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MODULE - II

FINITE AUTOMATA

It is a mathematical model to solve the problems. No memory is required. Here, the problems are solved based on the current input or situation.

Types:

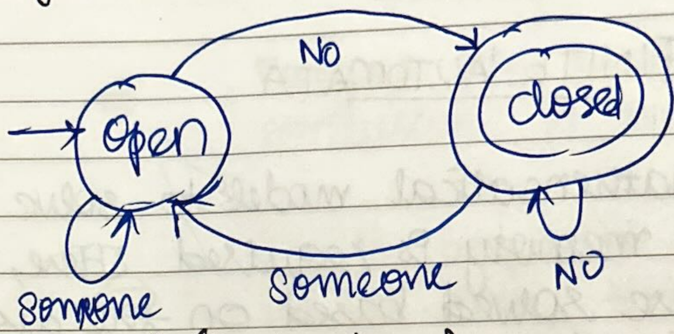
(i) Deterministic Finite Automata.
(ii) Non-deterministic Finite Automata.

Difference:

→ In deterministic finite automata, there will not be epsilon (ε) transition; vice versa in the other.
→ In deterministic finite automata, for the given input, there will be only one transition. In the other, there will be zero or more transitions.

5 Tuples of Finite Automata
$\{Q, \Sigma, \delta, q_0, F\}$
Q → Set of States, contains input ○
Σ → Set of input symbols
δ → Transition function
q_0 → Start State (→)
F → Final State ◎

Eg: automated door system



$Q \rightarrow \{open, close\}$
$\varepsilon \rightarrow \{someone, no\}$
$q_0 \rightarrow open$
$F \rightarrow closed$

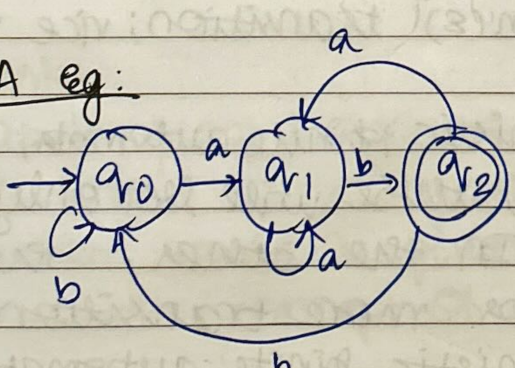
δ (State, Input) = output

δ (open, NO) = closed
δ (open, someone) = open
δ (closed, NO) = closed
δ (closed, someone) = open.

DFA: $\delta(q_0, a) = P$

NFA: $\delta(q_0, a) = \phi$ or $\{P_1, P_2, \dots P_n\}$

DFA eg:



$Q = \{q_0, q_1, q_2\}$
$\varepsilon = \{a, b\}$
$q_0 = q_0$
$F = q_2$

δ (State, Input) = output

$\delta(q_0, a) = q_1$
$\delta(q_0, b) = q_0$
$\delta(q_1, a) = q_1$
$\delta(q_1, b) = q_2$
$\delta(q_2, a) = q_1$
$\delta(q_2, b) = q_0$

Transition table

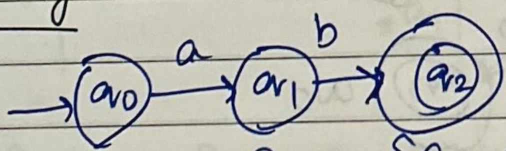
Q \ Σ	a	b
$\rightarrow q_0$	q_1	q_0
q_1	q_1	q_2
$*q_2$	q_1	q_0

DFA & NFA.

For a particular input, there will be only one transition from the given state.

For a particular input, there may be a transition or no transition

NFA eg :



NFA = $\{Q, \Sigma, \delta, q_0, F\}$

$Q \rightarrow \{q_0, q_1, q_2\}$

$\Sigma \rightarrow \{a, b\}$

$q_0 \rightarrow q_0$

$F \rightarrow q_2$

$\delta(q_0, a) = q_1$

$\delta(q_0, b) = \phi$

$\delta(q_1, a) = \phi$

$\delta(q_1, b) = q_2$

$\delta(q_2, a) = \phi$

$\delta(q_2, b) = \phi$

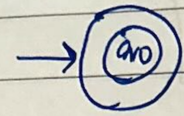
Q \ Σ	a	b
$\rightarrow q_0$	q_1	ϕ
q_1	ϕ	q_2
$*q_2$	ϕ	ϕ

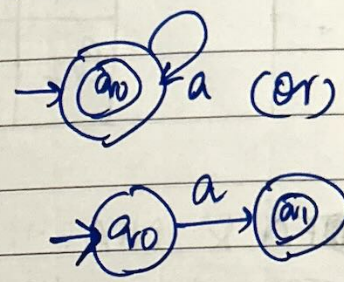
PROBLEM ①

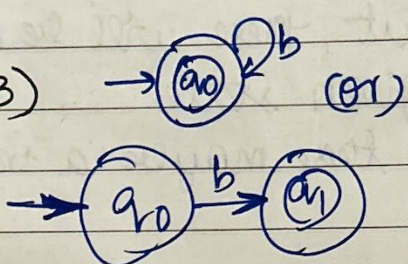
Construct NFA for the following language:

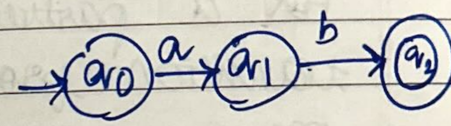
1) $L = \{\varepsilon\}$
2) $L = \{a\}$
3) $L = \{b\}$
4) $L = \{a, b\}$
5) $L = \{aa, ab\}$
6) $L = \{aaa, aba, abb\}$

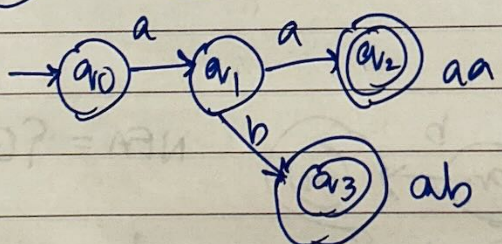
Solution

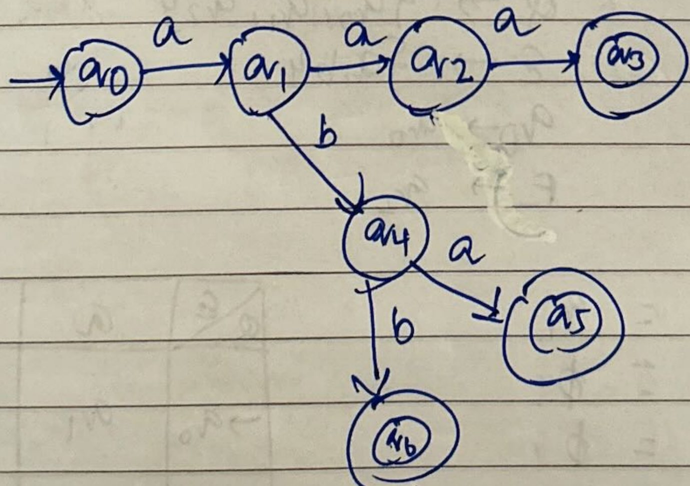
1) 

2) 

3) 

4) 

5) 

6) 

1) $Q \rightarrow \{q_0\}$

$\Sigma \rightarrow \varepsilon$

$q_0 \rightarrow q_0$

$F \rightarrow q_0$

$\delta(q_0, \varepsilon) = \phi$

	ε
q_0	ϕ

2) $Q \rightarrow \{q_0\}$ (or) $\{q_0, q_1\}$
$\varepsilon \rightarrow \{a\}$
$q_0 \rightarrow q_0$
$F \rightarrow q_0$ (or) q_1

