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MODULE - III

Regular expression:

A mathematical notation used to describe the regular language. It is formed by using three symbols.

* Dot operator (.) → a·b
* Union operator (+) → a+b
* Closure operator (*) → a^*

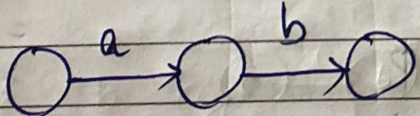
→ In dot operator, we are performing concatenation.

→ In union operator, it contains all transitions except epsilon (ε). Used to find atleast one occurence of the variable.

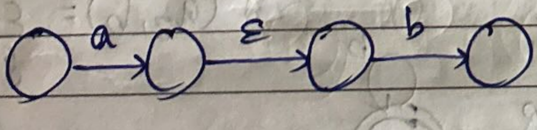
→ In closure operator, used to find zero or more occurence of the variable.

Converting the regular expression to ε-NFA using Thomson Construction Method.

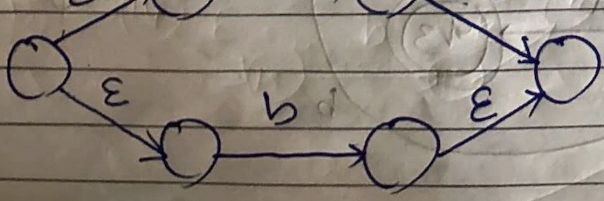
(i) ab (concatenation)



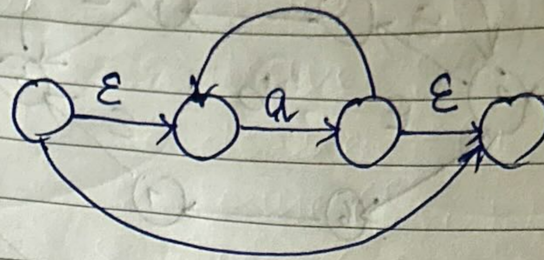
(OR)



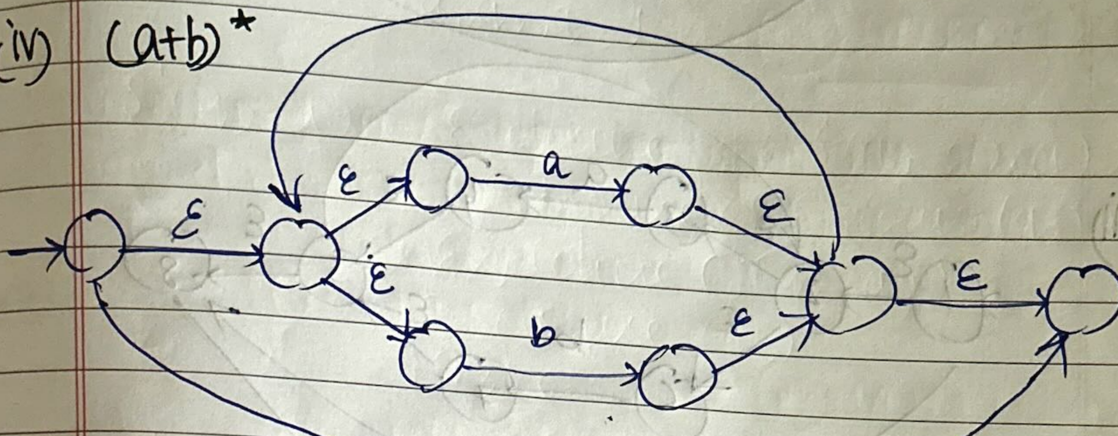
(ii) a+b (union)



