes three stages: the initial growth stage, the exponential growth stage, and the steady growth stage. These three stages are determined by two crossover point, which the tangent line of the inflection point intersects with $y = 0$ and $y = A$.

Richards equation can simulate these three stages by four parameters with biological meanings: the maximum specific growth rate μ_m, the lag time λ, the asymptotic value A and the shape parameter v, which can be written as below[11]:

$$y = A\left\{1 + v \cdot \exp(1+v) \cdot \exp\left[\frac{\mu_m}{A} \cdot (1+v)^{\left(1+\frac{1}{v}\right)} \cdot (\lambda - t)\right]\right\}^{\left(-\frac{1}{v}\right)} \tag{3.1}$$

In order to simplify the problem, **we suppose the dragon's shape satisfying geometrically similar with time going by**[12]. This can be shown as:

$$S = kV^{\frac{2}{3}} \tag{3.2}$$

S is the surface area of the dragon, V is the volume of the dragon and k is a constant. If Richards equation satisfies Eq. 3.2 , we could yield $v = -\frac{1}{3}$, and Eq. 3.1 only has three parameters that need to be determined, and it transforms to Bertalanffy equation. Considering the requirements in the problem that dragon's weight is 10 kg when hatched and $30 - 40$ kg after a year's growth, so we need another requirement. We tend to determine this by estimating A, which indicates the maximum weight of the dragon.

Since the dragon does not exist in real life, we use the data of real creatures to analogize the weight of a mature dragon. Tyrannosaurus Rex is a good choice whose shape is very similar to the dragon. After our investigation, we found that the largest T. rex weighs about 14.85 tons. The latest T. rex was found in Montana, USA, in 1987. It is the most complete specimen found, and it has a skull about five feet long, which is equal to about 1.5 m[13]. For the dragon, we refer to the description in *Game of Thrones*, *the biggest (skull of dragon) was the size of a carriage.* We set the carriage with a length of 4 m. Then, according to geometrical similarity, we can derive:

$$\frac{14.85 \times 10^3}{A} = \frac{1.5^3}{4^3}$$

From the equation above, we yield $A = 281.6$ tons.

3.2 The Model of Dragon's Energy Expenditure

After clarifying how the dragon grows, we need to determine the energy that the dragon needs in daily life. According to the researches in bioenergetics, we divide the individual biological energy flow into several categories, which is shown in Fig. 3.2.

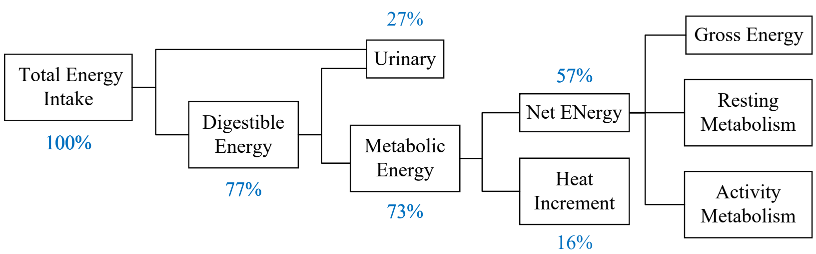


Figure 3.2: Flow Chart of Energy Distribution

To estimate the total energy intake of a dragon, we tend to determine net energy. By our analysis, net energy can be divided into five parts, which are: energy for basic metabolism, energy for growth, energy for flight, energy for breathing fire and energy for recovery of trauma. As we can see them located in Tab. 3.1.

Table 3.1: Energy Cost Activities We Need to Analyze

energy cost activity	category	influencing factor
energy for basic metabolism	basal metabolism	weight
energy for growth	growth energy	time
energy for flight	activity metabolism	location of food
energy for breathing fire	activity metabolism	amount of food
energy for recovery of trauma	activity metabolism	proportional to the energy for basic metabolism

From the consuming side, the total energy E_n for the dragon can be written as Eq. 3.3 below:

$$E_n = E_m + E_g + E_f + E_b + E_t \tag{3.3}$$

Next, we will determine each five part of E_n.

3.2.1 Energy for Basic Metabolism

Animals produce energy by breathing while they live on earth, which is called cellular respiration. Even though an animal remains static in a certain place, they need the energy for basic metabolism E_m. We can calculate E_m by the chemical equation below:

$$C_6H_{12}O_6 + 6H_2O + 6O_2 \rightarrow 6CO_2 + 12H_2O + energy \tag{3.4}$$

If we could determine the oxygen consumption, then we can get the energy released by the cellular respiration.

Oxygen consumption V_{o_2} increases progressively less with the increase in species' weight. V_{o_2}/kg is larger in small animals than in large animals, for small mammals will have a larger body surface area/body mass ratio than heavier mammals. Dragon is a huge organism, so they may have less V_{o_2}/kg than the human.

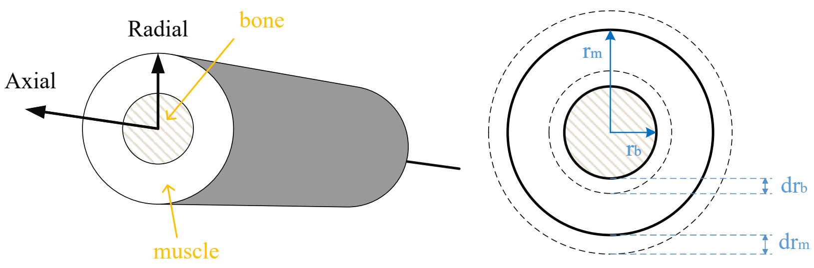
Let V_E to be V_{o_2}/(kg·min). By referring to the literature[14], we can get that V_E for an adult is 150 ml/(kg·min) in average, and we also can get that V_E for the mouse that is 600 ml/(kg·min) in average, almost four times of an adult. Because the scale comparison between a dragon and an adult is similar to that between an adult and a mouse, we can estimate V_E for dragon is 37.5 ml/(kg·min).

$$E_m = \frac{\rho_o m_d V_E t}{6\mu_o} E_o \tag{3.5}$$

ρ_o means the air density of oxygen, m_d means the weight of the dragon, μ_o represents the relative molecular mass of oxygen, and E_o means the energy provided by 1 mol oxygen through Eq. 3.4 to compose ATP.

3.2.2 Energy for Growth

The dragon's body mostly consists of bone and muscle, so the energy for dragon growing can be estimated by the bone growth and muscle growth. As we assume that the dragon's shape satisfying geometrically similar varying with time, the axial growth rate for these two components remains the same but becomes different in radial direction. This process can be shown by Fig. 3.3.



(a) 3D View

Figure 3.3: Sketch of Muscle and Bone

Let r_b be the external radius of bone, r_m be the external radius of muscle, so growth of bone volume dV_b, muscle volume dV_m, total volumn dV in time period t_d have the relationship:

$$\frac{dV_b}{dV_m} = \frac{2\pi r_b \cdot dr_b \cdot dl}{2\pi r_m \cdot dr_m \cdot dl} \tag{3.6}$$

$$dV = dV_b + dV_m \tag{3.7}$$

In Eq. 3.6, dl is the growth of axial length. In order to keep the ratio of bone and muscle in cross section remaining the same, we can derive:

$$\frac{\pi r_b^2}{\pi r_m^2} = \frac{\pi (r_b + dr_b)^2}{\pi (r_m + dr_m)^2} \approx \frac{\pi \left(r_b^2 + 2r_b dr_b\right)}{\pi \left(r_m^2 + 2r_m dr_m\right)} = \frac{2\pi r_b dr_b}{2\pi r_m dr_m} \tag{3.8}$$

Combining Eq. 3.6 and Eq. 3.8, we can derive:

$$\frac{dV_b}{dV_m} = \frac{r_b^2}{r_m^2} \tag{3.9}$$

Let dm_b be the bone's weight increment, dm_m be the muscle's weight increment. Suppose the dm_m is contributed by protein, dm_b is contributed by protein partially. E_p is the energy/kg protein contained, E_{bone} is the energy/kg bone contained. By referring to the literature, we can get bone consists of 10%-30% protein[15]. Other parts are water and bone mineral which do not contain energy. So we set $E_{bone} = 0.2E_p$:

$$E_{gm} = \rho_m dV_m E_p \tag{3.10}$$

$$E_{gb} = \rho_b dV_b E_{bone} \tag{3.11}$$

E_{gm} and E_{gb} are the growth energy of muscle and bone, while ρ_m and ρ_b are the density of muscle and bone. dm_d is the mass increment of dragon in time period t_d. and they also satisfy:

$$dm_d = \rho_m dV_m + \rho_b dV_b = \left(\rho_m + \rho_b \frac{r_b^2}{r_m^2}\right)\left(\frac{r_b^2}{r_b^2 + r_m^2}\right) dV \tag{3.12}$$

$$E_g = E_{gm} + E_{gb} = \left(\rho_m \frac{r_m^2}{r_m^2 + r_b^2} E_p + \rho_b \frac{r_b^2}{r_m^2 + r_b^2} E_{bone}\right) dV \tag{3.13}$$

We set $r_b = 0.8r_m$. Through Eq. 3.12, we can get dV . Then we could derive the energy of growth by Eq. 3.13.

