**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Solution: B

Z=(x-u)/ σ

Average times increases , so it becomes= x=45+10=55 and σ=8

probability that the service manager cannot meet his commitment= 1 – ((60-55)/8) =0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Solution: Z=(x-u)/ σ

Z for 44= (44-38)/6 =1 => 84.13%( from z score table)

Z for 38= (38-38)/6= 0 => 50%(from z score table)

Employees b/t 38 to 44= (84.13-50)= 34.13% = approx. 137 employees

Z for >44= 100 - 84.13= 15.87 = approx. 63 employees

Hence, above statement is false

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Solution: Z for <30= (30-38)/6 =-1.33 = 0.0915 = approx. 36

Hence, Statement is true.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Solution: *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) then,

2 *X*1 ~ N(2u, 4 σ2 )

*X*1 + *X*2 ~N (2u+2 σ2 )

( 2 *X*1 – (*X*1 + *X*2 )) = 2 σ2

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Solution: D

np.round(stats.norm.interval(0.99, loc = 100, scale = 20),1))

 [ 48.5 ,151.5]

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Solution:



