$R = Q + RP$

$= Q + (Q + RP)P$

$= Q + QP + RP^2$

$= Q + QP + (Q + RP)P^2$

$= Q + QP + QP^2 + RP^3$

- - - - -

$= Q + QP + QP^2 + \cdots QP^n + RP^{n+1}$

$= Q + QP + QP^2 + \cdots QP^n + QP^* P^{n+1}$

$= Q(\Lambda + P + P^2 + \cdots P^n + P^* P^{n+1})$

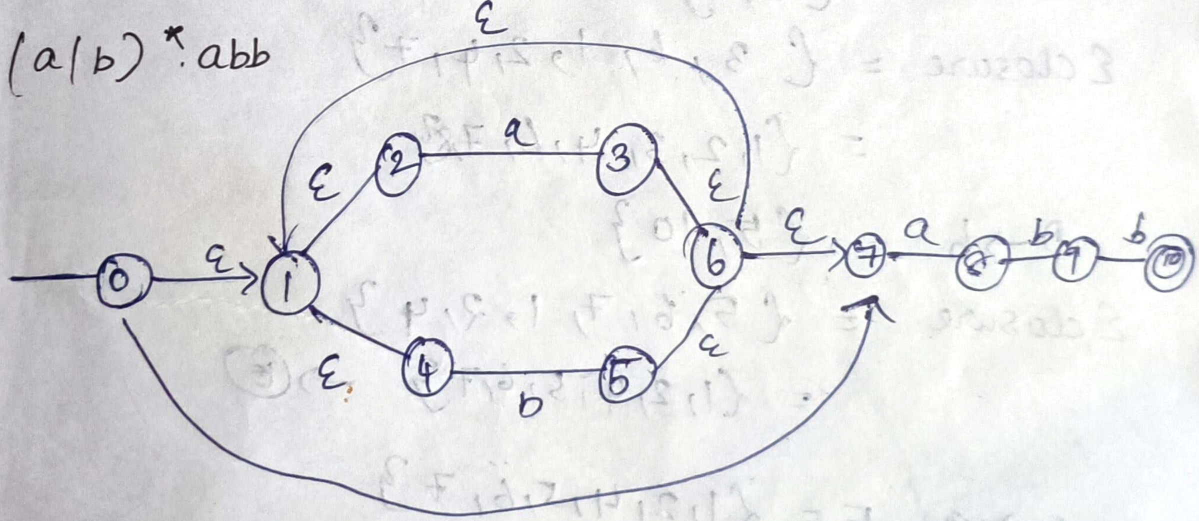
$= QP^*$

MINIMIZATION OF DFA :-

(or)

State reduction Method :- (or) formula Method :-

① $(a|b)^* . abb$



ε-closure $(0) = \{0, 1, 2, 4, 7\} \Rightarrow A$

$A \rightarrow a = \{3, 8\}$

ε closure $\{3, 8\} = \{3, 6, 7, 1, 2, 4, 8\}$

$= \{1, 2, 3, 4, 6, 7, 8\} \Rightarrow B$

$A \rightarrow b = \{5\}$

Eclosure $(5) = \{5, 6, 7, 1, 2, 4\}$

$= \{1, 2, 4, 5, 6, 7\} \Rightarrow C$

B = {1, 2, 3, 4, 6, 7, 8}

B → a = {3, 8}

ε closure = {1, 2, 3, 4, 6, 7, 8} ⇒ B

B → b = {5, 9}

ε closure = {5, 6, 1, 2, 4, 9, 7}

= {1, 2, 4, 5, 6, 7, 9} → D

C = {1, 2, 4, 5, 6, 7}

C → a = {3, 8}

ε closure = {1, 2, 3, 4, 6, 7, 8}

C → b = {5}

= {5, 6, 1, 2, 4} = {1, 2, 4, 5, 6

D → a =

D = {1, 2, 4, 5, 6, 7, 9}

D → a = {3, 8}

ε closure = {3, 6, 1, 2, 4, 7}

= {1, 2, 3, 4, 6, 7, 8}

D → b = {5, 10}

ε closure = {5, 6, 7, 1, 2, 4}

= {1, 2, 4, 5, 6, 7} → E

E → a E = {1, 2, 4, 5, 6, 7}

E → a = {3, 8}

ε closure = {1, 2, 3, 4, 6, 7, 8}

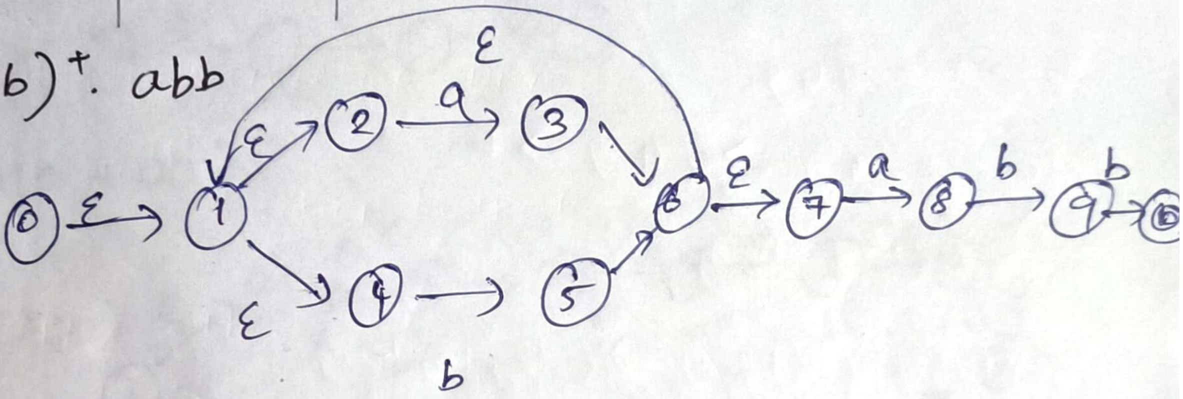
E → b = {5, 9}

= {5, 6, 1, 2, 4, 9, 7}

~~States~~

ip / States	a	b
A	B	C
B	B	D
C	B	C
D	B	E
E	B	C

2) $(a/b)^+ . abb$



ε- closure 0 :- $\{0, 1, 4\} \Rightarrow A$

$A \rightarrow a$:-

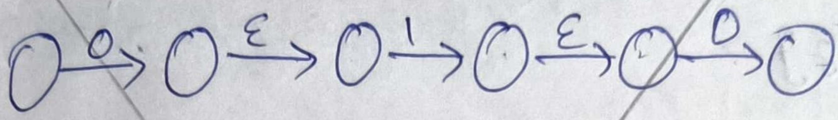
Construct a NFA for the following expression using thompson's construction method & then convert it into DFA.

