8.00

MODULE - 3

9.00

Regular Expression

10.00 **Regular Expression :-**

* A mathematical notation used to describe the regular language.

11.00

12.00 * It is formed by using 3 symbols.

1.00 i) Dot (•) Operator used for concatenation a.b
ii) Union Operator a+b
2.00 iii) Closure Operator a*

3.00

* In dot Operator we are performing concatenation.

4.00

5.00 * In union Operator the symbol used is '+'. It is used for finding atleast one occurance of the variable.

6.00

7.00 * In closure operator it is used for finding 0 or more occurance of the variable.

Notes

28 Saturday MARCH

FEBRUARY '20

S	M	T	W	T	F	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

2020

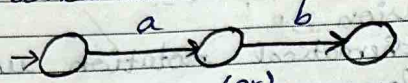
Wk 13 • 088-278

8.00 Converting the regular expression to ε-NFA by using Thomson Construction Method :-

9.00

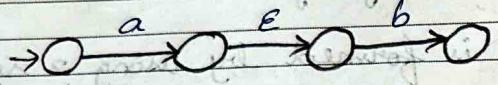
i) ab (Concatenation)

10.00



(or)

11.00

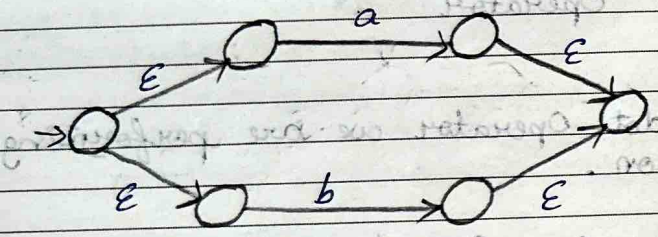


12.00

1.00

ii) a+b (Union)

2.00

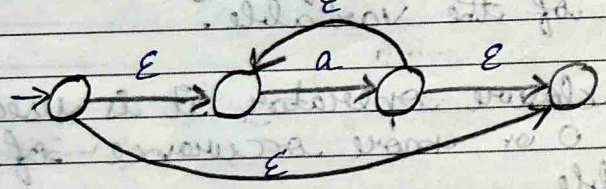


3.00

4.00

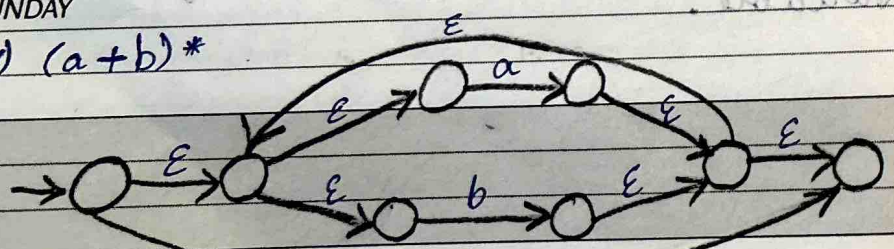
5.00

iii) a*



SUNDAY

iv) (a+b)*



2020

Wk 14 • 090-276

APRIL '20

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

Monday MARCH 30

8.00 : *Problem

9.00 1. Construct a regular ε-NFA for the given regular expression using thomson construction method.

