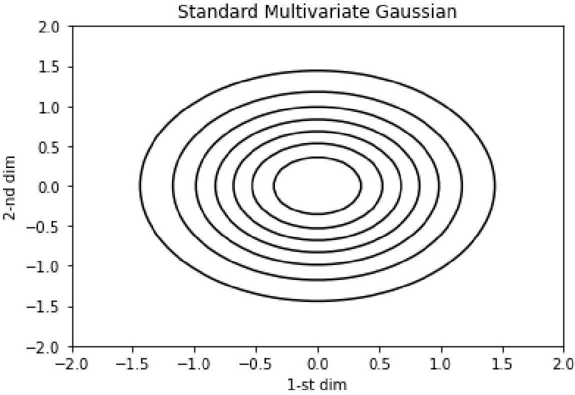
Deep Programming Probabilistic Languages

Vangjus Komini

Assignment 1

August 29, 2022

Gaussian distribution



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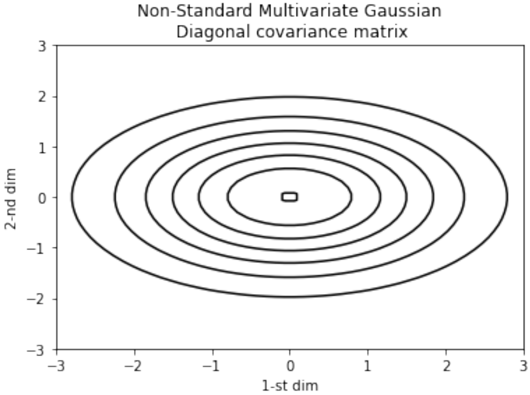

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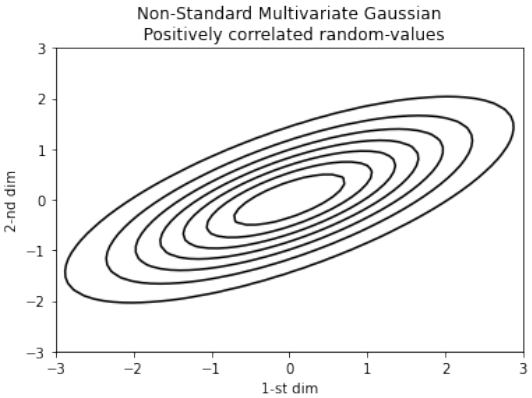

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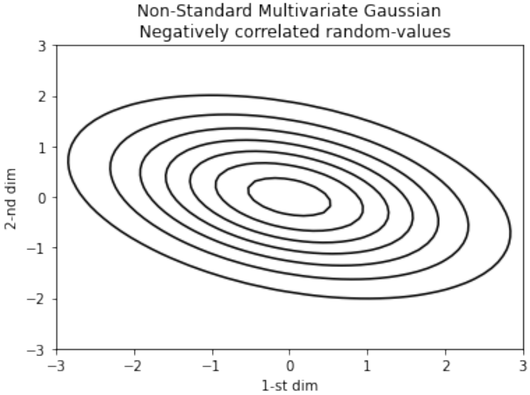

Likelihood Gaussian distribution

$$L(\mu, \Sigma|x) = -\frac{(x-\mu)^T\Sigma^{-1}(x-\mu)}{2} - \frac{1}{2}\{N * ln(2\pi) + ln(|\Sigma|)\}$$

The log-likelihood for both mean given the covariance matrix is:

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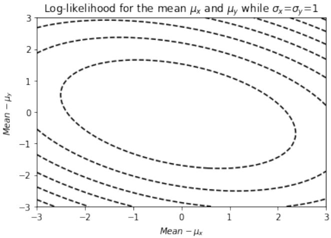

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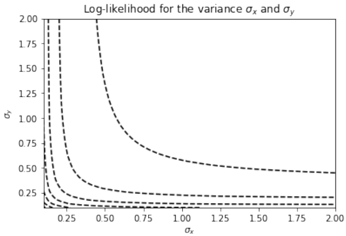