① $(010+00)^*(10)^*$

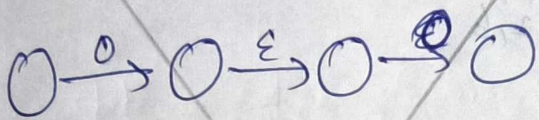
② $(a+b)^*a(a+b)$

① $(010+00)^*(10)^*$

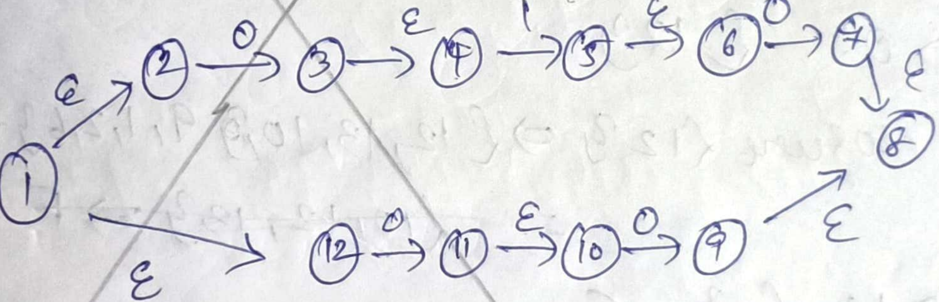
010:



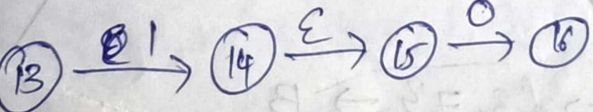
00:



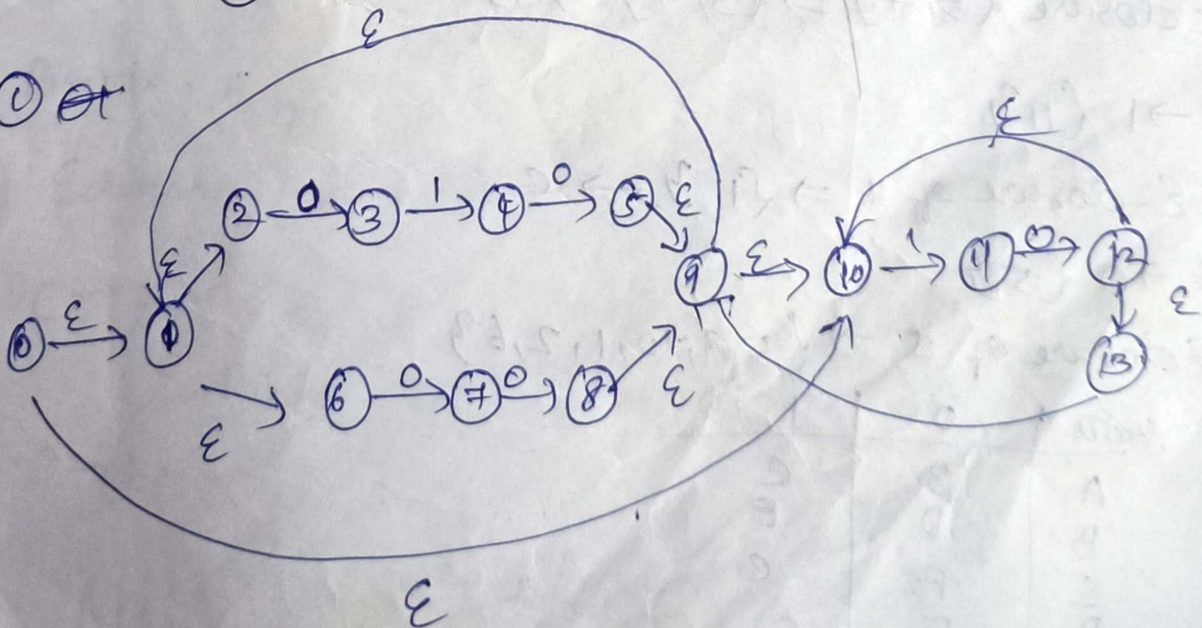
(010 + 00)



10:



① ~~01~~



ε closure (0) = {0, 1, 10, 2, 6}

= {0, 1, 2, 6, 10} ⇒ A

A → 0 :- ⇒ ~~{2, 3}~~, {3, 7}

ε closure {3, 7} ⇒ {3, 7} ⇒ B

A → 1 :- ~~{4}~~ {11}

ε closure {11} ⇒ {11} ⇒ C

B → 0 :- {8}

ε closure {8} ⇒ {8, 9, 10, 1, 2, 6} ⇒ D

B → 1 :- {4}

ε closure {4} ⇒ {4} ⇒ E

C → 0 :- {12}

ε closure {12} ⇒ {12, 13, 10, 9, 1, 2, 6} → F

~~= {10, 12, 13} ⇒ F~~

C → 1 :- {11} = ε

D → 0 :- {3, 7}

ε closure {3, 7} ⇒ {3, 7} → B

D → 1 {11}

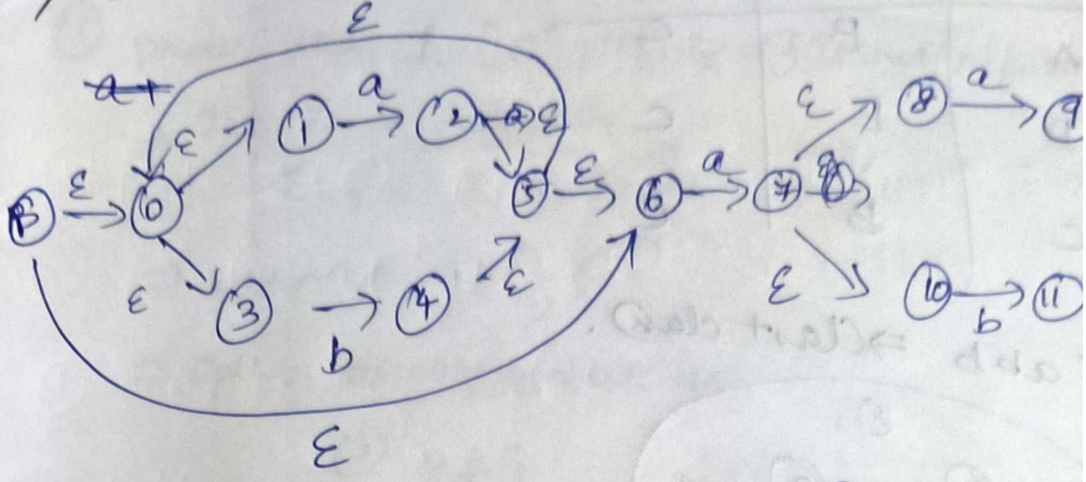
ε-closure of 11 ⇒ {11} → C

E → 0 {5}

ε closure of 5 → {5, 9, 10, 1, 2, 6}

States	0	1
A	B	C
B	D	E
C	F	C
D	B	C

2) $(a+b)^* a(a+b)$



ε- closure$(13) = \{13, 0, 1, 3, 6\}$

$= \{0, 1, 3, 6, 13\} \Rightarrow A.$

$A \to a : \{2, 7\}$

ε- closure$\{2, 7\} = \{2, 5, 6, 0, 1, 3\}$

$= \{0, 1, 2, 3, 5, 6\} \Rightarrow B$

$A \to b : \{4\}$

ε- closure$\{4\} = \{4, 5, 6, 0, 1, 3\}$

$= \{0, 1, 3, 4, 5, 6\} \Rightarrow C.$

$B \to a :- \{2, 7\}$

εclosure$\{2, 7\} = \{2, 5, 0, 1, 3, 6\}$

$= \{0, 1, 2, 3, 5, 6\} \Rightarrow B.$

$B \to b : \{4\}$

εclosure$\{4\} = \{0, 1, 3, 4, 5, 6\} \Rightarrow C.$

$C \to a :- \{2, 7\}$

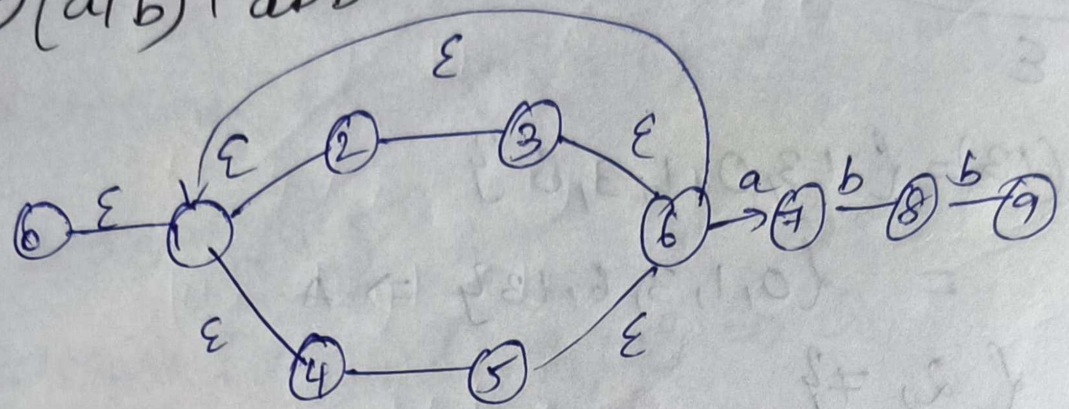
εclosure$\{2, 7\} = \{0, 1, 2, 3, 5, 6\} \Rightarrow$

$C \to b : \{4\}$

εclosure$\{4\} = \{0, 1, 3, 4, 5, 6\} \to$

States	a	b
A	B	C
B	B	C
C	B	C

3) $(a|b)^* abb$ ⇒ (last class).



ε closure $0 = \{0, 1, 2, 4\} \rightarrow A$

$A \rightarrow a \{3\}$

ε closure $(3) = \{3, 6, 1, 2, 4\} \rightarrow B$

$A \rightarrow b \{5\}$

εclosure $\{5\} = \{5, 6, 1, 2, 4\} \rightarrow C$

$B \rightarrow a \{3, 7\}$

ε closure $\{3, 6, 1, 2, 4\} \rightarrow B$

$C \rightarrow a \{3, 7\} \rightarrow B$

$C \rightarrow b \{5\} \rightarrow C$

States	a	b
A	B	C
B	B	C
C	B	C

PUMPING LEMMA METHOD

① prove that $L = \{0^n 1^n \mid n \geq 1\}$ is not regular

$\Rightarrow$ Let L is regular

$$L = \{\overset{n=1}{01}, \overset{n=2}{0011}, \overset{n=3}{000111}, \overset{n=4}{00001111}, \ldots\}$$

$\Rightarrow$ $w = 0^n 1^n$, $|w| \geq n$

$\Rightarrow$ Split $w = xyz$ such as

(i) $y \neq \varepsilon$ $\quad |y| \geq 1$

(ii) $|xy| \leq n$

(iii) $xy^k z \in L \ \forall \ k \geq 1$

$$\underset{x}{0}\ \underset{y}{0}\ \underset{z}{11} \qquad x = 0 \quad y = 0 \quad z = 11$$

(ii) $|xy| = |00| = 2 \leq 2$

(iii) $xy^k z \in L \ \forall \ k \geq 1$

$xy^k z \quad k \geq 1$

$k = 1$, $xyz = 0011 \in L$

$k = 2$, $xy^2 z \Rightarrow 00^2 11 \Rightarrow 00011 \notin L$

$\therefore L$ is not a regular lan...

2) $L = a^n b^n c^n,\ n \geq 1\}$

$L = \{\overset{n=1}{abc}, \overset{n=2}{aabbcc}, \overset{n=3}{aaabbbccc}, \dots\}$

Split the w.

aabbcc

$x = aa$

$y = bb$ ~~Or~~ $z = cc$

(i) $y \neq \varepsilon$ $|y| \geq 1$

(ii) $|xy| \leq n$

$|aabb| \leq 2$

$4 \leq 2$ Condition false

(iii) $xy^k z \in L$

$k=1,\ xyz = aabbcc \in L$

$k=2,\ aabbbbcc \notin L$

3) $L = \{a^i b^j \mid i < j\}$

$L = \{b, abb, aabbb, \dots\}$ $L = \{a^n b^{n+1}, n \geq 1\}$

$w = abb$

$x = a$ $y = b$ $z = b$

(i) $y \neq \varepsilon$ $|y| \geq 1$

(ii) $|xy| \leq n$

$|ab| \leq 1$

$2 \leq 1$ false

(iii) $xy^k z \in L$

$k=1,\ xyz = abb \in L$

$k=2,\ xy^2z = abbb \notin L$

4) $L = \{a^n b a^n, n \geq 1\}$

$n=1$, $L = \{\overset{n=1}{aba}, \overset{n=2}{aabaa}, aaabaaa, \ldots\}$

$w = aabaa$

$x = aa \quad y = b \quad z = aa.$

(i) $y \neq \varepsilon \quad |y| \geq 1$

(ii) $|xy| \leq n$

$|aab| \leq 2$

$3 \leq 2$ false.

(iii) $xy^k z \in L$

$k=1, \quad xyz = aabaa \in L$

$k=2, \quad xy^2 z = aabbaa \notin L$

5) $L = \{0^i 1^j, i > j\}$

$i=1 \; j=0. \quad i = n+1 \; j = n$

$L = \{0, 001, \underline{00011}, 0000111, \ldots\}$

$w = 00011$

$x = 0 \quad y = 00 \quad z = 11$

(i) $y \neq \varepsilon \quad |y| \geq 1$

(ii) $|xy| \leq n$

$|000| \leq 2$, false.

(iii) $xy^k z \in L$

$k=1 \Rightarrow xyz = 00011 \in L$

$k=2 \Rightarrow xy^2 z = 0000011 \notin L$

6) $L = \{0^{2n} ; n \geq 1\}$

$n = 1, 2, 3 \ldots$

$L = \{00, 0000, 000000, \ldots$

split, 0000

$x = 0 \quad y = 00 \quad z = 0$

(ii) $|xy| \leq n \Rightarrow 3 \leq 2 \rightarrow$ false

(iii) $xy^Kz \in L$

$K=1,\ xyz = 0000 \in L$

$K=2,\ xy^2z = 000000 \in L$

PRODUCTION RULES

$G = (V, T, P, S)$

V ⇒ variables (non terminals)
A, B, C, D, ...

T ⇒ terminals
a, b, c, d, ..., 0, 1, 2, 3, ... #, $, ?

P ⇒ set of productions

S ⇒ starting ~~rule~~. symbol.

1) $A \to 0A$ $\quad B \Rightarrow \#$
$A \to B$
$V = \{A, B\}$ $\quad P = 3$
$T = \{0, 1, \#\}$ $\quad S = \{A\}$

2) $S \to AB \mid ba$
$A \to a0$ $\quad B \to b \mid \varepsilon$
$V = \{A, B\}$
$T = \{a, b, 0, \varepsilon\}$
P ⇒
1) $S \to AB$
2) $S \to ba$
3) $A \to a0$
4) $B \to b$
5) $B \to \varepsilon$

$S \Rightarrow \{S\}$

3) $S \to aSa$
$S \to bSb$
$S \to c$
$V = \{S\}$
$T = \{a, b, c, S\}$
$P = 3$
$S = \{S\}$

left most defirnition

right most definition

4) $S \to aS$

$S \to \varepsilon$

deriene the String "aaaaaa"

$S \to aS$

$\to aaS$

$\to aaaS$

$\to aaaS$

$\to aaaaS$

$\to aaaa\cancel{S}\varepsilon$

$\to aaaaaa$

5) $E \to E+E \,/\, E*E \,/\, (E) \,/\, id$ derieve the string "id + id * id"

$E \to E+E$

$E \to E*E$

$E \to E$

$E \to id$

$E \to E+E$

$\to id + E$

$\to id + E*E$

$\to id + id * id$

6) $S \to aB \,/\, bA$

$A \to a \,/\, aS \,/\, bAA$

$B \to b \,/\, bS \,/\, aBB$

"aabbabba"

$S \to aB$ $\quad$ $A \to bAA$

$S \to bA$ $\quad$ $B \to b$

$A \to a$ $\quad$ $B \to bS$

$A \to aS$ $\quad$ $B \to aBB$

~~S → aB~~
~~→ aaB~~
~~→ aabb~~
~~→ aaBB~~
~~→ aab~~

A → aS
→ aaB
→ aabS
→ aabbA
→ aabbaS
→ aabbabA
→ aabbabbA
→ aabbabba

aabbabba

S → aB
→ aaBB
→ aabbS
→ aabbaB
→ aabbabS
→ aabbabbA
→ aabbabba.

7) S → aAS|a
A → SbA|SS|ba
generate :- "aabbaa"

S → aAS
S → a
A → SbA
A → SS
A → ba

S → aAS
→ aSbAS
aa bb aa.

