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MODULE - I

Theory of computation

- **Automata Theory**
  - Finite Automata
  - Pushdown Automata
  - Linear Bound Automata
  - Turing machine
- **computability** (whether can be solved or not)
  - Decidable
  - Undecidable
- **Complexity**
  - Time
  - Space

Formal language in theory of computation is a set of strings with well-defined rules for determining whether the particular string is accepted by the string or not.

Theory of computation is also known as automata theory. It is a theoretical branch of computer science and mathematics which deals with a set of computation for the given input.

Formal languages:

* Regular grammar
* context-free grammar
* context-sensitive grammar
* Unrestricted grammar
* Recursive
* Recursively enumerable

Formal Languages

Regular Grammar

Non-terminal ← $X \rightarrow a$ → Terminal.

$X \rightarrow aY$

Above is an example of regular grammar (Non-terminal followed by terminal or terminal and a non-terminal)

Context free grammar.

$X \rightarrow a$

Above is an example of context-free grammar.
(Non-terminal followed by terminal, or more terminals)

Context-sensitive grammar

$X \rightarrow Y$

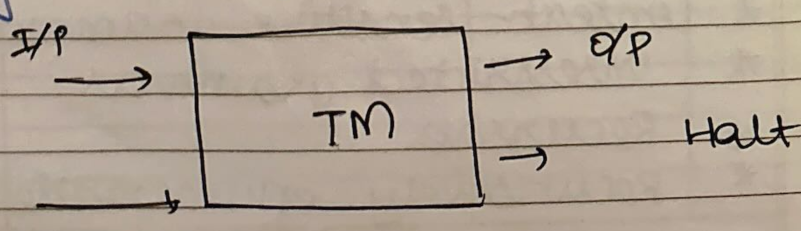
Above is an example of context-sensitive grammar
(Non-terminal followed by a non-terminal).

Unrestricted grammar

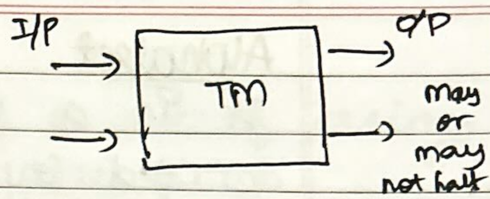
It can be anyway. There is not restriction

Recursive

If a string has to be accepted by a turing machine, it halts for the input.



Recursively enumerable



If a string has to be accepted by a turing machine, it may or may not hat

Automata Theory

Finite Automata.

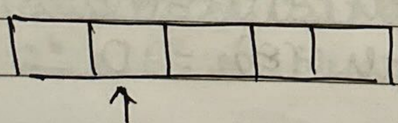
In finite automata, depending on the input given the current state acts. No memory unit.
Eg: Automated door system

Push down Automata.

In pushdown automata, stack is the memory used by the system. In this case, random access is not possible.

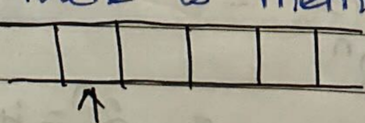
Linear-Bound Automata

In linear bound automata, finite tape is the memory unit. In this system, size of the tape cannot be altered.



Turing machine

In the turing machine, infinite tape is the memory unit. Random access is possible, and there is memory used up a lot.



Alphabet

It is a finite, non-empty set of strings, denoted by Σ.

$\Sigma = \{0, 1\} \rightarrow$ Binary

$\Sigma = \{A, B, \dots Z\} \rightarrow$ capital

$\Sigma = \{a, b, \dots z\} \rightarrow$ small.

Strings

$W = 0100101$

$\Sigma = \{0, 1\}$

$W = abaabcc$

$\Sigma = \{a, b, c\}$

Empty string $\quad \varepsilon$ (or) λ

Reverse string $\quad x = x^{a}$

concatenate

$x \quad y \Rightarrow x \cdot y$ (or) xy

Length of String

length(automata) = 8

length (ε) = 0

Substring

$y = w\underline{x}z$

Power Set

2^n

$A = \{1, 2\} \quad = 2^{a} = 4$

$\{\phi, 1, 2, (1, 2)\}$.

Language

Language of automata is a set of strings that is accepted by the automata.

Eg: Set of ~~odd~~ odd length strings over an alphabet $\{0,1\}$

Set of even length strings over an alphabet $\{a,b\}$

Set Notation form of the language : L

$L = \{ w \mid w$ contains equal no. of 0's and 1's $\}$

complement of the language : $\bar{L}$

$\bar{L} = \{ w \mid w$ does not contain equal no. of 0's and 1's $\}$.

Proof

(i) Deductive

If hypothesis then conclusion.

Eg: If $x > 4$, $2^x > x^2$; For $x \le 2$

$x = 5, 6, \dots$

$x = 5 \Rightarrow 2^5 > 5^2 \Rightarrow 32 > 25$ True

$x = 6 \Rightarrow 2^6 > 6^2 \Rightarrow 64 > 36$ True

$x = 4 \Rightarrow 2^4 > 4^2 \Rightarrow 16 > 16$ False

$x = 3 \Rightarrow 2^3 > 3^2 \Rightarrow 8 > 9$ False

$x = 2 \Rightarrow 2^2 > 2^2 \Rightarrow 4 > 4$ False

$x = 1 \Rightarrow 2^1 > 1^2 \Rightarrow 2 > 1$ True.

