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A THEORETICAL ANALYSIS OF THE MOVING OBSERVER METHOD

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1. INTRODUCTION

THE ORIGINAL "moving observer method" was proposed by Wardrop and Charlesworth (1954) as a way of estimating the average flow and journey time of traffic travelling in either direction over a road link solely from measurements made from a moving vehicle. Observers in a test car are required to travel along the road in the direction of the stream considered, counting the number n_s of slower vehicles overtaken and the number n_f of faster vehicles which overtake them, and recording their journey time τ_i. A run is then made in the opposite direction, counting the number n_a of (opposing) vehicles met and again recording the journey time τ_a. On the assumption that the traffic consists of a number of sub-streams, in each of which the vehicles move with constant, uniform spacing and speed, Wardrop and Charlesworth showed that the average flow Q and journey time T of the traffic stream are respectively given by

$$Q = (n_i + n_a)/(\tau_i + \tau_a), \tag{1}$$

$$T = (n_a \tau_i - n_i \tau_a)/(n_i + n_a), \tag{2}$$

where the quantity $n_i = n_f - n_s$ is often called the "tally count".

In practice, the speed and flow of traffic tend to fluctuate randomly from time to time and from place to place, and it was proposed that several runs should be made and the average values $\bar{n}_i$, $\bar{n}_a$, $\bar{\tau}_i$, and $\bar{\tau}_a$ of the respective counts and journey times should be used to determine estimates $\hat{Q}$ and $\hat{T}$ of the average flow and journey time thus:

$$\hat{Q} = (\bar{n}_i + \bar{n}_a)/(\bar{\tau}_i + \bar{\tau}_a), \tag{3}$$

$$\hat{T} = (\bar{n}_a \bar{\tau}_i - \bar{n}_i \bar{\tau}_a)/(\bar{n}_i + \bar{n}_a). \tag{4}$$

It is commonly believed that the standard error of the flow estimate is about the same as the standard error of an equivalent estimate made from flow observations at a fixed point on the road and over the same total period of time that the moving observer spends in transit. This has been confirmed empirically by Mortimer (1956) and Williams and Emmerson (1961) for light traffic. The properties of stationary counts in light traffic are well known, and consequently it is often possible to predict in advance the precision of a moving observer flow estimate for a given number of runs.

On the other hand, little is known about the standard error of the journey time estimate. A table of suggested numbers of runs needed to obtain a specified precision in urban and rural conditions generally was given by Dawson (1968).

Although moving observer runs can theoretically be carried out at almost any speed, for reasons of safety and efficiency it is usual for the driver to regulate his progress systematically in one of several possible ways. In the "floating car" method, he drives (in the direction of the stream considered) in such a way that the number of vehicles he overtakes is equal to the number of vehicles which overtake him; his tally count n_i is then always zero. In the "average run" method, he drives at what he judges to be the average speed of the surrounding traffic. While the latter method is said to require fewer runs, it will be shown that the element of subjective judgement involved can theoretically lead to biassed results, whereas any bias in the floating car method arises independently of the driver's behaviour.

Based on a model of an ideal random traffic stream, theoretical expressions are derived in this paper for the variances of the flow and journey time estimates for three versions of the moving observer method. The effect of turning traffic is briefly examined, and a method is suggested for obtaining flow and journey time estimates from traffic counts at the endpoints of a road link.

2. THE VALIDITY OF THE MOVING OBSERVER METHOD

Flow estimates obtained by a moving observer as compared with fixed point traffic counts

Many authors have made numerical comparisons between moving observer flow estimates and counts made by a stationary observer during the same period. Among others, Blensly (1956) and O'Flaherty and Simons (1970) seem to have regarded the fixed-point counts as the "true" values of the flow—both authors specifically refer to any discrepancies as "errors". This would be reasonable if one were interested only in the value of the flow at the particular point where the count was made and for the particular duration of the experiment, but if one were interested in the road as a whole the moving observer estimate would be a more appropriate figure. In fact, when the runs in opposite directions are carried out simultaneously, and provided there are no errors of observation, it can easily be shown that the estimate given by equation (3) is exactly equal to the mean of the flows actually occurring at the endpoints of the road section during the period of the runs. [See equation (43) below.]

However, the purpose of taking flow measurements is more often to estimate the long term average flow as distinct from the flow occurring during a particular period on a particular day; properly applied, both methods yield estimates which are subject to roughly the same amount of random error and in this respect there is little to choose between them. On roads on which there is a moderate amount of turning traffic the moving observer method may in fact be more efficient, since it may not be necessary to take explicit account of the turning movements. This possibility is considered in section 4 below.

The effect of variations in the speeds of vehicles along the road

It is not always realised that moving observer estimates are unaffected by local variations in the speed of the observer and the observed traffic. When the observer is travelling in the direction of the stream considered, any driver who enters the section later than the observer and who leaves it sooner must overtake the observer one more time than the latter overtakes him. His contribution to the tally count $n_i=n_f-n_s$ must be $+1$. Conversely, any driver who enters the section sooner than the observer and leaves it later contributes -1. No other vehicles affect the tally count. These statements are true regardless of whether the individual drivers or the observer drive at fluctuating speeds along the route, provided that no vehicles enter or leave the road at any intermediate point. When the observer travels in the opposite direction to the stream considered he counts all the vehicles which were in the section when he started plus those which enter during his run; here, the speeds of the observed vehicles are altogether irrelevant.

For theoretical purposes therefore, we need only consider the overall journey speeds of the vehicles involved during the period of the observer's run; it is convenient to picture each vehicle in the stream as travelling at constant speed. A run by the observer of duration τ may be visualised either as a run at constant speed or even as a run at infinite speed followed by a wait, on reaching the end of the section, of period τ.

It also follows that *in practice* it is quite legitimate for a test driver to stop during the course of a run. For instance, in the floating car technique, where it is difficult for the driver to maintain a perpetual balance between the number of overtaking and overtaken vehicles, it may be preferable for him to travel at a relatively fast speed over the section but to stop at the end. He may then wait until sufficient vehicles have passed to bring the tally count to zero, and record his journey time as the run of the running time and the waiting time.

Bias in the estimates of average flow and journey time

Let the true average flow and journey time in the direction considered be Q and T respectively. In what follows it will be seen that the expectations of the estimates $\hat{Q}$ and $\hat{T}$ are not equal to Q and T so that in general the estimates are biassed. Consider an experiment in which M runs are made in both directions over a road link of length Δ. If M is large the differences between the observed means $\bar{n}_i$, $\bar{n}_a$, $\bar{\tau}_i$ and $\bar{\tau}_a$, and their true means or expectations $E(\bar{n}_i)$, $E(\bar{n}_a)$, $E(\bar{\tau}_i)$ and $E(\bar{\tau}_a)$ will be relatively small—we denote their order of magnitude by the symbol δ. The right-hand sides of equations (3) and (4) can therefore be expanded as Taylor series about the quantities $E(\bar{n}_i+\bar{n}_a)/E(\bar{\tau}_i-\bar{\tau}_a)$ and $[E(\bar{n}_a)E(\bar{\tau}_i)-E(\bar{n}_i)E(\bar{\tau}_a)]/E(\bar{n}_i+\bar{n}_a)$ respectively. The expansions are given in equation (A1) and (A2) in the Appendix. Taking expectations of both sides we obtain

$$E(\hat{Q}) = \frac{E(\bar{n}_i+\bar{n}_a)}{E(\bar{\tau}_i+\bar{\tau}_a)} + \frac{E(\bar{n}_i+\bar{n}_a)[\mathrm{var}(\bar{\tau}_i)+\mathrm{var}(\bar{\tau}_a)-2\,\mathrm{covar}(\bar{\tau}_i,\bar{\tau}_a)]}{E^3(\bar{\tau}_i+\bar{\tau}_a)} - \frac{[\mathrm{covar}(\bar{n}_i,\bar{\tau}_i)+\mathrm{covar}(\bar{n}_i,\bar{\tau}_a)+\mathrm{covar}(\bar{n}_a,\bar{\tau}_i)+\mathrm{covar}(\bar{n}_a,\bar{\tau}_a)]}{E^2(\bar{\tau}_i+\bar{\tau}_a)} + O(\delta^3), \quad (5)$$

$$E(\hat{T}) = \frac{E(\bar{n}_a)\,E(\bar{\tau}_i)-E(\bar{n}_i)\,E(\bar{\tau}_a)}{E(\bar{n}_i+\bar{n}_a)} + \frac{E(\bar{\tau}_i+\bar{\tau}_a)[E(\bar{n}_a)\,\mathrm{var}(\bar{n}_i)-E(\bar{n}_i)\,\mathrm{var}(\bar{n}_a)]}{E^3(\bar{n}_i+\bar{n}_a)} + \frac{\{E(\bar{n}_i)[\mathrm{covar}(\bar{n}_a,\bar{\tau}_a)+\mathrm{covar}(\bar{n}_a,\bar{\tau}_i)]-E(\bar{n}_a)[\mathrm{covar}(\bar{n}_i,\bar{\tau}_i)+\mathrm{covar}(\bar{n}_i,\bar{\tau}_a)]\}}{E^2(\bar{n}_i+\bar{n}_a)} + \frac{E(\bar{\tau}_i+\bar{\tau}_a)\,\mathrm{covar}(\bar{n}_i,\bar{n}_a)[E^2(\bar{n}_a)-E^2(\bar{n}_i)]}{E^4(\bar{n}_i+\bar{n}_a)} + O(\delta^3). \quad (6)$$

Now it is reasonable to assume that the test driver behaves consistently from run to run, and does not allow the results of a run to affect his driving in the next. If it is also stipulated that the runs with the stream take place at widely separated intervals of time, and similarly in the case of the runs against the stream, the variables n_i, n_a, τ_i, and τ_a can each be taken as independently and identically distributed from run to run. The product moments on the right-hand sides of equations (5) and (6) can then be expressed in terms of the corresponding product moments for single samples as follows: $\text{var}(\bar{n}_i)=\text{var}(n_i)/M$, $\text{covar}(\bar{n}_i,\bar{\tau}_a)=\text{covar}(n_i, \tau_a)/M$, and so on. It follows that the expectations of the estimates $\hat{Q}$ and $\hat{T}$ vary with the number of runs, and that in general they must be biassed. It should be stressed that this bias arises purely from the way in which the sampled data are aggregated—i.e. according to equations (3) and (4). It has not been found to be significant in practice. Furthermore, as the number of runs M is increased the terms containing the product moments vanish; if one can ensure that the residual terms $E(\bar{n}_i+\bar{n}_a)/E(\bar{\tau}_i+\bar{\tau}_a)$ and $[E(\bar{n}_a)E(\bar{\tau}_i)-E(\bar{n}_i)E(\bar{\tau}_a)]/E(\bar{n}_i+\bar{n}_a)$ are always equal to Q and T respectively then the estimates will be asymptotically unbiassed. Noting that $E(\bar{n}_i)=E(n_i)$, $E(\bar{n}_a)=E(n_a)$, $E(\bar{\tau}_i)=E(\tau_i)$ and $E(\bar{\tau}_a)=E(\tau_a)$, these conditions can be arranged in the form

$$E(n_i) = Q[E(\tau_i) - T], \tag{7}$$

$$E(n_a) = Q[E(\tau_a) + T]. \tag{8}$$

It will be shown in the following section [see equations (13)] that if the test driver fixes in advance the durations τ_i and τ_a of his runs with and against the stream for every run, and succeeds in meeting his schedule, then equations (7) and (8) will necessarily be satisfied. With certain provisos, it is possible to demonstrate a similar result for the case where "target" values of the counts n_i and n_a are fixed instead; for the journey with the stream the particular case $n_i=0$ corresponds to the floating car method, for which it is well known that $E(\tau_i)=T$ and hence equation (7) is satisfied. [Whether equation (8) is also satisfied will be discussed later.] On the other hand, when using the average run method no target is set and the driver is allowed a certain amount of freedom; if he consistently misjudges the average speed of the traffic the results may, in theory, be systematically biassed. To demonstrate this, consider a sequence of runs in moderately congested traffic. On any one run with the stream, the test driver's journey time will uniquely determine the value of the tally count. Two examples of the way in which n_i and τ_i are inter-related are graphically illustrated in Fig. 1; the solid curve refers to a run occurring during the passage of a dense platoon of slow vehicles, and the dotted curve to a run during a period of low flow with vehicles travelling relatively quickly. In either case the test driver will try to match his speed with those of the vehicles he can see, and his journey time will probably be greater than the overall average T when travelling with the dense platoon and less in the period of low flow. But, to take an extreme hypothetical case, he might act in such a contrary way as to obtain samples represented by the points A and B in Fig. 1; if he did this consistently we should have $E(n_i) < Q[E(\tau_i)-T]$ so that from equations (5) and (6), (ignoring small-order terms),

$$E(\hat{Q}) < Q,$$

$$E(\hat{T}) > T.$$

While there is no experimental evidence that this happens in practice, the example serves

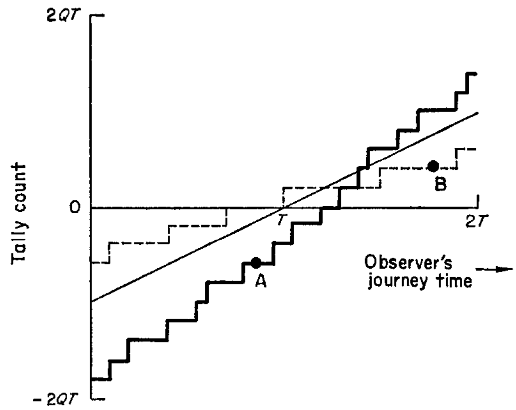

FIG. 1. Schematic illustration of the relationship between a moving observer's tally count and his journey time over a road section when travelling in the direction of the stream considered. The solid line represents a run in slow, dense traffic and the dotted line a run in fast, light traffic. Using the average run method, the driver can theoretically "force" his tally count to conform to any point on the appropriate curve, and the results will be biassed unless the mean tally count and journey time for the sequence of runs both lie on the narrow straight line.

to illustrate that the average run method neither *guarantees* accurate results nor is amenable to theoretical analysis.

A similar problem arises with the floating car method when the flows in the opposing streams are large enough to cause mutual interference. When "floating" against the stream considered, the quantities n_a and τ_a are intrinsically determined both by events in either stream and the interactions between them, and there is no guarantee that equation (8) will be satisfied. On the other hand, if the two streams can be assumed to behave independently, τ_a is effectively determined from a source external to the stream under consideration and can be taken as equivalent to a preset "target". We shall use this assumption in the subsequent analysis. Note also that in the floating car method the average journey time estimate $\hat{T}$ is independent of $\bar{n}_a$ and $\bar{\tau}_a$, and is always unbiassed.

