

SpacePhaser: Phase Space Embedding Visual Analytics

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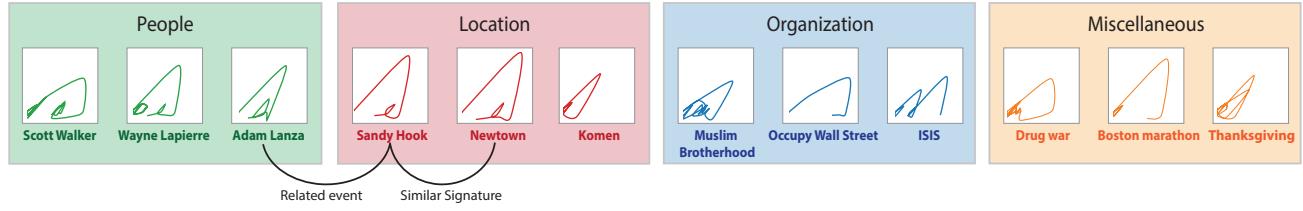


Fig. 1. *SpacePhaser* visualization for the *Huffington Post*. Terms are color-coded by category: *person*, *location*, *organization*, and *others*.

Abstract—In the paper, we focus on the statistical detection and understanding of signaling behavior via various media outlets that are associated with a violent hate crime. Following a body of theory that identifies the links between hate rhetoric with violence, we explore textual patterns of hate speech in traditional and non-traditional media outlets, including television news shows, social media, political blogs, and political forums, to detect an association with hate crime events per time and place. Applying the dynamical system theory, we can characterize the dynamic behaviors of essential keywords. Grouping keyword with similar temporal signatures enables us to highlight important information/topics corresponding to emerging events. We also introduce *SpacePhaser*, an interactive visual tool for textual pattern analysis. To show the usefulness of *SpacePhaser*, we demonstrate the results of its applications on various real-world datasets.

Index Terms—Phase Space, Dynamical System Theory, Time Series Visualization, Visual Analytics, Dynamic Graphs, Social Media, Entity Recognition, Topic Modeling

I. INTRODUCTION

Over the last couple of years, we have suffered from a significant rise in hate crime, with many of those instances, parallel domestic terrorism and involving mass casualties, for instance, the Pittsburgh synagogue shooting, the Charleston church shooting, or the Orlando nightclub shooting. While relevant for the current American discourse and setting, this trend has more far-reaching implications outside of the United States as well. Political signaling that triggers violent action is a common practice in numerous countries and can lead to dramatic results, from voters' violent suppression during election season to ethnocide.

In this paper, we propose an approach to extract *textual* and *temporal* aspects of the media outlets with the hate crime data,

harnessing the mathematical methods of phase space [1], [2] and attractor reconstruction [3], [4] from the dynamical system theory and text analysis [5], [6] for quickly perceiving the most prominent concept streams and emerging development trends in the temporally annotated textual datasets. We introduce *SpacePhaser*, an interactive visual tool for textual pattern analysis to fulfill the following tasks:

- **Task 1.** To develop an approach for discovery of topics, considering temporal proximity of timestamps, as well as *proximity of terms and topics* extracted by using natural language processing and structured prediction methods [7], [8].
- **Task 2.** To design and implement an architecture embedding of the textual time series into phase space of a dynamical system associated with the diverse and often disparate data sets.

While focusing on this study on the links between hate speech and hate crime, we believe the same process, with proper adjustments, can be applied to other settings, including political mobilization and other application domains. We will highlight this extension in the last use case in Section V.

The remainder of the paper is organized as follows: we first describe the background and related research in this direction. We explain the design motivations of this project in Section III. We then present the project roadmap and give the detailed exposition of each proposed step concluding with the outcome. Then, we offer the use cases of our *SpacePhaser*. Finally, we conclude the paper with future work.

II. RELATED WORK

A. Phase space models

Phase space [2] is used to describe the orbit of one particle. It can even be used to describe a large number of collections of N particles, where N itself is a large number. In other words, phase space can be used to describe the probability distributions of how collections of N particles behave if they are allowed to exchange particles and energy with the universe.