① For generating a language that generate equal no. of a's and b's in the form $a^n b^n$ create CFG

$S \to aAb$

$A \to aAb / \varepsilon$

$L = \{ab, aabb, aaabbb, \dots\}$

Consider aaabbb,

$V = \{S, A\}$

$T = \{a, b, \varepsilon\}$

$P = S \to aAb$

$A \to aAb$ $\quad P \Rightarrow \{3\}$

$A \to \varepsilon$

$P \Rightarrow \{3\}$

$S \Rightarrow \{S\}$

$S \to aAb$

$\to aaAbb$

$\to aaaAbbb$

$\to aaa\varepsilon bbb$

$\to aaabbb$

② Construct CFG for the language having any no. of a's ~~and~~ over the set $\Sigma = \{a\}$

$S \Rightarrow aS$

$S \Rightarrow \varepsilon$

$L = \{a, aa, aaa, aaaa, aaaaa, \dots\}$

Consider aaa,

$V = \{S\}$

$T = \{a, \varepsilon\}$

$P = \{2\}$

$S = \{S\}$

$S \to aS$

$\to aaS$

$\to aaaS$

$\to aaa\varepsilon$

$\to aaa.$

3) construct CFG for the language $L = \{w c w\}$ where $w \in (a,b)$

$S \to aSa$ i/p :- "abbcbba"

$S \to bSb$

$S \to c$

$V = \{S\}$

$T = \{a, b, c\}$

$P = \{3\}$

$S = \{S\}$

$S \to aSa$

$\to abSba$

$\to abbSbba$

$\to abbcbba$

PARSE TREE (OR) DERIVATION TREE (OR) RIGHT MOST (OR) LEFT MOST TREE

$S \to bSb / a / b$

i/p string "bbabb"

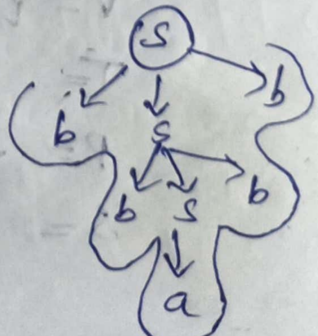
S

$\downarrow$

bSb

$bbSbb$

$bbabb$



$S \rightarrow AB \mid \varepsilon$

$A \rightarrow aB$

$B \rightarrow Sb$ i/p "aabbb"

$V = \{S, A, B\}$

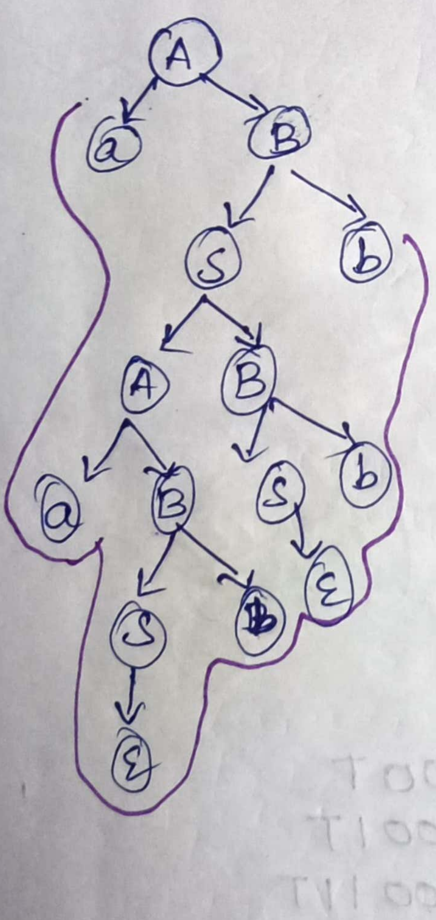
$T = \{a, b\}$

$P = \{4\}$

$S = \{S\}$

$\cancel{S \rightarrow AB \rightarrow aBB \rightarrow aSbSb \rightarrow aA}$

$A \rightarrow aB \rightarrow aSb \rightarrow aABb \rightarrow aaBBb \rightarrow aaSbBb \rightarrow aaSbSbb \rightarrow aabbb$



3) $E \rightarrow E+E$

$E \rightarrow E*E$

$E \rightarrow a|b|c$ i/p: "a+b*c"

$V = \{E\}$

$T = \{a, b, c\}$

$P = \{5\}$

$S = \{E\}$

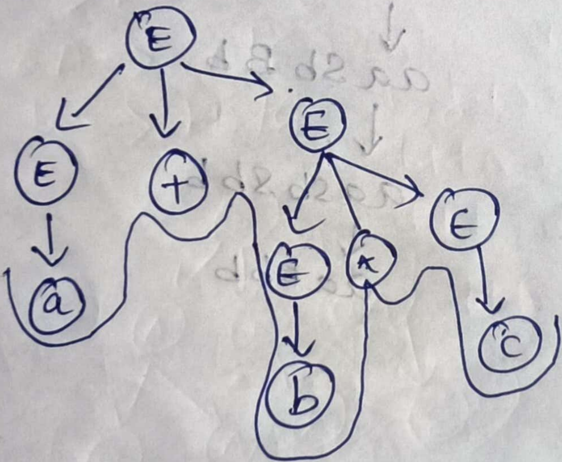
$E \rightarrow E+E$

$\rightarrow a+E$

$\rightarrow a+\cancel{Bb}E*E$

$\rightarrow a+b*E$

$\rightarrow a+b*c$



4) $S \rightarrow T00T$

$T \rightarrow 0T$

$T \rightarrow 1T$ input: "1000111"