3.2.3 Energy for Flying

Dragons are aggressive, so they do not have any natural enemy. **We suppose that dragons always fly in the speed by consuming minimum power** because they will not escape. According to the literature[16], we can get the relationship between dragon's flying speed v_d and its weight :

$$v_d = 5.70 m_d^{0.16} \tag{3.14}$$

Let L_d be the total distance that the dragon fly to the preys, E_v be the cost energy /(kg·hour) which can be estimated by the literature [17]. We can get the energy for flight below:

$$E_f = m_d \frac{L_d}{v_d} E_v \tag{3.15}$$

3.2.4 Energy for Breathing Fire

The dragon needs to roast preys, for they have to eat them. The breathing fire can be depicted by flame. For analysis, we cannot and do not need the whole temperature field's distribution accurately, but using the Solid Flame Model (SFM) to simplify the flame[9].

In the SFM, we suppose **the heat transfer ratio keeps the same for each direction**, which can be depicted by Fig. 3.4.

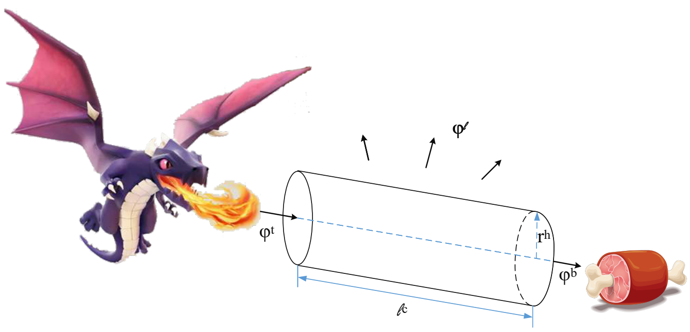


Figure 3.4: Sketch of Heat Transfer

ϕ_b is the heat flux at the bottom of the cylinder, which outputs to the prey. ϕ_l is the heat flux transferring to the side lateral area of the cylinder, which indicates the lost. ϕ_t is the heat flux provided by the dragon. r_h is the radius of the dragon's mouth, l_c is the length of the cylinder.

We set ϕ_l and ϕ_b are proportional to the transferring area, and heat transfer ratio q as a constant, which can be determined by Eq. 3.16:

$$\frac{\phi_l}{2\pi r_h l_c} = \frac{\phi_b}{\pi r_h^2} = q \tag{3.16}$$

By referring to Eq. 3.16, we can get the equality which shows the conservation of the heat energy:

$$\phi_t = \phi_l + \phi_b = \phi_b \left(1 + \frac{2l_c}{r_h}\right) \tag{3.17}$$

We set the heat capacity of the prey as c_p, prey's weight as m_p, heating time as t_p, environment temperature as T_e. We also set $l_c = 4r_h$, for the dragon will roast food in a comfortable position while the dragon grows. The prey's temperature needs to be appropriate for the dragon to eat, and we set this as T_p. So we can derive:

$$\phi_b t_p = c_p m_p (T_p - T_e) \tag{3.18}$$

We let n be the number of the preys. According to the Eq. 3.16, Eq. 3.17 and Eq. 3.18, we can get the energy for breathing fire E_b:

$$E_b = \phi_t t_p = n c_p m_p \left(1 + \frac{2l_c}{r_h}\right)(T_p - T_e) \tag{3.19}$$

3.2.5 Energy for Recovery of Trauma

As we mentioned above, the dragons are aggressive, so they seldom get hurt from the preys. We suppose this part of energy is proportional to the energy for basic metabolism, and the coefficient for the possibility to get hurt ξ is small. So the energy for recovery of trauma can be written as:

$$E_t = \xi E_m \tag{3.20}$$

3.2.6 Algorithm for Calculating the Total Energy Expenditure

Eq. 3.3 determines E_n from the consuming side. In the aspect of absorbing side, we can calculate the net energy by Eq. 3.21 and Eq. 3.22 below:

$$E_n = 0.57\eta E_a \tag{3.21}$$

$$E_a = \sum_i n_i m_{pi} E_{pi} \tag{3.22}$$

Not all parts of the preys can be eaten by the dragon. η is the efficiency which indicates the proportion of preys that the dragon eats. E_a is the absorbed energy and E_q is the energy per kg for the preys. From Fig. 3.2, we can get E_n is 0.57 times of the absorbed energy.

From Eq. 3.15, we can get that E_f is proportional to the flying distance. From Eq. 3.19, we can know that E_b is proportional to the number of preys. So these two parts of energy are associated with a single prey's energy and the preys' distribution, and this will set the total energy expenditure uncertain. In order to calculate the total energy expenditure with the time period t_d, we develop an algorithm and **keep the category of the preys and their distribution type fixed**. We also find the population density of these preys in Australia online [18, 19] for a better simulation.

Our algorithm supposes that **the dragon will find the nearest prey relating to its position** because this corresponds the lowest energy expenditure. We define this as **Dragon Hunting Algorithm**, whose steps are shown in Algorithm 1.

After that, we can calculate the energy expenditure E_n with the time period t_d for each age through this algorithm, and get the curve of E_n versus age. Then we can estimate the minimum area for the living of three dragons. This is similar to the famous 'Cattle Grazing Problem' proposed by Newton. Dragons are eating the preys and the preys will reproduce, and they would reach a transient balance.

Let B_i be the adding number of prey type i by considering breeding among time period t_b, t_d be the time period for the dragon's predation. Then the absorbing energy producing by prey type i in period t_b in supported area $S(age)$ can be written as:

$$E_{vi} = \frac{B_i}{2\frac{t_b}{t_d}} \cdot \eta m_{pi} E_{pi} \rho_{pi} S(age) \tag{3.23}$$

In Eq. 3.23 , the formula dividing 2 means that each newborn prey is bred by two mature preys. Also we can get Eq. 3.24 by transient energy balance:

$$E_n(age) = \sum_i E_{vi} \tag{3.24}$$

So we can derive $S(age)$ by Eq. 3.25:

$$S(age) = 2\frac{t_b}{t_d} \cdot \frac{E_n(age)}{\eta \sum_i B_i m_{pi} E_{pi} \rho_{pi}} \tag{3.25}$$

Algorithm 1 Dragon Hunting Algorithm

Step 1: Set a maximum of age age_{max} to be simulated. For the age of dragon age from 0 to $age_{max} - 1$, do the following steps:

Step 2: Initialize a square area according the preset side length a, and generate preys(sheep and cattle) according the area of the square and the density of each kind of prey by using random distribution. Initialize the number of preys caught by the dragon $n_{cattle} = n_{sheep} = 0$.

Step 3: Initialize the position of dragon (x_d, y_d) in the square area.

Step 4: Set the time period for predating $period$ =2 days. Calculate the growing speed μ and weight $weight$ at age age. Then calculate E_m, E_g, E_t, and initialize $E_f = E_b = 0$. Initialize absorbed energy $E_a = 0$ and $E_n = E_m + E_g + E_t$.

Step 5: While $0.57\eta E_a < E_n$, do the following steps:

Step 6: Search the nearest prey (x^*, y^*) in the square area.

$$L(x, y) = \sqrt{(x - x_d)^2 + (y - y_d)^2}$$

$$(x^*, y^*) = \arg\min_{(x,y)} L(x, y)$$

Step 7: Eat the prey at position (x^*, y^*) and update n_{cattle}or n_{sheep}, and update $(x_d, y_d) = (x^*, y^*)$. Calculate the energy for flying E_f and energy for breathing fire E_b, together with updating the energy consumed $E_n = E_n + E_f + E_b$. Update the energy absorbed $E_a = E_a + E_{prey}$where the E_{prey} is the energy of the prey eaten.