δ (State, Input) = Output
$\delta(q_0, a) = q_0$
(or)
$\delta(q_0, a) = q_1$

	a
$*q_0$	q_0

(or)

	a
$\rightarrow q_0$	q_1
$*q_1$	ϕ

3) $Q \rightarrow \{q_0\}$ (or) $\{q_0, q_1\}$
$\varepsilon \rightarrow \{b\}$
$q_0 \rightarrow q_0$
$F \rightarrow q_0$ (or) q_1

$\delta(q_0, b) = q_0$
(or)
$\delta(q_0, b) = q_1$

	b
q_0	q_0

(or)

	b
$\rightarrow q_0$	q_1
$*q_1$	ϕ

4) $Q \rightarrow \{q_0, q_1, q_2\}$
$\varepsilon \rightarrow \{a, b\}$
$q_0 \rightarrow q_0$
$F \rightarrow q_2$

$\delta(q_0, a) = q_1$
$\delta(q_0, b) = \phi$
$\delta(q_1, a) = \phi$
$\delta(q_1, b) = q_2$
$\delta(q_2, a) = \phi$
$\delta(q_2, b) = \phi$

	a	b
$\rightarrow q_0$	q_1	ϕ
q_1	ϕ	q_2
$*q_2$	ϕ	ϕ

5) $Q \rightarrow \{q_0, q_1, q_2, q_3\}$
$\varepsilon \rightarrow \{a, b\}$
$q_0 \rightarrow q_0$
$F \rightarrow q_2, q_3$

$\delta(q_0, a) = q_1$
$\delta(q_0, b) = \phi$
$\delta(q_1, a) = q_2$
$\delta(q_1, b) = q_3$
$\delta(q_2, a) = \phi$
$\delta(q_2, b) = \phi$
$\delta(q_3, a) = \phi$
$\delta(q_3, b) = \phi$

	a	b
q_0	q_1	ϕ
q_1	q_2	q_3
q_2	ϕ	ϕ
q_3	ϕ	ϕ
q_4	ϕ	ϕ

6) $Q \to \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}$

$\Sigma \to \{a, b\}$

$q_0 \to q_0$

$F \to q_3, q_5, q_6.$

$\delta(q_0, a) = q_1$
$\delta(q_0, b) = \phi$
$\delta(q_1, a) = q_2$
$\delta(q_1, b) = q_4$
$\delta(q_2, a) = q_3$
$\delta(q_2, b) = \phi$
$\delta(q_3, a) = \phi$
$\delta(q_3, b) = \phi$

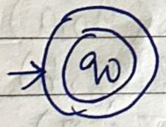
$\delta(q_4, a) = q_5$
$\delta(q_4, b) = q_6$
$\delta(q_5, a) = \phi$
$\delta(q_5, b) = \phi$
$\delta(q_6, a) = \phi$
$\delta(q_6, b) = \phi$

	a	b
$\to q_0$	q_1	ϕ
q_1	q_2	q_4
q_2	q_3	ϕ
$*q_3$	ϕ	ϕ
q_4	q_5	q_6
$*q_5$	ϕ	ϕ
$*q_6$	ϕ	ϕ

PROBLEM ②
Construct DFA for the same as before.

Solution

1)

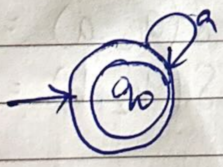


$Q \to \{q_0\}$
$\Sigma \to \varepsilon$
$q_0 \to q_0$
$F \to q_0$

$\delta(q_0, \varepsilon) = \phi$

	ε
q_0	ϕ

2)

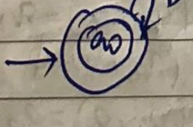


$Q \to \{q_0\}$
$\Sigma \to \{a\}$
$q_0 \to q_0$
$F \to q_0$

$\delta(q_0, a) = q_0$

	a
q_0	q_0

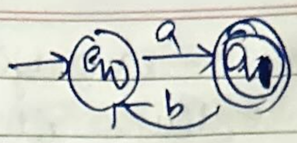
3)



$Q \to \{q_0\}$
$\Sigma \to \{b\}$
$q_0 \to q_0$
$F \to q_0$

$\delta(q_0, b) = q_0$

	b
q_0	q_0

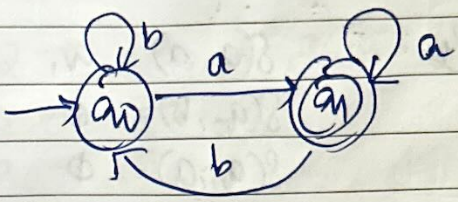
4) 

$Q \to \{q_0, q_1\}$ $q_0 \to q_0$

$\Sigma \to \{a, b\}$ $F \to q_1$

$\delta(q_0, a) = q_0$
$\delta(q_0, b) = \phi$
$\delta(q_1, a) = \phi$
$\delta(q_1, b) = q_0$

	a	b
$\to q_0$	q_1	ϕ
$*q_1$	ϕ	q_0

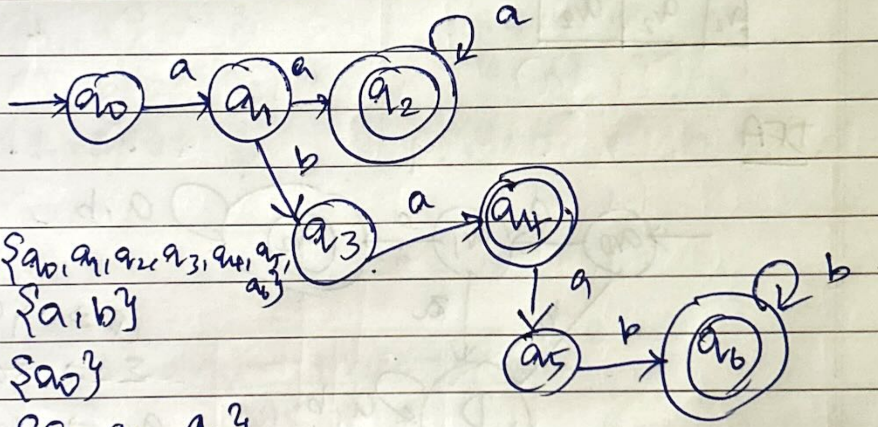
5) 

$Q \to \{q_0, q_1\}$ $q_0 \to q_0$

$\Sigma \to \{a, b\}$ $F \to q_1$

$\delta(q_0, a) = q_1$
$\delta(q_0, b) = q_0$
$\delta(q_1, a) = q_1$
$\delta(q_1, b) = q_0$

	a	b
$\to q_0$	q_1	q_0
$*q_1$	q_1	q_0.

6) 

$Q \to \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}$
$\Sigma \to \{a, b\}$
$q_0 \to \{q_0\}$
$F \to \{q_2, q_4, q_6\}$

$\delta(q_0, a) = q_1$
$\delta(q_0, b) = \phi$
$\delta(q_1, a) = q_2$
$\delta(q_1, b) = q_3$
$\delta(q_2, a) = q_2$
$\delta(q_2, b) = \phi$
$\delta(q_3, a) = q_4$
$\delta(q_3, b) = \phi$
$\delta(q_4, a) = q_5$
$\delta(q_4, b) = \phi$

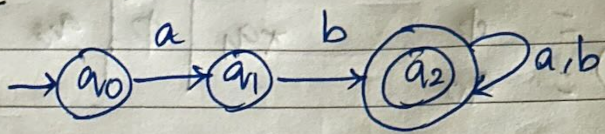
$\delta(q_5, a) = \phi$
$\delta(q_5, b) = q_6$
$\delta(q_6, a) = \phi$
$\delta(q_6, b) = q_6$

	a	b
$\to q_0$	q_1, ~~[illegible]~~	ϕ
q_1	q_2	q_3
$*q_2$	q_2	ϕ
q_3	q_4	ϕ
$*q_4$	q_5	ϕ
q_5	ϕ	q_6
$*q_6$	ϕ	q_6

PROBLEM 3

Construct NFA and DFA for the language over an alphabet $\Sigma = \{a, b\}$ that contains all strings starting with a, b.

