レポート課題

10.1 $E = 100\angle 0^\circ\,[V]$、$Z_1 = 40[\Omega]$、$Z_2 = -j10[\Omega]$、$Z_3 = j20[\Omega]$、$Z_4 = 10[\Omega]$

図 10.13から

$I_1 = I_2 + I_3 \Rightarrow I_1 - I_2 - I_3 = 0$

$S_a: I_1 Z_1 + Z_2 I_2 = E$

$S_b: I_2 Z_2 - (Z_3 + Z_4) I_3 = 0$

$$\begin{bmatrix} 1 & -1 & -1 \\ Z_1 & Z_2 & 0 \\ 0 & Z_2 & -(Z_3+Z_4) \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ E \\ 0 \end{bmatrix}$$ と書けます。

$$\begin{bmatrix} 1 & -1 & -1 \\ 40 & j10 & 0 \\ 0 & -j10 & -(j20+10) \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 0 \end{bmatrix}$$

左辺の det は

$\Delta = -200 + j100 - 40[j20 + 10 - j10] = -200 + j100 - j400 - 400$

$\Delta = -600 - j300$

$$I_1 = \frac{1}{-600-j300}\begin{vmatrix} 0 & -1 & -1 \\ 100 & -j10 & 0 \\ 0 & -j10 & -(j20+10) \end{vmatrix} = \frac{1}{-600-j300}[-100(j20+10-j10)]$$

$$I_1 = \frac{j1000 + 1000}{600 + j300} = \frac{j10 + 10}{6 + j3} = \frac{j20+40}{45} = \frac{j2+6}{3} = 2 + j0.667$$

$$I_2 = \frac{1}{-600-j300}\begin{vmatrix} 1 & 0 & -1 \\ 40 & 100 & 0 \\ 0 & 0 & -(j20+10) \end{vmatrix} = -\frac{1}{600+j300}[100(-j20-10)]$$

$$I_2 = \frac{j2000 + 1000}{600 + j300} = \frac{10 + j20}{6 + j3} = \frac{120 + j90}{45} = \frac{8+j6}{3} = 2.667 + j2$$

$$I_3 = \frac{1}{-600-j300}\begin{vmatrix} 1 & -1 & 0 \\ 40 & -j10 & 100 \\ 0 & -j10 & 0 \end{vmatrix} = -\frac{1}{600+j300}[-100(-j10)]$$

$$I_3 = \frac{-j1000}{600 + j300} = \frac{-j10}{6 + j3} = -\frac{30+j60}{45} = -0.667 - j1.333$$

10.2

$$(Z_1+Z_2)I_a + Z_2 I_b = E$$

$$Z_2 I_a + (Z_2+Z_3+Z_4) I_b = 0$$

$$\begin{bmatrix} Z_1+Z_2 & Z_2 \\ Z_2 & Z_2+Z_3+Z_4 \end{bmatrix}\begin{bmatrix} I_a \\ I_b \end{bmatrix} = \begin{bmatrix} E \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 40-j10 & -j10 \\ -j10 & 10+j10 \end{bmatrix}\begin{bmatrix} I_a \\ I_b \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 40-j10 & -j10 \\ -j10 & 10+j10 \end{vmatrix} = 400+j300+100+100 = 600+j300$$

$$I_a = \frac{1}{600+j300}\begin{vmatrix} 100 & -j10 \\ 0 & 10+j10 \end{vmatrix} = \frac{1}{600+j300}(1000+j1000) = \frac{10+j10}{6+j3}$$

$$I_a = \frac{90+j30}{45} = 2+j0.667$$

$$I_b = \frac{1}{600+j300}\begin{vmatrix} 40-j10 & 100 \\ -j10 & 0 \end{vmatrix} = \frac{j1000}{600+j300} = \frac{j10}{6+j3} = 0.667+j1.333$$

$$I_1 = I_a = 2+j0.667$$

$$I_2 = I_a + I_b = 2.667+j2$$

$$I_3 = -I_b = -0.667-j1.333$$

10.3.

