## 2.5 EQUIVALENCE OF DFA AND NFA

(i) As every DFA is an NFA, the class of languages accepted by NFA's includes the class of languages accepted by DFA's.

(ii) DFA can simulate NFA.

(iii) For every NFA, there exist an equivalent DFA.

**Theorem** **(Apr/May 2005), (Nov/Dec 20..)**

For every NFA, there exists a DFA which simulates the behavior of NFA. If L is the set accepted by NFA, then there exists a DFA which also accepts L.

**Proof**

Let M= (Q, $\Sigma$, $q_0$, F, $\delta$) be NFA accepting L

we construct DFA $M^1 = (Q^1, \Sigma, q_0^{\,1}, F^1, \delta^1)$, where

(i) $Q^1 = 2^Q$ (power set of Q)

(any state in $Q^1$ is denoted by $[q_1, q_2, .....q_i]$ where $q_1, q_2, .... q_i \in Q$)

(ii) $q_0^{\,1} = [q_0]$

(iii) $F^1$ is set of final states.

Before defining $\delta^1$, let us look at the construction of $Q^1$, $q_0^{\,1}$ and $F^1$.

M is initially at $q_0$. On application of an input symbol say $a$, M can reach any of the states $\delta(q_0, a)$. To describe M, just after application of the input symbol $a$, we require all the possible states that M can reach after the application of $a$. So, $M^1$, has to remember all these possible states at any instant of time.

As M (NFA) starts with inital state $q_0$. $q_0^{\,1}$ is defined as $[q_0]$.

In $M^1$ (DFA) the final state ($F^1$) can be subset of Q containing all final states of F.

Now we define

$\delta^1([q_1, q_2, ..... q_i], a) = \delta(q_1, a) \cup \delta(q_2, a) \cup .... \delta(q_i, a)$

*equivalently,*

$\delta^1([q_1, q_2, ....q_i], a) = [p_1, p_2, ...p_j]$

*if and only if*

$\delta(\{q_1, q_2, ....q_i\}, a) = \{p_1, p_2, ...p_j\}$

**Proof by Induction**

*Input string x*

$\delta^1(q_0^{\,1}, x) = [q_1, q_2, ...q_i]$

*if and only if*

$\delta(q_0, x) = \{q_1, q_2, ...... q_i\}$

**Basis**

The result is trivial if string length is 0 i.e., $|x| = 0$

since $q_0{}' = [q_0]$. $x$ must be $\varepsilon$

**Induction**

Suppose the hypothesis is true for inputs of length m.

Let $xa$ be a string of length m +1 with $a$ in $\Sigma$.

Then $\delta'(q_0{}', xa) = \delta'(\delta'(q_0{}',x), a)$

*By induction hypothesis*

$\delta'(q_0{}', x) = [p_1,p_2 \ldots\ldots\ldots\ldots p_j]$

*if and only if*

$\delta(q_0, x) = \{p_1,p_2 \ldots\ldots\ldots\ldots p_j\}$

*By definition of* $\delta'$

$\delta'([p_1,p_2 \ldots\ldots\ldots\ldots p_j], a) = [r_1,r_2 \ldots\ldots\ldots\ldots r_k]$

*if and only if*

$\delta(\{p_1,p_2 \ldots\ldots\ldots\ldots p_j\}, a) = \{r_1,r_2 \ldots\ldots\ldots\ldots r_k\}$

Thus

$\delta'(q_0{}', xa) = [r_1,r_2 \ldots\ldots\ldots\ldots r_k]$

*if and only if*

$\delta(q_0, xa) = \{r_1,r_2 \ldots\ldots\ldots\ldots r_k\}$

which establishes the inductive hypothesis.

Thus L(M) = L(M ')