NFA



$Q \rightarrow \{q_0, q_1, q_2\}$

$\Sigma \rightarrow \{a, b\}$

$q_0 \rightarrow q_0$

$F \rightarrow q_2$

$\delta(q_0, a) = q_1$

$\delta(q_0, b) = \phi$

$\delta(q_1, a) = \phi$

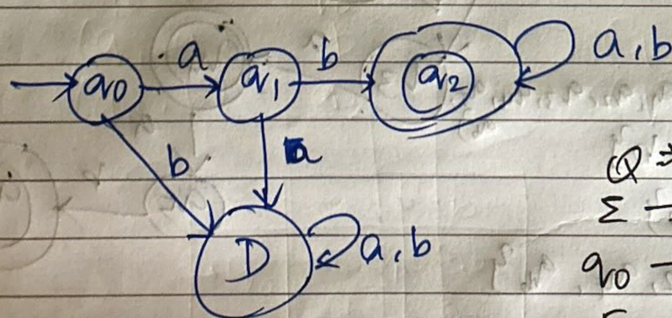
$\delta(q_1, b) = q_2$

$\delta(q_2, a) = q_2$

$\delta(q_2, b) = q_2$

Q \ Σ	a	b
q_0	q_1	ϕ
q_1	ϕ	q_2
q_2	q_2	q_2

DFA



$Q \Rightarrow \{q_0, q_1, q_2, D\}$

$\Sigma \rightarrow \{a, b\}$

$q_0 \rightarrow q_0$

$F \rightarrow q_2$

$\delta(q_0, a) = q_1$

$\delta(q_0, b) = D$

$\delta(q_1, a) = D$

$\delta(q_1, b) = q_2$

$\delta(q_2, a) = q_2$

$\delta(q_2, b) = q_2$

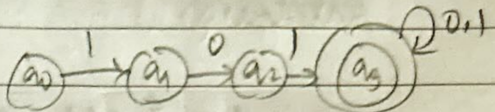
$\delta(D, a) = D$

$\delta(D, b) = D$

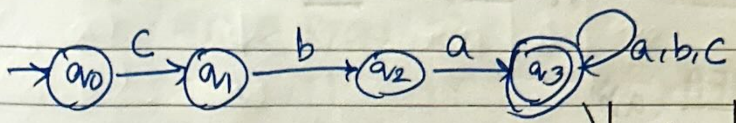
	a	b
q_0	q_1	D
q_1	D	q_2
q_2	q_2	q_2
D	D	D

PROBLEM 4

Construct NFA and DFA for the language over an alphabet $\Sigma \{a,b,c\}$ that contains all strings starting with c, b, a.



NFA



$Q \rightarrow \{q_0, q_1, q_2, q_3\}$

$\Sigma \rightarrow \{c, b, a\}$

$q_0 \rightarrow q_0$

$F \rightarrow q_3$

	a	b	c
$\rightarrow q_0$	ϕ	ϕ	q_1
q_1	ϕ	q_2	ϕ
q_2	q_3	ϕ	ϕ
$*q_3$	q_3	q_3	q_3

$\delta(q_0, a) = \phi$

$\delta(q_0, b) = \phi$

$\delta(q_0, c) = q_1$

$\delta(q_1, a) = \phi$

$\delta(q_1, b) = q_2$

$\delta(q_1, c) = \phi$

$\delta(q_2, a) = q_3$

$\delta(q_2, b) = \phi$

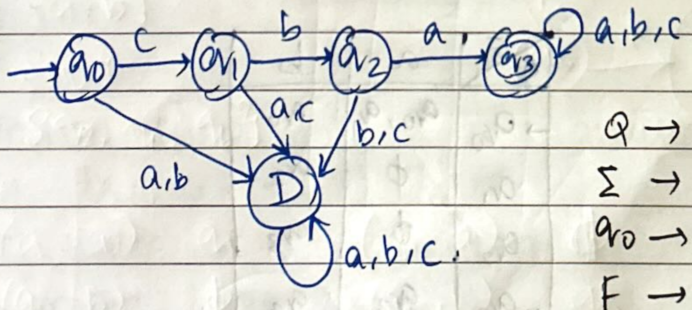
$\delta(q_2, c) = \phi$

$\delta(q_3, a) = q_3$

$\delta(q_3, b) = q_3$

$\delta(q_3, c) = q_3$

DFA



$Q \rightarrow \{q_0, q_1, q_2, q_3, D\}$

$\Sigma \rightarrow \{a, b, c\}$

$q_0 \rightarrow q_0$

$F \rightarrow q_3$

$\delta(q_0, a) = D$

$\delta(q_0, b) = D$

$\delta(q_0, c) = q_1$

$\delta(q_1, a) = D$

$\delta(q_1, b) = q_2$

$\delta(q_1, c) = D$

$\delta(q_2, a) = q_3$

$\delta(q_2, b) = D$

$\delta(q_2, c) = D$

$\delta(q_3, a) = q_3$

$\delta(q_3, b) = q_3$

$\delta(q_3, c) = q_3$

$\delta(D, a) = D$

$\delta(D, b) = D$

$\delta(D, c) = D$

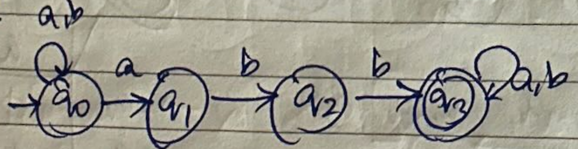
	a	b	c
$\rightarrow q_0$	D	D	q_1
q_1	D	q_2	D
q_2	q_3	D	D
$*q_3$	q_3	q_3	q_3
D	D	D	D

PROBLEM ⑤

Construct NFA and DFA for the language over alphabet $\Sigma = \{a, b\}$ that contains abb as substring.

$$L = \{ \underline{abb},\ a\underline{abb},\ \underline{abb}a,\ aa\underline{abb}a, \ldots \}$$

NFA



$$a^n b^n\ \underline{abb}\ a^n b^n$$

$$NFA = \{ Q, \Sigma, \delta, q_0, F \}$$

$Q \rightarrow \{ q_0, q_1, q_2, q_3 \}$, $q_0 \rightarrow q_0$

$\Sigma \rightarrow \{ a, b \}$, $F \rightarrow q_3$

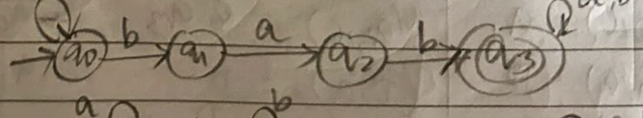
$\delta(q_0, a) = \{ q_0, q_1 \}$ $\quad$ $\delta(q_2, a) = \phi$

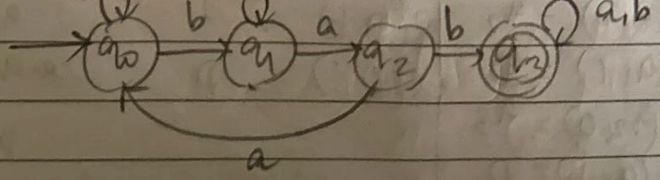
$\delta(q_0, b) = q_0$ $\quad$ $\delta(q_2, b) = q_3$

$\delta(q_1, a) = \phi$ $\quad$ $\delta(q_3, a) = q_3$

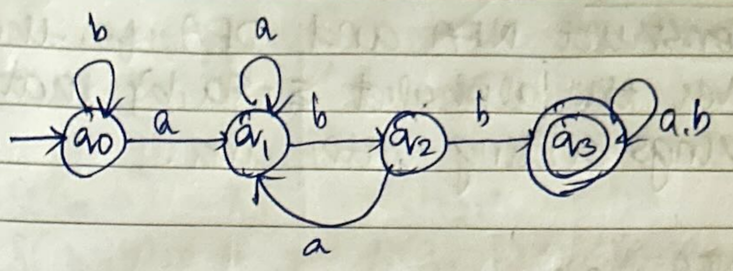
$\delta(q_1, b) = q_2$ $\quad$ $\delta(q_3, b) = q_3$

