

Regular expression

→ Regular expression (RE) is defined as

→ ϵ is RE corresponding to the

$$L = \{\epsilon\}$$

→ \emptyset is RE

$$L = \{\}$$

→ x is RE

$$L = \{x\}$$

regular language

→ If x is RE over language $L(x)$

union y is RE

$$L(y)$$

→ $x+y$ is RE corresponding to
 $L(x) \cup L(y)$

where $L(x \cup y) = L(x) \cup L(y)$

FEB 2022	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S
	•	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19

→ $x \cdot y$ is RE corresponding to
 $L(x) \cdot L(y)$

where $L(x \cdot y) = L(x) \cdot L(y)$

Closure

→ R^* is RE corresponding to
 $L(R^*) = (L(\epsilon))^*$

Applications

→ Data validation

e.g.: Compiler

Regular Language

↳ Language that can be defined
 by ~~expressions~~

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 \rightarrow DFA

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 \rightarrow Regular expression

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 \rightarrow RE denoting language with strings

have any number of a's over

1

$$\Sigma = \{a\}$$

2

$$RE = a^*$$

3

(1) any no's of 'a' ~~or~~ or any no's b's

4

$$RE = (a+b)^*$$

5

(2) RE denoting L with strings starts with 'a' and end with 'b'

6

$$RE = a(a+b)^*b$$

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If P and α are two RE over Σ ,
 and if P does not contain ϵ , then
 the following equation in R given by
 $R = \alpha + RP$ has a unique solution
 i.e $R = \alpha P^*$

sohn

$$R = \alpha + RP \rightarrow \textcircled{1}$$

$$= \alpha + \alpha P^* P$$

$$\boxed{R = \alpha P^*}$$

$$= \alpha (\epsilon + P^* P)$$

$$(\epsilon + P^* R = R^*)$$

$$= \alpha P^*$$

Hence Proved

$$R = \alpha + RP$$

$$= \alpha + [\alpha + RP]P$$

$$= \alpha + \alpha P + RP^2$$

$$= \alpha + \alpha P + [\alpha + RP]P^2$$

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DAY 054-311 WEEK 09

WEDNESDAY

2022

FEBRUARY

$$8 = Q + QP + QP^2 + RP^3$$

$$9 \quad \vdots \\ \vdots$$

$$10 = Q + QP + QP^2 + \dots + QP^n + RP^{n+1}$$

$$11 [R = QP^n]$$

$$12 = Q + QP + QP^2 + \dots + QP^n + QP^n P^{n+1}$$

$$2 = Q \underbrace{(E + P + P^2 + \dots + P^n + P^n P^{n+1})}_{\text{closure of } P}$$

$$3 R = QP^n$$

FEB	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S							
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FEBRUARY

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THURSDAY

24

Identity Rules for RE

* Simplify the RE

* $\phi \rightarrow$ null set

$\epsilon \rightarrow$ null string

If P, Q, R are
the RE then

$$\text{Rule 1} : \phi + \gamma = \gamma$$

$$2 : \phi \cdot \gamma = \gamma \cdot \phi = \phi$$

$$3 : \epsilon \cdot \gamma = \gamma \cdot \epsilon = \gamma$$

$$4 : \gamma + \gamma = \gamma$$

$$5 : \gamma^* \cdot \gamma^* = \gamma^*$$

$$6 : \gamma^* \gamma = \gamma \gamma^* = \gamma^*$$

$$7 : (\gamma^*)^* = \gamma^*$$

$$8 : \epsilon + \gamma^* \gamma = \epsilon + \gamma \gamma^* = \gamma^*$$

$$9 : (PQ)^* P = P(QP)^*$$

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DAY 056-309 WEEK 09

FRIDAY

2022

Conversion of Finite

FEBRUARY

automata to RE

$$I/P = FA$$

$$O/P = RE$$

→ Arden's method

$$R = Q + RP$$

↓

$$R = QP^*$$

→ state elimination method

eg: Conversion of RE from FA

Arden's method

→ equation for each state based

on incoming edges.

→ Add ϵ to the equation of initial state

→ Simplify the equation using Arden's

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SATURDAY

26

FEBRUARY

and find RE

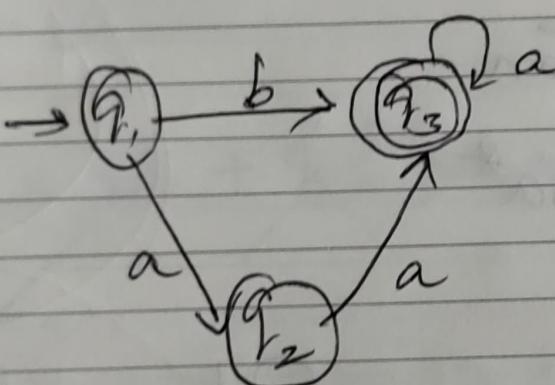
conditions

→ FA should not contain E

transition

→ FA should have only one initial state.