We apply these concepts into multidimensional data analysis. In particular, every degree of freedom is represented as an axis of a multidimensional space. For every observable state of the system, a point is included in the multidimensional space, so that the system's evolving state over time traces a phase space trajectory through the high-dimensional space. The shape of phase space trajectories can easily *elucidate qualities* of the system that might not be obvious otherwise. Moreover, the *Takens embedding theorem* [9], [10] allows us to unfold the *attractor* of a data dynamical system from observations of a single variable at different sampling times. We view the construction of vectors for attractor reconstruction as a problem in finding a *good coordinate system* to represent the data. In the delay embedding theorem [1], Takens had shown that a delayed signal $s(t + \tau)$ is functionally independent of $s(t)$, so that the delay coordinates is the natural choice for an embedding or attractor reconstruction [3], [4]. A reconstructed attractor is guaranteed to be equivalent (up to some unknown diffeomorphism) to the actual one, provided that the number of delay coordinates exceeds $d \geq 2D + 1$ (where D stands for the dimension of the attractor).

For example, a dynamic word cloud (of words w_1, \dots, w_M) is described by a multivariate time series $\{s_w(t)|w = 1, \dots, M\}$ sampled simultaneously at equally-spaced intervals, $t = 1, \dots, N$. The embedded vector (of d dimensions), $\mathbf{v}_{w_1, \dots, w_d}(t) = (s_{w_1}(t + \tau_1), s_{w_2}(t + \tau_2), \dots, s_{w_d}(t + \tau_d))$, where the $w_k \in \{1, \dots, M\}$ are various choices from the multivariate data and, in general, each τ_k is different for each component. To decide if we need to add another component to $\mathbf{v}(t)$, i.e. increase the dimension of the embedding space, we must test whether the new candidate component, $s_{w_{d+1}}(t + \tau_{d+1})$ is functionally independent of the previous d components. In this paper, we develop a statistical approach to phase space embedding to determine the coordinate system to represent the dynamic of texts in a corpus.

B. Topic Modeling

The early study of topic modeling is Latent Semantic Analysis (LSA), a theory and method for extracting and representing the contextual-usage meaning of words by statistical computations from encountering large samples of language. The underlying idea is that the aggregate of all the word contexts in which a given word does and does not appear provides a set of mutual constraints that largely determines the similarity of meaning of words and sets of words to each other [11].

In 2003, Blei et al. [8] proposed Latent Dirichlet Allocation (LDA), a flexible generative probabilistic model, which treats

documents as a random mixture of a certain number of topics, and categorizes the topics based on distribution over the words through a three-level hierarchical Bayesian model. LDA is very popular, as it can classify various kinds of documents in different areas, from financial topics modeling [12] to sentiment analysis [13]. After that, various LDA-based models developed. Dynamic Topic Model (DTM) is an extension of LDA that captures the evolution of topics over time in a sequentially organized corpus of documents [14]. Wang et al. [15] extended LDA into Spatial Latent Dirichlet Allocation (SLDA), which encodes spatial structures among visual words into the same topic. Iwata et al. [16] propose a dynamic topic modeling that works with multiple timescales. Hierarchical LDA builds a hierarchical topic model by combining the prior with the likelihood that is based on a hierarchical variant of latent Dirichlet allocation [17]. Structural Topic Model (STM) accommodates corpus structure through document-level covariates affecting the topical prevalence and topical content. The central idea is to specify the priors as generalized linear models through which people can estimate arbitrary observed data [18]. SparseLDA [19] is designed for faster sampling of topic modeling with utilizing less memory than traditional LDA-based systems.

Topic modeling enables us to summarize the large text corpora, and hence it is highly used for the visualization of the large data in a time efficient manner. In this paper, we apply the physics dynamical system theory to characterize the dynamic behaviors of important terms in text corpus and create a network of similar words (similar temporal behaviors) to highlight essential topics corresponding to emerging events.

C. Text Visualizations

Word clouds, in which font size is determined by the incidence of the words, provide a concise way to summarize the content of websites or text documents. First, implementations of word clouds include filtering out common words. Otherwise, the large font size of common words would attract more user attention than small tags [20]. Second, the further content-specific text parsing is required to identify a set of representative keywords, since the information-theoretic significance of a word w given by $-\log p(w)$ where $p(w)$ is the probability of finding it in the text [21] would be obviously small for the significant font size words and vice versa. This representation *contrariety between frequency and significance* of words makes it difficult to use word clouds for portraying temporal content evolution of a set of documents and deriving insights from a large collection of word clouds over time. Indeed, the clouds of representative keywords derived at different time stamps from the temporally ordered sets of documents vary one from another, lacking semantic coherence and spatial stability. In the proposed research effort, we develop an innovative approach that can substantially improve upon current analytic and visual tools in bringing insights into a variety of application domains.