(iii) a^*



(iv) $(a+b)^*$



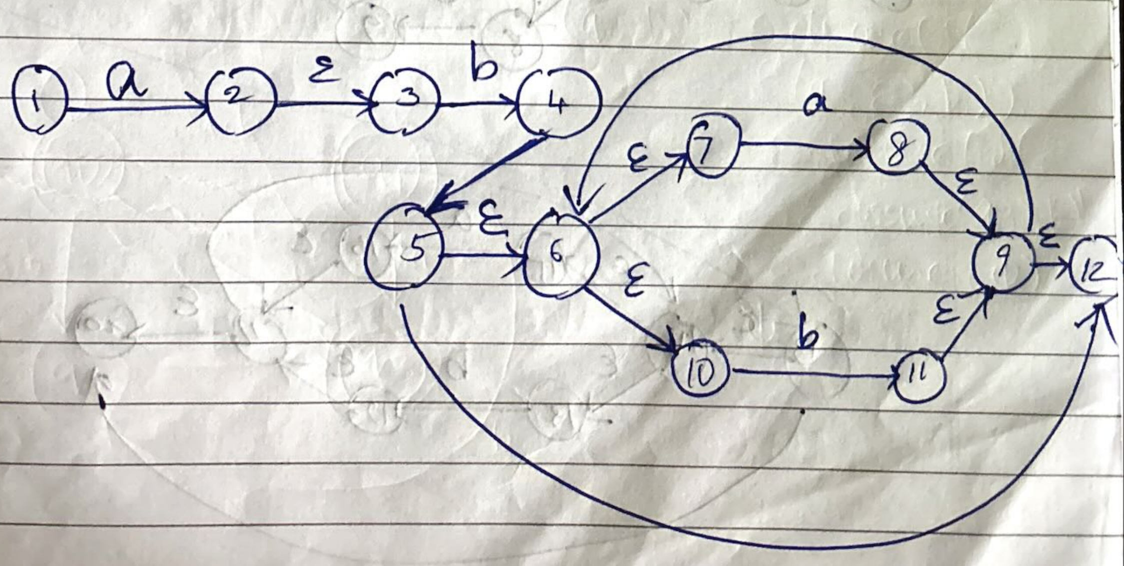
PROBLEM 22

Construct ε-NFA for the given regular expression using Thomson construction Method

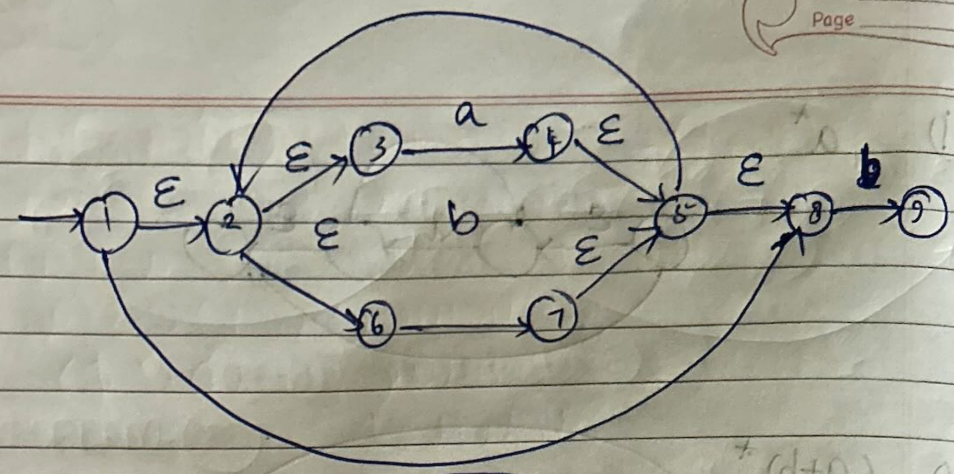
(i) $ab.(a+b)^*$	(v) $b.a(a+b)^*(a+b)$
(ii) $(a+b)^*.b$	(vi) $(a+b)(a+b)^*$
(iii) $(a+b)^*.ab.(a+b)^*$	(vii) $(ab)^*$
(iv) $(ab+ba).(a+b)^*$	(viii) $(a+b)ab(a+b)^*$
(ix) $(a+b)ba(a+b)$	(x) $a^*(ab)^*$

Solution:

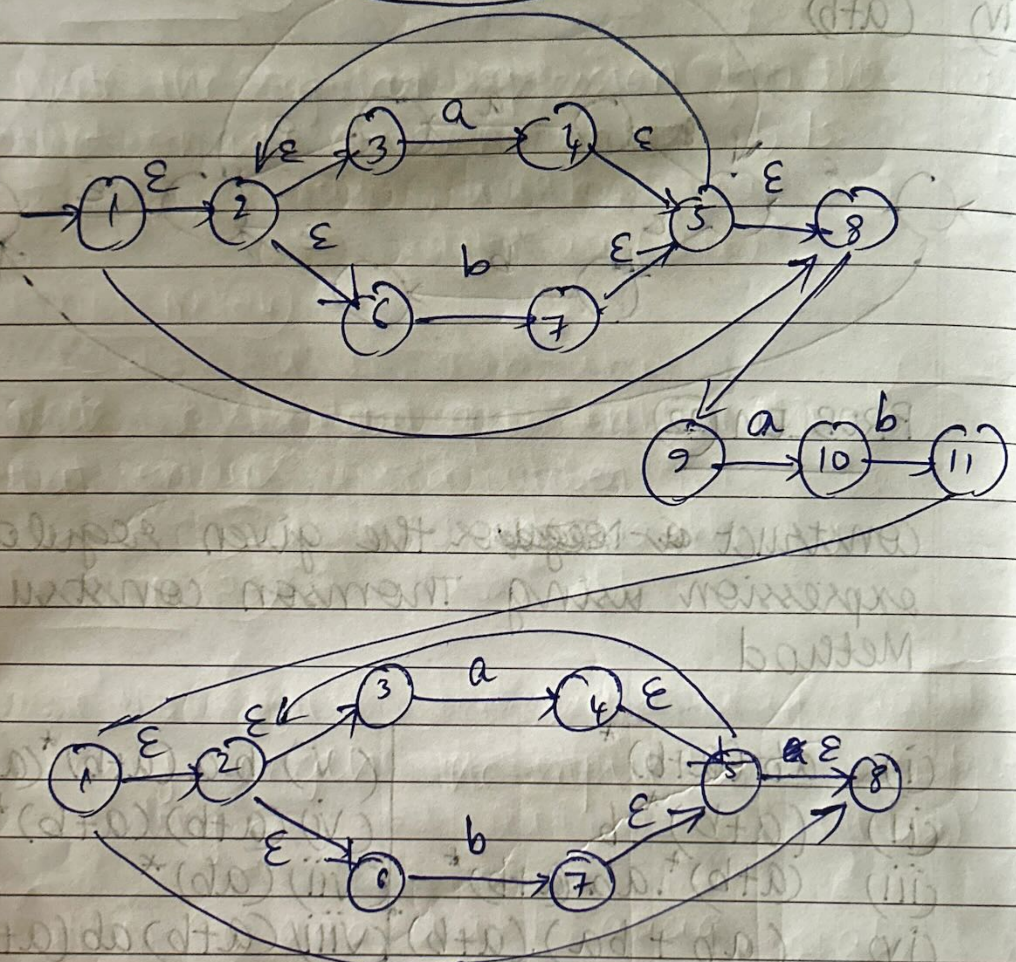
(i)