eg: 1



$$\varphi_1 = \epsilon \quad \text{--- } ①$$

$$\varphi_2 = \varphi_1 a \quad \text{--- } ②$$

$$\varphi_3 = \varphi_1 b + \varphi_2 a + \varphi_3 a \quad \text{--- } ③$$

Apply ① & ② in ③

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$$q_3 = \ell \cdot b + q_1 a a + q_2 a$$

$$q_3 = b + \ell \cdot a a + q_2 a$$

$$q_2 = (b + a a) + q_2 a$$

~~q_1~~ \downarrow \downarrow \downarrow

$$R = Q + R P$$

(Identity Rule)

$$\ell \cdot b = b$$

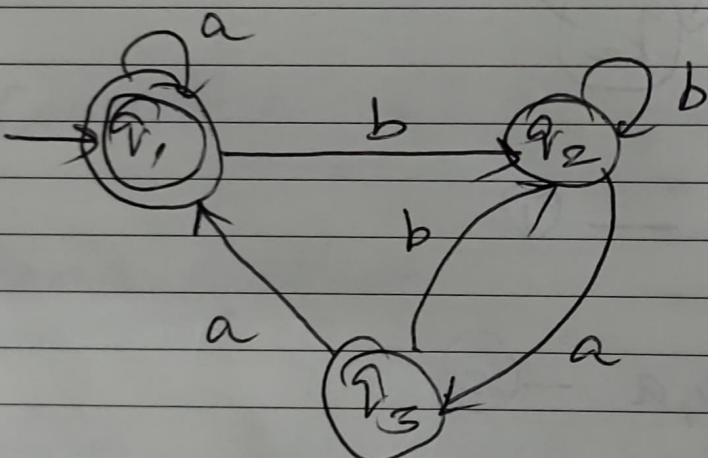
$$\ell \cdot a a = a a$$

$$R = Q P^*$$

unique solution

$$R = (b + a a) a^*$$

eg 2:-



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FEBRUARY

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MONDAY

28

$$q_1 = q_1 a + q_3 a + \epsilon - \textcircled{1}$$

$$q_2 = q_2 b + q_1 b + q_3 b - \textcircled{2}$$

$$q_3 = q_2 a - \textcircled{3}$$

$\textcircled{3}$ in $\textcircled{1}$

$$\cancel{q_2 = q_1 b + q_2 b + q_3 ab}$$

$$\cancel{q_2 =}$$

$$q_1 = q_1 a + q_2 a + \epsilon'$$

$$= q_1 a + (q_2 b + q_1 b + q_3 b) a + \epsilon$$

$$= q_1 a + q_2 ab + q_1 ab + q_3 ab + \epsilon$$

$$\cancel{q_1 = q_1 a + q_2 ab (q_1 + q_2 + q_3)}$$

$$\cancel{Q = Q + RP}$$

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③ in ②