3. THE EFFICIENCY OF SOME MOVING OBSERVER TECHNIQUES

We shall measure the efficiency of the estimators $\hat{Q}$ and $\hat{T}$ in terms of the dimensionless ratios $M\text{var}(\hat{Q})/Q^2$ and $M\text{var}(\hat{T})/T^2$; they are proportional to the numbers of runs needed to achieve any chosen level of confidence that the respective estimates will lie within any given percentage of the true means. Subtracting equation (5) from equation (A.1), squaring, and taking expectations of both sides we obtain that

$$\begin{aligned} E\{[\hat{Q} - E(\hat{Q})]^2\} &= \text{var}(\hat{Q}) = [\text{var}(\bar{n}_i) + \text{var}(\bar{n}_a) + 2\,\text{covar}(\bar{n}_i, \bar{n}_a)]/E^2(\bar{\tau}_i + \bar{\tau}_a) \\ &- 2E(\bar{n}_i + \bar{n}_a)[\text{covar}(\bar{n}_i, \bar{\tau}_i) + \text{covar}(\bar{n}_i, \bar{\tau}_a) + \text{covar}(\bar{n}_a, \bar{\tau}_i) \\ &+ \text{covar}(\bar{n}_a, \bar{\tau}_a)]/E^3(\bar{\tau}_i + \bar{\tau}_a) + E^2(\bar{n}_i + \bar{n}_a)[\text{var}(\bar{\tau}_i) + \text{var}(\bar{\tau}_a) \\ &+ 2\,\text{covar}(\bar{\tau}_i, \bar{\tau}_a)]/E^4(\bar{\tau}_i + \bar{\tau}_a) + O(\delta^3), \end{aligned} \tag{9}$$

and similarly,

$$E\{[\hat{T} - E(\hat{T})]^2\} = \mathrm{var}(\hat{T}) = E^2(\bar{\tau}_i + \bar{\tau}_a)[E^2(\bar{n}_i)\,\mathrm{var}(\bar{n}_a) + E^2(\bar{n}_a)\,\mathrm{var}(\bar{n}_i)$$
$$- 2E(\bar{n}_i)\,E(\bar{n}_a)\,\mathrm{covar}(\bar{n}_i, \bar{n}_a)]/E^4(\bar{n}_i + \bar{n}_a) + 2E(\bar{\tau}_i + \bar{\tau}_a)\{E(\bar{n}_i)\,E(\bar{n}_a)[\mathrm{covar}(\bar{n}_i, \bar{\tau}_a)$$
$$+ \mathrm{covar}(\bar{n}_a, \bar{\tau}_i)] - E^2(\bar{n}_a)\,\mathrm{covar}(\bar{n}_i, \bar{\tau}_i) - E^2(\bar{n}_i)\,\mathrm{covar}(\bar{n}_a, \bar{\tau}_a)\}/E^3(\bar{n}_i + \bar{n}_a)$$
$$+ [E^2(\bar{n}_a)\,\mathrm{var}(\bar{\tau}_i) + E^2(\bar{n}_i)\,\mathrm{var}(\bar{\tau}_a) - 2E(\bar{n}_i)\,E(\bar{n}_a)\,\mathrm{covar}(\bar{\tau}_i, \bar{\tau}_a)]/E^2(\bar{n}_i + \bar{n}_a) + O(\delta^3). \tag{10}$$

In what follows, we attempt to express these equations in terms of the basic parameters of the traffic stream for each of three versions of the moving observer method. For reasons noted earlier, only methods which are effectively preset target methods can be treated in this way.

Preset journey times

The concept of driving to a fixed time schedule does not correspond to any sampling method used in practice, but it appears to be practically feasible (it should be remembered that the driver may vary his speed *en route* and even stop if necessary) and bias-free. Suppose that on any run with the stream the driver completes his journey in the scheduled time $\tau_i = \xi$; ξ may have a different value for each run, which we sssume to be selected at random from a population with p.d.f. $h_i(\xi)$. We have immediately that $\mathrm{covar}(\bar{n}_i, \bar{\tau}_a) = \mathrm{covar}(\bar{n}_a, \bar{\tau}_i) = \mathrm{covar}(\bar{\tau}_i, \bar{\tau}_a) = 0$. Given the journey time target ξ, the conditional expectation of the tally count on a run with the stream is given by (in an obvious notation):

$$E(n_i \mid \tau_i = \xi) = E(n_f \mid \tau_i = \xi) - E(n_s \mid \tau_i = \xi).$$

Let the length of the road section be Δ. Integrating over the vehicle population we find that

$$E(n_i \mid \tau_i = \xi) = Q\left\{\int_{u=\Delta/\xi}^{\infty} \left(\xi - \frac{\Delta}{u}\right) f_0(u)\,\delta u - \int_{u=0}^{\Delta/\xi} \left(\frac{\Delta}{u} - \xi\right) f_0(u)\,\delta u\right\} = Q(\xi - T), \tag{11}$$

where $f_0(u)$ is the p.d.f. of the journey speeds of vehicles observed from a point on the road. Similarly,

$$E(n_a \mid \tau_a = \xi) = Q\int_{u=0}^{\infty} \left(\frac{\Delta}{u} + \xi\right) f_0(u)\,\delta u = Q(\xi + T). \tag{12}$$

It follows that the *a priori* expectation of the tally count before the target is set, given by the expectation of the right-hand side of equation (11), is

$$E(n_i) = Q[E(\tau_i) - T], \tag{13}$$

and similarly,

$$E(n_a) = Q[E(\tau_a) + T]. \tag{14}$$

Let the conditional p.d.f. of n_i when τ_i has the fixed value ξ be $p_i(.\,|\tau_i=\xi)$, and let the conditional p.d.f. of n_a when τ_a has the fixed value ξ be $p_a(.\,|\tau_a=\xi)$. Then

$$\operatorname{var}(n_i) = \int_{\xi=0}^{\infty} h_i(\xi) \sum_{m=0}^{\infty} m^2\, p_i(m \mid \tau_i = \xi)\, d\xi - E^2(n_i).$$

Substituting equation (13) for $E(n_i)$, and writing

$$V_i = \int_{\xi=0}^{\infty} h_i(\xi) \sum_{m=0}^{\infty} [m - E(n_i \mid \tau_i = \xi)]^2\, p_i(m \mid \tau_i = \xi)\, d\xi,$$

we obtain after some manipulation

$$\operatorname{var}(n_i) = V_i + Q^2 \operatorname{var}(\tau_i). \tag{15}$$

Similarly, writing

$$V_a = \int_{\xi=0}^{\infty} h_a(\xi) \sum_{m=0}^{\infty} [m - E(n_a \mid \tau_a = \xi)]^2\, p_a(m \mid \tau_a = \xi)\, d\xi,$$

where $h_a(.)$ is the p.d.f. of the journey times against the stream, we obtain

$$\operatorname{var}(n_a) = V_a + Q^2 \operatorname{var}(\tau_a). \tag{16}$$

Also, it can be shown that

$$\operatorname{covar}(n_i, \tau_i) = \int_{\xi=0}^{\infty} \xi h_i(\xi) \sum_{m=0}^{\infty} m p_i(m \mid \tau_i = \xi)\, d\xi - E(n_i)\, E(\tau_i) = Q \operatorname{var}(\tau_i), \tag{17}$$

and similarly

$$\operatorname{covar}(n_a, \tau_a) = Q \operatorname{var}(\tau_a). \tag{18}$$

For the moment, assume that the runs against the stream are made at different times from the runs with the stream, so that n_i and n_a are independent of each other and $\operatorname{covar}(n_i,n_a)=0$. Substituting for all the product moments and for $E(n_i)$ and $E(n_a)$ in equations (9) and (10) we obtain

$$\operatorname{var}(\hat{Q}) = \frac{V_i + V_a}{ME^2(\tau_i + \tau_a)} + O(\delta^3), \tag{19}$$

$$\operatorname{var}(\hat{T}) = \frac{V_a[E(\tau_i) - T]^2 + V_i[E(\tau_a) + T]^2}{MQ^2\, E^2(\tau_i + \tau_a)} + O(\delta^3). \tag{20}$$