Q \ Σ	a	b
$\rightarrow q_0$	q_0, q_1	q_0
q_1	ϕ	q_2
q_2	ϕ	q_3
$* q_3$	q_3	q_3





DFA



$$DFA = \{Q, \Sigma, \delta, q_0, F\}$$

$Q = \{q_0, q_1, q_2, q_3\}$ $\qquad q_0 = q_0$

$\Sigma = \{a, b\}$ $\qquad F = q_3$

$\delta(q_0, a) = q_1$ $\qquad \delta(q_2, a) = q_1$

$\delta(q_0, b) = q_0$ $\qquad \delta(q_2, b) = q_3$

$\delta(q_1, a) = q_1$ $\qquad \delta(q_3, a) = q_3$

$\delta(q_1, b) = q_2$ $\qquad \delta(q_3, b) = q_3$

$Q \backslash \Sigma$	a	b
$\rightarrow q_0$	q_1	q_0
q_1	q_1	q_2
q_2	q_1	q_3
$*q_3$	q_3	q_3

NOTE

(i) In NFA, there may be a transition or may not be a transition.

(ii) If there is a transition in NFA, it can be 0 or more transition.

(iii) In DFA, there will be exactly one transition from a state.

(iv) Dummy state is one represented by D and it can be used only once after checking all the possibilities, if not satisfied.

classmate

Date

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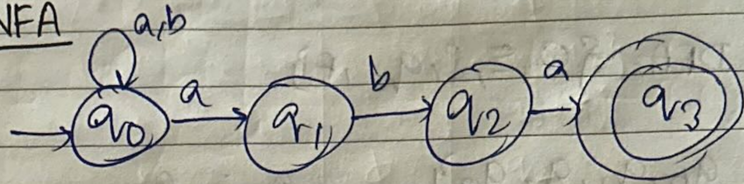
PROBLEM 6

Construct NFA and DFA for the language over the alphabet $\Sigma = \{a, b\}$ that contains all strings ending with aba.

NFA



$a^n b^n aba$

$$NFA = \{Q, \Sigma, \delta, q_0, F\}$$

$Q = \{q_0, q_1, q_2, q_3\}$ $\quad$ $q_0 = \{q_0, q_1\}$

$\Sigma = \{a, b\}$ $\quad$ $F = q_3$

$\delta(q_0, a) = \{q_0, q_1\}$ $\quad$ $\delta(q_2, a) = q_3$

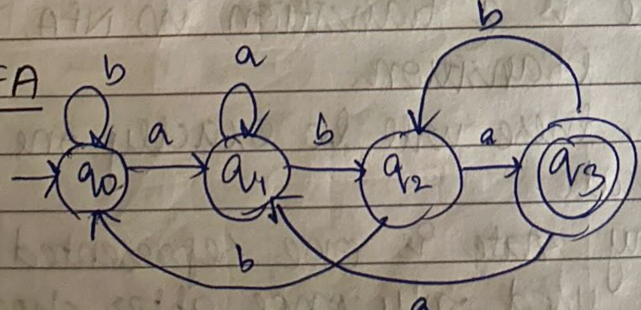
$\delta(q_0, b) = q_0$ $\quad$ $\delta(q_2, b) = \phi$

$\delta(q_1, a) = \phi$ $\quad$ $\delta(q_3, a) = \phi$

$\delta(q_1, b) = q_2$ $\quad$ $\delta(q_3, b) = \phi$

Q \ Σ	a	b
→ q_0	q_0, q_1	q_0
q_1	ϕ	q_2
q_2	q_3	ϕ
* q_3	ϕ	ϕ

DFA



aba | aba

aba | ba

$$DFA = \{Q, \Sigma, \delta, q_0, F\}$$

$Q = \{q_0, q_1, q_2, q_3\}$ $q_0 = q_0$
$\Sigma = \{a, b\}$ $F = q_3$

$\delta(q_0, a) = q_1$ $\delta(q_2, a) = q_3$
$\delta(q_0, b) = q_0$ $\delta(q_2, b) = q_0$
$\delta(q_1, a) = q_1$ $\delta(q_3, a) = q_1$
$\delta(q_1, b) = q_2$ $\delta(q_3, b) = q_2$

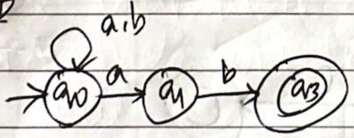
$Q \backslash \Sigma$	a	b
$\rightarrow q_0$	q_1	q_0
q_1	q_1	q_2
q_2	q_3	q_0
$*q_3$	q_1	q_2

16M

Conversion of NFA to DFA using subset construction method.

(i) Find the subset of the states using the formula 2^n.
(ii) Find the transition table for all the states in the subset.
(iii) Rename the states in the transition table.
(iv) Remove the unnecessary states in the transition table.
(v) Construct a transition diagram for DFA

PROBLEM (7)



Step 1: $Q = \{q_0, q_1, q_2\}$

$n = 3$

$2^n = 2^3 = 8$

$\{\phi, q_1, q_2, q_3, \{q_0, q_1\}, \{q_0, q_2\}, \{q_1, q_2\}, \{q_0, q_1, q_2\}\}$

Step 2 & 3:

		a	b
A	ϕ	ϕ A	ϕ A
→B	$\rightarrow q_0$	$\{q_0, q_1\}$ E	q_0 B
C	q_1	ϕ A	q_2 D
*D	$* q_2$	ϕ A	ϕ A
E	$\{q_0, q_1\}$	$\{q_0, q_1\}$ E	$\{q_0, q_2\}$ F
*F	$*\{q_0, q_2\}$	$\{q_0, q_1\}$ E	q_0 B
*G	$*\{q_1, q_2\}$	ϕ A	q_2 D
*H	$*\{q_0, q_1, q_2\}$	$\{q_0, q_1\}$ E	$\{q_0, q_2\}$ F

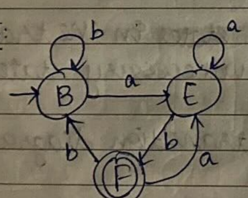
Step 4:

	a	b
A	A	A
→B	E	B
C	A	D
*D	A	A
*E	E	F
*F	E	B
*G	A	D
*H	E	F

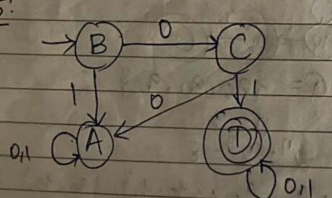
⇒

	a	b
→B	E	B
E	E	F
*F	E	B

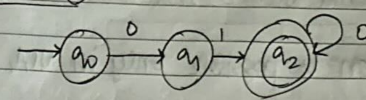
Step 5:



Step 5:



PROBLEM 8



Step 1: $Q = \{q_0, q_1, q_2\}$

$n = 3$

$2^n = 2^3 = 8$

$\{\phi, q_0, q_1, q_2, \{q_0, q_1\}, \{q_0, q_2\}, \{q_1, q_2\}, \{q_0, q_1, q_2\}\}$

Step 2 & 3.