Gamma and Poisson conjugacy

- Gamma distribution for hyper-parameter λ for a given α and β is:
- $$p(\lambda|\alpha,\beta) = \frac{\beta^{-\alpha}}{\Gamma(\alpha)}\lambda^{\alpha-1}e^{-\beta\lambda}$$
- The likelihood of the of the hyper-parameter λ for a given set of independent data $x_1, x_2, ..., x_N$ using a Poisson distribution is:
- $$P(x_1, x_2, ..., x_N|\lambda) = \prod_{i=1}^{N} \frac{\lambda^{x_i}e^{-\lambda}}{x_i!} = \frac{\lambda^{\sum_{i=1}^{N} x_i}e^{\sum_{i=1}^{N}(-\lambda)}}{\prod_{i=1}^{N} x_i!} = \frac{\lambda^{Nx_{mu}}e^{-N\lambda}}{\prod_{i=1}^{N} x_i!}$$
- Using Bayes it is possible to get the update on λ as:
- $$p(\lambda|x_1, x_2, ..., x_N) = \frac{p(x_1, x_2, ..., x_N|\lambda)p(\lambda)}{\int p(x_1, x_2, ..., x_N, \lambda)d\lambda} = \frac{p(x_1, x_2, ..., x_N|\lambda)p(\lambda)}{p(x_1, x_2, ..., x_N)}$$
- Since the evidence is not dependent on λ it is not important on the maximum a posterior:
- $$p(\lambda|x_1, x_2, ..., x_N) \propto p(x_1, x_2, ..., x_N|\lambda)p(\lambda)$$
- $$p(\lambda|x_1, x_2, ..., x_N) \propto \frac{\lambda^{Nx_{mu}}e^{-N\lambda}}{\prod_{i=1}^{N} x_i!}p(\lambda)$$

Gamma and Poisson conjugacy

- $$p(\lambda|x_1, x_2, ..., x_N) \propto \frac{\lambda^{Nx_{mu}} e^{-N\lambda}}{\prod_{i=1}^{N} x_i!} \frac{\beta^{-\alpha}}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda}$$
- $$p(\lambda|x_1, x_2, ..., x_N) \propto \frac{\beta^{-\alpha}}{\Gamma(\alpha) \prod_{i=1}^{N} x_i!} \lambda^{Nx_{mu}} e^{-N\lambda} \lambda^{\alpha-1} e^{-\beta\lambda}$$
- $$p(\lambda|x_1, x_2, ..., x_N) \propto \frac{\beta^{-\alpha}}{\Gamma(\alpha) \prod_{i=1}^{N} x_i!} \lambda^{Nx_{mu}+\alpha-1} e^{-N\lambda-\beta\lambda}$$
- Since $\frac{\beta^{-\alpha}}{\Gamma(\alpha) \prod_{i=1}^{N} x_i!}$ is not dependent on λ can be removed away from the right hand side.
- $$p(\lambda|x_1, x_2, ..., x_N) \propto \lambda^{Nx_{mu}+\alpha-1} e^{-\alpha(N+\beta)}$$
- $$p(\lambda|x_1, x_2, ..., x_N) \propto Gamma(Nx_{mu} + \alpha, N + \beta)$$

  Hence the Poisson and the Gamma distribution form a conjugacy.

MCMC

$$p(x_1, x_2) = e^{-\frac{(2x_1 + sin(2\pi x_1))^2}{2}} e^{-\frac{(x_2 - x_1^3)^2}{0.1}}$$