$T \rightarrow \varepsilon$

$S \rightarrow T00T$

$\rightarrow 1T00T$

$\rightarrow 10T00T$

$\rightarrow 10\varepsilon 00T$

$\cancel{\rightarrow 100T00T}$

$\cancel{\rightarrow 100}$

$\rightarrow 1000T$

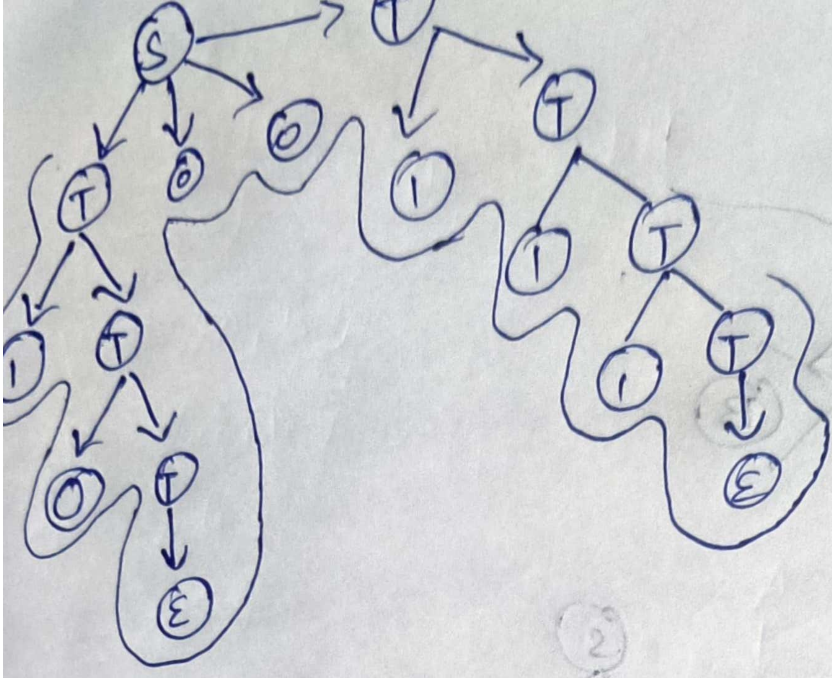
$\rightarrow 10001T$

$\rightarrow 100011T$

$\rightarrow 1000111T$

$\rightarrow 1000111\varepsilon$

$\rightarrow 1000111$

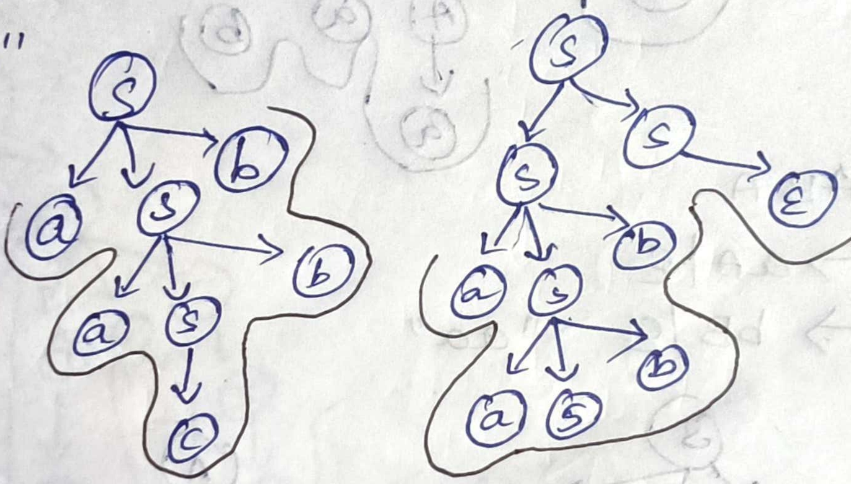


AMBIGOUS:

$S \rightarrow aSb / SS$

$S \rightarrow \varepsilon$

"aabb"

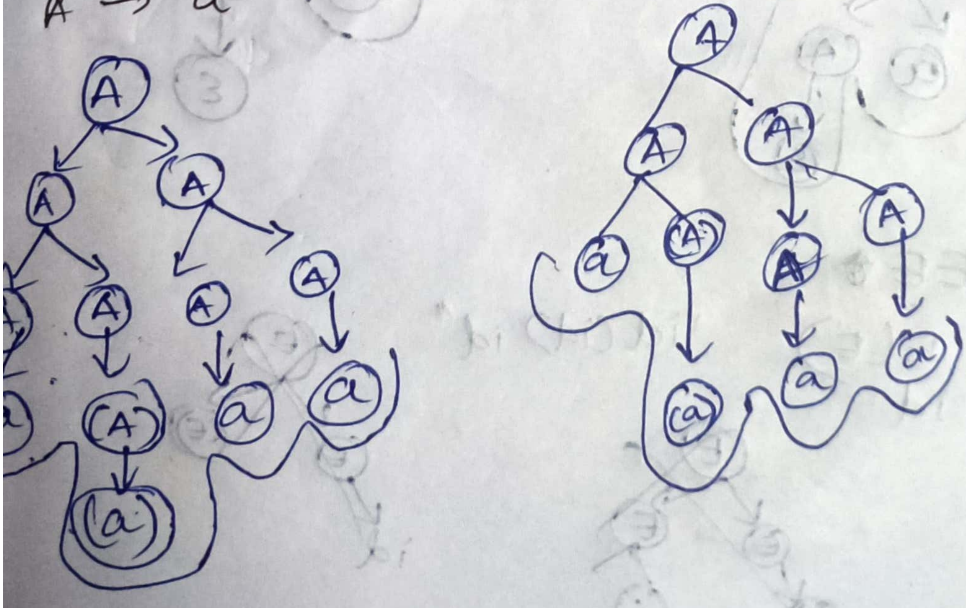


$A \rightarrow AA$

$A \rightarrow (A)$

$A \rightarrow a$

"a(a).aa"

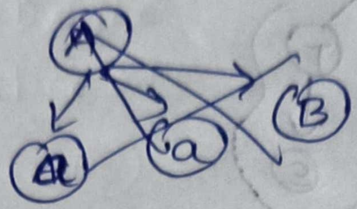


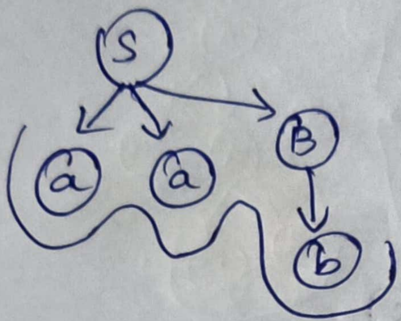
3) $S \to AB \mid aaB$

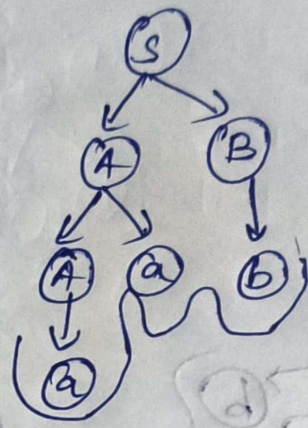
$A \to a \mid Aa$

$B \to b$

"aab"