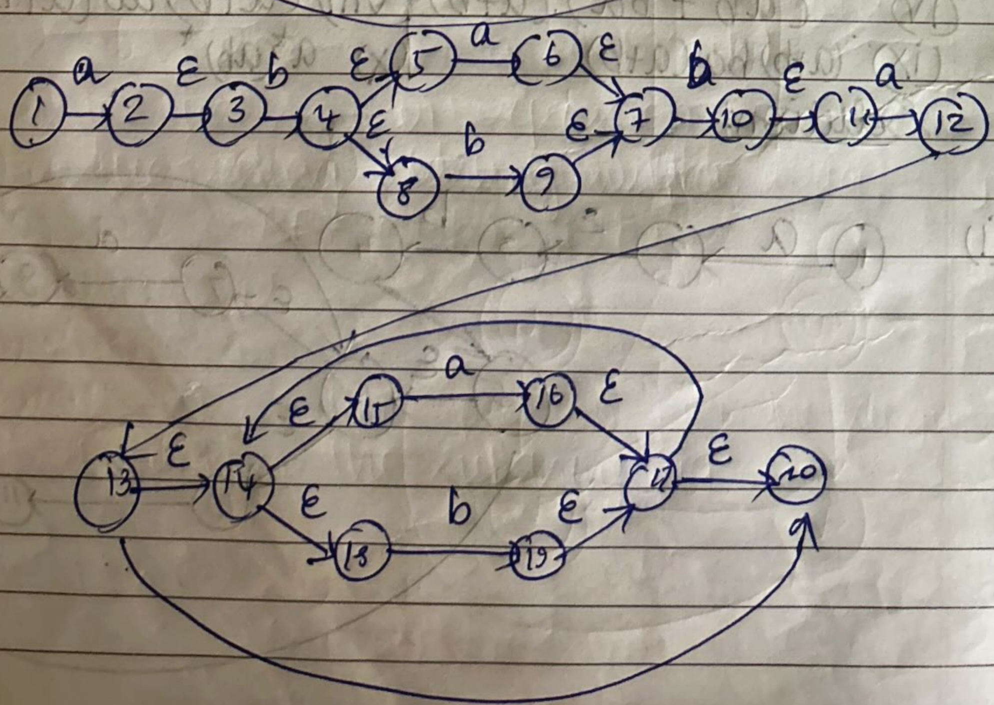
(ii)



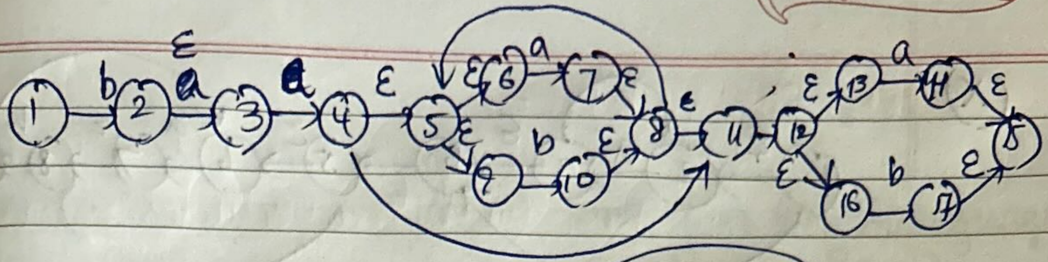
(iii)



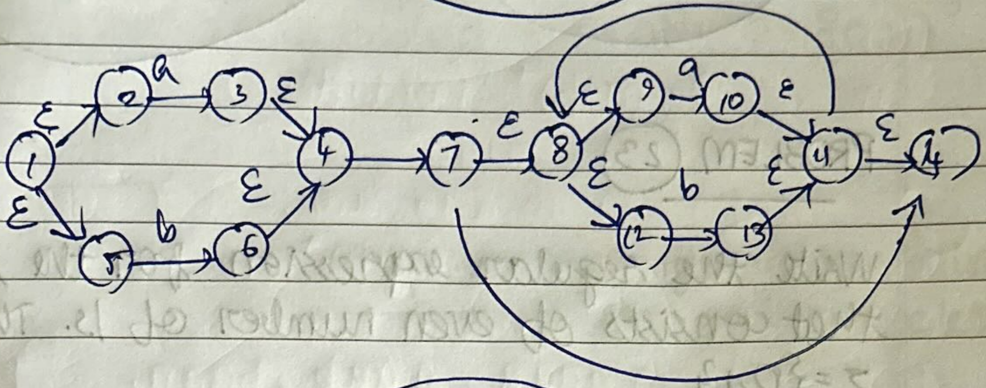
(iv)



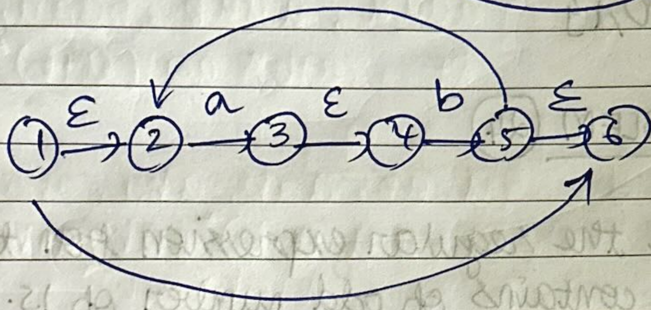
(v)