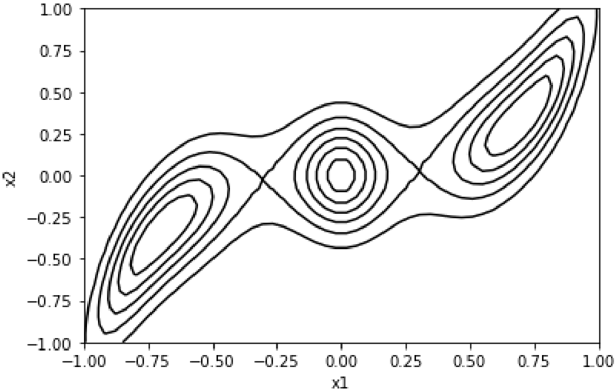


Figure: Target distribution

Metropolis Hasting with Gibbs

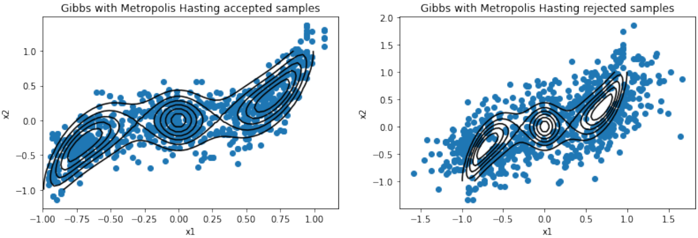


Figure: Left: Accepted samples. Right: Rejected samples

Metropolis Hasting

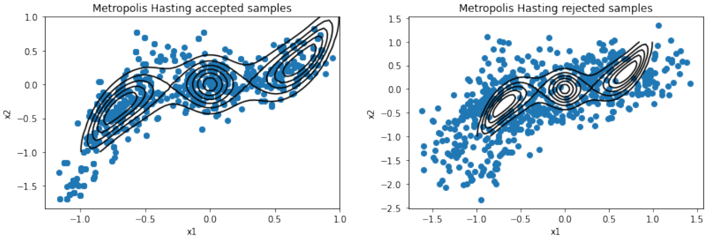


Figure: Left: Accepted samples. Right: Rejected samples

HMC

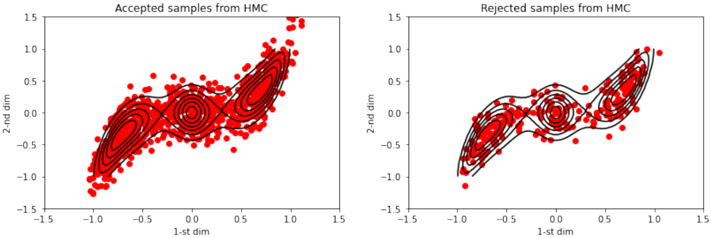
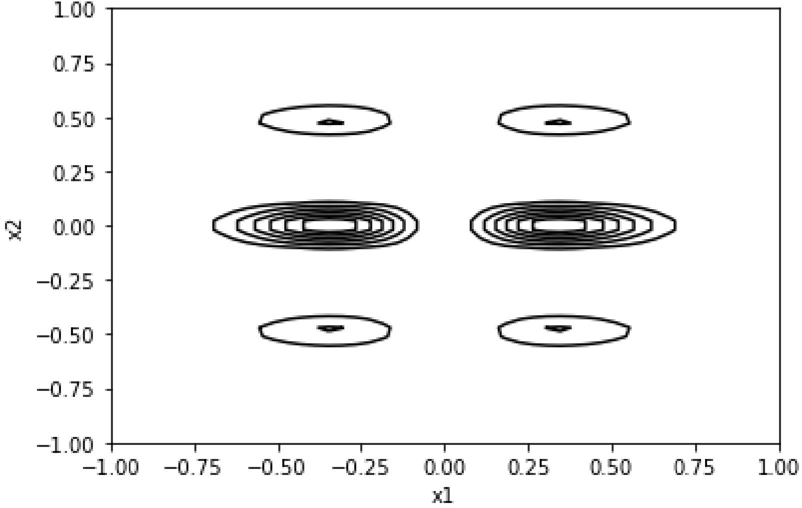


Figure: Left: Accepted samples. Right: Rejected samples

SMC

$$\pi(x) = \frac{1}{Z}\mathbb{1}(-1 \leq x_1 \leq 1)\mathbb{1}(-1 \leq x_2 \leq 1)\sin^2(x_1\pi)\cos^8(x_2 2\pi)\exp(-5(x_1^2 + x_2^2))$$