TIARA [22] includes keyword clouds embedded in the layers of a stacked graph, whose layers depict the different top-

ics. *TextFlow* [23] demonstrates evolutions and relationships among topics and their critical events: birth, death, split, merge. *Morphable Word Cloud* [24] specifies a sequence of shapes as boundaries of word clouds at each time step. These intermediate shapes of the sequence could also be automatically generated using interpolation from the specified first and the last shape. This is a series of the separated word clouds at different time steps with shapes as boundaries and still contains spaces between shapes. *Parallel Tag Clouds* [25] utilize the parallel coordinates to represent time constraint. At each time step terms are placed in alphabetical order or order of importance, based on term frequency. This technique also has a feature that is implemented in *SpacePhaser*, which is to display a stream when a term is selected. However, *Parallel Tag Clouds* is not space-efficient and cannot show the topic or overall evolution across time.

III. DESIGN DECISIONS FOR THE *SpacePhaser* VISUALIZATION

Inspired by the idea of *Phase space*, we apply this dynamical system theory [26] into dynamic behaviors of topics in the text corpus. In the analysis of complex multiple time series, we reconstruct a source of observed temporal series of data patches (e.g. *word clouds*, the *weighted lists* of words allocated accordingly their frequency and centrality). Phase space is a concept which unifies geometry with the analysis of system dynamics, and therefore can naturally be applied to the analysis of the dynamic behaviors of topics in the text corpus. For example, a tag in a cloud can be characterized by its usage frequency in a given day and by the daily rate of frequency change, so that time-series flow of tags embedded into a *phase plane* may give qualitative information about the topic dynamics of interest in real time. As the topic evolves, the tags would follow different trajectories on the phase diagram revealing the stable and unstable components of the topic. However, the estimation of derivatives is very sensitive to experimental noise, which precludes its use in this context.

IV. *SpacePhaser* ROADMAP

Following a body of theory that identifies the links between hate rhetoric with violence, we analyze dynamic patterns of the hate keywords from media outlets, such political forums, to highlight the association with hate crime events per time and place. Figure 2 shows the major stages in our project roadmap:

A. Data collecting and cleaning

Data collection process use hate speech taxonomy (such as shooting, bombing, and burning), obtaining data from various sources such as news outlets, social media platforms, and political blogs. As data comes from heterogeneous sources often in ungrammatical format, data cleaning is performed at the first step. Then, the input text are preprocessed into entities and further classified into different categories [7], such as people name, location, organization, time, and number. In many

cases, the term frequency might not convey user interests [27]. For example, the term “Trump” repeated numerous times in political blogs and news might not draw a lot of attention or interest [28]. To focus on the more significant terms, we use the *sudden change* measure (or delta frequency), referring to a sharp increase in frequency [29]. Let F_1, F_2, \dots, F_n be the frequency of an entity at n different time points. The sudden attention series (S_1, S_2, \dots, S_n) is computed by $S_t = \frac{(F_t+1)}{(F_{t-1}+1)}$. The output of this step is the list of significant terms associated to each timestamp.

B. Data processing and mining

The collected data is then divided into different categories [7]: person, location, organization, time, and number. Other natural language processing methods [8], [30] can be applied in combination to augmented *wisdom of crowds* on top of the processed data. The different categories are associated to different topic streams in our dynamic word cloud visualization as shown in the last panel of Figure 2.

C. Data visualization

For studying the *phase portrait* of the data set (representing the *streams of concepts* rather than words), we proceed to construct a complex network by considering each vector point in reconstructed phase space as a basic node of a *phase space network* [26], [31], [32] and using the phase space distance to determine network connection. Many methods for transforming time series into complex networks have been designed, such as cycle networks [33], recurrence networks [34], correlation networks [35], [36], and visibility graphs [37]. Our network connects terms with similar dynamic signature representing the evolution of emerging latent topics within the text corpus.