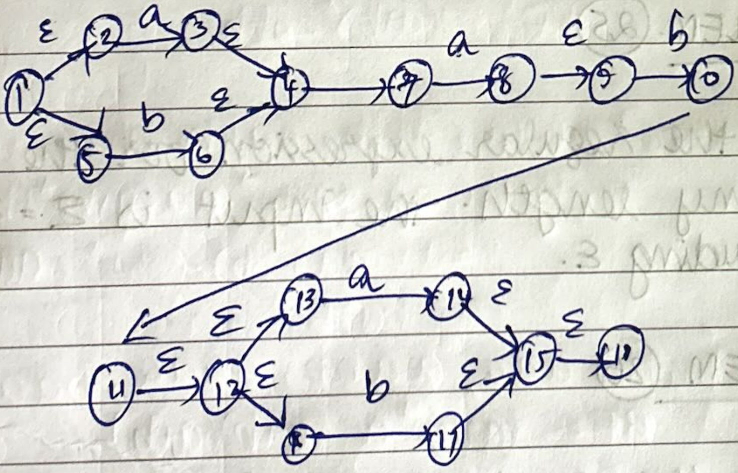
(vi)



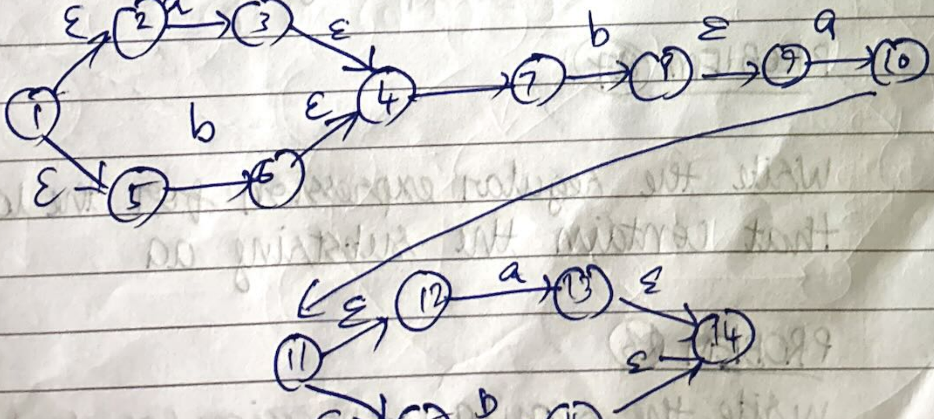
(vii)



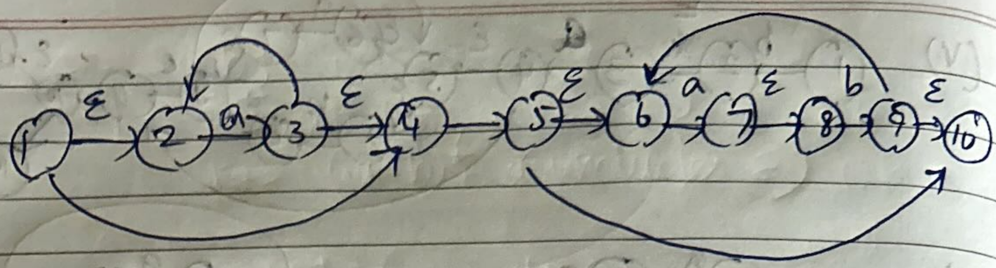
(viii)



(ix)



(X)



PROBLEM 23

Write the regular expression for the language that consists of even number of 1s. The input $\Sigma = \{0,1\}$

PROBLEM 24

Write the regular expression for the language that contains of odd number of 1s. The input same.

PROBLEM 25

Write the regular expression for the language
(i) of any length. the input is $\Sigma = \{a, b\}$.
(ii) excluding ε.