$$q_2 = q_1 b + q_2 b + q_2 ab$$

$$q_2 = q_1 b + q_2 (b+ab)$$

$$R = \cancel{q_1} + \cancel{R} P^* \rightarrow R = QP^*$$

$$\boxed{q_2 = q_1 b (b+ab)^*} \rightarrow ④$$

Apply ③ in ①

$$q_1 = q_1 a + q_2 aa + \epsilon \rightarrow ⑤$$

Apply ④ in ⑤

$$q_1 = q_1 a + q_1 b (b+ab)^* aa + \epsilon$$

$$q_1 = \epsilon + q_1 (a+b(b+ab)^* aa)$$

$$R = Q + RP \rightarrow R = QP^*$$

~~$$q_1 = \epsilon \cdot (a+b(b+ab)^* aa)^*$$~~

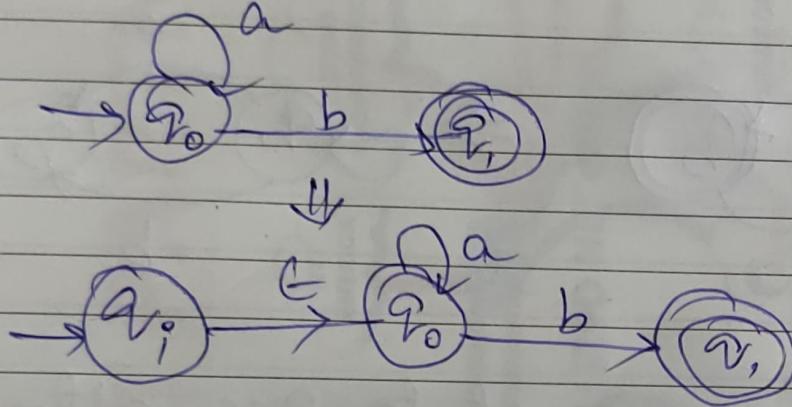
$$q_1 = (a+b(b+ab)^* aa)^*$$

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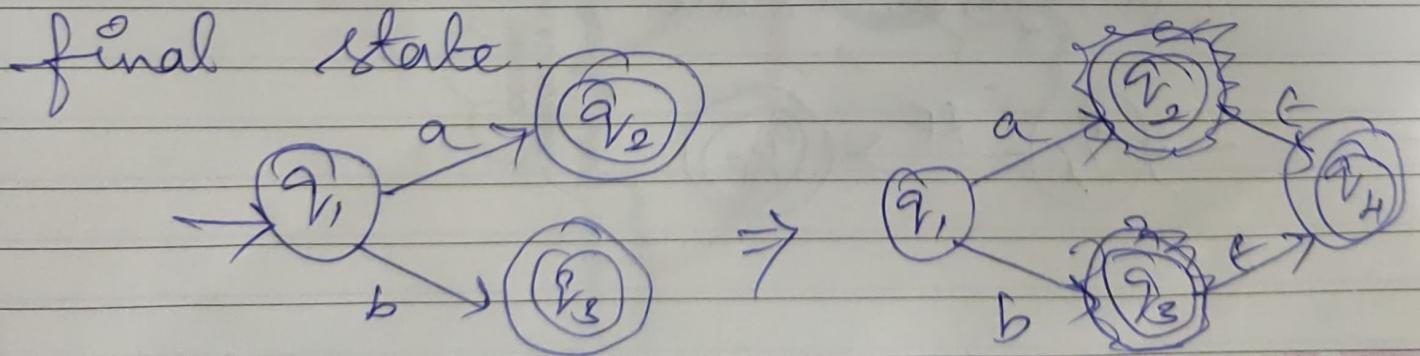
Conversion of FA to RE

(State eliminating method)

- ① Initial state should not have incoming edge. If exist create new state and make it as initial state.



- ② FA should have single final state
If there exist multiple final states, create new state and make it as final state

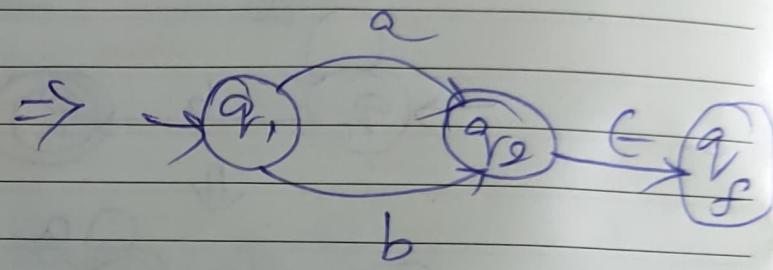
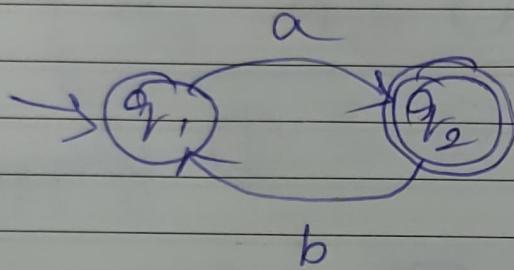


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(3)

→ Final state should not have outgoing edge. If exist create new state and make it as a final state.



(4)

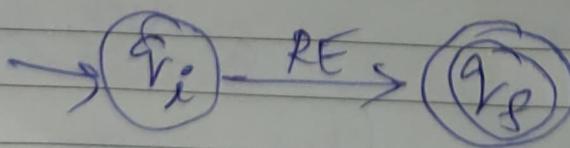
→ Eliminate every state one after another

(5)