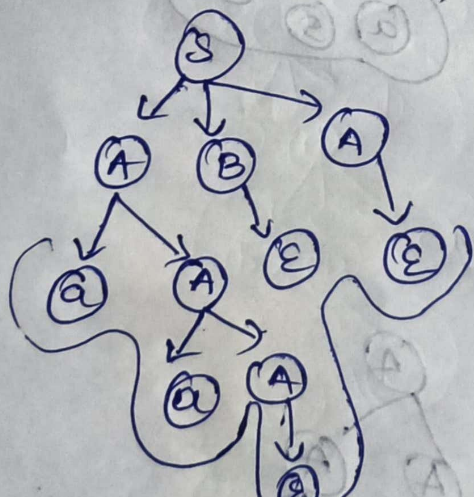


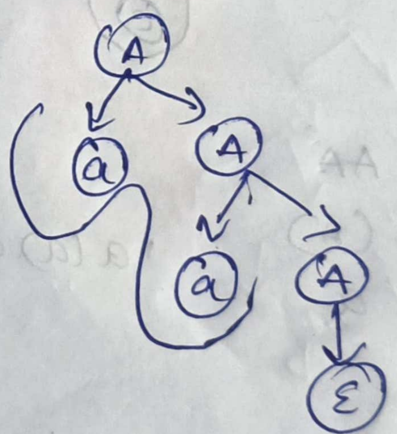


4) $S \to ABA$

$A \to aA \mid \varepsilon$

$B \to bB \mid \varepsilon$ "aa"

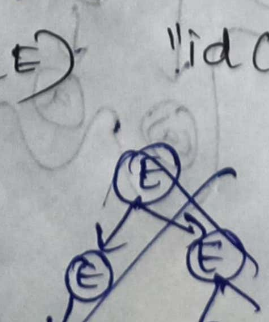


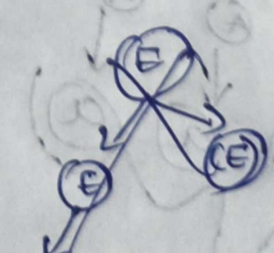


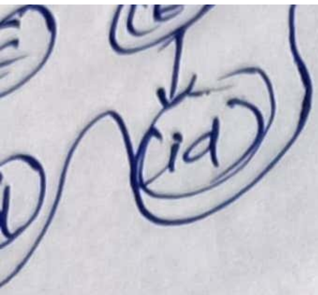
5) $E \to EE$

$E \to E(E)$ "id(id) id"

$E \to id$







unambig

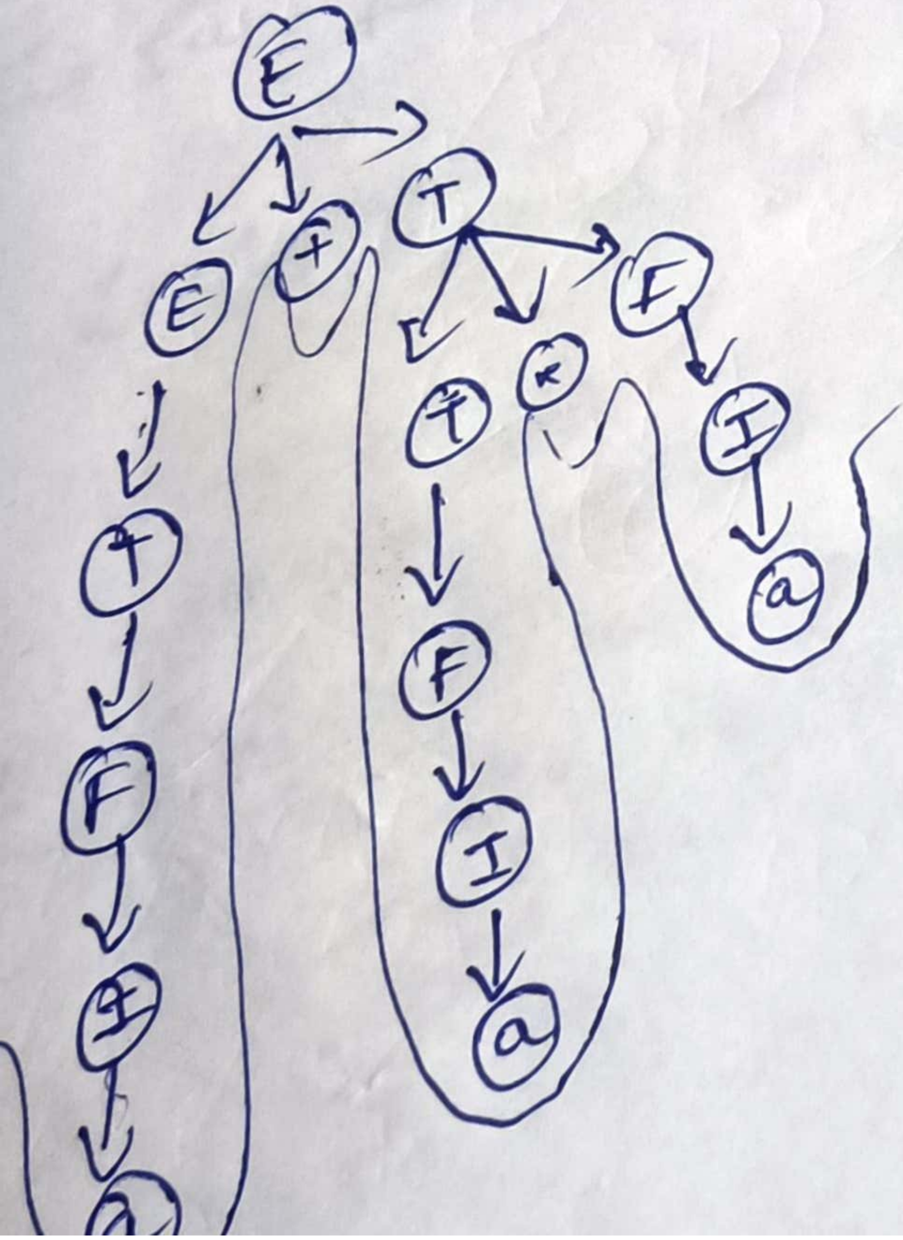
$I \to a / b / Ia / Ib / I0 / I1$

$F \to I / (E)$

$T \to F / T * F$

$E \to T / E + T$

"a+a*a".



