Calculus (MS0262) Seminar Sheet 18

(1) Find partial derivatives of the following functions:

(a) $z = \dfrac{x}{\sqrt{x^2+y^2}}$ (b) $z = x^y$

(c) $z = e^{\sin\frac{y}{x}}$ (d) $u = (xy)^z$

(2) Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\mathrm{d}z}{\mathrm{d}x}$ if $z = \arctan\dfrac{y}{x}$ and $y = x^2$.

(3) Show that the function $\omega = f(u,v)$, where $u = x + at$ and $v = y + bt$, satisfies the equation

$$\frac{\partial \omega}{\partial t} = a\frac{\partial \omega}{\partial x} + b\frac{\partial \omega}{\partial y}\,.$$

(4) Show that the function $z = y\phi(x^2 - y^2)$ satisfies the equation

$$\frac{1}{x}\frac{\partial z}{\partial x} + \frac{1}{y}\frac{\partial z}{\partial y} = \frac{z}{y^2}\,.$$

(5) Show that the function $z = e^y\,\phi\left(ye^{\frac{x^2}{2y^2}}\right)$ satisfies the equation

$$(x^2 - y^2)\frac{\partial z}{\partial x} + xy\frac{\partial z}{\partial y} = xyz\,.$$

Answers

(1)

(a) $z'_x = \dfrac{y^2}{(x^2+y^2)^{3/2}}\,, \quad z'_y = -\dfrac{xy}{(x^2+y^2)^{3/2}}$

(b) $z'_x = yx^{y-1}\,, \quad z'_y = x^y \ln x$

(c) $z'_x = -\dfrac{y}{x^2} e^{\sin\frac{y}{x}} \cos\dfrac{y}{x}\,, \quad z'_y = \dfrac{1}{x} e^{\sin\frac{y}{x}} \cos\dfrac{y}{x}$

(d) $u'_x = yz(xy)^{z-1}\,, \quad u'_y = xz(xy)^{z-1}\,, \quad u'_z = (xy)^z \ln(xy)$

(2)

$$\frac{\partial z}{\partial x} = -\frac{y}{x^2+y^2}\,, \quad \frac{\mathrm{d}z}{\mathrm{d}x} = \frac{1}{1+x^2}$$