(don't have any order)
FA should have only ~~one~~ two states

(i) initial state

(ii) final state



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01

DAY 060-305 WEEK 10

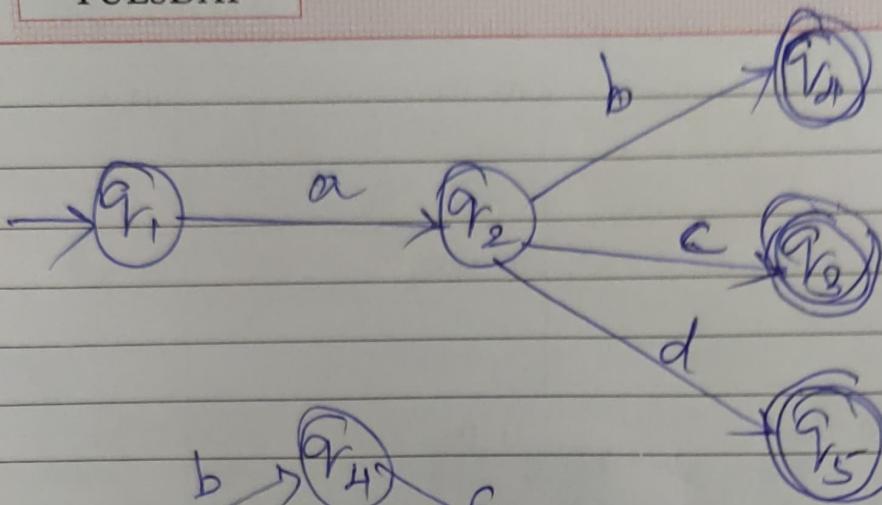
TUESDAY

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eg: 1

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Eliminating q_{r_4}

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 $a+b$

a

b

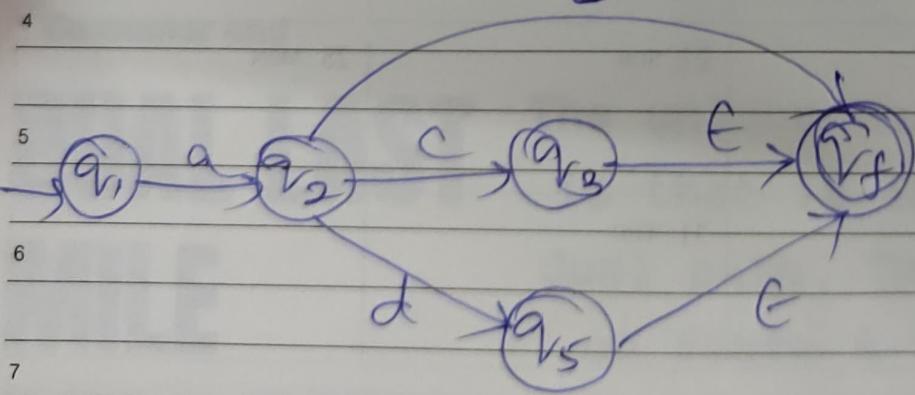
 a^*

a

 ab

a

b

 $b \cdot e$ 

MAR

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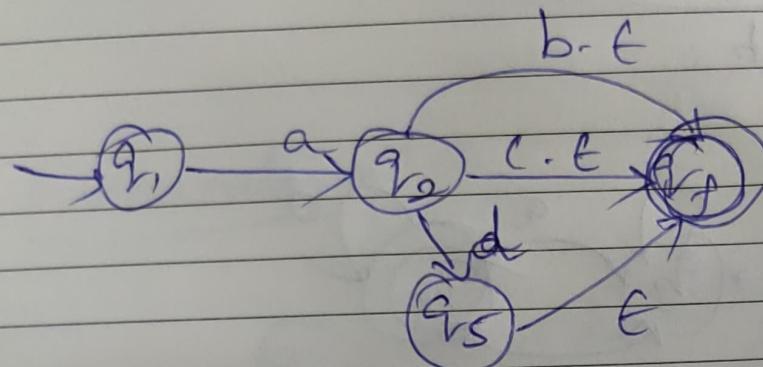
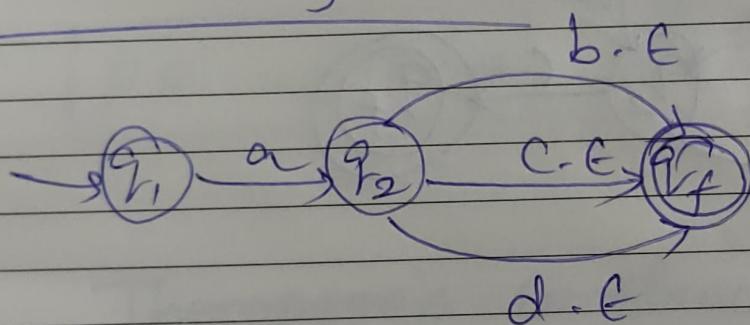
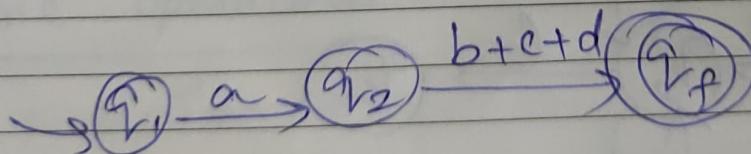
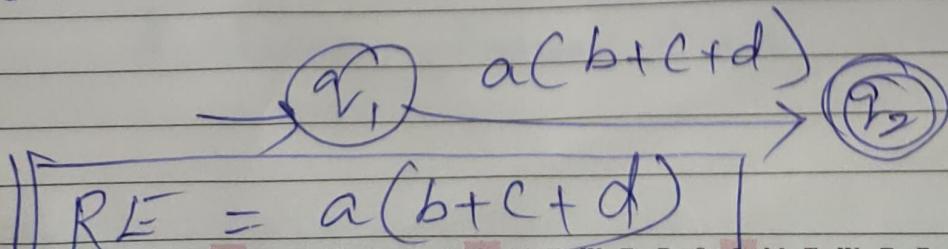
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DAY 061-304 WEEK 10

WEDNESDAY

02

Eliminating q_3 Eliminating q_5 EliminatingEliminating q_2 

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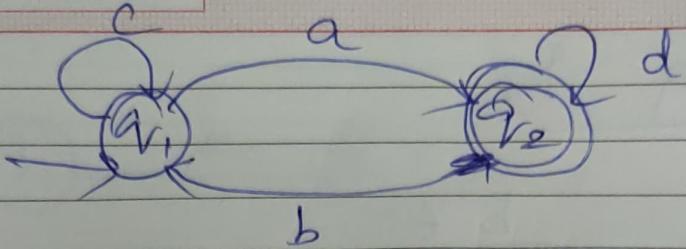
DAY 062-303 WEEK 10

THURSDAY

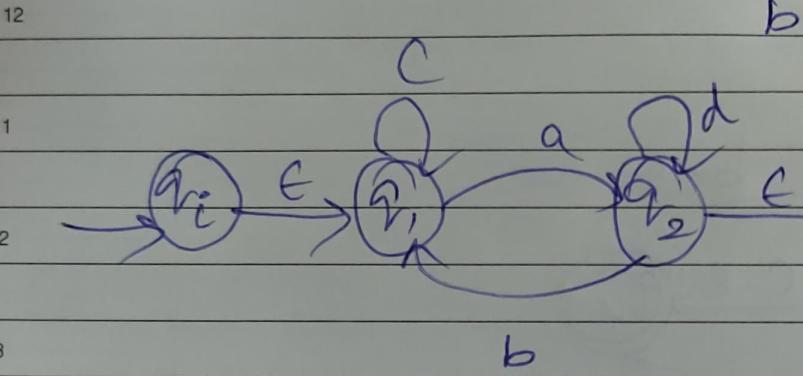
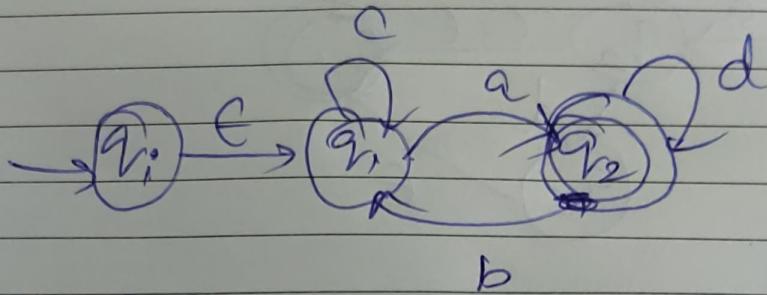
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MARCH

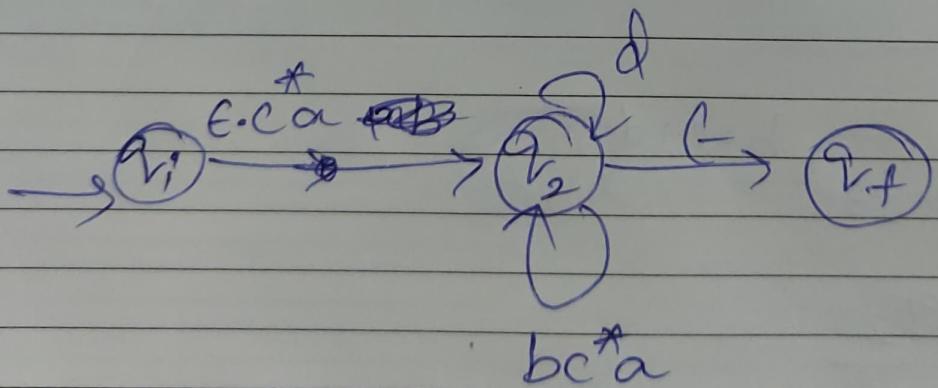
eg 2:-



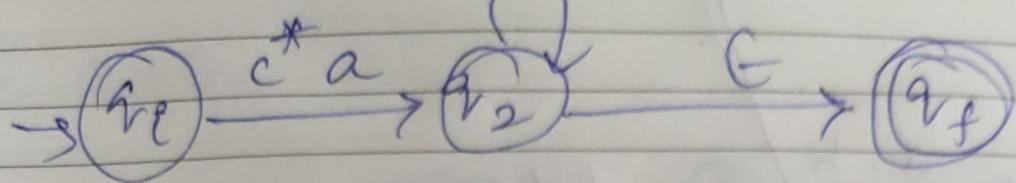
sohn



Eliminate q_1

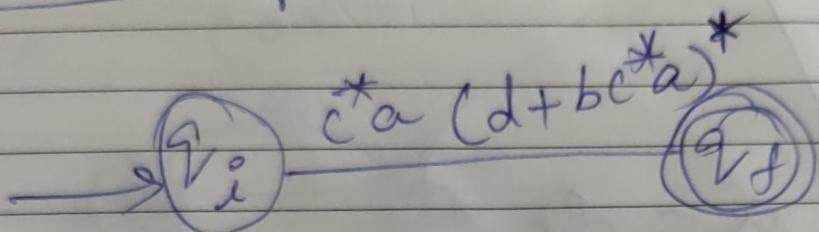


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9

10 Eliminating q_2



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$$1 \quad RE = c^* a (d + b c^* a)^*$$

3

4 Thompson's construction of convert

5

RE to ϵ -NFA

6

Basic operators

7

1. union (+)

2. concatenation (.)