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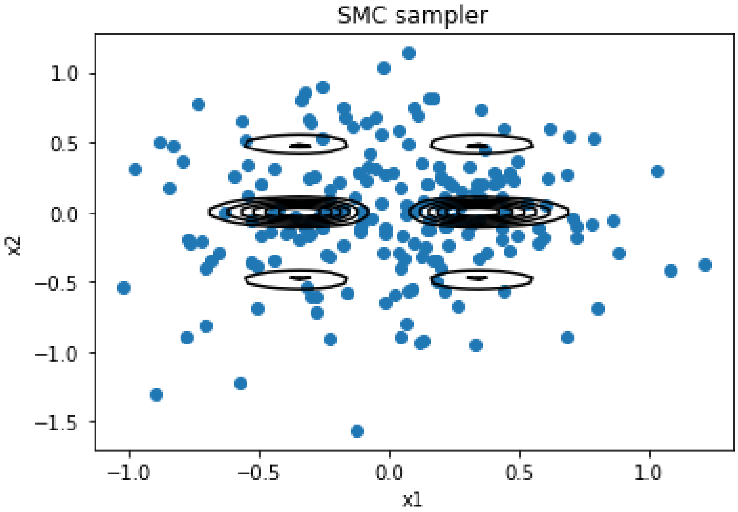

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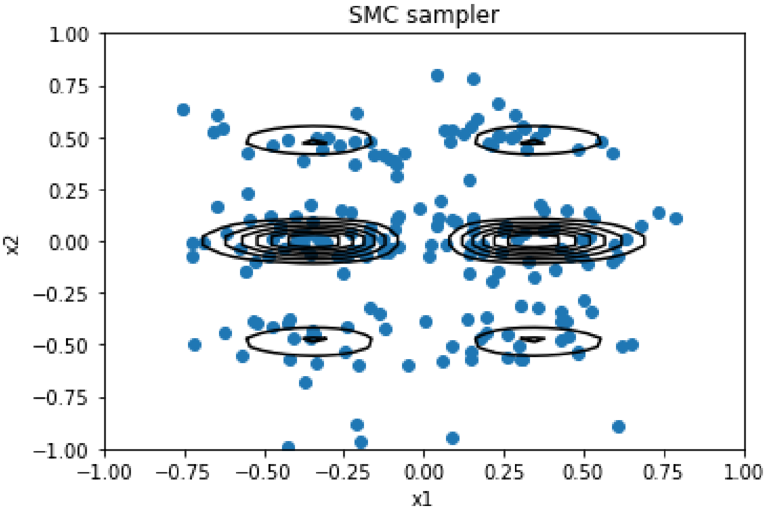

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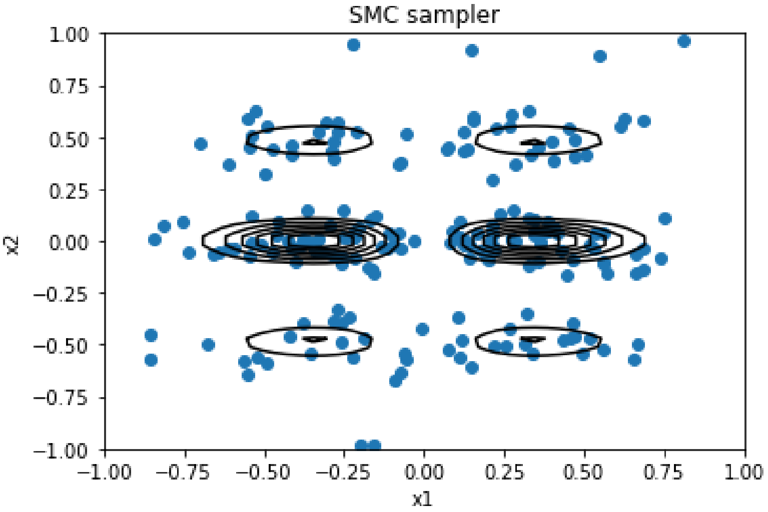