PROBLEM 26

Write the regular expression for the language for the string starting with a

PROBLEM 27

Write the regular expression for the language that contain the substring aa

PROBLEM 28

Write the regular expression for the language. Starting with either a or ab.

PROBLEM (29)

Write the regular expression for the language with the string ending with abb. $\Sigma = \{a, b\}$

PROBLEM (30)

Write the regular exp. for the language whose string begins and ends with double consecutive letters

PROBLEM (31)

Write the regular expression for the language in which the third symbol from the right is a.

PROBLEM (32)

Write the regular expression for the language in which the second symbol from left is b.

Solutions:

(23) $\Sigma = \{0, 1\}$

$L = \{11, 101, 110, 1100 \dots\}$

$RE = (11)^*$

$= (00)^*$ (Even zeroes)

(24) $\Sigma = \{0, 1\}$

$L = \{1, 100, 1000, \dots\}$

$RE = (11)^* . 1$

$= (00)^* . 1$ (Odd zeroes)

(25) (i) $\Sigma = \{a, b\}$

$L = \{\varepsilon, ab, ab, a, b, \ldots\}$

$RE = (a+b)^*$ (includes ε)

(ii) $RE\ (a+b)^+$ (excludes ε)

(26) $L = \{aa, ab, aaa, aab, \ldots\}$

$RE = a \cdot (a+b)^+$

(27) $L = \{aaaa, aaab, baaa, \ldots\}$

$RE = (a+b)^* aa (a+b)^*$

(28) $L = \{a, ab, aab, aba, abaa, \ldots\}$

$RE = a(a+b)^* + ab(a+b^*)$

$= (a+ab)(a+b)^*$

(29) $L = \{aabb, ababb, aaabb, \ldots\}$

$RE = (a+b)^* \cdot abb$

(30) $L = \{aaaa, bbbb, aabb, bbaa, aaabbb, \ldots\}$

$RE = aa(a+b)^*bb + bb(a+b)^*aa$

$= (aa+bb)(a+b)^*(aa+bb)$

(31) $L = \{abb, aaa, aabb, aaaa, baab, \ldots\}$

$RE = (a+b)^* a (a+b)(a+b)$

(32) $L = \{aba, abb, bba, bbb, abaa, \ldots\}$

$RE = (a+b) b (a+b)^*$

Structural Induction:

It is used to prove some proposition $p(x)$ holds for all x of some sort of recursively defined structure such as formulas, list or trees. A well defined partial order is defined on the structure (or) sub formula for formula, sub-list, sub-tree for tree.

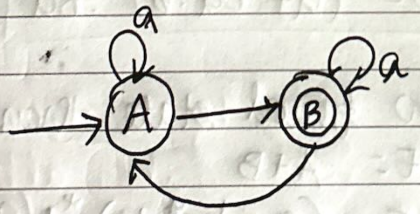
Arden's theorem

Step 1: Find the equation for all the states ~~that~~ is represented in the transition diagram by taking incoming edges of the state.

Step 2: Add ε to the initial state of the given transition table.

Step 3: Frame the regular expression by taking final state in the transition diagram.

PROBLEM 33



If $R = Q + RP$ then

$R = QP^*$

Step 1: Find equation

$A = A \cdot a + B \cdot b + \cancel{\varepsilon} \rightarrow (1)$

$B = B \cdot a + A \cdot b \rightarrow (2)$

Step 2: Epsilon add for initial state.

$A = A \cdot a + B \cdot b + \varepsilon \rightarrow (1)$

$B = Ba + A \cdot b \rightarrow (2)$.

Step 3: Regular expression

$R = Q + Rp$

$\boxed{B = A \cdot b + B \cdot a}$

$R = B$
$Q = A \cdot b$
$P = a$

$R = Qp^*$

$\boxed{B = A \cdot b\, a^*} \rightarrow (3)$.

Substitute (3) in (1),

$(1) \rightarrow \quad A = A \cdot a + B \cdot b + \varepsilon$

$A = A \cdot a + A \cdot ba^* \cdot b + \varepsilon$

$A = \varepsilon + A \cdot a + A \cdot ba^* \cdot b$

$\boxed{R = Q + Rp}$

$A = \varepsilon + A(a + ba^* \cdot b)$

$R = Q + Rp$

$R = Qp^*$

$\boxed{A = (a + ba^* b)^*} \rightarrow (4)$.