(ii) Inductive

(a) Basis. $n = 0$ (or) 1 initially depending on condition

LHS and RHS should be equal after

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substituting n.
then $f(n)$ is considered to be true.

(b) Inductive assumption.
If $f(n)$ is true, then $f(n+1)$ also true.

PROBLEM ①

$$1+2+3+\dots+n = \frac{n(n+1)}{2}$$

Basis

LHS: $n=1$

RHS: $\frac{n(n+1)}{2} = \frac{1(1+1)}{2} = \frac{1(\cancel{2})}{\cancel{2}} = 1$

$\therefore$ LHS = RHS
Hence, $f(n)$ is true.

Inductive assumption

$$n = n+1$$

LHS:

$$1+2+3+\dots+n = \sum_{i=1}^{n} i$$

$$\Downarrow$$

$$\sum_{i=1}^{n} i = \sum_{i=1}^{n+1} i$$

$$= \sum_{i=1}^{n} i + (n+1)$$

$$= \frac{n(n+1)}{2} + (n+1)$$

$$= \frac{n(n+1) + 2(n+1)}{2}$$

$$= \frac{n^2+n+2n+2}{2}$$

$$= \frac{n^2+3n+2}{2}$$

RHS:

$$\frac{n(n+1)}{2} = \frac{(n+1)((n+1)+1)}{2}$$

$$= \frac{(n+1)(n+2)}{2}$$

$$= \frac{n^2+2n+n+2}{2}$$

$$= \frac{n^2+3n+2}{2}.$$

$\therefore$ LHS = RHS

If $f(n)$ is true, then $f(n+1)$ also true.

HW

PROBLEM 1

If x is a sum of squares of four positive integers, then 2^x should be greater than x^2.

PROBLEM 2

For all $n>0$,

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Soln:

1. x is sum of squares of 4 positive integers.

Let $x = 1^2+2^2 = 1+4 = 5$

$2^x > x^2$

$2^5 > 5^2 \Rightarrow 32>25$ True

$x = 2^2+3^2 = 4+9 = 13$

$2^{13} > 13^2$ True.

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(2) Basis:

LHS: $n = 1^2 = 1$

RHS: $\frac{1(1+1)(2(1)+1)}{6} = \frac{1(2)(3)}{6} = 1$

LHS = RHS , then f(n) is true.

<u>Inductive Assumption</u>

$n = n+1$

LHS: $\sum_{i=1}^{n} i^2 = \sum_{i=1}^{n+1} i^2$

$= \sum_{i=1}^{n} i^2 + (n+1)^2$

$= \frac{n(n+1)(2n+1)}{6} + (n+1)^2$

$= (n+1)\left(\frac{2n^2+n+6n+6}{6}\right)$

$= \frac{2n^3+n^2+6n^2+6n+2n^2+n+6n+6}{6}$

$= \frac{2n^3+9n^2+13n+6}{6}$

RHS: $\frac{n(n+1)(2n+1)}{6}$

$= \frac{(n+1)((n+1)+1)(2(n+1)+1)}{6}$ $(n = n+1)$

$= \frac{(n+1)(n+2)(2n+3)}{6}$

$= \frac{(n^2+2n+n+2)(2n+3)}{6}$

$= \frac{(n^2+3n+2)(2n+3)}{6}$

$= \frac{2n^3+9n^2+13n+6}{6}$ $\Rightarrow$ LHS = RHS

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Kleen closure $\Sigma = \{0,1\}$

$\Sigma^* = \{\varepsilon, 0, 01, 011, \dots\}$ (zero must be in odd numbers)

Kleen plus (or) Positive closure

$\Sigma^+ = \{0, 01, 011, 0001, \dots\}$ (ε or λ mustn't be added)

Important Laws

I) Set Identities

1) $A \cap U = A$ Identity laws
$A \cup \phi = A$

2) $A \cup U = U$ Domination laws
$A \cap \phi = \phi$

3) $A \cup A = A$ Idempotent laws
$A \cap A = A$

4) $\overline{(\overline{A})} = A$ Complementation law

5) $A \cup B = B \cup A$ Commutative laws
$A \cap B = B \cap A$

6) $A \cup (B \cup C) = (A \cup B) \cup C$ Associative laws
$A \cap (B \cap C) = (A \cap B) \cap C$

7) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ Distributive laws
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

8) $\overline{A \cap B} = \overline{A} \cup \overline{B}$ De Morgan's laws
$\overline{A \cup B} = \overline{A} \cap \overline{B}$

9) $A \cup (A \cap B) = A$
$A \cap (A \cup B) = A$ Absorption laws

10) $A \cup \bar{A} = U$; $U' = \phi$
$A \cap \bar{A} = \phi$; $\phi = U$ complement laws