10.00

i) a.b.(a+b)*

11.00

ii) (a+b)*.b

12.00

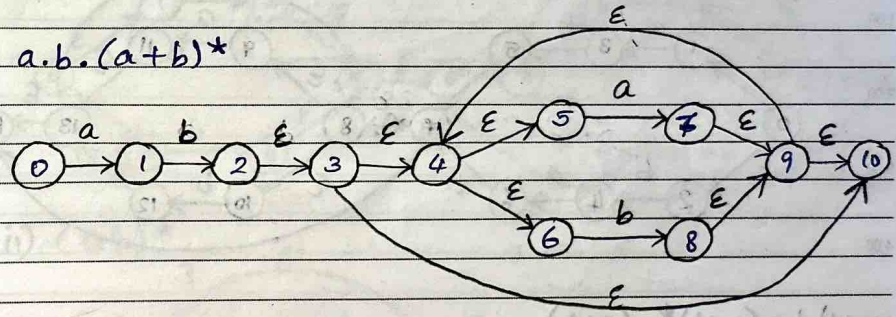
iii) (a+b)*.ab.(a+b)*

iv) (ab+ba).(a+b)*

1.00

i) a.b.(a+b)*

2.00



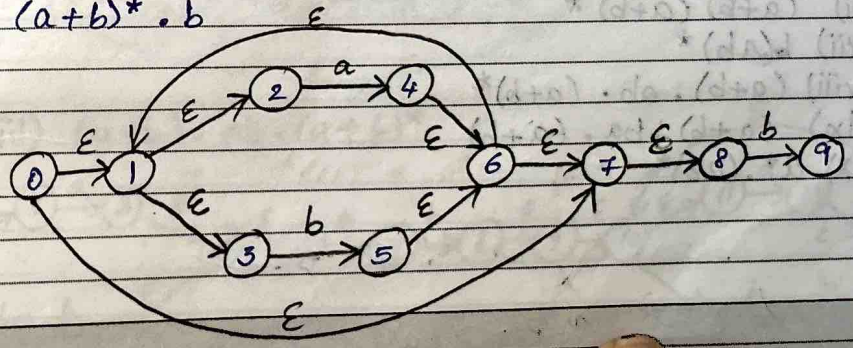
3.00

4.00

5.00

ii) (a+b)*.b

6.00



7.00

Notes

31 Tuesday MARCH

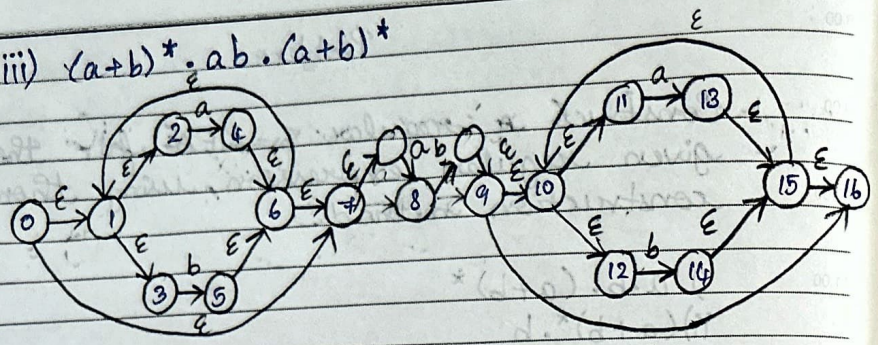
FEBRUARY '20

S	M	T	W	T	F	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

2020

Wk 14 • 091-275

8.00 iii) $(a+b)^* . ab . (a+b)^*$

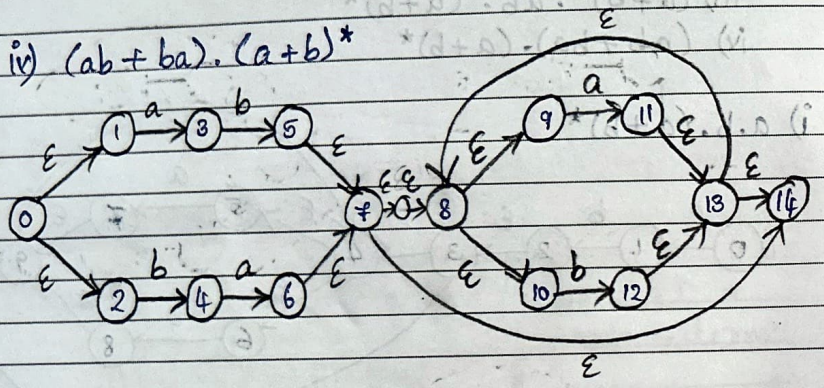


9.00

10.00

11.00

12.00 iv) $(ab + ba) . (a+b)^*$



1.00

2.00

3.00

4.00

5.00 v) $b.a(a+b)^* . (a+b)$

vi) $(a+b)(a+b)^*$

6.00 vii) $b(ab)^*$

viii) $(a+b) . ab . (a+b)^*$

7.00 ix) $(a+b) . ba . (a+b)$

x) $a^* . (ab)^*$

Notes

2020

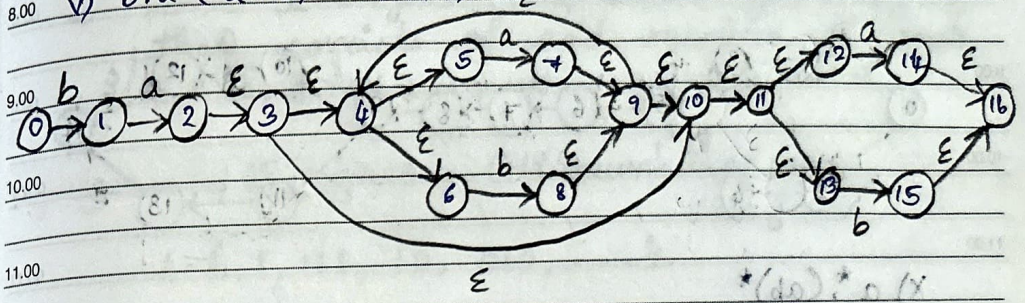
Wk 14 • 092-274

MAY '20

S	M	T	W	T	F	S
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

Wednesday APRIL 01

8.00 v) $b.a(a+b)^* . (a+b)$

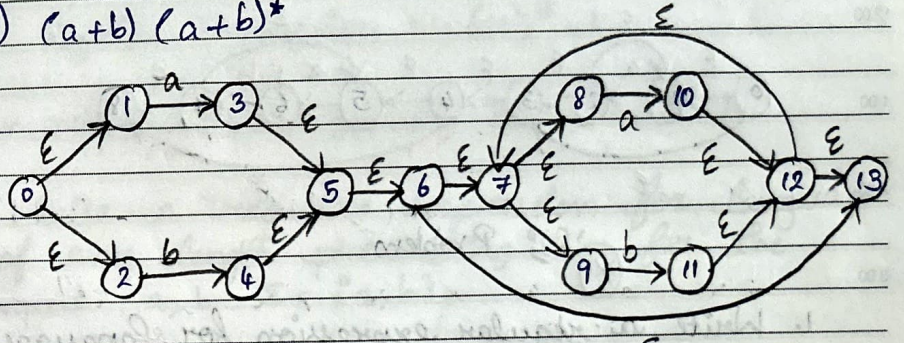


9.00

10.00

11.00

12.00 vi) $(a+b)(a+b)^*$

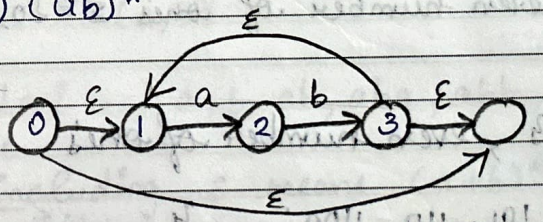


1.00

2.00

3.00

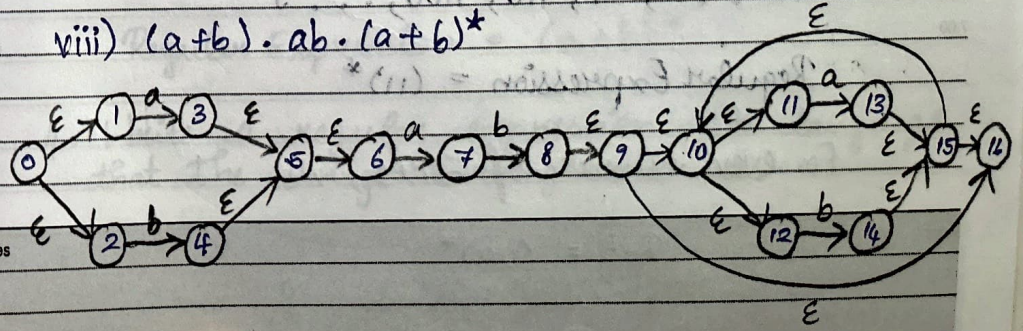
4.00 vii) $(ab)^*$



5.00

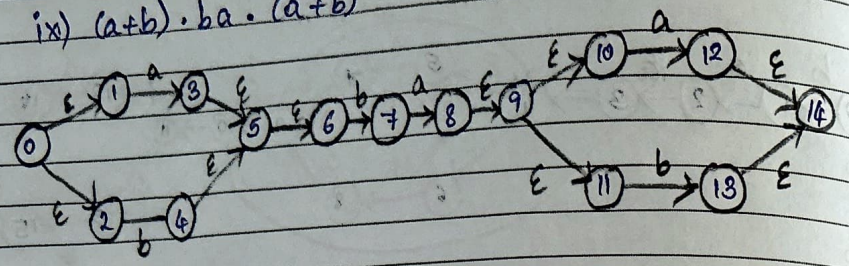
6.00

7.00 viii) $(a+b) . ab . (a+b)^*$



Notes

8.00 ix) $(a+b) \cdot ba \cdot (a+b)$



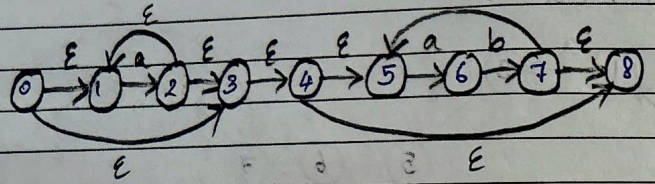
9.00

10.00

11.00

x) $a^*(ab)^*$

12.00



1.00

2.00

2nd Problem

3.00

1. Write a regular expression for language that consists of even number of one's $\Sigma = \{0, 1\}$ as input.

4.00

$\Sigma = \{0, 1\}$, even number of one's

5.00

$L = \{11, 101, 110, 1100, \ldots\}$

6.00

Regular Expression = $(11)^*$

.00

For even number of zeros,

es

R.E = $(00)^*$

8.00 2. Write a regular expression for language that consists of odd number of one's $\Sigma = \{0, 1\}$ as input...