3. Kleene closure (*)

APR

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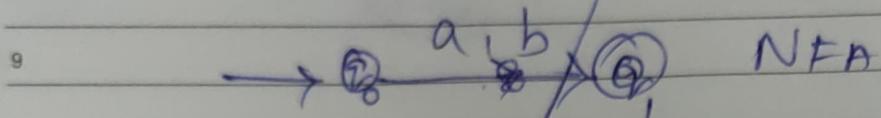
MARCH

05

DAY 064-301 WEEK 10

SATURDAY

8 17 a+b



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E-NFA

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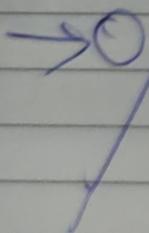
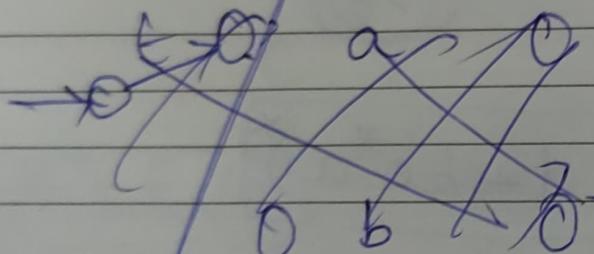
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It's also known as McNaughton

Yamada-Thompson Algorithms, is a method of transferring a regular expression into an equivalent

NFA / E-NFA and then one can convert to

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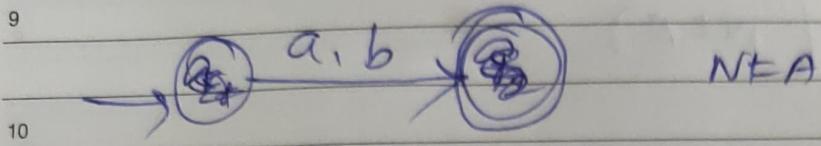
2022

MARCH

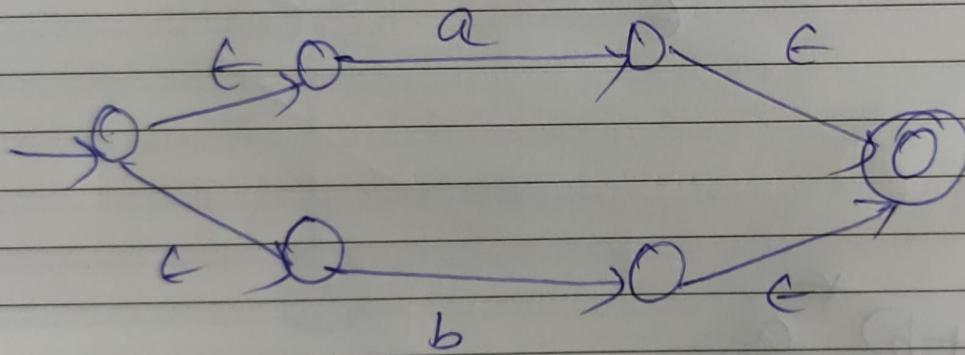
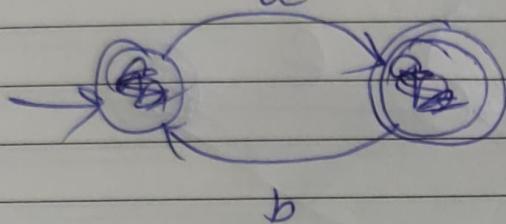
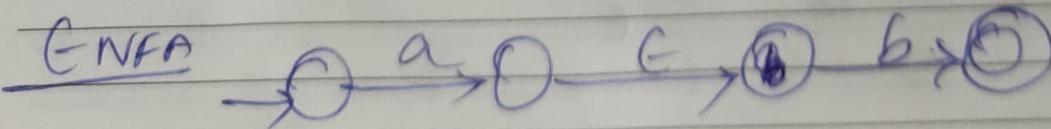
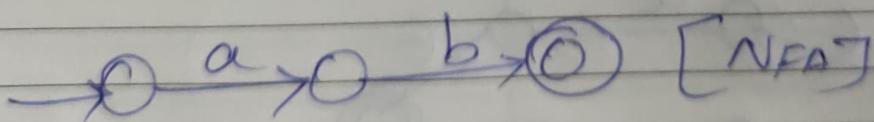
DAY 065-300 WEEK 10

SUNDAY

06

8 1) $a+b$ 

11 E-NFA

6 ② ~~(NFA)~~ ab

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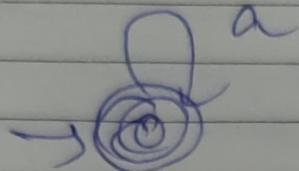
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DAY 066-299 WEEK 11

