

# Yauibach Normal form.

Defn:- A CFG is in GNF if the production are

(\*) A non terminal generating a terminal

(\*) A non terminal  $A \rightarrow b$  generates a terminal followed by any number of NT

$$A \rightarrow bc_1c_2c_3 \dots$$

conversion of a CFG into GNF.

## Rules

- 1> check for unit and null productions and remove them.
- 2> check whether CFG is CNF, convert if not.
- 3> change the names of NT symbols into some  $A_i$  ascending order of  $i$ .
- 4> Alter the rules so that NT are in ascending order, such that if the production is of the form  $A_i \rightarrow A_j x$  then  $i < j$  and should never be  $i \geq j$ .
- 5> Left recursion.

Null Production.

$$A \rightarrow \epsilon$$

Eg

$$\begin{aligned} S &\rightarrow ABAC \\ A &\rightarrow aA\epsilon \\ B &\rightarrow bB\epsilon \\ C &\rightarrow c \end{aligned}$$

TO eliminate  $\epsilon$

$$\begin{aligned} S &\rightarrow ABAC \quad (A \rightarrow \epsilon); \quad S \rightarrow ABC \\ A &\rightarrow BAC \\ A &\rightarrow Bc \\ a &\rightarrow aA \\ a &\rightarrow a \end{aligned}$$



TO remove  $\epsilon$  for B

New Grammar:  $S \rightarrow ABAC \mid AB\epsilon \mid BAC \mid B\epsilon$

$A \rightarrow aA \mid a$

$B \rightarrow bB \mid \epsilon$

$C \rightarrow c$

Now replace B with  $\epsilon$

$S \rightarrow AAC \mid AC \mid C$

$B \rightarrow b$

New productions:

$S \rightarrow ABAC \mid AB\epsilon \mid BAC \mid B\epsilon \mid AAC \mid AC \mid C$

$A \rightarrow aA \mid a$

$B \rightarrow bB \mid b$

$C \rightarrow c$



## unit production

Any production rule of the form  $A \rightarrow B$  where  $A, B \in \text{Non-terminal}$  is called unit production.

How to remove?

(1) Add  $A \rightarrow x$  rule when  $B \rightarrow x$  occurs in the grammar.  
( $x \in \text{Terminal/ (and Null)}$ )

(2) Delete  $A \rightarrow B$  from the grammar.

(3) Repeat steps.

(eg)  $S \rightarrow xy, x \rightarrow a, y \rightarrow z|b, z \rightarrow M, M \rightarrow N, N \rightarrow a$

Soln  $y \rightarrow z, z \rightarrow M, M \rightarrow N.$

$\therefore N \rightarrow a,$   
 $M \rightarrow a \quad (M \rightarrow N)$

$\therefore S \rightarrow xy, x \rightarrow a, y \rightarrow z|b, z \rightarrow M, M \rightarrow a, N \rightarrow a$

Since  $M \rightarrow a, z \rightarrow M$   
So that  $z \rightarrow a$

Such that

$S \rightarrow xy, x \rightarrow a, y \rightarrow z|b, (z \rightarrow a, M \rightarrow a, N \rightarrow a)$

$\downarrow$   
 $y \rightarrow a.$

$\downarrow$   
remove as they  
are unreachable.



### (3) Chomsky Normal form.

In Chomsky Normal form (CNF) we have a restriction on the length of RHS, which is elements in RHS should either be 2 variables or a terminal.

$$\bullet) A \rightarrow a \quad (\checkmark)$$

$$\bullet) A \rightarrow BC \quad (\checkmark)$$

$A, B, C \rightarrow$  Non-terminals  
 $a \rightarrow$  Terminal.

Example: Convert the CFG into CNF.

$$S \rightarrow cA|BB$$

$$B \rightarrow b|SB$$

$$C \rightarrow b$$

$$A \rightarrow a.$$

Soln :

(1) <sup>on</sup> checking, we know that there is no unit production or Null production.

(2) It is in CNF.

(3) Replace

$$S \rightarrow A_1$$

$$C \rightarrow A_2$$

$$A \rightarrow A_3$$

$$B \rightarrow A_4$$

(Arrange only in ascending order).

Now

$$A_1 \rightarrow A_2 A_3 | A_4 A_4$$

$$A_4 \rightarrow b | A_1 A_4$$

$$A_2 \rightarrow b$$

$$A_3 \rightarrow a$$



4)  $A_i \rightarrow A_j x$ , then  $i < j$  & never be  $i \geq j$

So,  $A_1 \rightarrow A_2 A_3$  |  $A_1 \rightarrow A_4 A_4$   
 $\downarrow$  |  $\downarrow$   
 $i$  |  $j$   
 $1 < 2$  (✓) satisfied. |  $1 < 4$  (✓)

•  $A_4 \rightarrow b$  (already in GNF).

$A_4 \rightarrow A_1 A_4$   
 $4 > 1$  (x)

$A_4 \rightarrow b | \underline{A_1} A_4$

$A_4 \rightarrow b | \underline{A_2} A_3 A_4 | A_4 A_4 A_4$

$A_4 \rightarrow b | b A_3 A_4 | A_4 A_4 A_4$   
 $\downarrow$   
 $i = j$

↓ left recursion.

(5) Remove left recursion.

↳ introduce a new variable

$A_4 \rightarrow b | b A_3 A_4 | \textcircled{A_4} A_4 A_4$

$Z \rightarrow A_4 A_4 Z | A_4 A_4$

$A_4 \rightarrow b Z | b A_3 A_4 Z | b | b A_3 A_4$

✓ Normal form

Now grammar is

$$A_1 \rightarrow A_2 A_3 / A_4 A_4$$

$$A_4 \rightarrow b A_3 A_4 / b z / b A_3 A_4 A_2$$

$$z \rightarrow A_4 / A_4 / A_4 A_4 z$$

$$A_2 \rightarrow b$$

$$A_3 \rightarrow a.$$

Now replace to get GNF form

$$A_1 \rightarrow b A_3 / b A_4 / b A_3 A_4 A_4 / b z A_4 / b A_3 A_4 A_2$$

$$A_4 \rightarrow b / b A_3 A_4 / b z / b A_3 A_4 A_2$$

$$z \rightarrow b A_4 / b A_3 A_4 A_4 / b z A_4 / b A_3 A_4 A_4 /$$
$$b A_4 z / b A_3 A_4 A_4 z / b z A_4 A_2 / b A_3 A_4 A_2 A_4 z$$

$$A_2 \rightarrow b$$

$$A_3 \rightarrow a.$$