9.00

$\Sigma = \{0, 1\}$, Odd number of ones.

10.00

$L = \{1, 111, 10, 010, \ldots\}$

11.00

Regular Expression = $(11)^* \cdot 1$

12.00

For odd number of zeros, RE = $(00)^* \cdot 1$

1.00

2.00 3. Write a regular expression for language of any length precluding 'ε' for the input a, b $\Sigma = \{a, b\}$.

3.00

$\Sigma = \{a, b\}$

4.00

$L = \{\ldots, a, b, ab, aba, abb, \ldots\}$

5.00

pre
including ε means $(a+b)^*$, without ε $(a+b)^+$.

6.00

Regular Expression = $(a+b)^*$

7.00

4. Write a regular expression for language that the string starting with a.

Notes

$\Sigma = \{a, b\}$, Starting with a.

8.00 $L = \{aa, ab, aaa, aab, \ldots\}$

9.00 Regular Expression = $a.(a+b)^*$

10.00 5. Write the regular expression for the language that contains a substring of a.a.

11.00

12.00 Substring is aa ; $\Sigma = \{a, b\}$

1.00 $L = \{baab, bbaab, aaab, \ldots\}$

2.00 Regular Expression = $(a+b)^* aa (a+b)^*$

3.00 6. Write the regular expression for the language that starts with either a or ab.

4.00

Starts with either a or ab.

5.00 $L = \{a, ab, aa, aba, abb, \ldots\}$

6.00 Regular Expression = $(a+ab).(a+b)^*$

a or ab

7.00

05 SUNDAY 7. Write the regular expression for the language with the string end with abb.
Input $\Sigma = \{a, b\}$.

Notes

String end with abb.

8.00 $L = \{babb, aabb, ababb, baabb, bbabb, \ldots\}$

9.00 Regular expression = $(a+b)^*.abb$.

10.00 8. Write the regular expression for the language string begins and ends with double consecutive letters. (Eg : aabb).

11.00

12.00 Double consecutive letters.

1.00 $L = \{aabb, aabbb, aababb, \ldots\}$

2.00 Regular Expression = $(aa+bb).(a+b)^*(aa+bb)$

3.00 9. Write the regular expression for the language in which the third symbol from right end is 'a'. $\Sigma = \{a, b\}$.

4.00

5.00 3rd symbol from right end is 'a'.

6.00 $L = \{babb, baba, baab, abb, aaa, \ldots\}$

Regular expression = $(a+b)^* a.(a+b)(a+b)$
(3rd: a; a or b; a or b)

7.00

10. Write the regular expression for the language in which the second symbol from left is 'b' $\Sigma = \{a, b\}$

2nd symbol from left is b.

Notes

$L = \{aba, abba, bba, ab, \ldots\}$

Regular expression = $(a+b).b.(a+b)^*$
(a or b; 2nd)

07 Tuesday APRIL

MARCH '20

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

Wk 15 • 098-268 2020

8.00 Important Questions

9.00 1. Constructing regular expression using Thomson Construction Method.

10.00

2. Constructing regular expression using Arden's theorem.

11.00

12.00 3. Prove Arden's theorem.

1.00

Induction

2.00 2nd Structural Intension :-

(x) Cat-1

3.00 * Structural Induction is used to prove that some proposition P(x) holds for all x of some sort of recursively defined structure, such as formulas, list or trees.

* A well-founded partial order is defined on the structures (or "sub-formula" for formula, "sublist" for list and "subtrees" for trees).

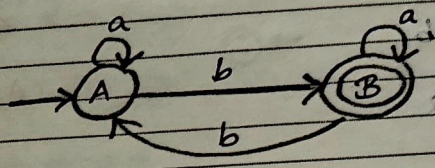
08 Wednesday APRIL

MAY '20

S	M	T	W	T	F	S
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

2020 Wk 15 • 099-267

8.00 ARDEN'S THEOREM :-



9.00 1.

Cat-1

10.00 *

11.00

If $R = Q + RP$ then $R = QP^*$.

12.00

This is the rule,

$R = Q + RP$ then $R = QP^*$

1.00

2.00 Arden's theorem Procedure :-

3.00 STEP-1:- Find the equation for all the states that is represented in the transition

4.00 diagram by taking the incoming edges of the table.

5.00

STEP-2:- Add ε to the initial state of the

6.00 transition diagram.

7.00 STEP-3:- Frame the regular expression by taking final states in the diagram.

Notes

APRIL

March '20

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

Wk 15 • 100-266 2020

$A = A.a + B.b + \varepsilon \rightarrow (1)$

$B = B.a + A.b \rightarrow (2)$

Incoming edges

Add ε in (1)

$B = A.b + B.a$

$R = Q + R.P$

$B = R$

$Q = A.b$

$P = a$

$R = QP^*$

$B = A.ba^* \rightarrow (3)$

Sub (3) in (1)

$A = A.a + B.b + \varepsilon$

$A = A.a + A.ba^*.b + \varepsilon$

$A = \varepsilon + A.a + A.ba^*b$

$A = \varepsilon + A(a + ba^*b)$

$R = Q + R.P$

$R = A$

$Q = \varepsilon$

$P = a + ba^*b$

$R = Q + RP$

$R = QP^*$

May '20

S	M	T	W	T	F	S
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

Friday 10 APRIL

2020 Wk 15 • 101-265

$A = \varepsilon(a + ba^*b)^* \rightarrow (4)$

$A = \varepsilon a + ba^*b)^*$

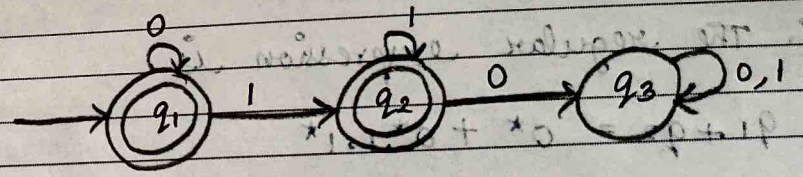
Sub (4) in (3)

$B = A.ba$

$B = (a + ba^*b)^*.ba^*$

If A is also the final state, then the Regular expression is,

$A + B = (a + ba^*b)^* + (a + Ba^*b)^*.ba^*$



$q_1 = q_1.0 + \varepsilon \rightarrow (1)$

$q_2 = q_1.1 + q_2.1 \rightarrow (2)$

$q_3 = q_2.0 + q_3.0 + q_3.1 \rightarrow (3)$

$q_2 = q_1.1 + q_2.1$

$R = Q + R.P$

$q_1 = q_1.0 + \varepsilon$

$q_1 = \varepsilon + q_1.0$

Notes

$q_2 = R$ $P = 1$ $R = q_1$ $P = 0$

$Q = q_1.1$ $Q = \varepsilon$

8.00 $R = QP^*$

$q_2 = q_1 \cdot 1 \cdot 1^*$ $\quad q_1 = \varepsilon 0^* 1 = 0^* \rightarrow (4)$

9.00 Sub (4) in (2)

10.00 $q_2 = 0^* \cdot 1 + q_2 \cdot 1$

11.00 $R = Q + RP$

12.00 $R = q_2$

$Q = 0^* \cdot 1$

1.00 $P = 1$

2.00 $R = QP^*$

$q_2 = 0^* 1 \cdot 1^* \rightarrow (5)$

3.00 ∴ The regular expression is

4.00 $q_1 + q_2 = 0^* + 0^* 1 \cdot 1^*$

5.00

6.00

7.00

NDAY

Problem

8.00 1. Construct a regular expression to accept all possible combination of A's and B's over the input A, B.