3.3 The Model of the Climate Condition

Climate condition changes may have several influences on our model, mostly altering the temperature and the humidity of the environment. We tend to analyze this influence for two parts: the influence on the dragon itself and the influence on the preys' distribution.

3.3.1 The Influence on the Dragon

After referring to the literature, we find that the higher environment temperature causes the more energy consuming for basic metabolism[20] and we suppose that the tendency of this phenomenon is similar to a bird. Then we can get Eq. 3.26 by averaging the consumption in daytime and at night:

$$\frac{E_m(T_e)}{E_m(T_r)} = \frac{V_E(T_e)}{V_E(T_r)} = 1.82 - 0.033T_e \tag{3.26}$$

T_r is the reference temperature, which was defined to be 25 °C. $E_m(T_e)$ is the energy consuming for basic metabolism at T_e while $E_m(T_r)$ is that at T_r .

Obviously, the temperature of environment will also influence E_b by affecting T_e. From Eq. 3.18 we can see: while T_e is lower, E_b becomes greater.

There will be no obvious influence on the dragon while the humidity of the environment varies, which agrees with our basic knowledge.

3.3.2 The Influence on the Preys' Distribution

Different climates such as an arid region, a warm temperate region, and an arctic region, also affect the distribution of the dragon's preys. We still take the Australia continent as an example. According to the website [21], we can find the states with different temperature and humidity in Australia. According to the website [18, 19], we could determine the number of sheep and cattle in different states.

We use data of Queensland (Qld) and New South Wales (NSW) to compare different temperature conditions. We use Western Australia (WA) and South Australia (SA) to compare different humidity conditions. The detailed data is shown in the Tab. 3.2.

Table 3.2: Statistics for the Australian States

State	Qld	NSW	WA	SA
Relative Temperature	low	high	$\times$	$\times$
Relative Humidity	$\times$	$\times$	high	low
Area ($\times 10^4$ km^2)	172.72	80.16	252.55	104.35
Number of Sheep (million)	2.1	26.9	14.2	11.5
Number of Cattle (million)	11.1	5.3	2.1	1.1
Density of Sheep (/km^2)	1.21	33.56	5.62	11.02
Density of Cattle (/km^2)	6.43	6.61	0.83	1.05

It can be seen from Tab. 3.2 that different climates affect the distribution density of sheep and cattle, thus changing the input parameters of our models.

3.4 Model of Interaction with Environment

In this part, we tend to determine which type of preys will be affected mostly by the dragon. We define three types of preys in our model, which are different in distribution density and meat energy. The detailed data of them is shown in Tab. 3.3.

Table 3.3: Detailed Data of Three Preys

Preys	Distribution Density (/km^2)	Meat Energy (kcal/100g)
Sheep	9.4514 (high)	118 (middle)
Cattle	3.4130 (middle)	125 (high)
Hare	2.426 (low)	102 (low)

According to biological principle, it is impossible for a species to contain high individual energy as well as high distribution density.

To determine the ecological impact of the dragon, we can input the species in Tab. 3.3 into our model, and statistic the changes in distribution out of each species.

4 Results

4.1 Results of the Model of Dragon's Growth

4.1.1 Determination of λ and μ_m

According to Eq. 3.1, the dragon's weight y is a function of time t. As $A = 281.6$ tons is a constant, there are only two undetermined parameters: λ and μ_m. By using the constraints given by the problem description:

$$\begin{cases} y(0) = 10 \\ y(1) = b \quad\quad 30 \le b \le 40 \end{cases}$$

we solve the value of λ and μ_m using SciPy's scipy.optimize.fsolve function. Because the result obtained by SciPy is related to the given initial value, we traverse all the initial values using step size 10 from 1 to 1000 for λ and μ_m, and then substitute the obtained solution into the original equation to measure its accuracy, and finally find the optimal solution. According to the range of b, we take three special values 30, 35, and 40 to solve the parameters, and the final result is shown in Tab. 4.1.

Table 4.1: Solving results λ and μ_m

$y(1)$	λ	μ_m
30	26.31	1895.42
35	22.42	2224.25
40	19.76	2523.83

From Tab. 4.1, we can conclude that when A and $y(0)$ is a constant, the larger $y(1)$, the smaller λ, because the faster the growth rate, the shorter the lag time; the larger $y(1)$, the larger μ_m, because the dragon can grow bigger in a short time, and then it will grow faster when bigger, which forms a virtuous cycle.

4.1.2 Determination of Growth Stages

Once λ and μ_m are determined, we can draw the tangent of y at the inflection point, which can be written as follows:

$$y = \mu_m(t - \lambda)$$

Then we can derive the three essential points and stages of the growth curve by

- the intersection of a vertical line which passes the intersection of the tangent and the horizontal line $y = 0$ and the original y-curve,
- the intersection of the tangent and the original y-curve,
- and the intersection of a vertical line which passes the intersection of the tangent and the horizontal line $y = A$ and the original y-curve.

When $b = 40$, the results of our solution are shown in Fig. 4.1.

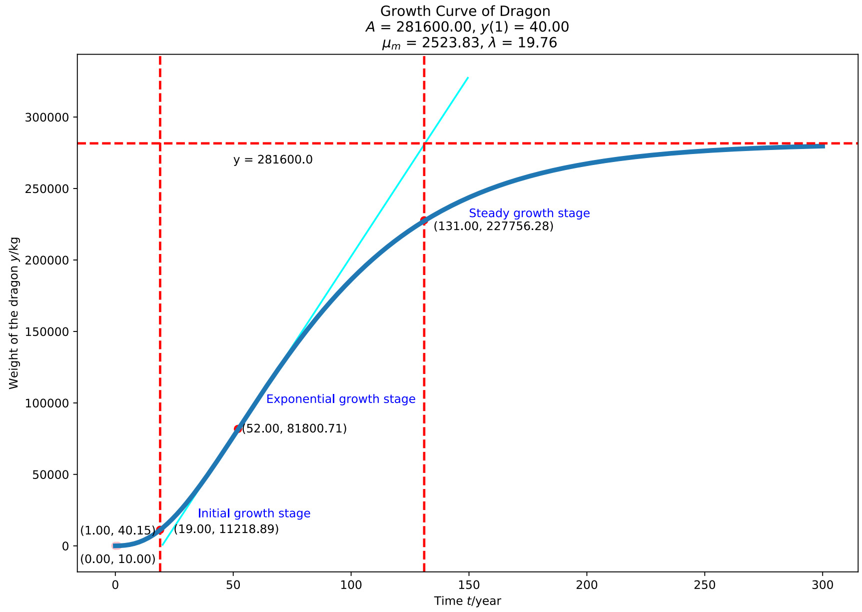


Figure 4.1: Groth curve of a dragon when $y(1) = 40$

Form Fig. 4.1, we can conclude that, when $y(1) = 40$, the dragon's asymptotic weight value A can be reached after nearly 280 years, which is a long time. The periods of the three stages of a dragon are:

1. The nitial growth stage: 0 - 19 years old
2. The exponential growth stage: 19 - 131 years old
3. The steady growth stage: > 131 years old

The time when the dragon reaches maximum growth rate μ_m is around 52 years old, and it weighs 81,800.71 kg.

4.2 Results of the Model of Dragon's Energy Expenditures

By applying Dragon Hunting Algorithm, we keep the preset parameters unchanged, run and output the results for further analysis:

- We simulate the age of dragon form 0 to 500 years old.
- The dragon hunts once every t_d =2 days.
- We assume $y(1) = 40$ and make use of the result of Fig. 4.1. If using other value of $y(1)$, the results will be similar.
- The side of the square area is empirically set to 10 meters to meet the dragon's hunting demand.
- The dragon starts hunting at position $(0, 0)$, which is the left-bottom of the square.
- Values of constant in Subsection 2.2 we use are shown in Tab. 8.1.

4.2.1 Simulation Results

By running Algorithm 1, we get the output as Fig. 4.2, Fig. 4.3 and Fig. 4.4.