Now, the values of the terms V_i and V_a depend on the characteristics of the traffic stream, for which some statistical model is required. We shall consider in detail only the case of random traffic, in which it is supposed that the arrivals of vehicles at any point on the road form a Poisson process in time, with journey speeds which are independently distributed. As noted earlier, we may visualise each vehicle as travelling at constant speed; then the instances of faster vehicles overtaking the observer and the instances of the observer overtaking slower vehicles also form Poisson processes in time. It follows that for journeys with the stream of fixed duration ξ the variances of n_f and n_s are respectively equal to their means. Thence

$$V_i = \int_{\xi=0}^{\infty} h_i(\xi)\,\mathrm{var}(n_i \mid \tau_i = \xi)\,d\xi = \int_{\xi=0}^{\infty} h_i(\xi)[\mathrm{var}(n_f \mid \tau_i = \xi) + \mathrm{var}(n_s \mid \tau_i = \xi)]\,d\xi$$

$$= \int_{\xi=0}^{\infty} h_i(\xi)\,E(n_f + n_s \mid \tau_i = \xi)\,d\xi,$$

and similarly,

$$V_a = \int_{0}^{\infty} h_a(\xi)\,E(n_a \mid \tau_a = \xi)\,d\xi = Q\int_{0}^{\infty} h_a(\xi)\,(\xi + T)\,d\xi = Q[E(\tau_a) + T].$$

Now the conditional expectation $E(n_s+n_f/\tau_i=\xi)$ is equal to

$$Q\left\{\int_{u=\Delta/\xi}^{\infty}\left(\xi - \frac{\Delta}{u}\right) f_0(u)\,du + \int_{u=0}^{\Delta/\xi}\left(\frac{\Delta}{u} - \xi\right) f_0(u)\,du\right\}.$$

This quantity has been plotted in Fig. 2 for the case where the distribution of the journey speeds of vehicles observed at an instant of time is Pearson Type III, for various values of coefficient of variation ν (ratio of standard deviation to mean). It can be shown that it has a minimum when the observer's journey speed is equal to the median of the distribution of

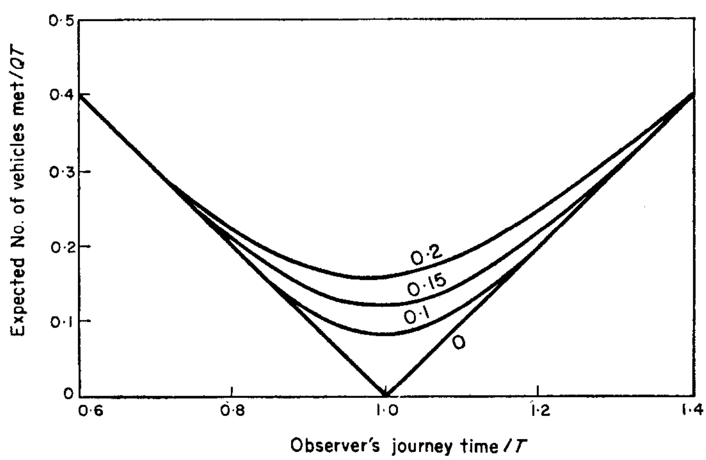

FIG. 2. The expected number of vehicles overtaken by and overtaking the observer on a run with the stream plotted as a function of his journey time, for various values of the coefficient of variation of the journey speeds of vehicles in the stream. The latter are taken as having a Pearson Type III distribution.

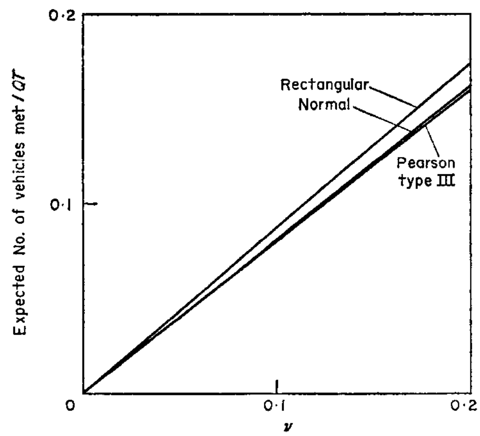

FIG. 3. The expected number of vehicles overtaken by and overtaking the observer on a run with the stream at the same speed as the average journey speed of vehicles in the stream, plotted as a function of the coefficient of variation of their journey speeds for three types of journey speed distribution.

speeds of vehicles observed at a point on the road, a value which is fairly close to the space-mean speed. The values of the minima have been plotted in Fig. 3 against ν both for the Pearson Type III distribution and also for normal and rectangular populations. In the first two cases the minima are given to a close approximation by the expression $0{\cdot}8\nu QT$, whereas in the latter case the appropriate expression would be $0{\cdot}9\nu QT$. It can be taken for practical purposes that if the observer travels within half a standard deviation of the space-mean speed of the vehicle population,

$$E(n_s + n_f \mid \tau_t = \xi) \sim 0{\cdot}8\nu\, QT, \tag{21}$$

with an error of up to about 10 per cent, and if he travels exactly at the space-mean speed, the error will be rather less. We can now write

$$\operatorname{var}(\hat{Q}) \sim \frac{Q[0{\cdot}8\nu T + E(\tau_a) + T]}{ME^2(\tau_t + \tau_a)}, \tag{22}$$

$$\operatorname{var}(\hat{T}) \sim \frac{\{[E(\tau_t) - T]^2 + 0{\cdot}8\nu T[E(\tau_a) + T]\}\,[E(\tau_a) + T]}{MQ\, E^2[\tau_t + \tau_a]}. \tag{23}$$

For runs in which the journey time with and against the stream are chosen to be approximately equal to T the above equations can be written approximately

$$\operatorname{var}(\hat{Q}) \sim (1 + 0{\cdot}4\nu)\, Q/2MT, \tag{24}$$

$$\operatorname{var}(\hat{T}) \sim 0{\cdot}8\nu T/MQ. \tag{25}$$

The similarity between equation (24) and the corresponding expression, $Q/2MT$, for the variance of a flow estimate derived from a fixed-point count over a total period $2MT$ should be noted. In comparison, the moving observer estimate is marginally less reliable—as has been found in practice.

Now consider the effect of having two test vehicles making runs simultaneously with and against the stream. The counts n_i and n_a will no longer be independent: the observer travelling against the stream will meet all the vehicles (n_s+n_f in number) encountered by the observer travelling with the stream, plus an additional number n_z, say. The quantities n_z and (n_s+n_f) will be independent Poisson variates. Therefore,

$$\begin{aligned}\text{covar}(\bar{n}_i, \bar{n}_a) &= \text{covar}(n_i, n_a)/M \\ &= \text{covar}[(n_f - n_s), (n_f + n_s + n_z)]/M \\ &= [\text{var}(n_f) - \text{var}(n_s)]/M \\ &= [E(n_f) - E(n_s)]/M \\ &= E(n_i)/M \\ &= Q[E(\tau_i) - T]/M. \end{aligned} \tag{26}$$

Now, the term $\text{covar}(\bar{n}_i,\bar{n}_a)$ does not occur in equation (9), so that any effect on the precision of the flow estimate must be of order δ^3 or less. On the other hand, from equation (10) it can be seen that the variance of the average journey time estimate is increased by the amount $-2E^2(\bar{\tau}_i+\bar{\tau}_a)E(\bar{n}_i)E(\bar{n}_a)\text{covar}(\bar{n}_i,\bar{n}_a)/E^4(\bar{n}_i+\bar{n}_a)$. Suppose that the test driver consistently exceeds the average journey time T when travelling with the stream by the amount T'; then $\text{covar}(\bar{n}_i,\bar{n}_a)=QT'/M$ and $\text{var}(\hat{T})$ will be decreased roughly by the amount T'^2/MQT. Conversely, if his journey time falls short of the average by the amount T', $\text{var}(\hat{T})$ will be increased by the amount T'^2/MQT. Most test drivers in private cars are more likely to fall in the second category and it would therefore appear that the practice of making simultaneous runs with this method would impair rather than improve the precision of the results.