3) $S \rightarrow AB \mid C$

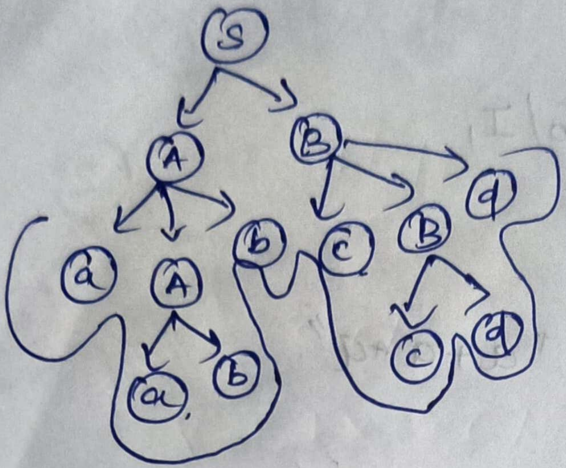
$A \rightarrow aAb \mid ab$

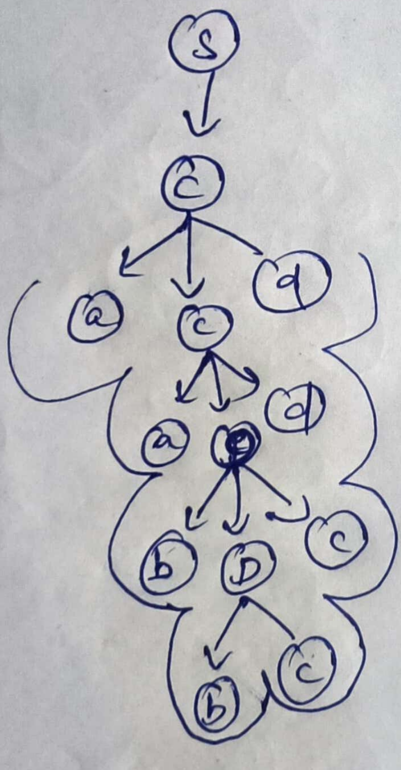
$B \rightarrow cBd \mid cd$

$C \rightarrow aCd \mid aDd$

$D \rightarrow bDc \mid bc$

"aabbccdd"





CONTEXT FREE GRAMMAR

1) $L = \{ w = \text{palindrome of string } w \in (a+b)^* \}$

$L = \{aa, bb, abba, baab, aabbaa, \dots\}$

$S \to a|b|\varepsilon$

$S \to aSa|bSb$

2) $L = \{ w = \text{palindrome of String of even length}, w \in (a+b)^* \}$

$L = \{aa, bb, aabb, abba, bbaa, aabbaa, \dots\}$

$S \to \cancel{a|b|\varepsilon}\ a|b|\varepsilon$

$S \to aSa|bSb$

$S \to aSa \,|\, bSb$

$S \to a|b|\varepsilon$

3) $L = \{ w = \text{palindrome of String of odd length } w \in (a+b)^* \}$

$L = \{a, b, aba, bab, aabaa, bbabb, \dots\}$

$S \to a|b|\varepsilon$

$S \to aSa|bSb.$

4) $L = \{ wcw^R, w \in (a+b)^* \}$

$L = \{acb, acbb, acaa, abcba, bbacabb, \dots\}$

$S \to aSc|bSb$ or $S \to SCS$

$S \to c$ $\quad S \to a|b|c$.

5) $L = \{ w = a^n b^n, w \in (a+b)^* \}$

$L = \{\varepsilon, ab, aabb, aaabbb, \dots\}$

$S \to aSb|\cancel{a}$

$S \to a|b|\varepsilon$

6) $L = \{ w = a^n b^{2n}, \ w \in (a+b)^* \}$

$L = \{abb, aabbbb, aaabbbbbb, \ldots$

$S \to aSbb$

$S \to a|b|\varepsilon$

7) $L = \{ w = a^n b^n c^k, \ k = 2n + 3n, \}$

$L = \{abccccc, aabbcccccc \ldots\}$

$S \to$

$S \to a|b|c|\varepsilon$

or

$S \to aScc | X$

$X \to bXccc | \varepsilon$

CHROMSKY NORMAL FORM

Special form of context free grammar

3 Rules :-

$A \to BC$

$A \to a$

$S \to \varepsilon$

A, B, C - non Terminal

a - terminal

ε - Empty symbol

S - start symbol

PROPERTIES :-

- Standardisation
- Binary structure - $A \to BC$
- NO Mixed terminals $\to$ $A \to aBX$

Ques-1 :-

$S \to AB$

$A \to aA | a$

$B \to bB | b$

STEP-1 :- Introduce new non-terminals.

$X \to a$

$Y \to b$

STEP-2:

$S \rightarrow AB$

$A \rightarrow XA / X$

$B \rightarrow YB / Y$

STEP-3: CNF

$S \rightarrow AB$ ✓

$A \rightarrow XA / X$ ✓

$B \rightarrow YB / Y$ ✓

$X \rightarrow a$ ✓

$Y \rightarrow b$ ✓

Theorem:-

Any context free language is generated by a context free grammar in CNF.