9.00

10.00 2. Construct a regular expression that contains any number of A's any number of B's and any number of C's for the input a, b and c.

11.00

12.00

1.00 3. Construct the regular expression that contains atleast 1 A, 1B and 1C for the input ABC.

2.00

4. Construct the regular expression that contains atleast 2 B's for the input A and B.

3.00

4.00 5. Construct the regular expression that contains exactly 2B's only for the input A and B.

5.00

6.00 1. $\Sigma = \{a, b\}$

$L = \{\varepsilon, ab, aab, abb, aabb, \ldots\}$

7.00 Regular Expression = $(a+b)^*$

2. $\Sigma = \{a, b, c\}$

$L = \{\varepsilon; abc, aabc, abbc, abcc, \ldots\}$

Notes

Regular Expression = $a^* b^* c^*$ any number $a^* b^* c^*$

3. $\Sigma = \{a, b, c\}$
$L = \{abc, aabc, abbc, abcc, \ldots\}$

Regular Expression = $a^+b^+c^+$ atleast

4. $\Sigma = \{a, b\}$
$L = \{abab, abba, abbb, \ldots\}$

Regular Expression = $(a+b)^* b (a+b)^* b (a+b)^*$

5. $\Sigma = \{a, b\}$
$L = \{abb, abab, aabb, \ldots\}$

Regular Expression = $a^* b a^* b a^*$

Inductive Proof

1. $0^2+1^2+2^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{2}$

Same as $1^2+2^2+3^2+\ldots n^2$ sum.

2. $1+4+7+\ldots(3n-2) = \frac{n(3n-1)}{2}$

Basis :-

LHS :- $n=1$, $3(1)-2 = 3-2 = 1$

RHS :- $\frac{n(3n-1)}{2} = \frac{1(3-1)}{2} = \frac{1(2)}{2} = 1.$

LHS = RHS $f(n)$ is true.

Inductive assumption :-

LHS :- $1+4+7+\ldots(3n-2)$

$n = n+1$

$1+4+7+\ldots(3n-2)+(3(n+1)-2)$

$= \frac{n(3n-1)}{2} + (3n+3-2)$

$= \frac{n(3n-1)}{2} + (3n+1) = \frac{3n^2-n+2(3n+1)}{2}$

Notes

$= \frac{3n^2-n+6n+2}{2} = \frac{3n^2+5n+2}{2}.$

16 Thursday APRIL

March '20

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

2020

Wk 16 • 107-259

8.00 RHS :- $\frac{n(3n-1)}{2}$ Sub $n = n+1$

9.00 $= \frac{(n+1)(3(n+1)-1)}{2}$

10.00 $= \frac{(n+1)(3n+3-1)}{2}$

11.00 $= \frac{(n+1)(3n+2)}{2} = \frac{3n^2+2n+3n+2}{2}$

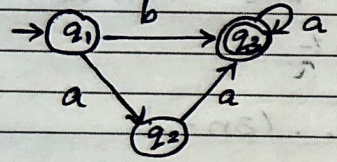
12.00 $= \frac{3n^2+5n+2}{2}$

LHS = RHS

∴ P(n+1) is true.

Arden's theorem

1.



$q_1 = q_2.a + q_3.b + \varepsilon$

$q_2 = q_3.a$

$q_3 = q_3.a$

$q_1 = \varepsilon \rightarrow (1)$

$q_2 = q_1.a \rightarrow (2)$

$q_3 = q_1.b + q_2.a + q_3.a \rightarrow (3)$

$R = Q + RP$

$R = q_3$ $P = \cancel{q_3}.a$

$Q = q_1.b + q_2.a$

2020

Wk 16 • 108-258

May '20

S	M	T	W	T	F	S
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

Friday 1. APRIL

8.00 $R = QP^*$

9.00 $q_3 = (q_1.b + q_2.a).a^*$

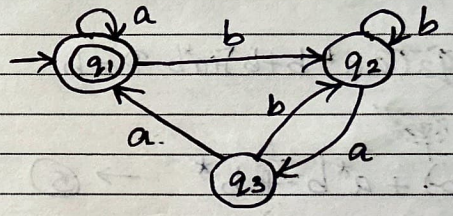
10.00 $= (q_1.b + (q_1.a)a).a^*$

11.00 $= (q_1(b+aa)).a^*$

12.00 $= \varepsilon(b+aa).a^*$

1.00 $q_3 = (b+aa)a^*$ → Regular Expression.

