MODULE 5

CONVERSION OF PDA TO CFG

Q) $M = (\{q_0, q_1\}, \{a, b\}, \{a, z_0\}, \delta, q_0, z_0, \phi)$

$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, b, a) = (q_1, a)$

$\delta(q_1, a, a) = (q_1, \epsilon)$

soln:

Tuples of PDA :

$$M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$$

CFG is defined by FOUR TUPLES :

$$G = \{V, T, S, P\}$$

Where,

V → Set of non-terminals (A to Z)

Σ or T → Terminals (a-z, operators, Numbers, Special symbols)

S → start symbol.

P → Production rule.

Variable, V is constructed using two rules.

1) Special symbol S

2) [PxQ] where P, Q are states in Q & x is in Γ

Variable is constructed by combining all the possible combinations of variable in Γ with Q ($\Gamma \times Q$)

$$(\{q_0, q_1\} \times \{a, z_0\})$$

$$V = (S, [q_0 a q_0], [q_0, a q_1,], [q_1 a q_0], [q_1 a q_1],$$
$$[q_0, z_0 q_0], [q_0 z_0 q_1] [q_1 z_0 q_0], [q_1 z_0 q_1])$$

Variable is constructed.

Terminals, T

Write the terminal symbols given in Question directly.

$$T = \{a, b\}$$

Now, Terminals also constructed for CFG

Start symbol S

$S \rightarrow$ (Initial state, TOP symbol of stack, set of states)

Here,

Initial state = q_0

TOP symbol of stack = Z_0

Set of states = $\{q_0, q_1\}$

$S \rightarrow [q_0 Z_0 q_0]$

$S \rightarrow [q_0 Z_0 q_1]$

} Start symbol.

Productions.

$\delta(q_0, a, Z_0) = (q_0, \underbrace{aZ_0}_{2})$

NOTE: There are two symbols in stack, so need to construct four production rules.

$\delta(q_0, a, Z_0) = (q_0, aZ_0)$

(Since 2 symbols in the stack, Right side of P. rule contains 2 Brackets).

$[q_0 Z_0 q_0] \rightarrow a [q_0 a q_0] [q_0 Z_0 q_0]$

$[q_0 Z_0 q_0] \rightarrow a [q_0 a q_1] [q_1 Z_0 q_0]$ (same)

$[q_0 Z_0 q_0] \rightarrow a [q_0 a q_0] [q_0 Z_0 q_1]$

$[q_0 Z_0 q_1] \rightarrow a [q_0 a q_1] [q_1 Z_0 q_1]$

$\delta(q_0, b, a) = (q_1, a)$

(Stack contains only one symbol, so 2 production rules can be formed)

$\delta(q_0, b, a) = (q_1, a)$

$[q_0\ a\ q_0] \rightarrow b\,[q_1\ a\ q_0]$

$[q_0\ a\ q_1] \rightarrow b\,[q_1\ a\ q_1]$

(since stack it contains only one symbol, Right side of rule has 1 bracket)

$\delta(q_1, a, a) = (q_1, \epsilon)$ → (since epsilon, only one stack contains production)

$[q_1\ a\ q_1] \rightarrow a$

The Tuples of CFG are,

(After finding every tuples separately, atlast (finally write it like this))

(This part is Important)

$G = \{V, T, S, P\}$

$V = \{S, [q_0 a q_0], [q_0 a q_1], [q_1 a q_0], [q_1 a q_1], [q_0 Z_0 q_0], [q_0 Z_0 q_1], [q_1 Z_0 q_0], [q_1 Z_0 q_1]\}$

$T = \{a, b\}$

$S \rightarrow [q_0\ Z_0\ q_0]$

$S \rightarrow [q_0\ Z_0\ q_1]$

Production Rules : (P)

$[q_0 Z_0 q_0] \rightarrow a\,[q_0 a q_0]\,[q_0 Z_0 q_0]$

$[q_0 Z_0 q_0] \rightarrow a\,[q_0 a q_1]\,[q_1 Z_0 q_0]$

$[q_0 Z_0 q_1] \rightarrow a\,[q_0 a q_0]\,[q_0 Z_0 q_1]$

$[q_0 Z_0 q_1] \rightarrow a\,[q_0 a q_1]\,[q_1 Z_0 q_1]$

$[q_0 a q_0] \rightarrow b\,[q_1 a q_0]$

$[q_0 a q_1] \rightarrow b\,[q_1 a q_1]$

$[q_1 a q_1] \rightarrow a$