Energy expenditures of a dragon and preys eaten in two days vary with age is shown in Fig. 4.2. From Fig. 4.2, we can see that:

- The energy a dragon need is also S-shaped. This is because E_m and E_t increase as y increases.
- The curve is steeper than the growth curve Fig. 4.1 at the inflection age (52 years old). This is because E_g increases as μ increases, not y, the former increases faster than the latter.
- A mature dragon needs 178 sheep and 470 cattle, which is 140,000 million calories of energy, per two days.

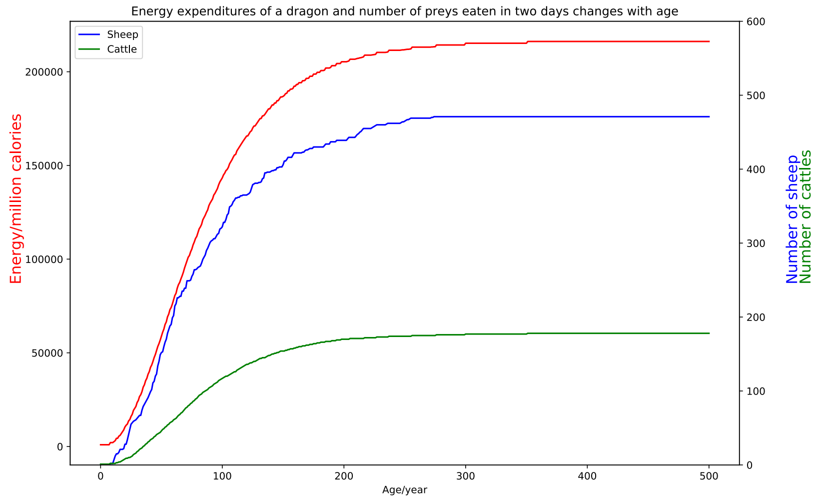


Figure 4.2: Energy expenditures of a dragon and preys eaten in two days$-age$ curve

Minimum area required to support the three dragons vary with age is shown in Fig. 4.3. As for minimum area required to support the three dragons, according to Eq. 3.25, we can determine the minimum area to support a dragon. Then the minimum area of three dragons is three times of $S(age)$ obtained by Eq. 3.25, which is shown in Fig. 4.3. We can see that:

- The trajectory is the same as that of Fig. 4.2. This is because the area required is proportional to the preys eaten.
- We need at least about 14, 000 km^2 area to support three mature dragons.

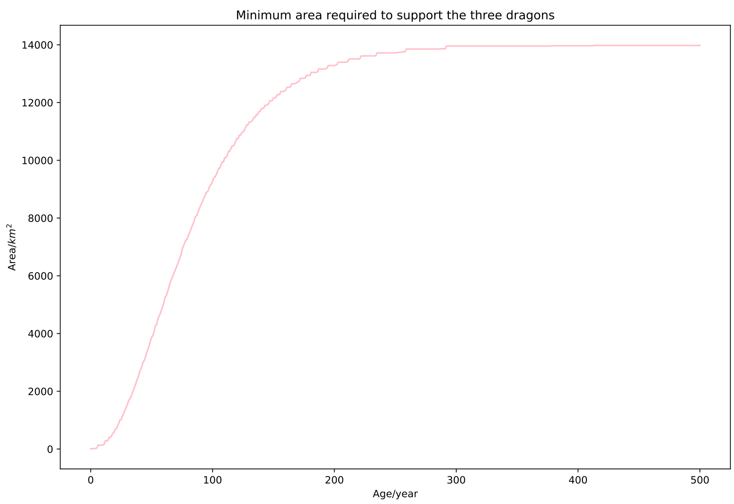


Figure 4.3: Minmum area required to support the three dragons$-age$ curve

Energy percentage of a dragon in different stages is shown in Fig. 4.4. Through Fig. 4.4, we could get the information below:

- The energy for basic metabolism E_m takes the most percentage of the total energy expenditure E_n during the whole life of the dragon, while the energy for growth, the energy for flying and the energy for recovery of trauma only take a low proportion. Because the dragon's growth causes increment on its weight, which will make E_m become greater.

- The energy for breathing fire E_b first takes a high percentage of E_n when the dragon is young. After it becomes mature, the percentage of E_b decreases a little and then become stable, for the reason that E_b is proportional to the number of predated preys n, which raises as the weight of the dragon m_d growing.

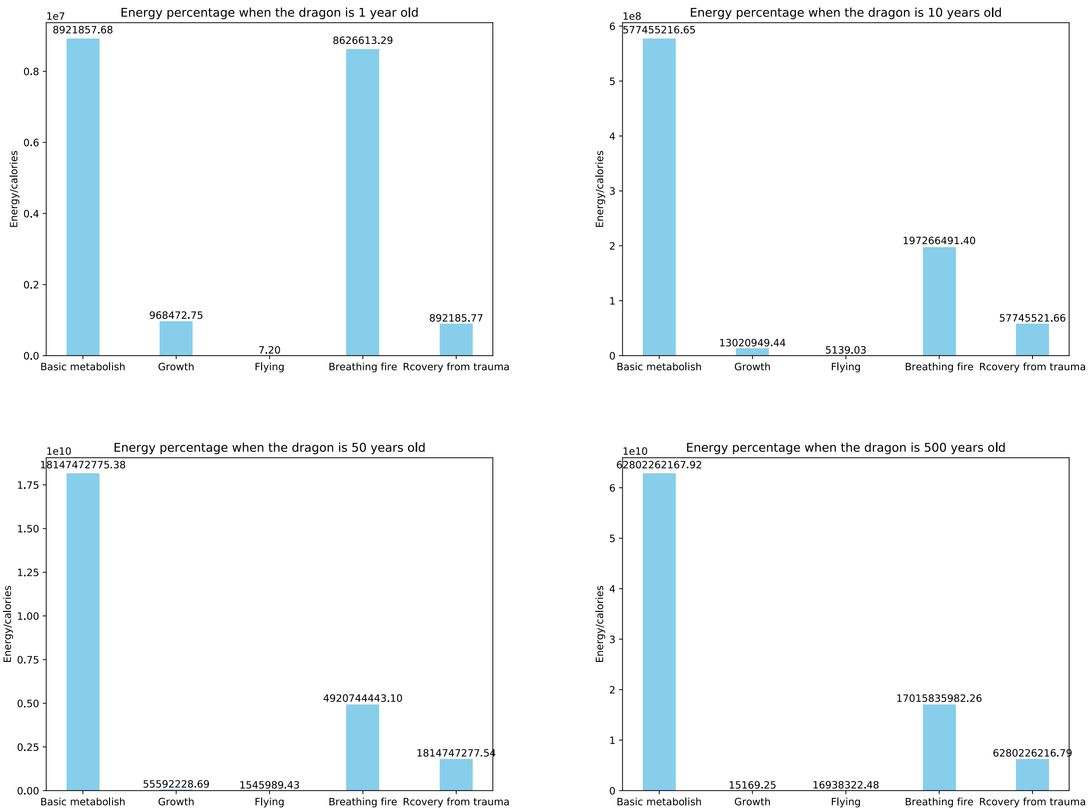
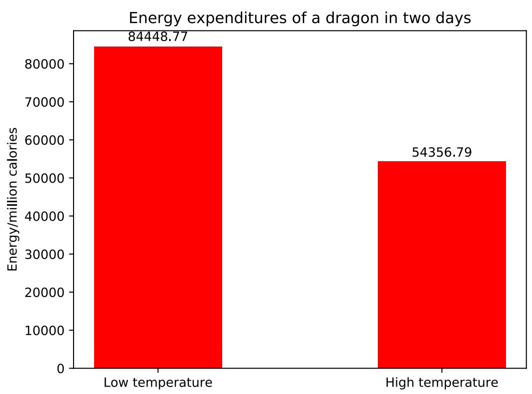


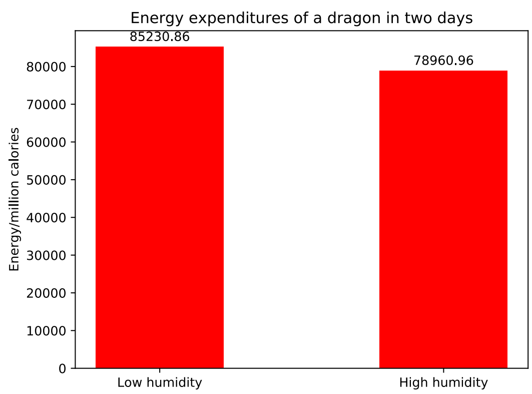
Figure 4.4: Energy percentage of a dragon$-age$ histogram

4.3 Results of the Model of the Climate Condition

We fix the dragon's age at 280 years old. Using the data of sheep and cattle density in Qld, NSW, WA, and SA. The simulation results are shown in Fig. 4.5.