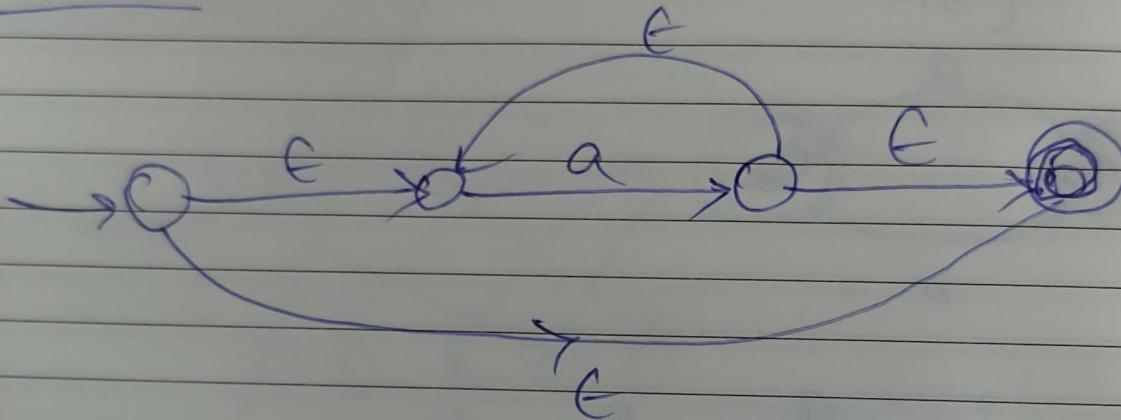
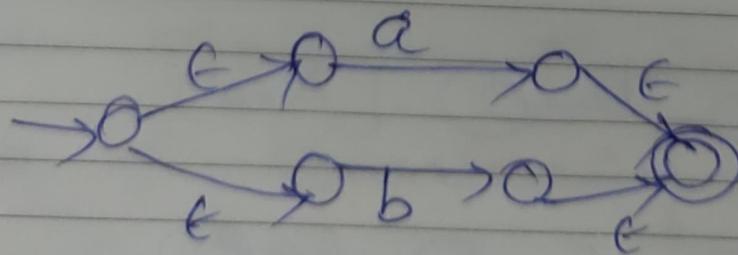
MONDAY

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(3) a^* 

[NFA]

E-NFA~~Ex 1~~ $(a+b)^* a$ $(a+b)$ 

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DAY 067-298 WEEK 11

TUESDAY

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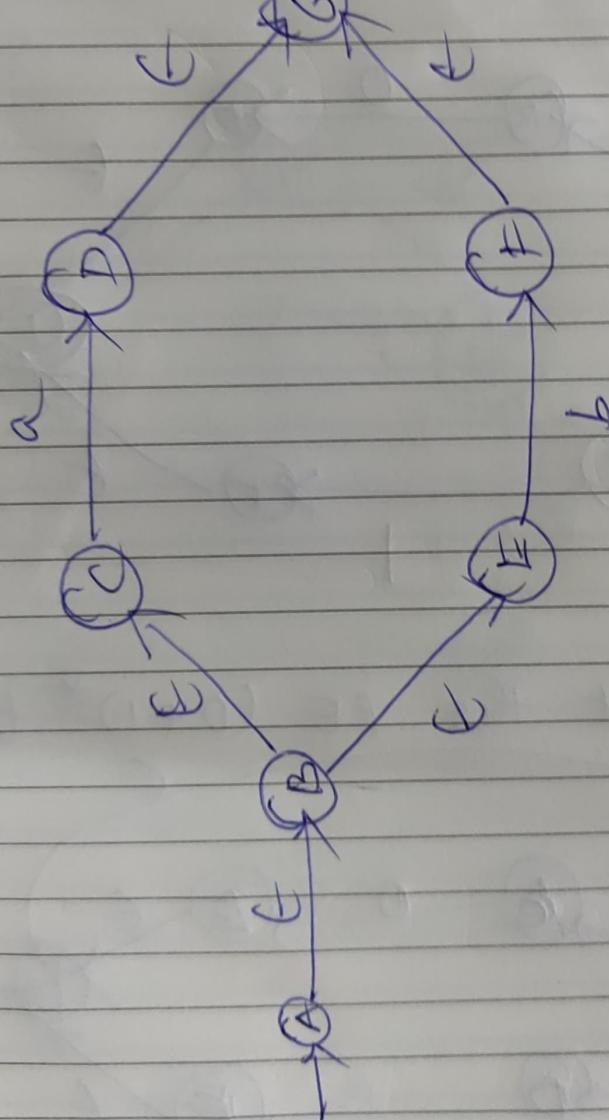
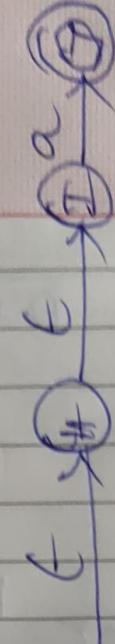
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DAY 068-297 WEEK 11