Preset tally count

We consider only the floating car method, for which $n_i=0$ for every run with the stream. Then

$$E(\bar{n}_i) = \text{var}(\bar{n}_i) = \text{covar}(\bar{n}_i, \bar{\tau}_i) = \text{covar}(\bar{n}_i, \bar{\tau}_a) = \text{covar}(\bar{n}_i, \bar{n}_a) = 0.$$

Again, we are restricted to random traffic, and as before

$$\begin{aligned}\text{covar}(\bar{n}_a, \bar{\tau}_i) &= \text{covar}(\bar{\tau}_i, \bar{\tau}_a) = 0; \\ \text{covar}(\bar{n}_a, \bar{\tau}_a) &= \text{covar}(n_a, \tau_a)/M = Q\,\text{var}(\tau_a)/M; \\ \text{var}(\bar{n}_a) &= \text{var}(n_a)/M = \{Q[E(\tau_a) + T] + Q^2\,\text{var}(\tau_a)\}/M.\end{aligned}$$

Substituting in equation (9) we obtain

$$\text{var}(\hat{Q}) \sim \frac{Q\left\{1 + \dfrac{Q\,\text{var}(\tau_i)}{[T + E(\tau_a)]}\right\}}{M[T + E(\tau_a)]}, \tag{27}$$

and since for this method $\hat{T}=\bar{\tau}_i$ we can write immediately

$$\text{var}(\hat{T}) = \text{var}(\tau_i)/M. \tag{28}$$

Now, the requirement $n_i=0$ means that the test driver has to reach the end of the section at some instant between the times of arrival of two particular consecutive vehicles on each run with the stream. His journey time is therefore controlled only to within a margin corresponding roughly to the average time headway between consecutive vehicles in the stream. We shall suppose for theoretical purposes that he chooses a particular value of journey time ξ with a frequency proportional to the probability that he would have obtained a tally count of zero if he had decided to complete his journey in time ξ beforehand. This probability is given by

$$\begin{aligned}\text{Prob}(n_i = 0 \mid \tau_i = \xi) &= \sum_{j=O}^{\infty} p_f(j \mid \tau_i = \xi)\, p_s(j \mid \tau_i = \xi)\\ &= I_0\{2[E(n_f \mid \tau_i = \xi)\, E(n_s \mid \tau_i = \xi)]^{\frac{1}{2}}\} \exp\{-[E(n_f \mid \tau_i = \xi) + E(n_s \mid \tau_i = \xi)]\}\end{aligned} \tag{29}$$

where $p_f(.)$ and $p_s(.)$ are the (Poisson) p.m.f.s of the numbers of vehicles overtaking and overtaken by the observer, and where $I_0(.)$ is a modified Bessel function of order zero. Now from equation (11) we have $E(n_i|\tau_i=\xi)=E(n_f-n_s|\tau_i=\xi)=Q(\xi-T)$ which can be re-arranged in the forms $E(n_s|\tau_i=\xi)=\frac{1}{2}[E(n_s+n_f|\tau_i=\xi)-Q(\xi-T)]$ and $E(n_f|\tau_i=\xi)=\frac{1}{2}[E(n_s+n_f/\tau_i=\xi)+Q(\xi-T)]$, so that equation (29) can be written

$$\text{Prob}(n_i = O \mid \tau_i = \xi) = I_0\left\{QT\left[\theta^2 - \left(\frac{\xi}{T} - 1\right)^2\right]^{\frac{1}{2}}\right\}\exp(-\,\theta QT),$$

where $\theta=E(n_s+n_f/\tau_i=\xi)/QT$. Let the p.d.f. of ξ be denoted by $g(\xi)$. Using the asymptotic expansion $\mathscr{L}t_{x\to\infty}\, I_0(x)=e^x/(2\pi x)^{\frac{1}{2}}$ we can then write

$$\mathscr{L}t_{T\to\infty}\, g(\xi) = C\exp\left(-\,\theta QT\left\{1 - \left[1 - \frac{1}{\theta^2}\left(\frac{\xi}{T} - 1\right)^2\right]^{\frac{1}{2}}\right\}\right)\Big/(2\pi\theta QT)^{\frac{1}{2}}\left[1 - \frac{1}{\theta^2}\left(\frac{\xi}{T} - 1\right)^2\right]^{\frac{1}{2}},$$

where C is a constant. Replacing the terms $[1 - 1/\theta^2(\xi/T - 1)^2]^{\frac{1}{2}}$ and $\left[1 - \frac{1}{\theta^2}\left(\frac{\xi}{T} - 1\right)^2\right]^{\frac{1}{2}}$

by appropriate binomial expansions we get

$$\mathscr{L}t_{\substack{T\to\infty\\ \xi\to T}}\, g(\xi) = C\exp\left[-\,\tfrac{1}{2}\cdot\frac{(\xi - T)^2}{(\theta T/Q)}\right]\Big/(2\pi)^{\frac{1}{2}}\,(\theta QT)^{\frac{1}{2}}. \tag{30}$$

The expression on the right-hand side of equation (30) is a normal p.d.f., implying that ξ is asymptotically normally distributed with mean T and variance $\theta T/Q$ for long road links carrying non-zero flow. Note also that since the coefficient of variation of the distribution of ξ tends to zero as $T\to\infty$, the test driver's journey times will fluctuate by asymptotically small relative amounts about the mean journey time of the traffic stream. We can therefore use the approximation of equation (21); putting $\theta\sim 0{\cdot}8\nu$ and substituting for $\text{var}(\tau_i)$ in equation (28) we obtain finally

$$\mathscr{L}t_{T\to\infty}\, \text{var}(\hat{T}) \sim 0{\cdot}8\nu T/MQ. \tag{31}$$

Note that the error of approximation in the result (31) arises only from the assumption of equation (21); it can therefore be compared directly with the expression for $\text{var}(\hat{T})$ using the preset journey time method [equation (25)]. It can be seen that the efficiency of the floating car method approaches that of the present journey time method for long roads.

In the general case it is necessary to evaluate $\text{var}(\hat{T})$ by a combination of graphical and numerical methods. The results will be referred to later.

Following a vehicle at random

The essential feature of the preset journey time method is that the journey times are fixed independently of traffic conditions on each run. In random traffic, every vehicle moves independently of every other, so that if the test driver were to choose vehicles at random and follow them along the road section he would effectively be using preset journey times whose distribution would duplicate that of the vehicle population.

For a traffic stream whose distribution of speeds at instants of time is normal with a coefficient of variation of ν, Wardrop (1952) has shown that the total average number of overtakings per unit distance and per unit time is (in our notation) $\nu Q^2 T/\Delta(\pi^{\frac{1}{2}})$. It follows that the average number of vehicles encountered by the test driver on a run with the stream $E(n_s+n_f)$ is equal to $[\nu Q^2 T/\Delta(\pi)^{\frac{1}{2}}]\times 2\Delta/Q$. This result can also be shown to be approximately true if the vehicle speeds follow a Pearson Type III distribution. The remainder of the analysis follows that for the preset journey time method and we quote only the results:

$$\text{var}(\hat{Q}) \sim (1 + 0{\cdot}56\nu)\ Q/M[E(\tau_a) + T]; \tag{32}$$

$$\text{var}(\hat{T}) \sim 1{\cdot}12\nu T/MQ. \tag{33}$$

If the test driver's average journey times with and against the stream are approximately equal, equation (32) can be written approximately as

$$\text{var}(\hat{Q}) \sim (1 + 0{\cdot}56\nu)\ Q/2MT. \tag{34}$$

The comparative efficiency of the methods discussed

The quantities $M\text{var}(\hat{Q})/Q^2$ and $M\text{var}(\hat{T})/T^2$ have been plotted respectively in Figs. 4 and 5 against $2QT$ for values of ν equal to 0·1 and 0·2, for the floating car method, the preset journey time method, and the "following a car at random" method. In each case it has been assumed that the traffic is sufficiently light to be regarded as random, and that the test driver's journey times with and against the stream have approximately equal means. In the preset journey time method it is supposed in addition that the test driver's journey times are roughly equal to the average for the whole traffic stream in the direction considered.