11) $A - B = A \cap B'$ Set Difference law

12) $(A \cap B) \subseteq A$
$(A \cap B) \subseteq B$ Inclusion for Intersection

13) $A \subseteq (A \cup B)$
$B \subseteq (A \cup B)$ Inclusion for Union

14) $(A \subseteq B) \wedge (B \subseteq C) \rightarrow (A \subseteq C)$ Transitive Property of Subsets

PROBLEM 2

$$1^3 + 2^3 + 3^3 + \cdots\cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Basis:

LHS: $n^3 = (1)^3 = 1$

RHS: $\left(\frac{n(n+1)}{2}\right)^2 = \left(\frac{1(1+1)}{2}\right)^2 = \left(\frac{1(2)}{2}\right)^2 = 1^2 = 1$

$\therefore$ LHS = RHS

Hence, $f(n)$ is true.

Inductive Assumption

$n = n+1$

LHS:

$$\sum_{1}^{n+1} i^3 = \sum_{1}^{n} i^3 + (n+1)^3$$

$$= \left(\frac{n(n+1)}{2}\right)^2 + (n+1)^3$$

$$= \left(\frac{(n^2+n)}{2}\right)^2 + (n+1)^3$$

$$= \frac{n^4+n^2+2n^3}{4} + n^3+1+3n^2+3n$$

$$= \frac{(n^4+n^2+2n^3)+(4n^3+4+12n^2+12n)}{4}$$

$$= \frac{n^4+6n^3+13n^2+12n+4}{4}$$

RHS:

$$\left(\frac{n(n+1)}{2}\right)^2 = \left(\frac{(n+1)((n+1)+1)}{2}\right)^2$$

$$= \left(\frac{(n+1)(n+2)}{2}\right)^2$$

$$= \frac{(n+1)^2(n+2)^2}{4}$$

$$= \frac{(n^2+2n+1)(n^2+4n+4)}{4}$$

$$= \frac{n^4+4n^3+4n^2+2n^3+8n^2+8n+n^2+4n+4}{4}$$

$$= \frac{n^4+6n^3+13n^2+12n+4}{4}$$

$\therefore$ LHS = RHS

If $f(n)$ is true, then $f(n+1)$ also true.

PROBLEM 3

$$1+3+5+\dots+(2n-1) = n^2$$

Basis:

LHS: $(2n-1) = 2(1)-1 = 2-1 = 1$

RHS: $n^2 = (1)^2 = 1$ $\quad\therefore$ LHS=RHS

Hence, $f(n)$ is true.

Inductive Assumption

$n = n+1$

LHS: RHS:

$1+3+5+\dots+(2n-1) = n^2$

$[1+3+5+\dots+(2n-1)] + 2(n+1)-1 = (n+1)^2$

$\Downarrow$

$n^2+2n+1 = n^2+2n+1$

$\therefore$ LHS = RHS

If $f(n)$ is true, then $f(n+1)$ is true

PROBLEM (4)

$2+4+6+\dots+2n = n^2+n$

Basis

LHS: $2n = 2(1) = 2$

RHS: $n^2+n = (1)^2+1 = 1+1 = 2$

$\therefore$ LHS = RHS

Hence, $f(n)$ is true.

Inductive Assumption

LHS: RHS:

$2+4+6+\dots+2n = n^2+n$

$[2+4+6+\dots+2n] + (2n+2) = n^2+n$

$\downarrow$

$(n^2+n) + (2n+2) = (n+1)^2 + (n+1)$

$n^2+3n+2 = (n^2+2n+1)+(n+1)$

$n^2+3n+2 = n^2+3n+2$

$\therefore$ LHS = RHS

If $f(n)$ is true, then $f(n+1)$ also true.

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Deductive proof:

Statement 1:
All students are active
Statement 2:
Ram is active.
conclusion:
Ram is a student.

Premises & $S1, S2, \ldots n$
⇓
Conclusion

The statements & conclusion define the argument.

Additional forms of proof:

(i) Proof by sets

(ii) Proof by contradiction

(iii) Proof by counter examples

PROOF:
It is a valid argument that establishes the truth of mathematical statement

(i) Proof of sets:

* The union of two sets A and B is equal to union of B and A.

$$A \cup B = B \cup A$$

* The intersection of two sets A and B is equal to intersection of B and A.

$$A \cap B = B \cap A.$$

(ii) Proof by contradiction

* If a number is divisible by 2 and 3, then the number is divisible by 6.
* If a number is not divisible by 2 and 3, then the number is not divisible by 6.

(iii) Proof by counter example

* All prime numbers are odd numbers.

2, 3, 5, 7, ...

* All students are lazy.

Validness of a Statement:

⇒ : Premises are true, therefore, conclusion is true

⇔ : It is impossible for all premises to be true, therefor conclusion is false.

Important Questions

2 Marks

1) Define TOC.
2) Types of TOC
3) Define formal languages
4) Types of formal languages
5) Kleen closure, Kleen Positive

16 marks

1) What is proof? Types of proof.
2) Write any 4 laws (8 mark)
3) A problem would be given, prove it.
4) Explain the concept of automata theory.