(a) Energy expenditures of a dragon in two days$-T_e$ histogram



(b) Energy expenditures of a dragon in two days$-$Humidity histogram

Figure 4.5: Simulation Results of the Model of the Climate Condition

We can conclude that:

- The higher T_e, the less energy the dragon consumes. This is because according to Eq. 3.19 and Eq. 3.26, the higher T_e, the less E_b and E_m. And other energy expenditure components do not change with T_e, so the energy required by the dragon is reduced. At the same time, the higher T_e, the higher the density of preys. Thus, the dragon can obtain a prey with less flight distance.
- The higher the humidity, the lower the energy the dragon needs. This is because prey is often densely distributed in areas with high humidity, making it easier for dragons to predate.
- The influence of temperature on the dragon is greater than humidity.

4.4 Results of the Model of Interaction with Environment

We preset the age of the dragon to 100 years old, and then run step 2 - 7 of the Algorithm 1, drawing the distribution changes with dragon's hunting contrast without dragon's hunting as Fig. 4.6.

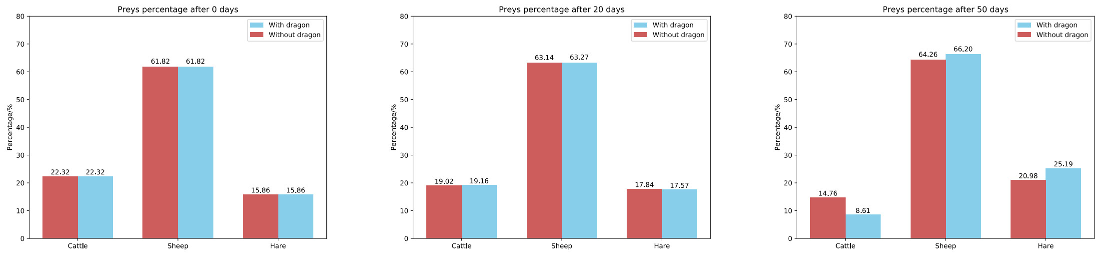


Figure 4.6: Preys' percentage changes when the dragon is 100 years old$-age$ histogram

From the figure, we can conclude that, with dragon hunting, the proportion of cattle is smaller than that of normal breeding, and other animals are the opposite. And the increase in the proportion of hare is greater than that of sheep. In short, cattle are most affected, followed by sheep, and hares have the least impact. Therefore, we can draw general conclusions: the dragon has a great impact on the prey with high energy and dense distribution but has little impact on prey with low energy and sparse distribution.

The predating path of the dragon is shown in Fig. 4.7.

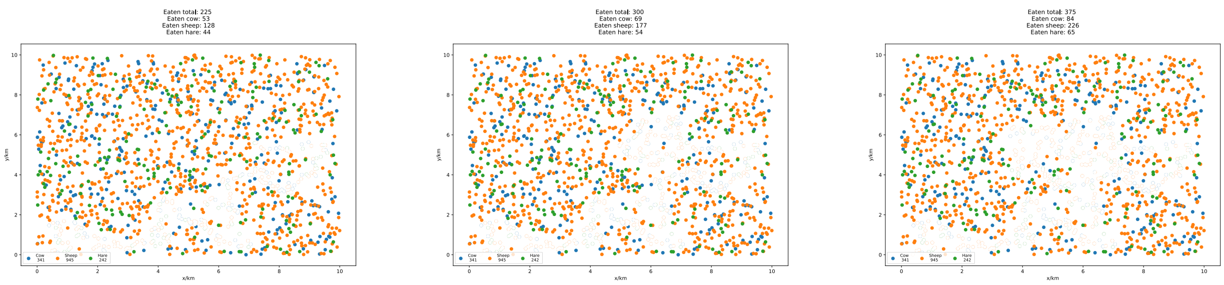
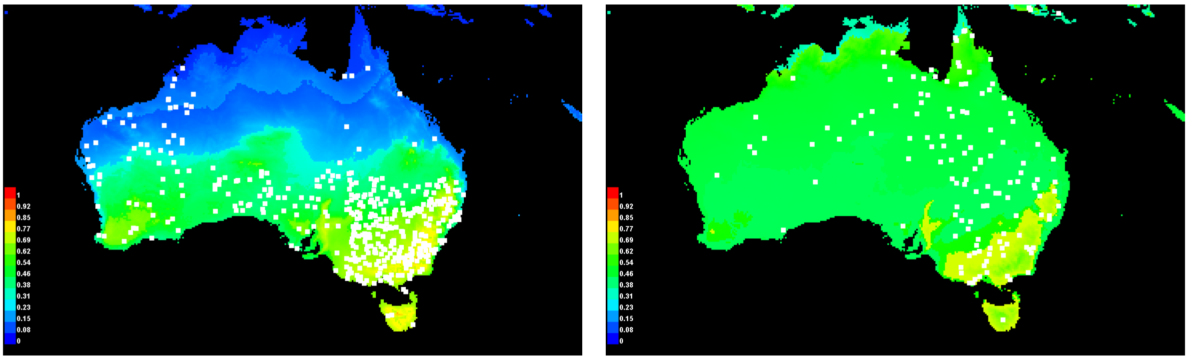


Figure 4.7: Predating Path of Dragon

5 Model Expansion

Our previous model is based on the assumption that species are subject to random distribution. Under this assumption, we ignore the differences in the environment. However, in the real world, the environment of a region cannot be the same everywhere.

In order to get closer to the realistic situation of species distribution, we use Maxent (Maximum entropy)[22] to model the species geographic distributions. The Maxent software is implemented in Java and available at the website [23]. In maximum entropy density estimation, we input the species distribution samples' data (species name, longitude, latitude)[18, 19] of sheep, cattle and climate data [24] in Australia, and then the true distribution of a species is simulated as a probability distribution π over the set X of sites in the study area. Thus, π assigns a non-negative value to every site x and the values $\pi(x)$ sum to one. The weather constraints are expressed in terms of simple functions of the environmental variables, called features. The mean of each feature is required to be close (within some error bounds) to the empirical average over the union of the presence sites. As a result, the species distribution map for survival appropriation in Australia is shown in the Fig. 5.1.



(a) Sheep distribution model (b) Cattle distribution model

Figure 5.1: Maxent species distribution modeling result for sheep and cattle in Australia
Warmer colors indicates areas with high probability of suitable conditions for the species, while white dots show the presence of locations utilized for training.

It can be reasonably assumed that the more suitable the species to survive, the greater the distribution density of the species. So we use the species distribution map for survival appropriation to represent the distribution density of the species. Replacing the random distribution with this distribution density will make our model more realistic.

6 Sensitivity Analysis

6.1 Parameters for Calculating the Net Energy

From Tab. 6.16.2, we can see that parameter ξ, $\frac{r_b}{r_m}$ will not influence the net energy E_n a lot. So these parameters are not sensitive to the results, for the energy for growth E_g and recovery of trauma E_t only take a low proportion of E_n.

Table 6.1: The turbulence of ξ

$\Delta\xi$	2%	4%	6%	8%
ΔE_n	0.251%	0.397%	0.543%	0.784%
$\Delta\xi$	-2%	-4%	-6%	-8%
ΔE_n	-0.146%	-0.292%	-0.543%	-0.729%

Table 6.2: The turbulence of $\frac{r_b}{r_m}$

$\frac{r_b}{r_m}$	2%	4%	6%	8%
$\Delta E_n(\times 10^{-4})$	-7.779%	-1.505 %	-2.185 %	-2.823 %
$\frac{r_b}{r_m}$	-2%	-4%	-6%	-8%
$\Delta E_n(\times 10^{-4})$	8.331 %	1.726 %	2.685 %	3.715 %

6.2 Parameters for Model of Interaction with Environment

From the Tab. 6.3 , we can get that energy are not sensitive to the density of sheep, because the varying density only causes the change of the energy for flying E_f through the flying distance, which also takes a little part of E_n.