The curves illustrate three important properties of moving observer methods:

(i) in general, the number of runs required to obtain any given level of confidence that either the flow estimate or the average journey time estimate lie within a given percentage of the true mean is inversely proportional to the product of the true average flow and the true average journey time;

(ii) but the attainment of a given level of precision in the flow estimate requires roughly $1/2\nu$ times as many runs as the attainment of the same level of precision in the average journey time estimate;

(iii) the number of runs needed to obtain a given level of precision in the average journey time estimate varies appreciably with the experimental procedure used, whereas in the case of the average flow estimate the choice of method has little effect.

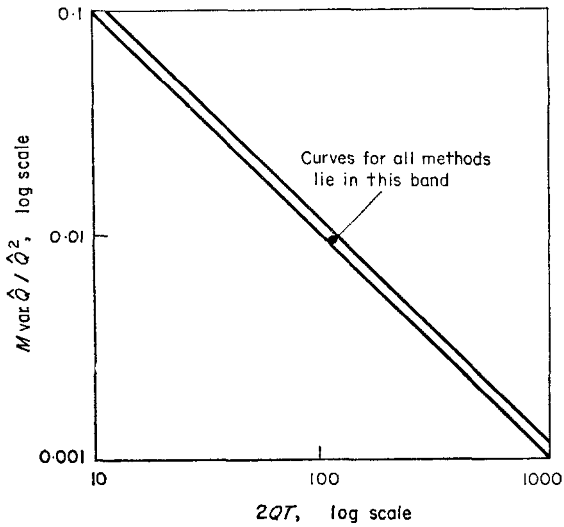

FIG. 4. The variances of average flow estimates obtained by using different versions of the moving observer method plotted as a function of the average number of vehicles entering the road (in the direction considered) during a two-way run. It is assumed that the test driver travels approximately at the average speed of the traffic stream and that the traffic is random. The units are standardised.

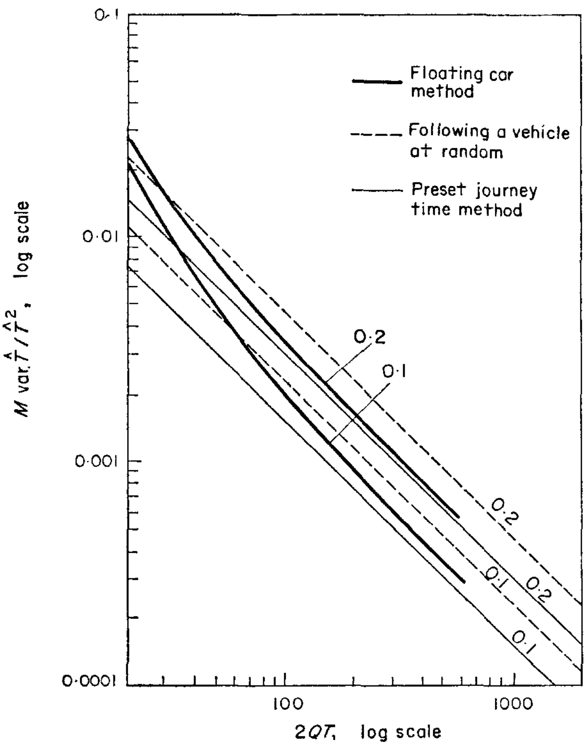

FIG. 5. The variances of average journey time estimates obtained by using different versions of the moving observer method plotted as a function of the average number of vehicles entering the road (in the direction considered) during a two-way run, under the same conditions applying to Fig. 4.

The curves also show that the preset journey time method is more efficient than the other methods at all values of QT; in comparison the method of following a vehicle at random requires about 50 per cent more runs to achieve the same level of precision, whereas the floating car method is the least efficient at low flow levels but compares more favourably with the other methods at high values of QT.

The precision of estimates made from runs in congested traffic

In moderately congested traffic on rural roads and urban arterials, vehicles interfere with one another's progress and they tend to gather into platoons or bunches. The arrivals of vehicles at a point on the road and the instances of vehicles overtaking or being overtaken by an observer moving at constant speed are no longer random events, the journey speeds of consecutive vehicles tend to be correlated, and the assumptions of the previous analysis break down. In particular, the variances of the numbers of arrivals or overtakings in consecutive time intervals tend to exceed the means, as compared to a Poisson or random process in which they are theoretically equal; for given values of ν, Q and T the quantities var(n_i) and var(n_a) will be relatively greater than those for random traffic. (The floating car method is an exceptional case, since var$(n_i)=O$ necessarily in all types of traffic. However, there are reasons for believing that the quantity var(τ_i) will increase instead, as a result of what might be called a degree of symmetry between the behaviour of the variables n_i and τ_i when either one or the other is fixed). Assuming that the variances of n_i, n_a, τ_i, and τ_a are the dominant terms in equations (9) and (10), it seems probable that in congested traffic the variances of the estimates of average flow and journey time will be increased, and their precision correspondingly reduced, whichever method is used.

In congested urban conditions, one has the additional factor that vehicles are subject to delay from external sources, their journey times tend to be more variable, and the value of ν is effectively very high. Thus (as has been found in practice) one would expect moving observer estimates of the average journey time in congested city streets to be rather unreliable.

4. SOME POSSIBLE LINES OF DEVELOPMENT OF THE MOVING OBSERVER METHOD

Application to roads on which there is turning traffic

In cases where a lot of traffic enters or leaves the road link under consideration at intermediate junctions or access roads, it is usual to carry out separate runs in each sub-section. If the junctions are controlled by signals one has no alternative because the periodic delays involved at the signals must be examined separately. However, if the junctions are uncontrolled one may be prepared to accept estimates of the average journey time *per mile* and the average flow in *vehicle-miles per hour* for the whole link, rather than treating each subsection as a separate problem. Under certain conditions, these can be obtained using standard moving observer methods applied to the link as a whole. To illustrate the possibilities, consider a link which is divided by an intermediate junction into two sub-sections of length Δ_1 and Δ_2, on which the average flows and journey times (in the direction considered) are repectively Q_1, T_1 and Q_2, T_2. We use the previous notation, adding the subscript 1 or 2 to distinguish quantities referring to either subsection. Quantities without a subscript refer to the whole link. Firstly, from equations (5) and (6), we have

$$\mathscr{L}t_{M\to\infty}\ E(\hat{Q}) = \frac{E(\bar{n}_i + \bar{n}_a)}{E(\bar{\tau}_i + \bar{\tau}_a)} = \frac{E(\bar{n}_{i1} + \bar{n}_{i2} + \bar{n}_{a1} + \bar{n}_{a2})}{E(\bar{\tau}_{i1} + \bar{\tau}_{i2} + \bar{\tau}_{a1} + \bar{\tau}_{a2})}; \tag{35}$$

$$\begin{aligned}\mathscr{L}t_{M\to\infty}\ E(\hat{T}) &= \frac{E(\bar{n}_a)\,E(\bar{\tau}_i) - E(\bar{n}_i)\,E(\bar{\tau}_a)}{E(\bar{n}_i + \bar{n}_a)} \\ &= \frac{E(\bar{n}_{a1} + \bar{n}_{a2})\,E(\bar{\tau}_{i1} + \bar{\tau}_{i2}) - E(\bar{n}_{i1} + \bar{n}_{i2})\,E(\bar{\tau}_{a1} + \bar{\tau}_{a2})}{E(\bar{n}_{i1} + \bar{n}_{i2} + \bar{n}_{a1} + \bar{n}_{a2})}.\end{aligned} \tag{36}$$

Assuming that a preset target method is used we have from equations (13) and (14)

$$\begin{aligned}E(n_{i1}) &= Q_1[E(\tau_{i1}) - T_1], \\ E(n_{i2}) &= Q_2[E(\tau_{i2}) - T_2], \\ E(n_{a1}) &= Q_1[E(\tau_{a1}) + T_1], \\ E(n_{a2}) &= Q_2[E(\tau_{a2}) + T_2].\end{aligned}$$