2. 2.00



3.00

4.00

5.00 $q_1 = q_1.a + q_3.a + \varepsilon \rightarrow (1)$

6.00 $q_2 = q_1.b + q_2.b + q_3.b \rightarrow (2)$

7.00 $q_3 = q_2.a \rightarrow (3)$

From (1), $q_1 = q_3.a + \varepsilon + q_1.a$

$R = Q + RP$

$R = q_1$

$Q = q_3.a + \varepsilon$

$P = a$

$R = QP^*$

$q_1 = (q_3.a + \varepsilon).a^* \rightarrow (4)$

18 Saturday APRIL

Wk 16 • 109-257 2020

8.00 Sub ④ in ②

9.00 $q_2 = ((q_3.a+\varepsilon).a^*).b + q_2.b + q_3.b$

10.00 $= (q_3.aa^* + a^*).b + q_2.b + q_3.b$

11.00 $= (q_3.aa^*b + a^*b) + q_2.b + q_3.b$

12.00 $= q_3.aa^*b + q_3.b + a^*b + q_2.b$

1.00 $q_2 = q_3(aa^*b+b) + a^*b + q_2.b$

2.00 $R = Q + RP$

$R = q_2 \quad Q = q_3(aa^*b+b) + a^*b \quad P = b$

3.00 $R = QP^*$

$q_2 = q_3(aa^*b+b) + a^*b + b^* \rightarrow ⑤$

$q_3 = q_2.a$

$= [q_3(aa^*b+b) + a^*b + b^*].a$

$= q_3(aa^*b+b)a + aa^*b + ab^*$

$q_3 = aa^*b + ab^* + q_3(aa^*b+b)a$

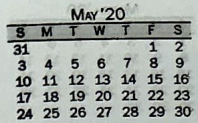
SUNDAY

$R = q_3$

$Q = aa^*b + ab^*$

$P = (aa^*b+b)a$

2020 Wk 17 • 111-255



Monday APRIL 20

8.00 $R = QP^*$

9.00 $q_3 = (aa^*b + ab^*)[(aa^*b+b)a)]^* \rightarrow ⑥$

10.00 $q_1 = (q_3.a+\varepsilon).a^*$

11.00 $= [[(aa^*b+ab^*)[(aa^*b+b)a]^*].a+\varepsilon].a^*$

12.00 $= [[(aa^*b+ab^*)(aaa^*b+ab)^*]aa^* + a^*$

—x—

1.00

2.00 Sub ③ in ②

3.00 $q_2 = q_1.b + q_2.b + q_2.a.b$

4.00 $q_2 = q_1.b + q_2(b+a.b)$

5.00 $R = Q + RP$

6.00 $R = q_2, \quad Q = q_1.b, \quad P = b+a.b$

7.00 $R = QP^*$

$q_2 = (q_1.b)(b+ab)^* \rightarrow ④$

Sub ③ in ①

Notes $q_1 = q_1.a + q_2.aa + \varepsilon \rightarrow ⑤$

Sub ④ in ⑤,

$q_1 = q_1.a + [(q_1.b)(b+ab)^*]aa + \varepsilon$

$= \varepsilon + q_1.a + q_1.b(b+ab)^*aa$

$q_1 = \varepsilon + q_1[a+b(b+ab)^*aa]$

$R = Q + RP$

$R = q_1$

$Q = \varepsilon$

$P = a+b(b+ab)^*aa$

$R = QP^*$

$q_1 = \varepsilon.[a+b.(b+ab)^*aa]^*$

$q_1 = (a+b(b+ab)^*aa)^*$

ARDEN'S THEOREM :-

Proof :-

If P and Q are two Regular Expression over Σ, and if P does not contains ε, Then the following equation in R is given by $R = Q + RP$ has a unique solution that is $R = QP^*$.

(So, now $R = QP^*$ will be the solution)

Solution :-

$R = Q + RP \rightarrow ①$

$R = Q + (QP^*)P$

$R = Q + QP^*P$

$R = Q(\varepsilon + P^*P)$

$R = QP^* \rightarrow ②$

Hence Proved.

Sub $\therefore R = QP^*$ in ①

$\therefore \varepsilon + P^*P = P^*$

Note :-

$\varepsilon + R^*R = R^*$

$\varepsilon + P^*P = P^*$

$\varepsilon + A^*A = A^*$

Sub $R = Q + RP$ in ①

$R = Q + [Q+RP]P$

$= Q + QP + RP^2$

$= Q + QP + [Q+RP]P^2$

$= Q + QP + QP^2 + RP^3 + \ldots$

$\vdots$

$R = Q + QP + QP^2 + \ldots + QP^n + RP^{n+1}$ (QP* over the last two terms)

$= Q[\varepsilon + P + P^2 + \ldots + P^n + P^*P^{n+1}]$

↳ closure of P

$R = QP^* \rightarrow ③$

$\therefore \varepsilon + P + P^2 + \ldots + P^n + P^*P^{n+1} = P^*$

Notes

and create clusters with union ...

23 Thursday APRIL 2020

March '20

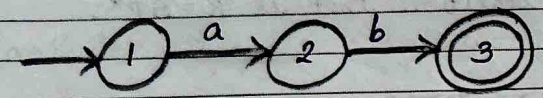
S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

Wk 17 • 114-252

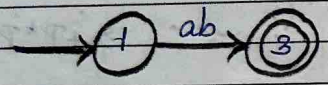
Equation ② and the Equation ③ are same. It states that whenever the equation is $R = Q + RP$, then we can say that $R = QP^*$.

Convert Finite Automata to RE using State Elimination method :-

1.



We need to eliminate the intermediate state.



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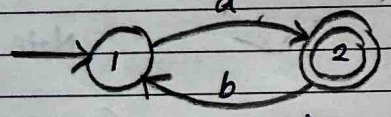

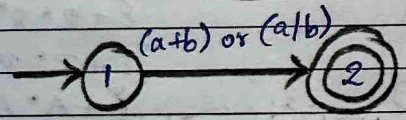

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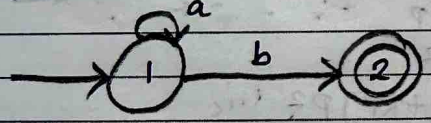
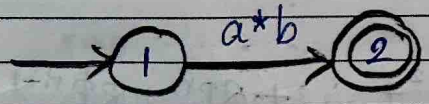




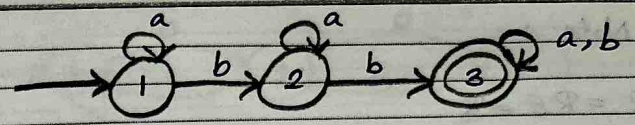
2020 Wk 17 • 115-251

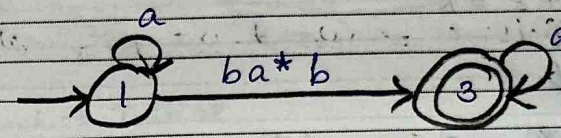
May '20

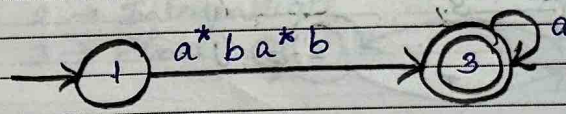
S	M	T	W	T	F	S
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

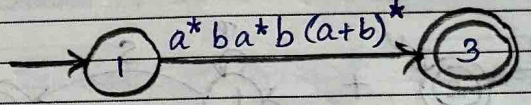
Friday 24 APRIL

4.





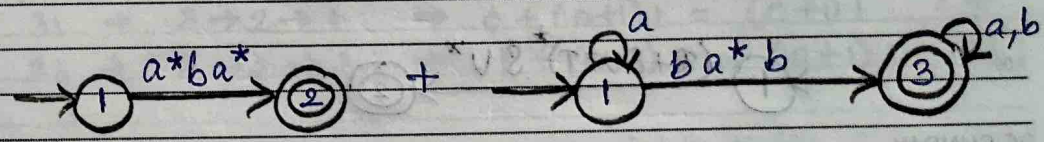


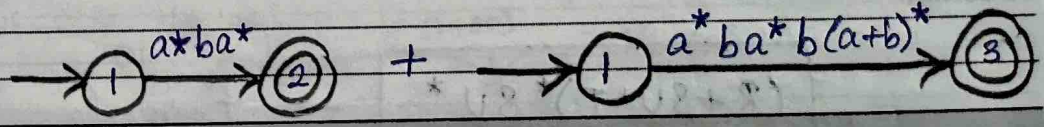


∴ (a,b) ⇒ (a+b)
Self loop ⇒ (a+b)*

5.

When we have 2 final states we have to split.