WEDNESDAY

2022

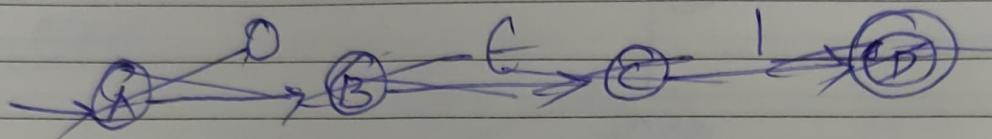
MARCH

eg 2: Convert RE $(0+1)^*$ to FA

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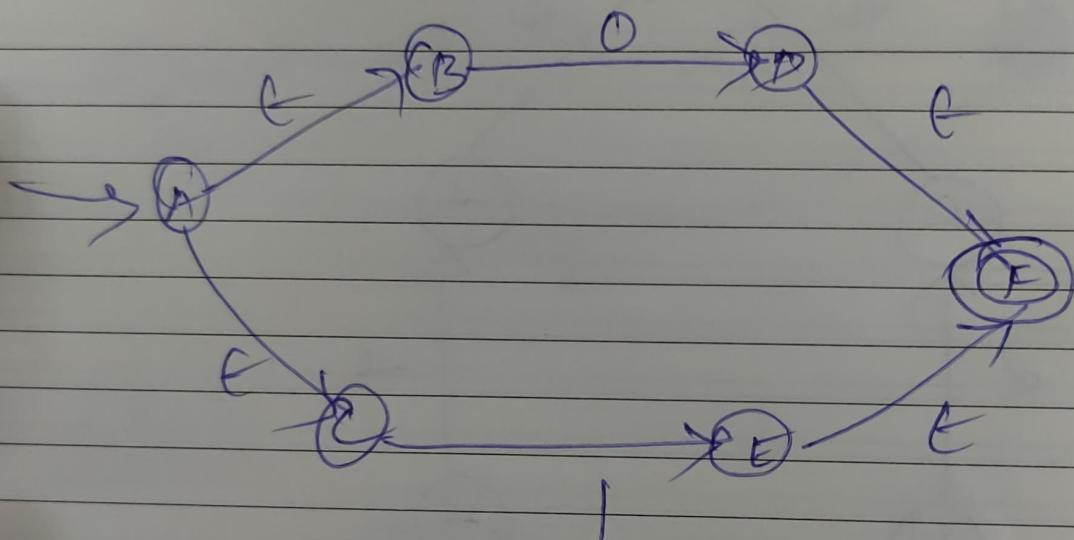
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MAR 2022 M T W T F S S M T W T F S S M T W T F S S M T W T F S S S

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2022

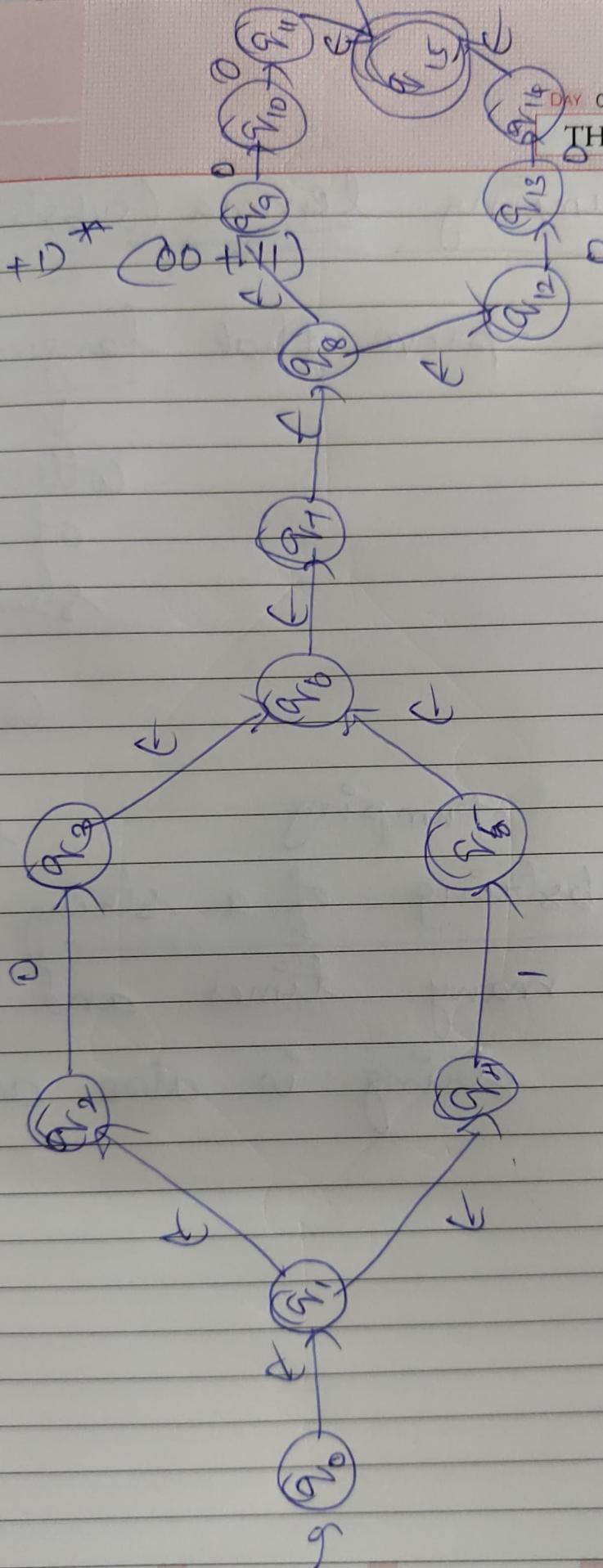
MARCH

DAY 069-296 WEEK 11

THURSDAY

10

8) $(O+D)^*$ $(O+T)$



M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S								
•	•	•	•	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	•	APR 2022

APR

MAY

JUN

11

DAY 070-295 WEEK 11

FRIDAY

2022

MARCH

Pumping lemma (substring)

→ used to prove that language is
not RE

↓
collection
of
strings

↓
Collection

of
symbols.

→ if substring of a string is
repeated many times and if the
resultant string is also available

MAR	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S						
2022	•	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	•	•