Table 6.3: The turbulence of density of sheep

Density	2%	4%	6%	8%
ΔE_n	0.0423%	0.0277%	0.0257%	0.1470%
Density	-2%	-4%	-6%	-8%
ΔE_n	-0.0099%	-0.0458%	0.0018%	0.0693%

7 Conclusion

7.1 Strengths and Weaknesses

- **Strength**
  - Our model simplifies the whole predating and growing process of the dragon. It is easy for us to calculate the total energy expenditure, which will make the analysis clearer. For example, our model simplify the analysis of the energy for flying and recovery of trauma, while the results exactly prove that these two parts take a low proportion.
  - Our model uses some convincing data in reality and achievements in the literature, which will make the results be more realistic and in accordance with our basic experience. For example, the energy for breathing fire will increase while the dragon is in frigid zone comparing to the tropic zone.
- **Weakness**
  - While comparing the manifestation between different climates, our model chooses the data of climates and the distribution of sheep and cattle in Australia to simulate. Nevertheless, due to the lackness of data, the maximum discrepancy in temperature or humidity between two states in Australia that we gets are not prominent enough. If the discrepancy is more distinctive, the results will be more meaningful.
  - As we determine the minimum area needed for the growth of three dragons, we suppose that there are only sheep and cattle for the dragons to predate. However, the preys' distribution will be more complex, and competition effect among the dragons and other predators will exist. Our model cannot analyze this situation.

7.2 Future Work

- We could consider the categories of preys will vary with districts under different climates. For example, the dragon can predate polar bears and walrus in the Arctic while penguins in the Antarctic, but these animals do not exist in the tropical area.
- We could consider the different predating difficulties provided by the environment. For example, the landform is grass in temperate zone causing an easier predating, while that is forest causing a hurdle for predating because of the predator's hiding.
- We could extend our model to the condition of predating on the sea.

7.3 Our Conclusion

Through our model, we derive that the dragon's growth curve is S-shaped. It takes 280 years for a dragon to reach the upper limit of weight, which is 281.6 tons. The dragon is in the initial growth stage before the age of 19. After the age of 131, the dragon enters the steady growth stage. The age from 19 to 131 is exponential growth stage, while the dragon grew fastest at the age of 52.

By analyzing the energy composition required by a dragon, our model figured out that a dragon's energy expenditure consists of five categories, which are: energy for basic metabolism, energy for growth, energy for flight, energy for breathing fire and energy for recovery of trauma. The curve of the energy a dragon's need vary with time is also S-shaped. A mature dragon requires about 140000 million calories each two days, which includes a community of 178 sheep and 470 cattle. The energy for breathing fire takes a high percentage of the total energy expenditure when the dragon is young. The energy for basic metabolism takes the most percentage of the total energy expenditure during the whole life of the dragon. And it needs at least about 14000 km^2 area to support three mature dragons to live.

As for the influence of climate conditions, we consider the factors of the temperature and the humidity. Our results show that the higher the temperature, the less energy the dragon consumes. The higher the humidity, the lower the energy the dragon needs. And the influence of temperature on the dragon is greater than that of humidity.

To determine what is the ecological impact of a dragon, we simulated an ecosystem with three different types of preys. The results reveal the dragon has a great impact on the prey with high energy and dense distribution, while has little impact on prey with low energy and sparse distribution.

Besides, if we suppose the dragons breed through sexual multiplication, it will not be stable for three dragons to make up of a population. By referring to the literature, we discover that the number of the individual that consists of a stable population will be more than 500 [25]. According to our model, it will take approximately 2.33×10^6 km^2 area for these dragons to live, almost 30% of Australia. In summary, the dragon can be alive on the earth.

7.4 Insight for Other Realms

Our model focuses on the dragon, which is a fictional creature that does not exist in the real world. Nevertheless, our model has great inspiration for biological species research. Researchers can use our model with a few modifications to make it suitable for:

1. Study the growth of individual species.
2. Study the energy consumption of individual species, the energy conversion relationship between several species.
3. Determine the impact of new species on the environment and whether it can adapt to the local environment.
4. Predict the distribution of various species.
5. Forecasting biological invasions.

For example, we need to analyze the endangered species. We can simulate the survival of these species according to the environmental conditions, and judge whether these species can continue to survive. If they can't survive, we need to make helpful changes to its preys and living environment to help them survive, which can be derived by our model.

8 Letter to George R.R. Martin

Dear Martin,

We are a group taking part in the competition of MCM, and we also fascinate the TV series *Game of Throne*s, which is adapted from your famous novel *A Song of Ice and Fire*. We are especially interested in the three dragons appearing in your novel. Dragon is a mysterious and formidable creature, which raises our curiosity for deeply research. We present our findings in this letter, hoping these will be helpful for your writing.

We take the biggest dragon Balerion, the Black Dread, as an example. According to the description in your novel, Balerion's head is as large as a carriage. We use the body of T. rex to analogize, estimating that the mature Barlerion's weight will be 281.6 tons. By referring to the biological law, we determine that it takes 280 years for Barlerion to reach its upper limit weight. Its growing curve can be depicted as 'S' trend. The dragon is in the initial growth stage before the age of 19. After the age of 131, the dragon enters the steady growth stage. The exponential growth stage comes from age 19 till 131. The dragon grows fastest at the age of 52.

For the total energy expenditure for a dragon, we do not consider the case with magic, which means the dragon cannot create energy by itself. The total energy expenditure for the dragon can be divided into five parts: the energy for the basic metabolism, growth, flying, breathing fire and recovery from trauma. We suppose that the dragon will fly and breath fire only while it is predating, and the time interval for its predation lasts for two days. It will consume almost 140 billion calories. This means the dragon has to eat 178 sheep and 470 cattle per two days, which is tremendous. The total energy expenditure for dragon mostly consists of the energy for basic metabolism in its whole life. The energy for breathing fire first takes a high percentage of total energy expenditure when the dragon is young, and then decreases until become steady.

Besides, if the dragon exists in real life, this will obviously bring a great impact on the environment. We consider a situation of an area including three preys: sheep, cattle, and hare, which differs a lot in distribution density and meat energy. Then we stimulate the condition within the time interval for a two days' predation. As a result, cattle are most affected, followed by sheep, and hares take the least. That is to say, the dragon has a great impact on the prey with high energy and dense distribution, and has little impact on prey with low energy and sparse distribution.

Then, our model considers that dragons might travel to different regions of the world under different climates. The climate changes will both influence the dragon and the preys' distribution. We choose four states in Australia to simulate, of which the temperature and humidity are different. Our results show that the higher the temperature, the less energy the dragon consumes. The higher the humidity, the lower the energy the dragon needs. And the influence of temperature on the dragon is greater than humidity.

Besides, if we suppose the dragons breed through sexual multiplication, it will not be stable for three dragons to make up of a population. According to scientific researches, we discover that the number of individuals which consists of a stable population will be more than 500. After modeling, we find it will take approximately 2.33×10^6 km^2 area for these dragons to live, almost 30% of Australia. In summary, it is possible for the dragon to be alive on the earth theoretically, but require a lot of resources and territory.