$R = A$
$Q = \varepsilon$
$P = (a + ba^* b)$

Substitute (4) in (3)

$B = A \cdot ba^*$

$B = \varepsilon (a + ba^* b)^* \cdot ba^*$

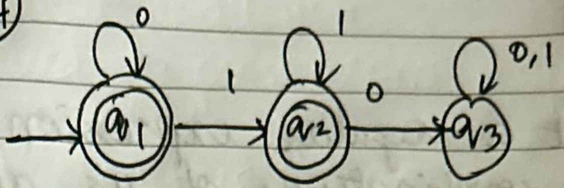
$\boxed{B = (a + ba^* \cdot b)^* \cdot ba^*}$

If 'A' also final state then,

$R.E = A + B$

$R.E = (a + ba^* \cdot b)^* + (a + ba^* \cdot b)^* \cdot ba^*$

PROBLEM 34



Step 1:

$$q_1 = q_1 \cdot 0 + \varepsilon$$
$$q_2 = q_2 \cdot 1 + q_1 \cdot 1$$
$$q_3 = q_3 \cdot 0 + q_3 \cdot 1 + q_2 \cdot 0$$

Step 2:

$$q_1 = q_1 \cdot 0 + \varepsilon \rightarrow (1)$$
$$q_2 = q_2 \cdot 1 + q_1 \cdot 1 \rightarrow (2)$$
$$q_3 = q_3 \cdot 0 + q_3 \cdot 1 + q_2 \cdot 0 \rightarrow (3).$$

$$R = Q + Rp$$
$$q_1 = \varepsilon + q_1 \cdot 0$$
$$R = Q + Rp$$
$$R = Qp^*$$
$$\boxed{q_1 = \varepsilon 0^*}$$

$R = q_1$
$Q = \varepsilon$
$p = 0$

Sub (4) in (2)

$$q_2 = q_2 \cdot 1 + q_1 \cdot 1$$
$$q_2 = q_2 \cdot 1 + \varepsilon 0^* \cdot 1$$
$$q_2 = 1(q_2 + \varepsilon 0^*)$$

$R = q_2$
$Q = \varepsilon 0^* \cdot 1$
$p = 1$

$$R = Q + Rp$$
$$q_2 = \varepsilon 0^* \cdot 1 + q_2 \cdot 1$$
$$R = Qp^*$$
$$q_2 = 0^* \cdot 1 \cdot 1^* \rightarrow (5)$$

Then,

$$q_1 + q_2 = 0^* + 0^* \cdot 1 \cdot 1^*$$

PROBLEM 35

Construct a regular expression to accept all possible combinations of a's and b's over input a,b.

$\Sigma = \{a, b\}$

$L = \{\varepsilon, ab, aab, abb, aabb, \ldots\}$

$R \cdot E = (a+b)^*$

PROBLEM 36

Construct a regular expression that contains any number of a's, b's and c's for input a,b,c.

$\Sigma = \{a, b, c\}$

$L = \{\varepsilon, abc, aabc, abbc, abcc, \ldots\}$

$R \cdot E = a^* b^* c^*$

PROBLEM 37

construct a regular expression that contains atleast one 'a', one 'b', one 'c' for the input a,b,c.

$\Sigma = \{a, b, c\}$

$L = \{abc, aabc, abbc, abcc, \ldots\}$

$R \cdot E = a^* b^* c^*$

PROBLEM 38

Construct a regular expression that contains atleast 2b's for the input a,b.

$\Sigma = \{a, b\}$

$L = \{\varepsilon, abb, aabaabaab, \ldots\}$

$RE: (a+b)^* b (a+b)^* b (a+b)^*$

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PROBLEM (39)

construct a regular expression that contains exactly 2 b's only for the input a,b.

$\Sigma = \{a, b\}$

$L = \{\varepsilon, abb, bab, \ldots\}$

$RE = a^* b a^* b^*$