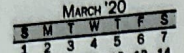




Notes

∴ $a^*ba^* + a^*ba^*b(a+b)^*$

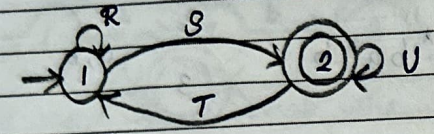
25 Saturday APRIL

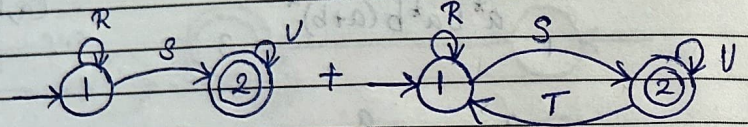


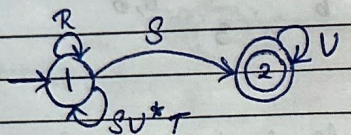
Note :-

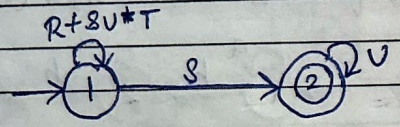
1. $\varepsilon R = R\varepsilon = R$
2. $\varepsilon + R = R + \varepsilon = R$
3. $\phi + R = R$
4. $\phi \cdot R = \phi$

6.

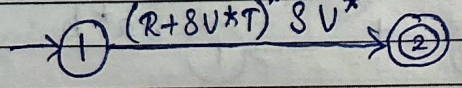






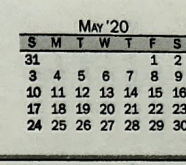


2 Self loop use union +



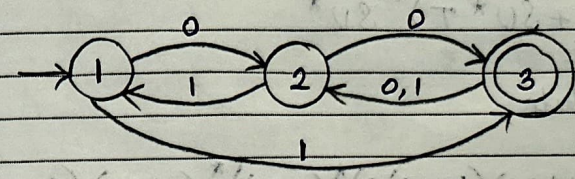
26 SUNDAY

$(R+SU^*T)^*SU^*$ → Formula.



Monday 27 APRIL

7.



1 → Initial
2 → Intermediate
3 → Final

Path = 2

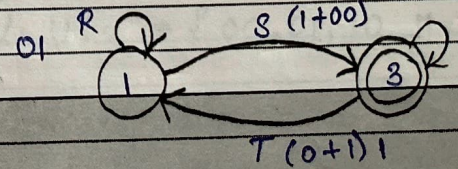
States + Path

1. Initial + Initial
2. Initial + final
3. final + Initial
4. final + final

0,1 ⇒ (0+1)

States + Path

11 ⇒ 1→2→1 ⇒ $\phi + 01 = 01$ = R
13 ⇒ 1→2→3 ⇒ $1 + 00$ = S
31 ⇒ 3→2→1 ⇒ $\phi + (0+1)1 = (0+1)1$ = T
33 ⇒ 3→2→3 ⇒ $\phi + (0+1)0 = (0+1)0$ = U



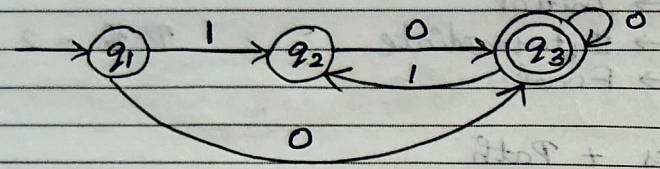
Notes

$(R+SU^*T)^*SV^*$

Substitute,

$(01+(1+00)((0+1)0)^*(0+1))^*(1+00)((0+1)0)^*$

8.



States + Path

$q_1q_1 \Rightarrow q_1 \to q_2 \to q_1 \Rightarrow \phi+1\phi = \phi = R$

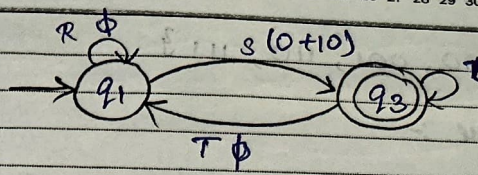
$q_1q_3 \Rightarrow q_1 \to q_2 \to q_3 \Rightarrow 0+10 = S$

$q_3q_1 \Rightarrow q_3 \to q_2 \to q_1 \Rightarrow \phi+1\phi = \phi = T$

$q_3q_3 \Rightarrow q_3 \to q_2 \to q_3 \Rightarrow 0+10 = U$

NOTE :-

1. $\varepsilon + R = R + \varepsilon = R$
2. $\varepsilon R = R\varepsilon = R$
3. $\phi + R = R$
4. $\phi R = \phi$



$(R+SU^*T)^*SU^*$

Substitute,

$(\phi+(0+10)(0+10)^*\phi)^*(0+10)(0+10)^*$

Operations of Regular Expression :-

1. Union
2. Concatenation
3. Kleen Closure.

1. Union :-

$L_1 = \{(0+1)\cdot(0+1)\}$

$L_1 = \{00, 01, 10, 11\}$

$L_2 = \{\varepsilon, 100\}$

$L_1 \cup L_2 = \{00, 01, 10, 11, \varepsilon, 100\}$

2. Concatenation :-

Notes

$L_1 = \{0, 1\}$

$L_2 = \{00, 11\}$

8.00 $L_1 . L_2 = \{000, 001, 110, 111\}$

9.00 3. Kleen Closure :-

10.00 $\Sigma = \{0, 1\}$

11.00 $L^* = \{\varepsilon, 0, 1, 00, 11, 01, 10, \ldots\}$

8m

12.00 Properties of Regular Language :-

1.00 1. Closure
2. Closure Law
2.00 3. Associativity
4. Identity
3.00 5. Annihilator
6. Commutative
4.00 7. Distributive (or) Distributed
8. Idempotent Law
5.00 9. Identities of regular Expression.

6.00 1. Closure :-

7.00 r^*
$r_1 + r_2$
$r_1 . r_2$

2. Closure Law :-

Notes

1. $(r^*)^* = r^*$
2. $\varepsilon + r^* = r^*$
3. $r^+ = r^* . r = r . r^*$
4. $\phi^* = \varepsilon$

8.00 3. Associativity :-

a b c

9.00 r_1, r_2, r_3

10.00 1. $r_1 + (r_2 + r_3) = (r_1 + r_2) + r_3$
2. $r_1 . (r_2 . r_3) = (r_1 . r_2) . r_3$

11.00

4. Identity :-

12.00 $r + X = r \Rightarrow X = \phi$
$r \cup \phi = r$ $r . X = r$ if $X = \varepsilon$
1.00 $r . \varepsilon = r$