Substituting in equations (35) and (36) and re-arranging we obtain

$$\mathscr{L}t_{M\to\infty}\ E(\hat{Q}) = \frac{Q_1\,E(\tau_{i1} + \tau_{a1}) + Q_2\,E(\tau_{i2} + \tau_{a2})}{E(\tau_{i1} + \tau_{i2} + \tau_{a1} + \tau_{a2})}; \tag{37}$$

$$\mathscr{L}t_{M\to\infty}\ E(\hat{T}) = \frac{(Q_1\,T_1 + Q_2\,T_2)\,E(\tau_{i1} + \tau_{i2} + \tau_{a1} + \tau_{a2})}{[QE(\tau_{i1} + \tau_{a1}) + Q_2\,E(\tau_{i2} + \tau_{a2})]}. \tag{38}$$

Let the observer's journey speeds in either sub-section have the same harmonic mean v. Then $E(\tau_{i1})=E(\tau_{a1})=\Delta_1/v$ and $E(\tau_{i2})=E(\tau_{a2})=\Delta_2/v$ so that after some re-arrangement,

$$Q_1\,\Delta_1 + Q_2\,\Delta_2 = (\Delta_1 + \Delta_2)\,[\mathscr{L}t_{M\to\infty}\ E(\hat{Q})]; \tag{39}$$

$$\frac{Q_1\,T_1 + Q_2\,T_2}{Q_1\,\Delta_1 + Q_2\,\Delta_2} = [\mathscr{L}t_{M\to\infty}\ E(\hat{T})]/(\Delta_1 + \Delta_2). \tag{40}$$

The quantities on the left-hand sides of equations (39) and (40) are respectively the rate of travel on the whole link, in vehicle-miles per hour, and the average journey time per mile. Asymptotically unbiassed estimates of these quantities will therefore be obtained by substituting numerical estimates $\hat{Q}$ and $\hat{T}$ for $E(\hat{Q})$ and $E(\hat{T})$ in equations (39) and (40).

These results also imply that small volumes of turning traffic on a road link can be virtually ignored. On well designed roads they are unlikely to be associated with large changes in the speed of the traffic stream and one would not expect the test driver's journey speeds to vary significantly between the sub-sections. Consequently $\hat{Q}$ and $\hat{T}$ can be taken as reasonably fair estimates of the average flow and journey time over the whole section—the important point to note is that they are relatively insensitive to turning movements, whereas fixed-point counting methods are not.

A method of obtaining estimates of the average flow and journey time from fixed-point traffic counts

Consider a sequence of M two-way runs on a road link of length Δ on which there is no turning traffic, in which the runs against the stream are made simultaneously with the runs with the stream, each pair of runs following immediately after the preceding pair and taking time τ. Substituting $E(\bar{\tau}_i)=E(\bar{\tau}_a)=\tau$ in equations (3) and (4) we have

$$\hat{Q} = (\bar{n}_i + \bar{n}_a)/2\tau; \tag{41}$$

$$\hat{T} = (\bar{n}_a - \bar{n}_i)\,\tau/(\bar{n}_i + \bar{n}_a). \tag{42}$$

Let a_m, b_m and c_m be the number of vehicles entering the section (in the direction considered) during the mth run, the number in the section when that run starts, and the number leaving the section during the run, respectively. The values of n_i and n_a that will be counted on the mth run will then be $a_m - b_{m+1}$ $(=c_m - b_m)$ and $c_m + b_{m+1}$ $(=a_m + b_m)$; substituting in equations (41) and (42) we find that

$$\hat{Q} = \sum_{m=1}^{M} (a_m + c_m)/2\tau; \tag{43}$$

$$\hat{T} = \tau\left[b_1 + b_{M+1} + 2\sum_{m=2}^{M} b_m\right]\Big/\sum_{m=1}^{M} (a_m + c_m). \tag{44}$$

In this type of experiment therefore, the information required can be expressed in the form of counts of vehicles at the ends of the section and counts of vehicles in the section at equally spaced instants of time. In principle, these measurements can be made with apparatus other than a moving vehicle. Furthermore, from the "law" of conservation of vehicles we have that

$$\begin{aligned} b_{M+1} &= a_M + b_M - c_M, \\ b_M &= a_{M-1} + b_{M-1} - c_{M-1}, \\ &\ \vdots \qquad\quad \vdots \\ b_2 &= a_1 + b_1 - c_1. \end{aligned}$$

In other words, if one counts the number of vehicles b_1 in the section at the beginning of the experiment and also the numbers of vehicles entering and leaving the section during successive intervals of period τ, one can deduce the values of $b_2, b_z, \ldots, b_{M+1}$ and hence calculate the estimates $\hat{Q}$ and $\hat{T}$ without any further measurements. The value of b_1 can easily be determined as follows: one observer makes a single run with the stream at any speed he wishes, making a tally count n_i in the normal way. When he reaches the end of the section he signals another observer, who remains at the beginning of the section and counts the number of vehicles which enter the section, n_e say, during the period of his partner's journey. At the instant of the signal, the number of vehicles in the section will be given by $n_e - n_i$. The two observers may then remain at their respective ends of the section, counting the numbers of vehicles which pass them in successive intervals τ.

While this method appears to offer advantages of economy both in terms of manpower and vehicle usage, it should be stressed that, unlike moving observer methods, the results may be misleading if any vehicles enter or leave the section at an intermediate junction. Errors in the counts may also cause disproportionate errors in the estimates. Properties of the estimates and formulae for them are to be discussed in a subsequent paper, [Wright *et al.* (1973)].

5. CONCLUSIONS

Theoretically, average flow and journey time estimates obtained with conventional moving observer methods are subject to a small degree of bias. In the average run method, for which an analytical model cannot be formulated because it is essentially indeterminate, the amount of bias depends on the skill of the driver, whereas in the floating car method it depends on the degree of interaction between the opposing traffic streams.

Local variations in the speeds of the observer and the observed traffic (as opposed to variations in their average speed over the whole section) have no effect on the tally count; the observer can speed up, slow down or even stop during a run. In principle, therefore, he is free to lose or make up time as necessary, and could use this facility to drive to a prearranged time schedule. It has been shown that this procedure is both relatively efficient and gives asymptotically unbiassed results.

Variations in flow along a road caused by turning traffic tend to be "smoothed out" in moving observer estimates, and if suitable precautions are taken quite large turning movements can be encompassed in a single run.

Further research is required as to the possibility of estimating average journey times on a road link by means of traffic counts at each end of the link.

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APPENDIX

Taylor series expansions of the formulae for the average flow and journey time estimates

From equation (3), treating $\hat{Q}$ as a function of the variables $\bar{n}_i$, $\bar{n}_a$, $\bar{\tau}_i$ and $\bar{\tau}_a$ and expanding about their expectations, we have

$$\hat{Q} = \frac{E(\bar{n}_i + \bar{n}_a)}{E(\bar{\tau}_i + \bar{\tau}_a)} + [\bar{n}_i - E(\bar{n}_i)] \frac{\partial \hat{Q}}{\partial \bar{n}_i} + [\bar{n}_a - E(\bar{n}_a)] \frac{\partial \hat{Q}}{\partial \bar{n}_a} + [\bar{\tau}_i - E(\bar{\tau}_i)] \frac{\partial \hat{Q}}{\partial \bar{\tau}_i}$$

$$+ [\bar{\tau}_a - E(\bar{\tau}_a)] \frac{\partial \hat{Q}}{\partial \bar{\tau}_a} + \frac{[\bar{n}_i - E(\bar{n}_i)]^2}{2!} \frac{\partial^2 \hat{Q}}{\partial \bar{n}_i^2} + \frac{[\bar{n}_a - E(\bar{n}_a)]^2}{2!} \frac{\partial^2 \hat{Q}}{\partial \bar{n}_a^2}$$