Yours sincerely,

Team 1911426

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Appendix A

Values of Constant

Table 8.1: Values of constant

Notation	Definition	Value
E_p	the energy producing by 1kg protein	17,130 kJ/kg
E_{bone}	the energy for growing 1kg bone	3,426 kJ/kg
E_o	the energy producing by 1mol O_2	193.5 kJ/mol
ρ_o	the air density of oxygen at 1atm	1.429 kg/m^3
ρ_b	the density of dragon's bone	1,230 kg/m^3
ρ_{p0}	the distribution density of cattle	9.4514 /km^2
ρ_{p1}	the distribution density of sheep	3.4130 /km^2
ρ_m	the density of dragon's muscle	1,120 kg/m^3
E_{p0}	the energy of 1kg cow	1,250 kcal/kg
E_{p1}	the energy of 1kg sheep	1,180kcal/kg
m_{p0}	the weight of a cow	753 kg
m_{p1}	the weight of a sheep	87.5 kg
η	the absorbing coefficient for prey	0.7
ξ	the trauma coefficient	0.1
c_{p0}	the heat capacity of cattle	1.47 kJ/(kg·°C)
c_{p1}	the heat capacity of sheep	1.50 kJ/(kg·°C)
T_r	the reference temperature	25 °C
T_e	the temperature of environment	25 °C
T_p	the temperature of prey able to eat	80 °C

Growth Curve for $y(1) = 30$ and $y(10) = 35$

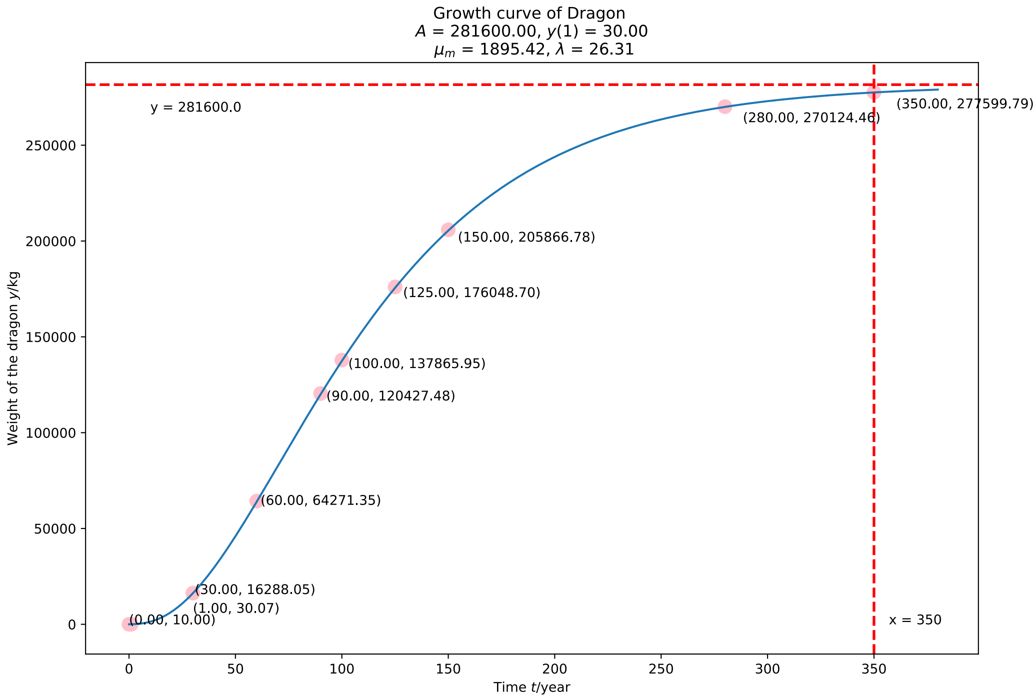

Figure 8.1: Groth curve of a dragon when $y(1) = 30$

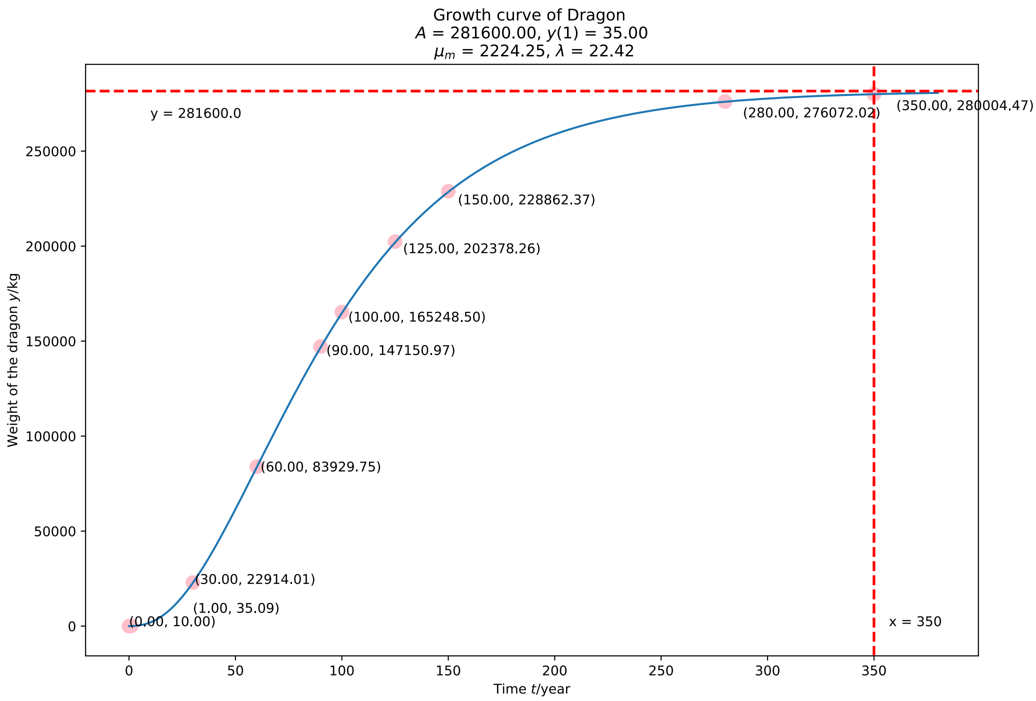

Figure 8.2: Groth curve of a dragon when $y(1) = 35$

Growth Curve Function

Table 8.2: Growth curve function

$y(1)$	$y(t)$	$y'(t)$
30	$2181600 \times (1 - 0.967e^{-0.0151t})^3$	$12373.57 \times (1 - 0.967e^{-0.0151t})^2 \, e^{-0.015t}$
35	$2181600 \times (1 - 0.967e^{-0.0178t})^3$	$14520.21 \times (1 - 0.967e^{-0.0178t})^2 \, e^{-0.0178t}$
40	$2181600 \times (1 - 0.967e^{-0.0202t})^3$	$16475.90 \times (1 - 0.967e^{-0.0202t})^2 \, e^{-0.0202t}$

Appendix B

Source Code of Dragon Hunting Algorithm

```python
# Name: Dragon Hunting Algorithm
# Author: Team 1911426 at MCM contest
# Time: 2019.1.28

# ### Important ###
# Units for functions parameters and global variables standard:
# kg for weight, calories for energy, km for area,  days for time

import numpy as np

# ##### Begin Global variables #####
# Species names
SPECIES_NAME = [
        ['Cattle', 'Sheep', 'Hare'],
        ['D', 'E', 'F'],
        ['G', 'H', 'J'],
]
# MASS of species in kg
MASS = [
        [753, 87.5, 3.94625],
        [0, 0, 0],
        [0, 0, 0]
]
# DENSITY of species in /km^2
DENSITY = [
        [3.4130, 9.4514, 0],
        [0, 0, 0],
        [0, 0, 0]
]
# ENERGY of species per MASS in calorie/kg
ENERGY_PER_MASS = [
        [1250000, 1180000, 1020000],
        [0, 0, 0],
        [0, 0, 0]
]
# Heat capacity of the meat of the species in calories/(kg * Celsius)
HEAT_CAPACITY = [
        [351.33843212, 358.50860421, 389.5793499],
        [0, 0, 0],
        [0, 0, 0]
]

# The side length of the square area
SIDE_LENGTH = 10.0
# The area of the square area
```

```
    AREA = SIDE_LENGTH ** 2

    # Times of hunting
    HUNTIMG_TIMES = 0

    # Number of species
    NUM_OF_SPECIES_PER_SPECIES = np.array(AREA * np.array(DENSITY),
 ↪  dtype=int)
    NUM_OF_SPECIES = np.sum(NUM_OF_SPECIES_PER_SPECIES)

    # Species used in our
    COW = SPECIES_NAME[0][0]
    SHEEP = SPECIES_NAME[0][1]
    # # Comment it out if you want to add more species
    # HARE = SPECIES_NAME[0][2]

    # Time period of dragon hunting
    DAYS = 2

    # The dragon's initial position in the area
    DRAGON_POS = np.array([
            [0],
            [0]
    ])
    # Not reachable area's position
    NOT_REACHABLE = np.inf

    # Net energy percentage
    NET_ENERGY_PERCENTAGE = 0.57
    # eta, see the paper
    ETA = 0.7
    # ##### End Global variables #####


    # ##### Begin Helper functions #####
    def get_mu_and_weight_at(age):
            """
            Get mu and weight at `age` using calculated S curve
 ↪  function.
            """
            mu_m = 2523.8311119211758
            lam = 19.76261573148329
            v = - 1/3
            A = 281.6 * 1000

            from sympy import symbols, exp

            t = symbols('t')

            temp1 = (mu_m / A) * ((1+v) ** (1 + 1/v)) * (lam - t)
            temp2 = exp(temp1)
```

```
        temp3 = v * exp(1 + v) * temp2
        temp4 = A * (1 + temp3)**(-1/v)

        y = temp4
        y_derivative = y.diff(t)

        return (y_derivative.subs(t, age), y.subs(t, age))

    def find_nearest(array, value):
        """
        Find the nearest element to `value`, return its index in `array`.
        """
        array = np.asarray(array)
        new_array = array - value
        norm_array = np.empty(len(new_array[0]))
        for i in range(len(new_array[0])):
            norm_array[i] = new_array[0][i] ** 2 + new_array[1][i] ** 2
        idx = norm_array.argmin()
        return idx

    def get_basic_metabolish_energy(weight):
        """
        Calculate E_m.
        """
        m_d = weight
        V_E = 2.25
        period = DAYS * 24
        V_O2 = m_d * V_E * period
        density_o2 = 1.429
        m_O2 = V_O2 * density_o2
        M_O2 = 32
        n_O2 = m_O2 / M_O2
        n_glucuse = n_O2 / 6
        energy = 277485.66 * n_glucuse
        return energy
    def get_growth_energy(mu):
        """
        Calculate E_g.
        """
        period = DAYS * 24
        dmd = mu / 365 / 24 * period
        rho_m = 1.12
        rho_b = 1.23
        r_b = 4
        r_m = 5
        coefficient_1 = rho_m + rho_b * (r_b ** 2) / (r_m ** 2)
        coefficient_2 = (r_b ** 2) / (r_b ** 2 + r_m ** 2)
        dS = dmd / coefficient_1 / coefficient_2