2.00

5. Annihilator :-

3.00 $r + X = r$
$r \cup X = X$
4.00 $r . X = X$ when $X = \phi$
$r = \phi$

When we do + operation the value will not change.
• Operation value will change

5.00 ↓ acts as Annihilator

6.00 6. Commutative :- • → has annihilator
+ → not have annihilator

a b
7.00 r_1, r_2

1. $r_1 + r_2 = r_2 + r_1$
2. $r_1 . r_2 \neq r_2 . r_1$

Notes

8.00 7. Distributive :-

9.00 1. $(R_1 + R_2) \cdot R_3 = R_1 \cdot R_3 + R_2 \cdot R_3$ (Right Distribution)
2. $R_1 \cdot (R_2 + R_3) = R_1 \cdot R_2 + R_1 \cdot R_3$ (Left Distribution)

10.00 3. $(R_1 \cdot R_2) + R_3 \neq (R_1 + R_3)(R_2 + R_3)$

11.00 8. Idempotent Law :-

12.00 1. $R_1 + R_1 = R_1 \Rightarrow R_1 \cup R_1 = R_1$
2. $R \cdot R \neq R$

1.00

9. Identities of regular expression :-

2.00

1. $\phi + R = R$

3.00 2. $\phi \cdot R = R \cdot \phi = \phi$
3. $\varepsilon \cdot R = R \cdot \varepsilon = R$

4.00 4. $\varepsilon^* = \varepsilon$ and $\phi^* = \varepsilon$
5. $R + R = R$

5.00 6. $R^* \cdot R^* = R^*$
7. $R^* \cdot R = R \cdot R^* = R^+$

6.00 8. $(R^*)^* = R^*$
9. $\varepsilon + R \cdot R^* = R^* = \varepsilon + R \cdot R^*$

7.00 10. $(P \cdot q)^* \cdot P = P \cdot (q \cdot P)^*$
11. $(P+q)^* = (P^* \cdot q^*)^* = (P^* + q^*)^*$

03 SUNDAY

12. $(P+q) \cdot R = P \cdot R + q \cdot R = R \cdot (P+q)$

Notes

$= R \cdot p + R \cdot q$.

8.00 Pumping Lemma for Regular language / Regular Expression :-

9.00 If 'L' is a regular language represented

10.00 with an automata with the maximum number of states denoted by 'n' then

11.00 there is a word or input denoted by 'z' in the language L such that length

12.00 of the word or input is given by $|z| \geq n$, further we may write

1.00 $z = uvw$ in such a way that $|uv| \leq n$ and $|v| \geq 1$, for all $i \geq 0$;

2.00

PROOF :-

3.00

Given :-

4.00 'L' → Regular Language
'n' → maximum number of states

5.00 'z' → word (or) Input

6.00 $|z| \geq n$
$z = uvw \quad |v| \geq 1$

7.00 $|uv| \leq n \quad i \geq 0.$

Let us consider an input for constructing an automata is the automata is having 'abb' in it.

Notes

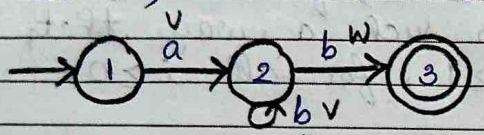
Now the value of z will be 'abb' and as per the condition given in the statement, $|z| \geq n$.

Here $|z| = 3$ so number of states 'n' is also taken as 3.

By using this the automata is constructed.

So of

$z = abb$, $|z| = 3$, $n = 3$



$z = uv^iw \in L$

So after constructing the automata as per the input split up $z = uvw$.

So, here the value 'b' that is denoted by v in the self-loop in the state 2 can be written as $z = uv^iw$, if i value is 0, v value will be 0. If i value is 1, v value will be 2. This will keep on go till we substitute the value for i.

Anyway for all the values of i, the input is satisfied by the automata, so we are declaring that $z = uv^iw \in L$

Notes

ab^*b abbbb
abb
abbb

1. Prove that $L = \{a^n b^n / n \geq 1\}$ is not regular.

Proof :-

1. Assume that given language 'L' is regular.
2. $z = uvw$ $|v| \geq 1$, $uv^iw \in L$ for all $i \geq 0$.

3. $z = \underbrace{aaa\ldots}_{uv} \underbrace{bbb\ldots}_{w}$	$z = \underbrace{aaa\ldots}_{u} \underbrace{bbb\ldots}_{vw}$	$z = \underbrace{aaa}_{u} \underbrace{bbb}_{v} \underbrace{bbb}_{w}$
$a^{(n-s)}\ a^{(s)}\ b^n$ (u v w)	$a^n\ b^{(s)}\ b^{(n-s)}$ (u v w)	$a^{n-1}\ ab\ b^{n-1}$ (u v w)
$uvw = n-s+s = n$	$uv^iw \notin L$	$uv^iw \notin L$

If $i = 2$ $uv^iw = n-s+2s$

$uv^2w = a^{n+s}b^n \notin L$

$\therefore uv^iw \notin L$

$\therefore L$ is not regular.

2. Prove that $L = \{1^p / p$ is a prime number$\}$ is not regular.

Proof :-

1. Assume that given language 'L' is regular.

Notes

2. $z = uvw$ $|v| \geq 1$, $uv^iw \in L$ for all $i \geq 0$.

07 Thursday MAY

April '20

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

Wk 19 • 128-238 2020

3. $|z| = p$ ($\because$ length of z is prime)

$|uvw| = p \Rightarrow |uv^i w| = p$

Let us take $i = p+1$

$|uv^i w| \Rightarrow |uv^{p+1} w| = |uvw| + |v^p| = p + p[|v|]$

$|uv^p v' w|$

$= p[1 + |v|]$

$= p[1+1]$

$= p \times 2$

All numbers when multiplied by 2 is not a prime number.

Eg :- $2 \times 2 = 4$.

$|uv^i w|$ is not a prime number.

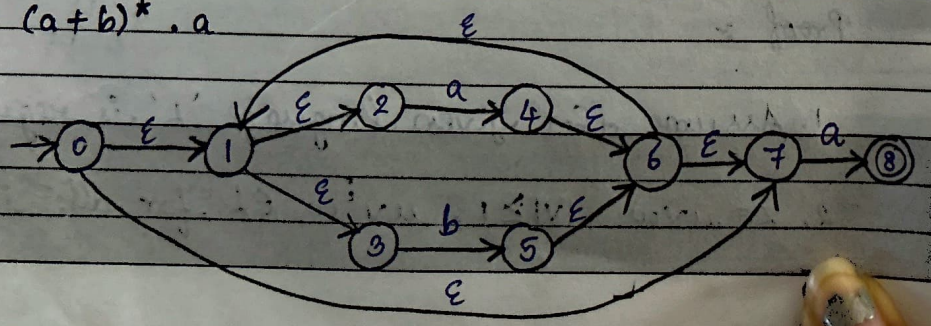
$\therefore L$ is not regular.

Converting regular expression to ε-NFA and then to NFA.

Converting regular expression to ε-NFA and then to DFA.