$$+ \frac{[\bar{\tau}_i - E(\bar{\tau}_i)]^2}{2!} \frac{\partial^2 \hat{Q}}{\partial \bar{\tau}_i^2} + \frac{[\bar{\tau}_a - E(\bar{\tau}_a)]^2}{2!} \frac{\partial^2 \hat{Q}}{\partial \bar{\tau}_a^2} + [\bar{n}_i - E(\bar{n}_i)] [\bar{n}_a - E(\bar{n}_a)] \frac{\partial^2 \hat{Q}}{\partial \bar{n}_i \, \partial \bar{n}_a}$$

$$+ [\bar{n}_i - E(\bar{n}_i)] [\bar{\tau}_a - E(\bar{\tau}_a)] \frac{\partial^2 \hat{Q}}{\partial \bar{n}_i \, \partial \bar{\tau}_a} + [\bar{n}_i - E(\bar{n}_i)] [\bar{\tau}_i - E(\bar{\tau}_i)] \frac{\partial^2 \hat{Q}}{\partial \bar{n}_i \, \partial \bar{\tau}_i}$$

$$+ [\bar{n}_a - E(\bar{n}_a)] [\bar{\tau}_i - E(\bar{\tau}_i)] \frac{\partial^2 \hat{Q}}{\partial \bar{n}_a \, \partial \bar{\tau}_i} + [\bar{n}_a - E(\bar{n}_a)] [\bar{\tau}_a - E(\bar{\tau}_a)] \frac{\partial^2 \hat{Q}}{\partial \bar{n}_a \, \partial \bar{\tau}_a}$$

$$+ [\bar{\tau}_i - E(\bar{\tau}_i)] [\bar{\tau}_a - E(\bar{\tau}_a)] \frac{\partial^2 \hat{Q}}{\partial \bar{\tau}_i \, \partial \bar{\tau}_a} + O(\delta^3),$$

where all the partial derivatives are evaluated at the expectations of $\bar{n}_i$, $\bar{n}_a$, $\bar{\tau}_i$ and $\bar{\tau}_a$, and the symbol $O(\delta^3)$ represents subsequent terms of order $[\bar{n}_i - E(\bar{n}_i)]^3$, $[\bar{\tau}_i - E(\bar{\tau}_i)]^3$ etc. Inserting expressions for the partial derivatives we find that

$$\begin{aligned}
\hat{Q} = {} & \frac{E(\bar{n}_i + \bar{n}_a)}{E(\bar{\tau}_i + \bar{\tau}_a)} + \frac{[\bar{n}_i - E(\bar{n}_i)] + [\bar{n}_a - E(\bar{n}_a)]}{E(\bar{\tau}_i + \bar{\tau}_a)} \\
& - \frac{E(\bar{n}_i + \bar{n}_a) \{[\bar{\tau}_i - E(\bar{\tau}_i)] + [\bar{\tau}_a - E(\bar{\tau}_a)]\}}{E^2(\bar{\tau}_i + \bar{\tau}_a)} \\
& + \frac{E(\bar{n}_i + \bar{n}_a) \{[\bar{\tau}_i - E(\bar{\tau}_i)] + [\bar{\tau}_a - E(\bar{\tau}_a)]\}^2}{E^3(\bar{\tau}_i + \bar{\tau}_a)} \\
& - \frac{\{[\bar{n}_i - E(\bar{n}_i)] + [\bar{n}_a - E(\bar{n}_a)]\} \{[\bar{\tau}_i - E(\bar{\tau}_i)] + [\bar{\tau}_a - E(\bar{\tau}_a)]\}}{E^2(\bar{\tau}_i + \bar{\tau}_a)} + O(\delta^3).
\end{aligned} \tag{A.1}$$

Similar treatment of the expression for the average journey time in equation (4) yields

$$\begin{aligned}
\hat{T} = {} & \frac{E(\bar{n}_a) E(\bar{\tau}_i) - E(\bar{n}_i) E(\bar{\tau}_a)}{E(\bar{n}_i + \bar{n}_a)} + \frac{E(\bar{\tau}_i + \bar{\tau}_a) \{E(\bar{n}_i) [\bar{n}_a - E(\bar{n}_a)] - E(\bar{n}_a) [\bar{n}_i - E(\bar{n}_i)]\}}{E^2(\bar{n}_i + \bar{n}_a)} \\
& + \frac{E(\bar{n}_a) [\bar{\tau}_i - E(\bar{\tau}_i)] - E(\bar{n}_i) [\bar{\tau}_a - E(\bar{\tau}_a)]}{E(\bar{n}_i + \bar{n}_a)} \\
& + \frac{E(\bar{\tau}_i + \bar{\tau}_a) \{E(\bar{n}_a) [\bar{n}_i - E(\bar{n}_i)]^2 - E(\bar{n}_i) [\bar{n}_a - E(\bar{n}_a)]^2\}}{E^3(\bar{n}_i + \bar{n}_a)} \\
& - \frac{\{[\bar{\tau}_i - E(\bar{\tau}_i)] + [\bar{\tau}_a - E(\bar{\tau}_a)]\} \{E(\bar{n}_a) [\bar{n}_i - E(\bar{n}_i)] - E(\bar{n}_i) [\bar{n}_a - E(\bar{n}_a)]\}}{E^2(\bar{n}_i + \bar{n}_a)} \\
& + \frac{[\bar{n}_i - E(\bar{n}_i)] [\bar{n}_a - E(\bar{n}_a)] [E^2(\bar{n}_a) - E^2(\bar{n}_i)] E(\bar{\tau}_i + \bar{\tau}_a)}{E^4(\bar{n}_i + \bar{n}_a)} + O(\delta^3).
\end{aligned} \tag{A.2}$$

Abstract—The moving observer method is a procedure commonly used to estimate the average flow and journey time of traffic on a road link through data collected from a moving vehicle. For each of several versions of the basic method, theoretical expressions are derived for the variances of the estimates on the assumption that the traffic flow is random, and the possibility of systematic bias is examined. Moving observer tests appear to be surprisingly insensitive both to fluctuations in the speeds of vehicles and variations in flow along the road, and, paradoxically, can sometimes be carried out without the use of a moving vehicle.

Résumé—La méthode de l'observateur mobile est un procédé couramment utilisé pour évaluer les flux moyens de circulation et les durées de trajet sur un tronçon de voirie à partir des données recueillies à bord d'un véhicule en déplacement. Pour chacune des différentes versions de la méthode de base sont proposées des relations donnant les variances des évaluations, basées sur l'hypothèse d'un flux de circulation aléatoire; en outre l'éventualité d'une erreur systématique est examinée. Les mesures effectuées par l'observateur mobile se révèlent curieusement insensibles à la fois aux variations de la vitesse des véhicules et aux variations du flux le long du tronçon; paradoxalement, ces mesures peuvent parfois être effectuées sans recourir à l'utilisation d'un véhicule en déplacement.

Zusammenfassung—Verkehrsbeobachtungen vom fahrenden Fahrzeug aus werden allgemein dazu benutzt, die durchschnittliche Verkehrsmenge und Reisegeschwindigkeit entlang eines Strassenabschnittes zu ermitteln. Für eine Reihe von Messverfahren, die auf einem gemeinsamen Grundkonzept aufbauen, werden theoretische Ausdrücke für die Varianz der Schätzwerte unter der Voraussetzung entwickelt, daß der Verkehrsablauf statistischer Natur ist. Auch wird die Möglichkeit des Auftretens systematischer Abweichungen untersucht. Es zeigt sich, daß Fahrzeugbeobachtungen überraschend unempfindlich sind gegenüber Änderungen der Fahrzeuggeschwindigkeiten und der Verkehrsmengen entlang des untersuchten Strassenabschnitts. Paradoxerweise können sie unter bestimmten Bedingungen auch ohne Benutzung eines Fahrzeuges durchgeführt werden.