

Pumping lemma for regular language

Pumping lemma is used to prove that a language is not regular.

Let L be a regular language then there is a constant ' n '
Select a string z such that $|z| \geq n$. Divide z into 3 parts $z = uvw$ in such a way that $|uv| \leq n$, $|v| \geq 1$ and for all $i \geq 0$, $uv^i w$ is in L .

Eg ① Prove $L = \{a^n b^n \mid n \geq 1\}$ is not Regular.

$$L = \{a^n b^n \mid n \geq 1\}$$

Sol

L be a Regular language

$$L = \{ab, aabb, aaabbb \dots\}$$

$$\text{Let } n = 4 \quad z = aabb$$

$$|z| = 4 \geq 4 \checkmark$$

$$\frac{aabb}{V \bar{V} W} \quad |UV| \leq n$$

$$|a \cdot ab| \leq n$$

$$3 \leq 4 \checkmark$$

$$|V| \geq 1 \quad UV^i W \circ$$

$$|ab| \geq 1 \quad a(ab) \cdot b$$

$$2 \geq 1 \quad \Rightarrow ab \in L$$

$$a(ab)^1 \cdot b \Rightarrow a \cdot ab \cdot b \in L$$

$$a(ab)^2 \cdot b$$

$$a \cdot ab \cdot ab \cdot b \notin L$$

↳ Contradiction

∴ It is not Regular.

∴ proved

Sg 2 prove that $L = \{a^p / p \text{ is a prime number}\}$
 is not regular. $|w| \geq n$

$$L = \{aa, aaa, aaaa \dots\}.$$

$$\text{let } n = 3$$

$z = aaa$ such that $|z| \geq n$
 $|aaa| \geq 3$

$$|3| \geq 3$$

Divide 'z' into 3 parts such that
 uvw such that $|uv| \leq n$ ✓
 $|v| \geq 1$ ✓

$$z = \overline{aaa} \quad |a \cdot a| \leq 3$$

$$2 \leq 3 \quad n$$

for $i \geq 0$, $uv^i w$ is in
 $|a| \geq 1 \Rightarrow i \geq 1$ ✓

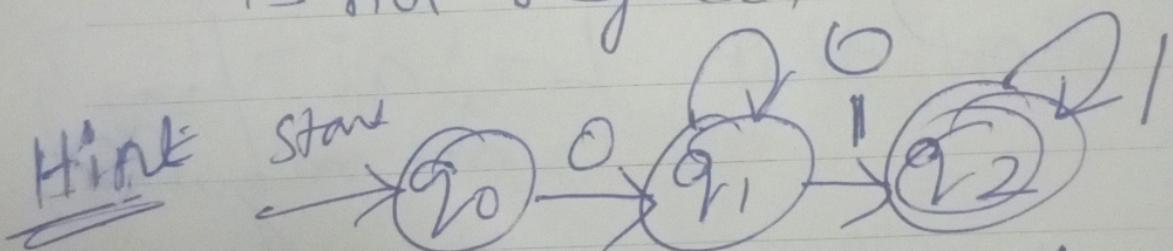
uv^iw

- $i=0 \Rightarrow a \cdot a^0 \cdot a \Rightarrow aaa \in L$
 - $i=1 \Rightarrow a \cdot a^1 \Rightarrow aaa \in L$
 - $i=2 \Rightarrow aaaa \notin L$
- \hookrightarrow contradiction

\therefore It is not regular
 \therefore proved.

Ex 3 prove that $L = \{0^n 1^n \mid n \geq 1\}$

is not regular



0001111

$\{01, 0011, 000111 \dots \}$

$$z = 0^n 1^n$$

$$n = 2 \quad z = 0011$$

uv^iw

Hence $L = \{0^n 1^n \mid n \geq 1\}$ is not regular

H.W Problem

1) $L = \{a^{2n} \mid n \geq 1\}$ is
not regular

2) $L = \{a^n \mid n \geq 1\}$ is not
regular

3) $L = \{a^i b^j c^k \mid k \geq i+j, i, j \geq 1\}$
Show that L is not regular

4) Show that $L = \{a^{i^2} \mid i \geq 1\}$
is not regular

5) $L = \{a^m b a^m \mid m \geq 1\}$ is not
regular