V. APPLICATIONS

To demonstrate the usefulness of *SpacePhaser*, two real-world datasets were used. The first dataset is the *Huffington Posts* with political posts collected from late 2010 until late 2014 and the second one is the real-time health status of high-performance computing systems.

A. Exploring the *Huffington Posts*

The *Huffington* dataset contains a total of 370,000 elements with 3,500 terms at most during one month in 2008. The blogs are preprocessed into entities, ranked by frequencies, and further classified into different categories [7]: people, places, organizations, and miscellaneous. Figure 3(a) shows the phase space graphs of all extracted terms plotted in a single view. The *x-axis* is term frequency while *y-axis* show its momentum (or delta temperature). A curve connects the 2D values of terms at different timestamps and color-coded by the term category. Attractor (average phase space) is the black curve in the middle of each plot. The circular patterns of curves confirm the dynamical system theory [38].

Figure 3(b) shows the network for phase space where links connect terms with similar dynamic signatures [39]. Users can

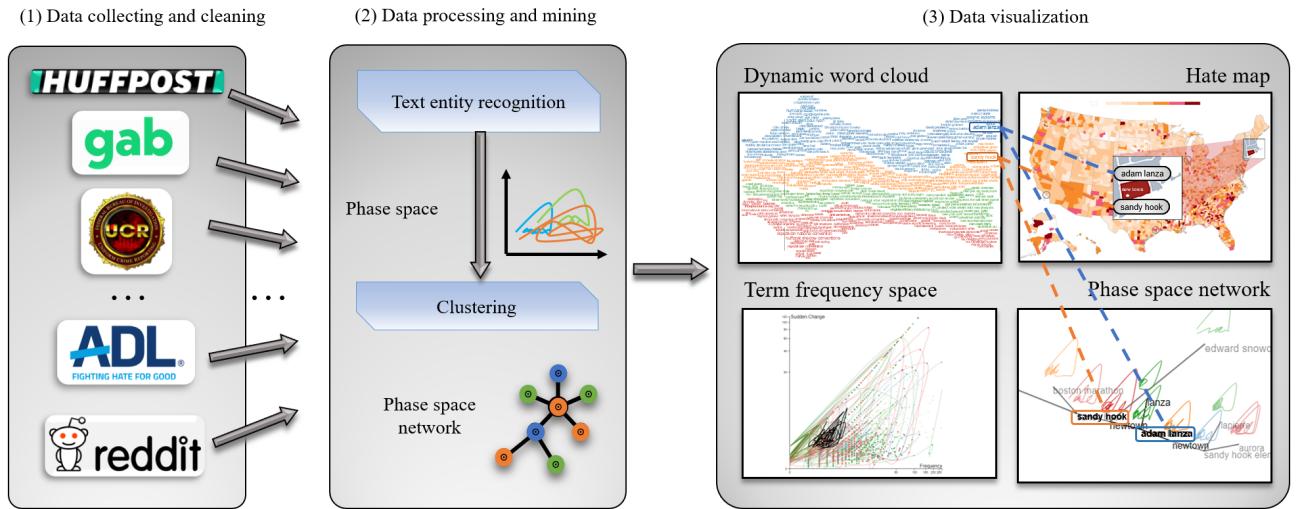


Fig. 2. A schematic overview of our project: (1) Data collecting (2) Data processing and (3) Data visualization.

adjust the similarity threshold (to define the network links) as well and the time period to refine into political events of a particular year. As depicted in Figure 3(b), we can quickly spot major political events in the past 10 years.

- **Region A:** The Sandy Hook Elementary School shooting occurred on December 14, 2012, in Newtown, Connecticut, when 20-year-old Adam Lanza fatally shot 20 children. Sometime before 9:30 am, Lanza shot and killed his mother Nancy Lanza at their Newtown home with a .22-caliber Savage rifle. As we enlarge their phase spaces in Figure 1, Adam Lanza, Sandy Hook, and Newtown have very similar signatures.
- **Region B:** The Shooting of Trayvon Martin. On the night of February 26, 2012, in Sanford, Florida, George Zimmerman fatally shot Trayvon Martin, a 17-year-old African American high school student. George Zimmerman and Trayvon Martin also have similar signatures. Notice that we take into account the timestamps when performing phase space comparison (visually similar signatures can have very different peak times).