		0	1
A	ϕ	ϕ A	ϕ A
→B	$\rightarrow q_0$	q_1 C	ϕ A
C	q_1	ϕ A	q_2 D
*D	$* q_2$	q_2 D	q_2 D
E	$\{q_0, q_1\}$	q_1 C	q_2 D
*F	$*\{q_0, q_2\}$	$\{q_0, q_2\}$ F	q_2 D
*G	$*\{q_1, q_2\}$	q_2 D	q_2 D
*H	$*\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$ G	q_2 D

Step 4:

	0	1
A	A	A
→B	C	A
C	A	D
*D	D	D
E	C	D
*F	F	D
*G	D	D
*H	G	D

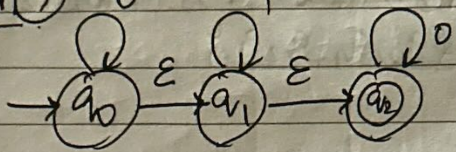
⇒

	0	1
→B	C	A
C	A	D
A	A	A
*D	D	D

Finite Automata with ε Transition

ε transition is possible only in NFA and not in DFA.
It is represented by ε-NFA.
For this ε-NFA, 5 tuples are same

PROBLEM 9



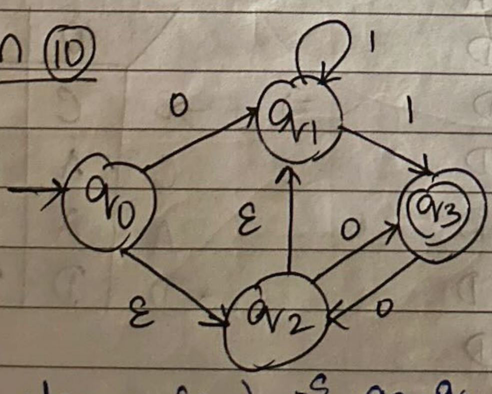
$Q = \{q_0, q_1, q_2\}$
$\Sigma = \{0, 1, \varepsilon\}$
$q_0 \rightarrow q_0$
$F \rightarrow q_2$

ε-closure$(q_0) = \{q_0, q_1, q_2\}$
ε-closure$(q_1) = \{q_1, q_2\}$
ε-closure$(q_2) = \{q_2\}$

$\delta(q_0, 0) = q_0$
$\delta(q_0, 1) = \phi$
$\delta(q_0, \varepsilon) = q_1$
$\delta(q_1, 0) = \phi$
$\delta(q_1, 1) = q_1$
$\delta(q_1, \varepsilon) = q_2$
$\delta(q_2, 0) = q_2$
$\delta(q_2, 1) = \phi$
$\delta(q_2, \varepsilon) = \phi$

	0	1	ε
$\rightarrow q_0$	q_0	ϕ	q_1
q_1	ϕ	q_1	q_2
$*q_2$	q_2	ϕ	ϕ

PROBLEM 10



$Q = \{q_0, q_1, q_2, q_3\}$
$\Sigma = \{0, 1, \varepsilon\}$
$q_0 \rightarrow q_0$
$F \rightarrow q_3$

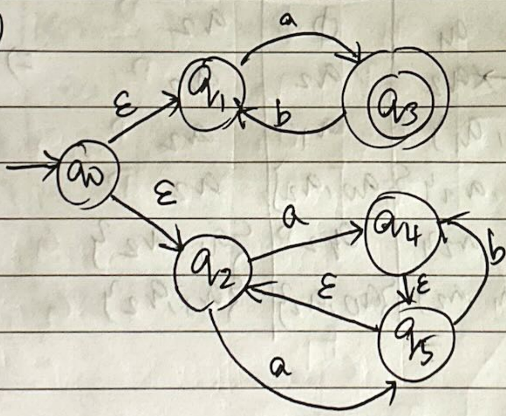
ε-closure$(q_0) = \{q_0, q_2, q_1\}$
ε-closure$(q_1) = \{q_1\}$
ε-closure$(q_2) = \{q_2, q_1\}$
ε-closure$(q_3) = \{q_3\}$

$\delta(q_0, 0) = q_1$
$\delta(q_0, 1) = \phi$
$\delta(q_0, \varepsilon) = q_2$
$\delta(q_1, 0) = \phi$
$\delta(q_1, 1)\ \{q_1, q_3\}$
$\delta(q_1, \varepsilon) = \phi$

$\delta(q_2, 0) = q_3$
$\delta(q_2, 1) = \phi$
$\delta(q_2, \varepsilon) = q_1$
$\delta(q_3, 0) = q_2$
$\delta(q_3, 1) = \phi$
$\delta(q_3, \varepsilon) = \phi$

	0	1	ε
$\rightarrow q_0$	q_1	ϕ	q_2
q_1	ϕ	$\{q_1, q_3\}$	ϕ
q_2	q_3	ϕ	q_1
$*q_3$	q_2	ϕ	ϕ

PROBLEM (11)



$Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$
$\Sigma = \{a, b, \varepsilon\}$
$q_0 \rightarrow q_0$
$F \rightarrow q_3$

ε- closure $(q_0) = \{q_0, q_1, q_2\}$
ε- closure $(q_1) = \{q_1\}$
ε- closure $(q_2) = \{q_2\}$
ε- closure $(q_3) = \{q_3\}$
ε- closure $(q_4) = \{q_4, q_5, q_2\}$
ε- closure $(q_5) = \{q_5, q_2\}$

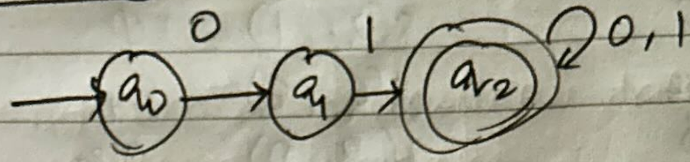
	0	1	ε
$\rightarrow q_0$	ϕ	ϕ	$\{q_1, q_2\}$
q_1	q_3	ϕ	ϕ
q_2	$\{q_4, q_5\}$	ϕ	ϕ
$*q_3$	ϕ	q_1	ϕ
q_4	ϕ	ϕ	q_5
$*q_5$	ϕ	q_4	q_2

$\delta(q_0, a) = \phi$
$\delta(q_0, b) = \phi$
$\delta(q_0, \varepsilon) = \{q_1, q_2\}$
$\delta(q_1, a) = \{q_3\}$
$\delta(q_1, b) = \phi$
$\delta(q_1, \varepsilon) = \phi$

$\delta(q_2, a) = \{q_4, q_5\}$
$\delta(q_2, b) = \phi$
$\delta(q_2, \varepsilon) = \phi$
$\delta(q_3, a) = \phi$
$\delta(q_3, b) = q_1$
$\delta(q_3, \varepsilon) = \phi$

$\delta(q_4, a) = \phi$
$\delta(q_4, b) = \phi$
$\delta(q_4, \varepsilon) = q_5$
$\delta(q_5, a) = \phi$
$\delta(q_5, b) = q_4$
$\delta(q_5, \varepsilon) = q_2$

PROBLEM (12) NFA TO DFA USING SUBSET CONST.