$Y_{BA} = \frac{1}{40} = 0.025[S]$, $Y_{AD} = \frac{1}{-j10} = j0.1[S]$, $Y_{AF} = \frac{1}{10+j20} = 0.02-j0.0$

点A, B, D, Fの電位を V_A, V_B, V_D, V_F とする。

ここで, $V_B = E$

$$I_1 = Y_{BA}(V_B - V_A) = 0.025(100 - V_A) = 2.5 - 0.025V_A \ [A]$$

$$I_2 = Y_{AD}(V_A - V_D) = j0.1 V_A \ [A]$$

$$I_3 = Y_{AF}(V_A - V_F) = (0.02 - j0.04) V_A \ [A]$$

また、図10.13から $I_1 = I_2 + I_3$ の式

$$2.5 - 0.025V_A = j0.1V_A + 0.02V_A - j0.04V_A$$

$$2.5 = V_A(0.045 + j0.06) \Rightarrow V_A = 20 - j26.667 \ [V]$$

$$I_1 = 2 + j0.667 \ [A]$$

$$I_2 = j0.1(20 - j26.667) = 2.667 + j2 \ [A]$$

$$I_3 = (0.02 - j0.04)(20 - j26.667) = -0.667 - j1.333 \ [A]$$

$$I_a = I_1 = 2 + j0.667 \ [A]$$

$$I_b = -I_3 = 0.667 + j1.333 \ [A]$$

10.4 $E_1 = 20\angle 0°\,[V]$、$E_2 = 20\angle 90°\,[V]$、$R_1 = 8[\Omega]$、$R_2 = 10[\Omega]$、$R_3 = 4[\Omega]$
$X_L = 6[\Omega]$、$X_C = 3[\Omega]$

I_a について
$S_a : (8+j6)I_a - 8I_b = 20\angle 0° = 20$
$S_b : (-8I_a + (8+10+4)I_b + 4I_c = 0$
$S_c : 4I_b + (4-j3)I_c = 20\angle 90° = j20$

$$\begin{bmatrix} 8+j6 & -8 & 0 \\ -8 & 22 & 4 \\ 0 & 4 & 4-j3 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \\ j20 \end{bmatrix}$$

$$\Delta = 8+j6[22(4-j3)-16] + 8[-8(4-j3)] = 716+j96$$

$$I_{R_2} = I_b = \frac{1}{716+j96} \begin{vmatrix} 8+j6 & 20 & 0 \\ -8 & 0 & 4 \\ 0 & j20 & 4-j3 \end{vmatrix}$$

$$I_{R_2} = \frac{1}{716+j96}[(8+j6)(-j80) + 8(20(4-j3))]$$

$$I_{R_2} = 1.33 - j1.74\,[A]$$

10.5 R_1、L_1 について $R_2 = 20[\Omega]$、$R_3 = 50[\Omega]$、$C_3 = 10[\mu F]$、$C_4 = 20[\mu F]$

$Z_1 = R_1 + j\omega L_1$、$Z_2 = R_2$、$Z_3 = R_3 + \frac{1}{j\omega C_3}$、$Z_4 = \frac{1}{j\omega C_4}$

$Z_1 = 20 + j\omega L_1[\Omega]$、$Z_2 = 20[\Omega]$、$Z_3 = 50 + \frac{1}{j\omega 10.10^{-6}}[\Omega]$、$Z_4 = \frac{1}{j\omega 20.10^{-6}}$

$Z_1 Z_4 = Z_2 Z_3$

$$(R_1 + j\omega L_1)\frac{1}{j\omega 20.10^{-6}} = 20\left[50 + \frac{1}{j\omega.10^{-5}}\right]$$

$$\frac{R_1 + j\omega L_1}{j\omega.10^{-5}.2} = 20\left[\frac{j50.\omega.10^{-5}+1}{j\omega.10^{-5}}\right]$$

$$R_1 + j\omega L_1 = j2\omega.10^{-2} + 40$$

両辺の実数部と虚部を等しくすると

$R_1 = 40[\Omega]$
$L_1 = 0.02[H]$