

First Order Logic

- In propositional logic, we can only represent the facts, which are either true or false. e.g. “It is raining”
- But Propositional logic is not sufficiently expressive to represent the following statements

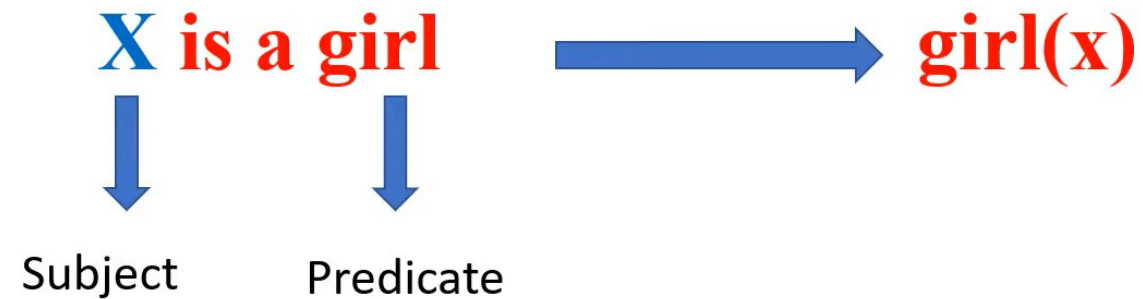
"Some humans are intelligent"

“All mangoes are sweet“

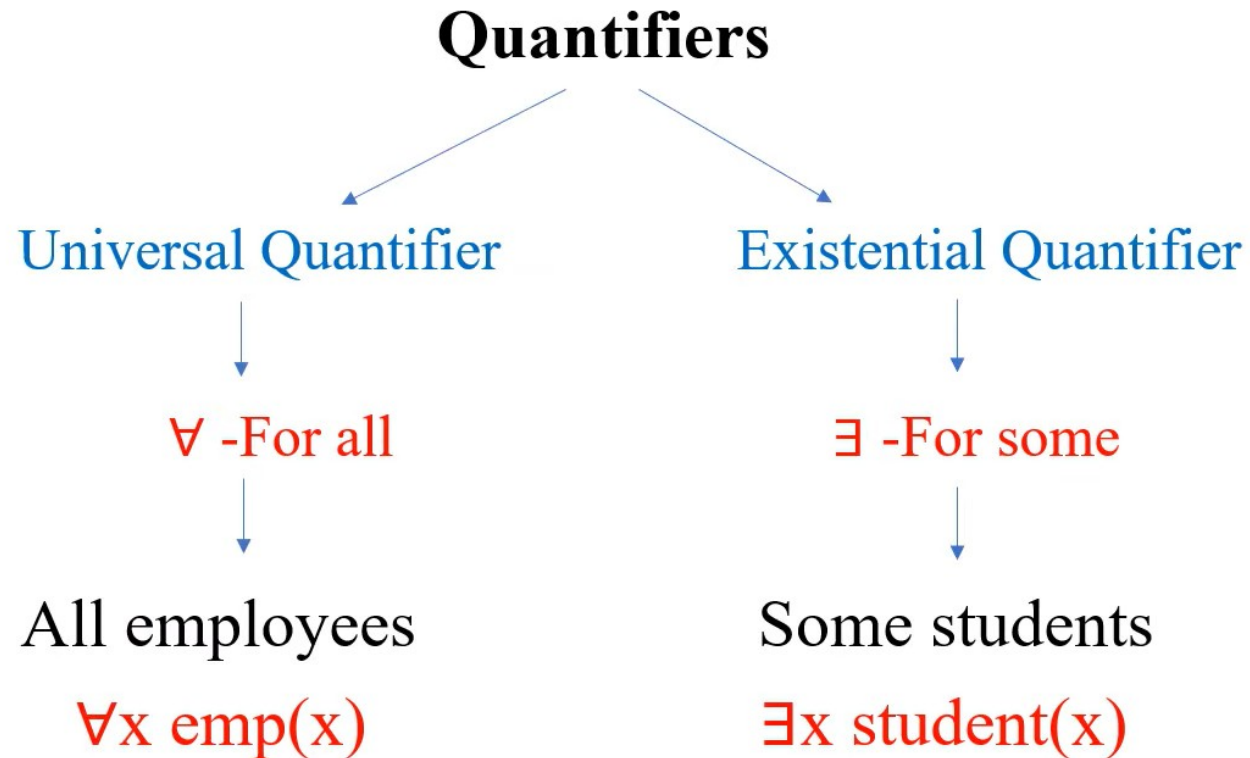
- First-order logic is capable of expressing facts about some or all objects in the universe.
- FOL specifies objects, relations and functions.
- First-order logic is also known as **Predicate logic**.

First Order Logic

- **Syntax:** function (term1, term 2, ..., term n)
Predicate (term1, term2,, term n)



Quantifiers



FOL-Example

- **Sam is tall**

$\text{tall}(\text{Sam})$

- **John likes cricket.**

$\text{likes}(\text{John}, \text{cricket}).$

- **Everyone likes cricket.**

$\forall x \text{ likes}(x, \text{cricket})$

- **All students like cricket.**

$\forall x (\text{student}(x) \rightarrow \text{like}(x, \text{cricket}))$

- **Some boys are intelligent**

$\exists x (\text{boys}(x) \wedge \text{intelligent}(x))$

- **Some integers are even and some are odd**

$\exists x \text{ even}(x) \wedge \exists x \text{ odd}(x)$

- **Not all students like both Mathematics and Science.**

$\neg \forall (x) (\text{student}(x) \rightarrow \text{like}(x, \text{Mathematics}) \wedge \text{like}(x, \text{Science}))$

\wedge	AND
\vee	OR
\sim	NOT
\rightarrow	IF..THEN
\Leftrightarrow	IF AND ONLY IF

\forall	\rightarrow
\exists	\wedge

Elements of FOL

Constant	1, 2, A, John, Mumbai, cat
Variables	x, y, z, a, b, etc.,
Predicates	Greater(5,3), Student(boy, girl).
Function	Mother-of(John) , Colour-of (Basket)
Connectives	\wedge , \vee , \neg , \Rightarrow , \Leftrightarrow
Equality	$=$
Quantifier	\forall , (universal quantifier) \exists , (existential quantifier)

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