$Q = \{q_0, q_1, q_2\}$

$n = 3$

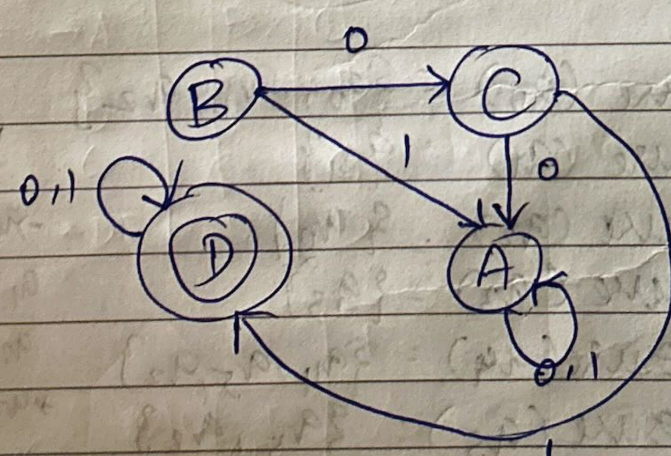
$2^3 = 8$

$\{\phi, q_0, q_1, q_2, \{q_0, q_1\}, \{q_0, q_2\}, \{q_1, q_2\}, \{q_0, q_1, q_2\}\}$

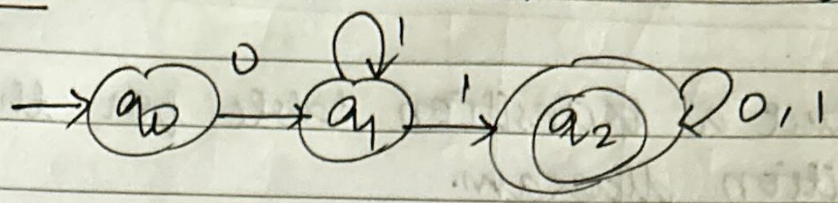
		0	1
A	ϕ	ϕ A	ϕ A
→B	→ q_0	q_1 C	ϕ A
C	q_1	ϕ A	q_2 D
*D	→* q_2	D q_2	q_2 D
E	$\{q_0, q_1\}$	q_1 C	q_2 D
*F	* $\{q_0, q_2\}$	$\{q_0, q_2\}$ F	q_2 D
*G	* $\{q_1, q_2\}$	q_2 D	$\{q_1, q_2\}$ G
*H	* $\{q_0, q_1, q_2\}$	$\{q_0, q_2\}$ F	$\{q_1, q_2\}$ G

⇒

	0	1
→B	C	A
A	A	A
C	A	D
*D	D	D



PROBLEM (13) NFA TO DFA USING SUBSET CONST.



$$Q = \{q_0, q_1, q_2\}$$

$$n = 3$$

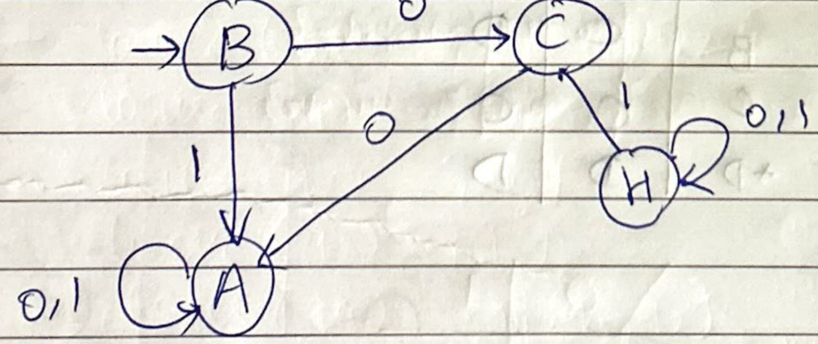
$$2^3 = 8$$

$$\{\phi, q_0, q_1, q_2, \{q_0, q_1\}, \{q_0, q_2\}, \{q_1, q_2\}, \{q_0, q_1, q_2\}\}$$

		0	1
A	ϕ	ϕ A	ϕ A
→B	→ q_0	q_1 C	ϕ A
C	q_1	[illegible] ϕ A	$\{q_1, q_2\}$ H
*D	* q_2	q_2 D	q_2 D
E	$\{q_0, q_1\}$	q_1 C	$\{q_1, q_2\}$ H
*F	* $\{q_0, q_2\}$	$\{q_1, q_2\}$ H	q_2 D
*G	* $\{q_1, q_2\}$	q_2 D	$\{q_1, q_2\}$ H
*H	* $\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$ H	$\{q_1, q_2\}$ H

⇒

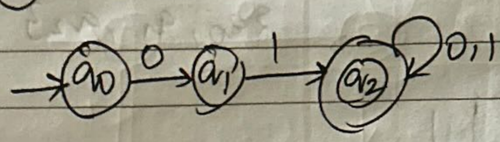
	0	1
→B	C	A
A	A	A
C	A	H
* H	H	H



Conversion of NFA to DFA using Lazy Method

Step 1) Construct a transition table for the given transition diagram.
Step 2) Rename the states
Step 3) Construct a transition diagram from step 2.

PROBLEM (14)



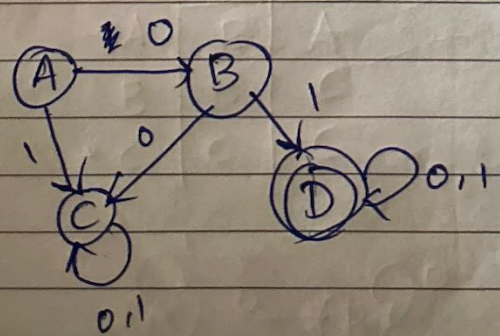
Step 1:

	0	1
$\rightarrow q_0$	q_1	ϕ
q_1	ϕ	q_2
ϕ	ϕ	ϕ
$* q_2$	q_2	q_2

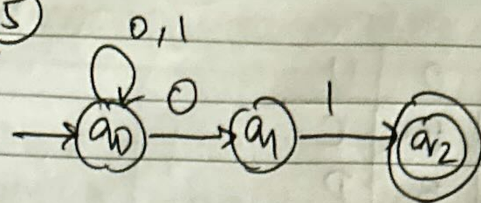
Step 2:

	0	1
$\rightarrow$A	B	C
B	C	D
C	C	C
*D	D	D

Step 3:



PROBLEM 15



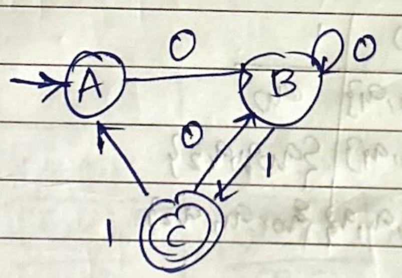
Step 1:

	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	q_0
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$*\{q_0, q_2\}$	$\{q_0, q_1\}$	q_0

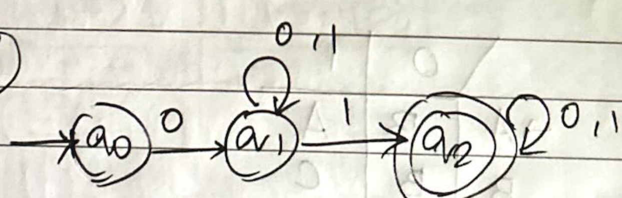
Step 2:

	0	1
$\rightarrow A$	B	A
B	B	C
C	B	A

Step 3:



PROBLEM 16



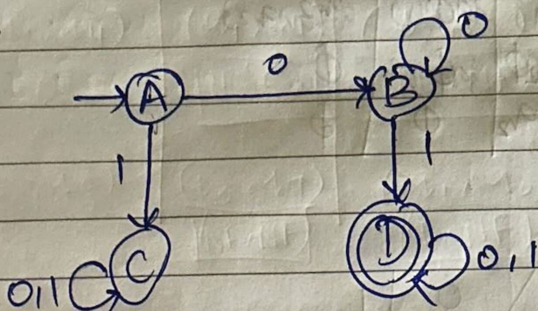
Step 1

	0	1
$\rightarrow q_0$	q_1	ϕ
q_1	q_1	$\{q_1, q_2\}$
$*\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_1, q_2\}$

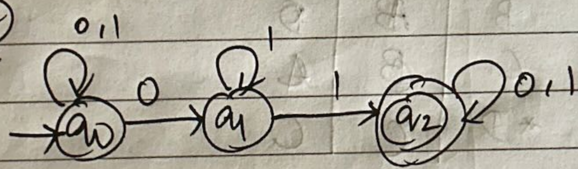
Step 2

	0	1
→A	B	C
B	B	D
C	C	C
*D	D	D

Step 3



PROBLEM (17)



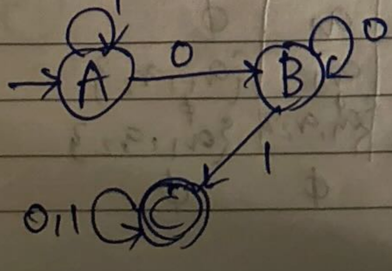
Step 1

	0	1
→ q_0	$\{q_0, q_1\}$	q_0
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$
* $\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$

Step 2

	0	1
→A	B	A
B	B	C
*C	C	C

Step 3

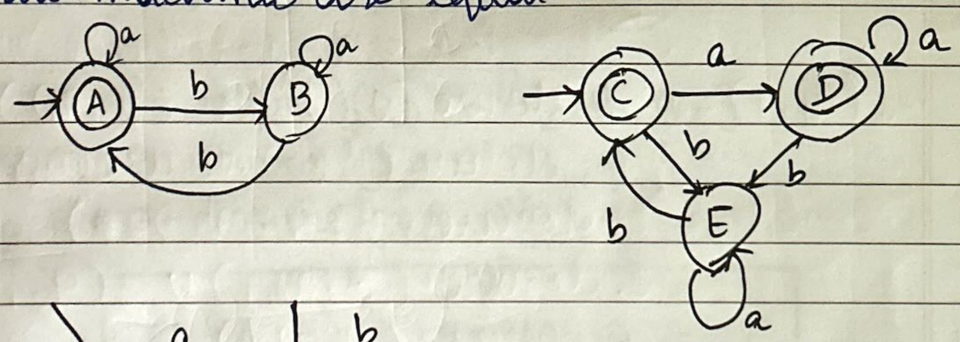


Theorem 1:

Equivalence of two Finite Automata

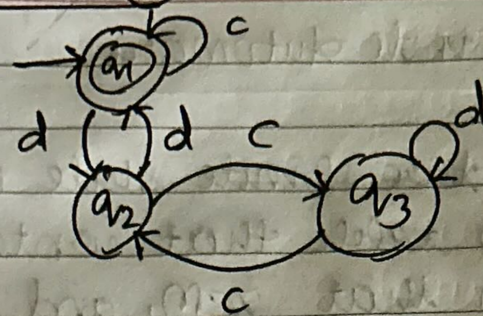
→ Let M_1 and M_2 be two finite state machine.
→ Construct transition table that contain pair of states (x, y) such that $x \in M_1$ and $y \in M_2$.
→ Start construction with pair of initial state.
→ If we get any pair in the form (F, NF) (OR) (NF, F), then stop the construction and declare two machines are not equal.
→ Continue the construction of table ~~for~~ every new pair in the form (F, F) and (NF, NF) and stop the construction when no pair occurs.
→ If transition table contains all the pairs of the form (F, F) and (NF, NF) then, two finite state machines are equal.

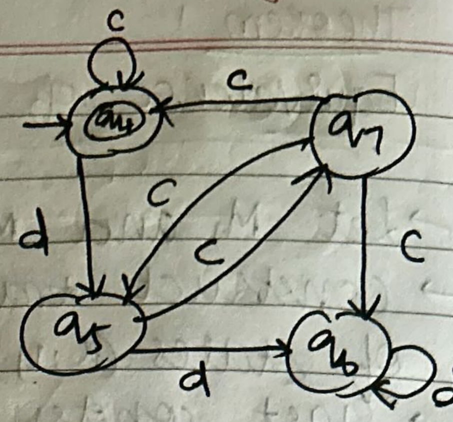
PROBLEM (18)



	a	b
(A, C)	(A, D) $\boxed{(F, F)}$	(B, E) $\boxed{(NF, NF)}$
(A, D)	(A, D) $\boxed{(F, F)}$	(B, E) $\boxed{(NF, NF)}$
(B, E)	(B, E) $\boxed{(NF, NF)}$	(A, C) $\boxed{(F, F)}$

PROBLEM (19)





	c	d
(q_1, q_4)	(q_1, q_4) $\boxed{(F, F)}$	(q_2, q_5) $\boxed{(NF, NF)}$
(q_2, q_5)	(q_3, q_7) $\boxed{(NF, NF)}$	(q_1, q_6) $\boxed{(F, NF)}$ → Stopped.

Given two machines aren't equal.

Converting ε-NFA to NFA

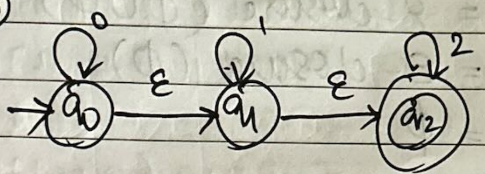
(i) Find out all the ε transitions from each state of Q, that will be called as ε-closure of (q_i) and $q_i \in Q$.

(ii) δ transition can be found (ie) ε-closure of each moves.

(iii) Repeat step 2 for each input symbol and for each state of given NFA.

(iv) By using the result, the transition table for equivalent NFA without ε can be built.

PROBLEM 20



$\varepsilon\text{-closure}(q_0) = \{q_0, q_1, q_2\}$

$\varepsilon\text{-closure}(q_1) = \{q_1, q_2\}$

$\varepsilon\text{-closure}(q_2) = \{q_2\}$

$$\boxed{\hat{\delta}(q_0, \varepsilon) = \varepsilon\text{-closure}(q_0)}$$

$$
\begin{aligned}
\hat{\delta}(q_0, 0) &= \varepsilon\text{-closure}(\delta(\boxed{\hat{\delta}(q_0, \varepsilon)}, 0)) \\
&= \varepsilon\text{-closure}(\delta(\varepsilon\text{-closure}(q_0), 0)) \\
&= \varepsilon\text{-closure}(\delta(q_0, q_1, q_2), 0)) \\
&= \varepsilon\text{-closure}(\delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0)) \\
&= \varepsilon\text{-closure}(q_0 \cup \phi \cup \phi) \\
&= \varepsilon\text{-closure}(q_0)
\end{aligned}
$$

$\delta'(q_0, 0) = \{q_0, q_1, q_2\}$

$$
\begin{aligned}
\hat{\delta}(q_0, 1) &= \varepsilon\text{-closure}(\delta(\hat{\delta}(q_1, \varepsilon), 1)) \\
&= \varepsilon\text{-closure}(\delta(\varepsilon\text{-closure}(q_1), 1)) \\
&= \varepsilon\text{-closure}(\delta(q_0, q_1, q_2), 1)) \\
&= \varepsilon\text{-closure}(\delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1) \\
&= \varepsilon\text{-closure}(\phi \cup q_1 \cup \phi) \\
&= \varepsilon\text{-closure}(q_1) = \{q_1, q_2\}
\end{aligned}
$$

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$$\delta'(q_0, 2) = \varepsilon\text{-closure}(\delta(\hat{\delta}(q_0, \varepsilon), 2))$$
$$= \varepsilon\text{-closure}(\delta(\varepsilon\text{-closure}(q_0), 2))$$
$$= \varepsilon\text{-closure}(\delta(q_0, q_1, q_2), 2))$$
$$= \varepsilon\text{-closure}(\delta(q_0, 2) \cup \delta(q_1, 2) \cup \delta(q_2, 2))$$
$$= \varepsilon\text{-closure}(\phi \cup \phi \cup q_2)$$
$$= \varepsilon\text{-closure}(q_2)$$
$$= \{q_2\}$$

$$\delta'(q_1, 0) = \varepsilon\text{-closure}(\delta(\hat{\delta}(q_1, \varepsilon), 0))$$
$$= \varepsilon\text{-closure}(\delta(\varepsilon\text{-closure}(q_1), 0))$$
$$= \varepsilon\text{-closure}(\delta(q_1, q_2), 0))$$
$$= \varepsilon\text{-closure}(\delta(q_1, 0) \cup \delta(q_2, 0))$$
$$= \varepsilon\text{-closure}(\phi \cup \phi)$$
$$= \varepsilon\text{-closure}(\phi)$$
$$= \phi$$

$$\delta'(q_1, 1) = \varepsilon\text{-closure}(\delta(\hat{\delta}(q_1, \varepsilon), 1))$$
$$= \varepsilon\text{-closure}(\delta(\varepsilon\text{-closure}(q_1), 1))$$
$$= \varepsilon\text{-closure}(\delta(q_1, q_2), 1)$$
$$= \varepsilon\text{-closure}(\delta(q_1, 1) \cup \delta(q_2, 1))$$
$$= \varepsilon\text{-closure}(q_1 \cup \phi)$$
$$= \varepsilon\text{-closure}(q_1)$$
$$= \{q_1, q_2\}$$

$$\delta'(q_1, 2) = \varepsilon\text{-closure}(\delta(\hat{\delta}(q_1, \varepsilon), 2))$$
$$= \varepsilon\text{-closure}(\delta(\varepsilon\text{-closure}(q_1), 2))$$
$$= \varepsilon\text{-closure}(\delta(q_1, q_2), 2)$$
$$= \varepsilon\text{-closure}(\delta(q_1, 2) \cup \delta(q_2, 2))$$
$$= \varepsilon\text{-closure}(\phi \cup q_2)$$
$$= \varepsilon\text{-closure}(q_2) = q_2$$