```

```
        E_p = 17130 * 1000 / 4.184
        E_b = 0.1 * E_p
        coefficient_3 = rho_m * (r_m ** 2) / (r_b ** 2 + r_m ** 2) * E_p
        coefficient_4 = rho_b * coefficient_2 * E_b
        E_g = (coefficient_3 + coefficient_4) * dS
        return E_g
    def get_fly_energy(weight, distance):
        """
        Calculate E_f.
        """
        m_d = weight
        # v_d is in m/s
        v_d = 5.70 * (m_d ** 0.16)
        # convert to m
        L_d = distance * 1000
        # convert to hours
        temp_time = L_d / v_d / 60 / 60
        E_v = 300 / 4.184
        E_f = m_d * E_v * temp_time
        return E_f
    def get_fire_energy(weight, x, y):
        """
        Calculate E_b.
        """
        c_p = HEAT_CAPACITY[x][y]
        m_p = MASS[x][y]
        constant = 5
        delta_T = 80 - 25
        return c_p * m_p * constant * delta_T

    def get_reproduction_res(index, now, period):
        """
        Index: 0-cattle,1-sheep,2-hare
        period: in days
        """
        per_day_animal = np.array([0.5, 4, 6]) / 365

        return int(now + now * per_day_animal[index] * period)

    def get_pos(idx):
        accr = np.cumsum(NUM_OF_SPECIES_PER_SPECIES)
        # Calculate species class from index in `total` (all the species)
        index1 = 0
        for i in range(len(accr)):
            if idx < accr[i]:
                index1 = i
                break
        x = index1 // 3
        y = index1 % 3
```

```python
        return (x, y)
    # ##### End Helper functions #####

    def hunting_at_age(age):
        """
        Main entraince of Dragon hunting algorithm.
        """
        global HUNTIMG_TIMES
        print(f'############### Hunting times: {HUNTIMG_TIMES} at age {age} ###############')
        HUNTIMG_TIMES += 1

        (mu, weight) = get_mu_and_weight_at(age)

        # Regenerate animals
        global NUM_OF_SPECIES_PER_SPECIES
        cow = np.random.rand(2, NUM_OF_SPECIES_PER_SPECIES[0][0]) * SIDE_LENGTH
        sheep = np.random.rand(2, NUM_OF_SPECIES_PER_SPECIES[0][1]) * SIDE_LENGTH
        # # Comment it out if you want to add more species
        # hare = np.random.rand(2, NUM_OF_SPECIES_PER_SPECIES[0][2]) * SIDE_LENGTH

        # Recovery dragon hunting animals
        total = np.append(cow, sheep, axis=1)
        # # Comment it out if you want to add more species
        # total = np.append(
        #     np.append(cow, sheep, axis=1),
        #     hare,
        #     axis=1
        # )

        # Generate dragon
        dragon_pos = DRAGON_POS

        energy_got = 0
        base_consumption = get_basic_metabolish_energy(weight)
        print('Base consumption:', base_consumption)
        growth_consumption = get_growth_energy(mu)
        print('Growth consumption:', growth_consumption)
        hurt_consumption = 0.1 * base_consumption
        print('Hurt consumption:', hurt_consumption)
        fly_consumption = 0
        fire_cos = 0
        energy_consumed = base_consumption + growth_consumption + hurt_consumption

        # Begin iteration
        iter_times = 0
        hunted_number = 0
```

```
        hunted_each = np.array([
            [0, 0, 0],
            [0, 0, 0],
            [0, 0, 0]
        ])
        global NUM_OF_SPECIES
        while hunted_number < NUM_OF_SPECIES:
            iter_times += 1
            print('======================')
            print(f'Iteration: {iter_times}')

            idx = find_nearest(total, dragon_pos)
            (x, y) = get_pos(idx)

            # ######### Begin hunting ##########
            hunted_number += 1
            hunted_each[x][y] += 1

            energy_got += ENERGY_PER_MASS[x][y] * MASS[x][y]

            temp_fire_energy = get_fire_energy(weight, x, y)
            fire_cos += temp_fire_energy
            energy_consumed += temp_fire_energy
            temp_fly_energy = get_fly_energy(
                weight, np.linalg.norm(dragon_pos-np.array([total[:,idx]]).T))
            fly_consumption += temp_fly_energy
            energy_consumed += temp_fly_energy

            print('Delta Energy this round: ',
                  ENERGY_PER_MASS[x][y] * MASS[x][y] - temp_fire_energy - temp_fly_energy)

            print(f'Nearest point {SPECIES_NAME[x][y]} at ({total[0][idx]}, {total[1][idx]}) hunted')
            dragon_pos = np.array([total[:, idx]]).T
            total[0][idx] = total[1][idx] = NOT_REACHABLE
            # ######### End hunting ##########

            if energy_got * NET_ENERGY_PERCENTAGE * ETA >= energy_consumed:
                print('Success got all energy')
                print('Animals before:\n', NUM_OF_SPECIES_PER_SPECIES)
                print('Animals hunted:\n', hunted_each)
                NUM_OF_SPECIES_PER_SPECIES -= hunted_each
                print('Animals left:\n', NUM_OF_SPECIES_PER_SPECIES)
                # Breeding Animals
                for i in range(3):
                    NUM_OF_SPECIES_PER_SPECIES[0][i]
```

```
= get_reproduction_res(i, NUM_OF_SPECIES_PER_SPECIES[0][i], DAYS)
                    print('Animals after reproduction:\n', NUM_OF_SPECIES_PER_SPECIES)
                    NUM_OF_SPECIES = np.sum(NUM_OF_SPECIES_PER_SPECIES)

            else:
                    print('energy_got * NET_ENERGY_PERCENTAGE * ETA', energy_got * NET_ENERGY_PERCENTAGE * ETA)
                    print('energy_consumed:', energy_consumed)
                    print('Still need these calories of energy:', energy_consumed - energy_got * NET_ENERGY_PERCENTAGE * ETA)
                    print('Energy got:', energy_got)
                    print('Fire energy need:', temp_fire_energy)
                    print('Fly energy need:', temp_fly_energy)

        print(hunted_each)
        print('##################################################################')
        return hunted_each

if __name__ == "__main__":
        hunting_at_age(100)
```