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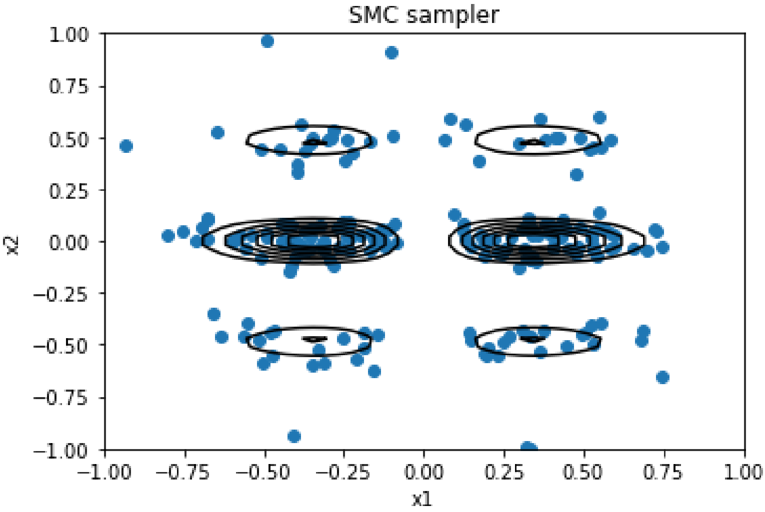

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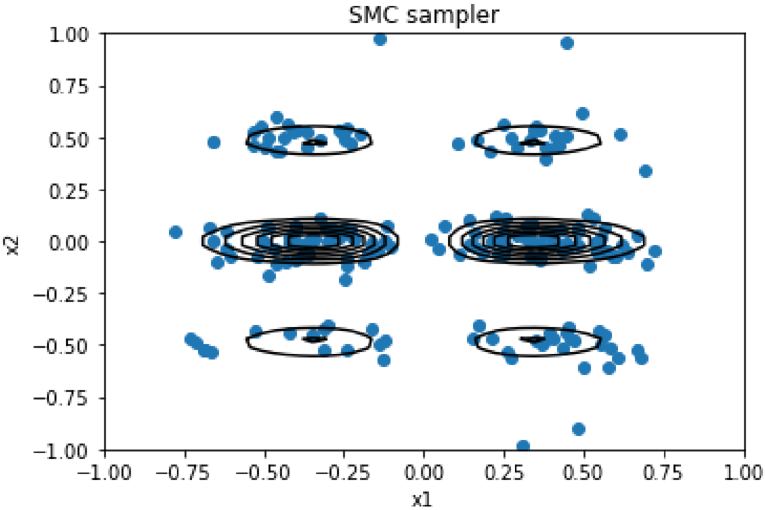

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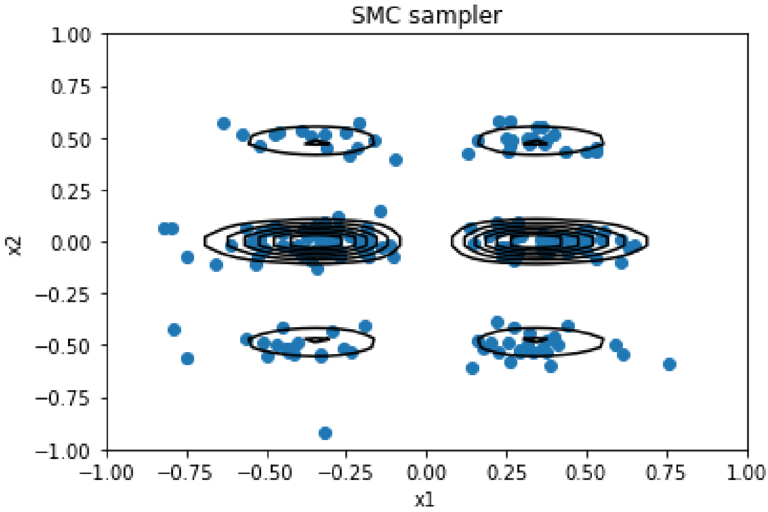