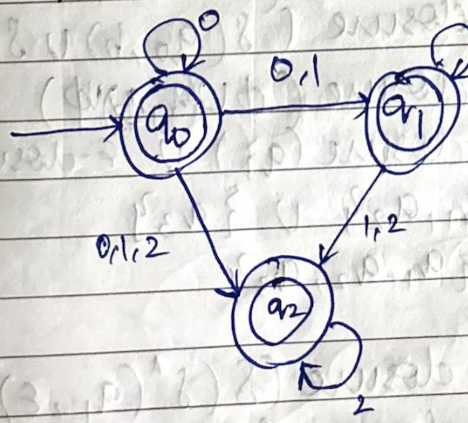
$$\delta'(q_2, 0) = \varepsilon\text{-closure}(\delta(\hat{\delta}(q_2, \varepsilon), 0)$$
$$= \varepsilon\text{-closure}(\delta(\varepsilon\text{-closure}(q_2), 0))$$
$$= \varepsilon\text{-closure}(\delta(q_2), 0)$$

$= \varepsilon\text{-closure}(\delta(q_2, 0))$
$= \varepsilon\text{-closure}(\phi)$
$= \phi$

$\delta'(q_2, 1) = \varepsilon\text{-closure}(\delta(\delta^\wedge(q_2, \varepsilon), 1))$
$= \varepsilon\text{-closure}(\delta(\varepsilon\text{-closure}(q_2), 1))$
$= \varepsilon\text{-closure}(\delta(q_2), 1)$
$= \varepsilon\text{-closure}(\delta(q_2, 1))$
$= \varepsilon\text{-closure}(\overset{\phi}{\cancel{q_1}})$
$= \cancel{\{q_1, q_2\}}\ \phi$

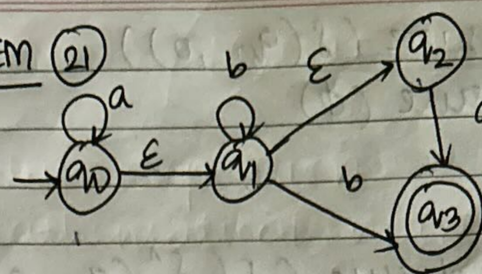
$\delta(q_2, 2) = \varepsilon\text{-closure}(\delta(\delta^\wedge q_2, \varepsilon), 2))$
$= \varepsilon\text{-closure}(\delta(\varepsilon\text{-closure}(q_2), 2))$
$= \varepsilon\text{-closure}(\delta(q_2), 2)$
$= \varepsilon\text{-closure}(\delta(q_2, 2))$
$= \varepsilon\text{-closure}(q_2)$
$= q_2$

	0	1	2
$\rightarrow q_0$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
q_1	ϕ	$\{q_1, q_2\}$	$\{q_2\}$
$* q_2$	ϕ	ϕ	$\{q_2\}$



(State from which ε move is present, declare as final state).

PROBLEM (21)



$\varepsilon\text{-closure}(q_0) = \{q_0, q_1, q_2\}$
$\varepsilon\text{-closure}(q_1) = \{q_1, q_2\}$
$\varepsilon\text{-closure}(q_2) = \{q_2\}$
$\varepsilon\text{-closure}(q_3) = \{q_3\}$

$\delta'(q_0, a) = \varepsilon\text{-closure}(\delta(\hat{\delta}(q_0, \varepsilon), a))$
$= \varepsilon\text{-closure}(\delta(\varepsilon\text{-closure}(q_0), a))$
$= \varepsilon\text{-closure}(\delta(q_0, q_1, q_2), a)$
$= \varepsilon\text{-closure}(\delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_2, a))$
$= \varepsilon\text{-closure}(q_0 \cup \phi \cup q_3)$
$= \varepsilon\text{-closure}(q_0) \cup \varepsilon\text{-closure}(q_3)$
$= \{q_0, q_1, q_2\} \cup \{q_3\}$
$= \{q_0, q_1, q_2, q_3\}$

$\delta'(q_0, b) = \varepsilon\text{-closure}(\delta(\hat{\delta}(q_0, \varepsilon), b))$
$= \varepsilon\text{-closure}(\delta(\varepsilon\text{-closure}(q_0), b))$
$= \varepsilon\text{-closure}(\delta(q_0, q_1, q_2), b))$
$= \varepsilon\text{-closure}(\delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_2, b)$
$= \varepsilon\text{-closure}(\phi \cup (q_1, q_3) \cup \phi)$
$= \varepsilon\text{-closure}(q_1) \cup \varepsilon\text{-closure}(q_3)$
$= \{q_1, q_2\} \cup \{q_3\}$
$= \{q_1, q_2, q_3\}$

$\delta(q_1, a) = \varepsilon\text{-closure}(\delta(\hat{\delta}(q_1, \varepsilon), a))$
$= \varepsilon\text{-closure}(\delta(\varepsilon\text{-closure}(q_1), a))$
$= \varepsilon\text{-closure}(\delta(q_1, q_2), a))$
$= \varepsilon\text{-closure}(\delta(q_1, a) \cup \delta(q_2, a))$
$= \varepsilon\text{-closure}(\phi \cup q_3)$
$= \{q_3\}$

$$\delta'(q_1, b) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_1, \epsilon), b))$$
$$= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_1), b))$$
$$= \epsilon\text{-closure}(\delta((q_1, q_2), b))$$
$$= \epsilon\text{-closure}(\delta(q_1, b) \cup \delta(q_2, b))$$
$$= \epsilon\text{-closure}((q_1, q_3) \cup \phi)$$
$$= \{q_1, q_2, q_3\}$$

$$\delta'(q_2, a) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_2, \epsilon), a))$$
$$= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_2), a))$$
$$= \epsilon\text{-closure}(\delta(q_2, a))$$
$$= \epsilon\text{-closure}(q_3)$$
$$= \{q_3\}$$

$$\delta'(q_2, b) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_2, \epsilon), b))$$
$$= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_2), b))$$
$$= \epsilon\text{-closure}(\delta(q_2, b))$$
$$= \phi$$

	a	b
$\rightarrow q_0$	$\{q_0, q_1, q_2, q_3\}$	$\{q_1, q_2, q_3\}$
q_1	$\{q_3\}$	$\{q_1, q_2, q_3\}$
q_2	$\{q_3\}$	ϕ
$*q_3$	ϕ	ϕ

$$\delta'(q_3, a) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_3, \epsilon), a))$$
$$= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_3), a))$$
$$= \epsilon\text{-closure}(\phi)$$
$$= \phi$$

$$\delta'(q_3, b) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_3, \epsilon), b))$$
$$= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_3), b))$$
$$= \epsilon\text{-closure}(\phi)$$
$$= \phi$$