In terms of users interaction, users can control the time span of displayed phase space by selecting the range of time series or explore specific details of a structure by hovering. When users drag and drop on the time axis, it will filter and redraw the phase spaces in range and update the corresponding network. By mouse over phase space, the corresponding data points in other time steps are highlighted and animated to appear sequentially by time step order.

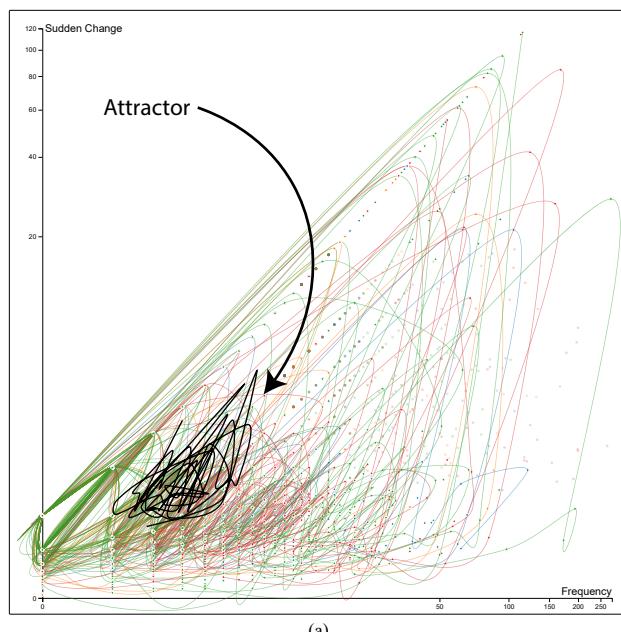
B. Monitoring High performing Computing Systems

Although originally focused on dynamic text analysis, we soon realized that *SpacePhaser* has more general applications. In the second use case of high-performance computing systems, the computer health status contains real-time data on several attributes, such as *CPU temperatures*, *Memory*

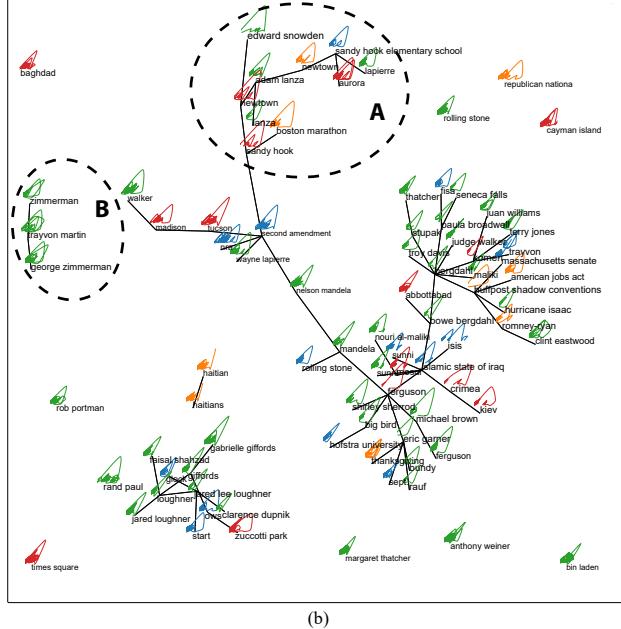
Usage, *Fan speed* and *Power consumption*. *SpacePhaser* can capture abnormal dynamic correlations between various health dimensions in the data center. Figure 4 illustrates *SpacePhaser* application to the health status of a data center at a university on September 26, 2018: The CPU on *Compute-3-13* suddenly became overheated. In this use case, the color scale can be changed basing on distance from attractor or mean-squared error, measuring the average difference between the two temperature series. In details, about 12 pm application was able to pick up the event and alerted system administrator to make CPU replacement for the malfunction CPU before it harms other neighboring CPUs. This abnormal event can be seen by the orange curve in Figure 4(a) and the isolated signature at the bottom of the network in Figure 4(b). As shown in Figure 4(c), fan speeds on other neighboring computers (*computer-3-10*, *computer-3-11*, and *computer-3-12*) had pumped the rate as they sensed the heat. These are outliers in the phase space network in Figure 4(d). Besides that, we can easily notice some isolated clusters with identical signatures.

We conduct informal user studies to gather qualitative responses about *SpacePhaser* from two experts, an expert from Dell Inc. and the high-performance computing center director. The study began with a brief description of *SpacePhaser*. Then the experts are free to use *SpacePhaser* and explore the visual tool before giving feedback. Both of them agree that the visualization is interesting and useful to convey the dynamic behaviors, and can be applied to various domains.

We discussed this thermal excursion through an informal interview with Dell experts. They value the diagnostics from our prototype and suggested thermal experts and hardware team to investigate this interesting correlation between CPU temperature and fan speed. The experts commented that “visual analytics provide an excellent opportunity to explore the correlation of hardware features” or “understanding the dynamic behaviors of health services is essential in our hardware



(a)



(b)

Fig. 3. *SpacePhaser* displays keywords in political blogs retrieved from the Huffington Post: (a) Phase Space graphs of all terms in a single view (b) Phase Space network of similar terms (representing similar dynamics). Curves (signatures) are colored by their text category. Attractors are in the middle of plot (a).

design process". *SpacePhaser* is currently being deployed with additional dimensions, such as real-time I/O bandwidth among other integrations in this on-going collaboration.

C. Implementation

Our visual interfaces are built within the web environment. Coordinated multiple views [40] is adapted to show a different

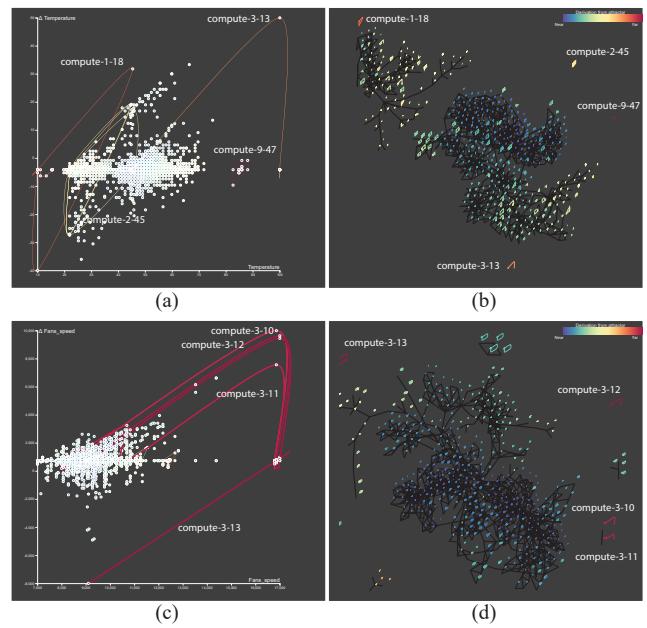


Fig. 4. Monitoring the health status of High-Performance Computing Systems at a university on September 26, 2018: (a) and (b) are for CPU temperature (c) and (d) for CPU fan speed. In this use case, the phase space curves of computers are colored based on how far they are from the attractor (the white curve in the middle): red for abnormal dynamics (strong variances) and blue for more stable computers.

perspective of dynamic topics. In particular, *SpacePhaser* is implemented using D3.js [41]. The demo video, online demo, more examples, and source code of the visualization can be found at the Github page <https://vixlab.github.io/SpacePhaser/>.

VI. CONCLUSION

This paper presents *SpacePhaser* an interactive visual prototype for textual pattern analysis based on the dynamical system theory. It aims to characterize the extracted entities from a text corpus, compare their dynamic behaviors, and group similar entities in a force-directed layout to formalize topic space. Within the phase space network, every vertex is a dynamic signature of popular terms which are interconnected based on their visual similarity and color-coded based on their classification: person name, location, organization, or miscellaneous. While initially focusing text analysis for social media, we present *SpacePhaser* applications on other application domains, such as monitoring the CPU temperature in a high-performance computing center.

Future work will focus on predictive analysis: building a multiscale hierarchical physical model based on scale-dependent anisotropic random walks [42] to incorporate into generative codes. This generative approach will be used to detect emerging concept streams. We also plan to conduct a formal study to validate our approach on different populations of users from various application domains

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