1. $(a|b)^*.a$ (or) (Same)

$(a+b)^*.a$



Notes

2020 Wk 19 • 129-237

June '20

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

Friday MAY 08

ε-closure (0) = {0, 1, 2, 3, 7}

ε-closure (1) = {1, 2, 3}

ε-closure (2) = {2}

ε-closure (3) = {3}

ε-closure (4) = {4, 6, 1, 2, 3, 7}

ε-closure (5) = {5, 6, 1, 2, 3, 7}

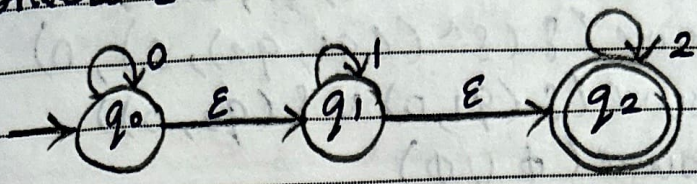
ε-closure (6) = {6, 1, 2, 3, 7}

ε-closure (7) = {7}

ε-closure (8) = {8}

Notes

2. Convert ε-NFA to DFA.



ε-closure $(q_0) = \{q_0, q_1, q_2\} = A$ $\quad \phi = D$

ε-closure $(q_1) = \{q_1, q_2\} = B$

ε-closure $(q_2) = \{q_2\} = C$

$\delta'(A, 0) = \varepsilon\text{-closure}(\delta(\delta^\wedge(A, \varepsilon), 0)$

$= \varepsilon\text{-closure}(\delta(\delta^\wedge((q_0, q_1, q_2), \varepsilon), 0)$

$= \varepsilon\text{-closure}(\delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0))$

$= \varepsilon\text{-closure}(q_0 \cup \phi \cup \phi)$

$= \varepsilon\text{-closure}(q_0)$ $\quad (A, 0) = A$

$= \{q_0, q_1, q_2\} = A$

$\delta'(A, 0) = A.$

$\delta'(A, 1) = \varepsilon\text{-closure}(\delta(\delta^\wedge(A, \varepsilon), 1)$

$= \varepsilon\text{-closure}(\delta(\delta^\wedge((q_0, q_1, q_2), \varepsilon), 1)$

$= \varepsilon\text{-closure}(\delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1))$

$= \varepsilon\text{-closure}(\phi \cup q_1 \cup \phi)$

$= \varepsilon\text{-closure}(q_1)$

$= \{q_1, q_2\}$

$\delta'(A, 1) = B.$

$\delta'(A, 2) = \varepsilon\text{-closure}(\delta(\delta^\wedge(A, \varepsilon), 2)$

$= \varepsilon\text{-closure}(\delta(\delta^\wedge((q_0, q_1, q_2), \varepsilon), 2)$

$= \varepsilon\text{-closure}(\delta(q_0, 2) \cup \delta(q_1, 2) \cup \delta(q_2, 2))$

$= \varepsilon\text{-closure}(\phi \cup \phi \cup q_2)$

$= \varepsilon\text{-closure}(q_2)$

$= \{q_2\}$

$\delta'(A, 2) = C.$

Notes

8.00 $\delta'(B,0) = \varepsilon\text{-closure}(\delta(\delta^\wedge(B,\varepsilon),0)$
$= \varepsilon\text{-closure}(\delta(\delta^\wedge((q_1,q_2),\varepsilon),0)$
9.00 $= \varepsilon\text{-closure}(\delta(q_1,0) \cup \delta(q_2,0))$
$= \varepsilon\text{-closure}(\phi \cup \phi)$
10.00 $= \varepsilon\text{-closure}(\phi)$
$= \{D\}$
11.00 $\delta'(B,0) = D.$

12.00 $\delta'(B,1) = \varepsilon\text{-closure}(\delta(\delta^\wedge(B,\varepsilon),1)$
$= \varepsilon\text{-closure}(\delta(\delta^\wedge((q_1,q_2),\varepsilon),1)$
1.00 $= \varepsilon\text{-closure}(\delta(q_1,1) \cup \delta(q_2,1))$
$= \varepsilon\text{-closure}(q_1 \cup \phi)$
2.00 $= \varepsilon\text{-closure}(q_1)$
$= \{q_1,q_2\}$
3.00 $\delta'(B,1) = B$

4.00 $\delta'(B,2) = \varepsilon\text{-closure}(\delta(\delta^\wedge(B,\varepsilon),2)$
$= \varepsilon\text{-closure}(\delta(\delta^\wedge((q_1,q_2),\varepsilon),2)$
5.00 $= \varepsilon\text{-closure}(\delta(q_1,2) \cup \delta(q_2,2)$
$= \varepsilon\text{-closure}(\phi \cup q_2)$
6.00 $= \varepsilon\text{-closure}(q_2)$
$= \{q_2\}$
7.00 $\delta'(B,2) = C$

17 SUNDAY $\delta'(C,0) = \varepsilon\text{-closure}(\delta(\delta^\wedge(C,\varepsilon),0)$
$= \varepsilon\text{-closure}(\delta(\delta^\wedge(q_2),0)$
$= \varepsilon\text{-closure}(\delta(q_2,0))$
$= \varepsilon\text{-closure}(\phi)$
$\delta'(C,0) = D.$

Notes

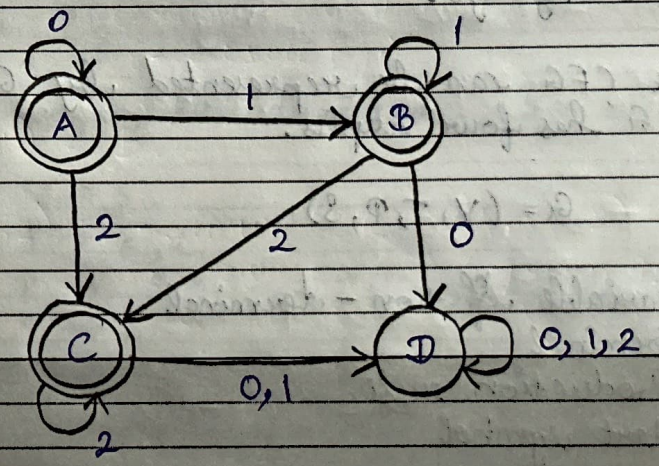
8.00 $\delta'(C,1) = \varepsilon\text{-closure}(\delta(\delta^\wedge(C,\varepsilon),1)$
$= \varepsilon\text{-closure}(\delta(\delta^\wedge(q_2),1)$
9.00 $= \varepsilon\text{-closure}(\delta(q_2,1))$
$= \varepsilon\text{-closure}(\phi)$
10.00 $\delta'(C,1) = D$

11.00 $\delta'(C,2) = \varepsilon\text{-closure}(\delta(\delta^\wedge(C,\varepsilon),2)$
$= \varepsilon\text{-closure}(\delta(\delta^\wedge(q_2),2)$
12.00 $= \varepsilon\text{-closure}(\delta(q_2,2))$
$= \varepsilon\text{-closure}(q_2)$
1.00 $= \{q_2\}$
$\delta'(C,2) = C$
2.00

$\delta'(D,0) = D$
3.00 $\delta'(D,1) = D$
$\delta'(D,2) = D.$
4.00

Transition Diagram (DFA) :-

5.00
6.00
7.00



Notes