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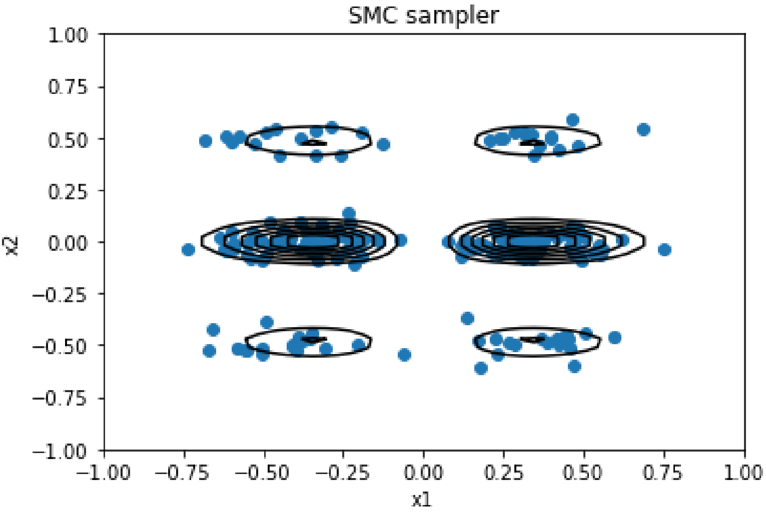

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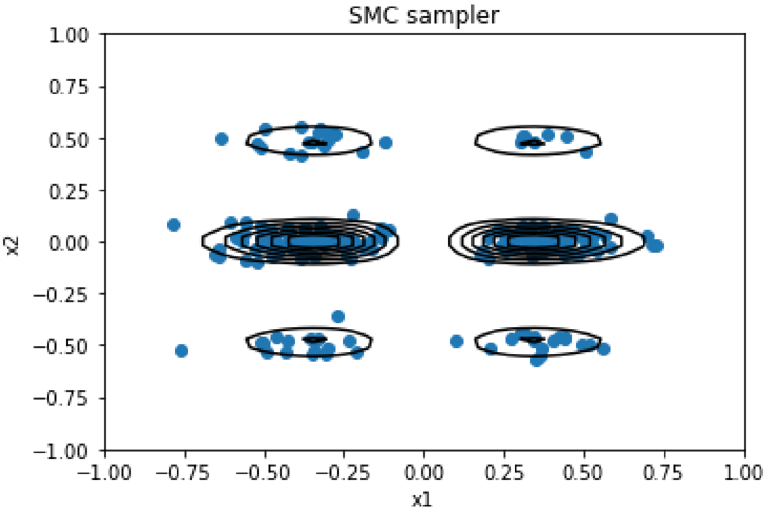

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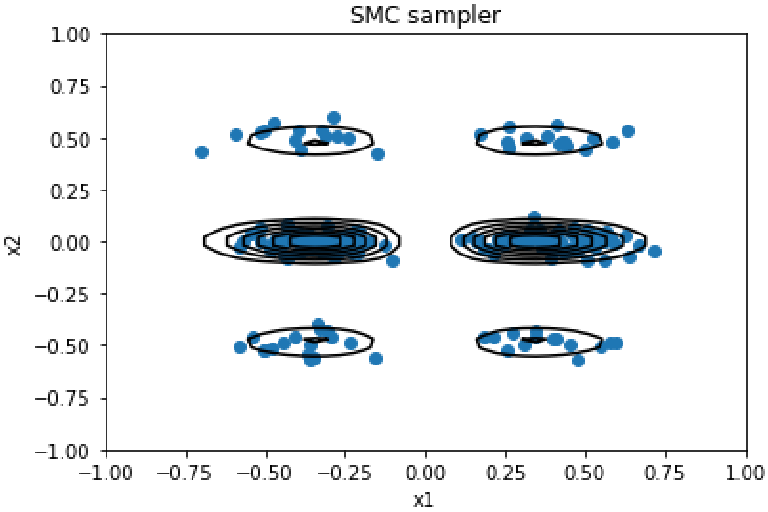

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