Programming in Prolog Search Space, Unification, Recursion

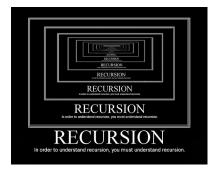
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Logic and Al Programming (Course 518)

What you will learn in this lecture



How does Prolog generate answers?



How does recursion work in Prolog?

How does Prolog generate answers?

A query with various calls

```
?- person(ada), quit(bob), quit(B), B \setminus= bob
```

- try to prove person(ada), then try to prove quit(bob), ...
 - ⇒ try to prove every call **from left to right**
 - ⇒ depth-first search
- try to match (unify) current call with head of a clause, then replace it with body of chosen clause
 - ⇒ try clauses from top to bottom
- call succeeds if it is unified with a fact or it evaluates to true
 (5 = 5); fails otherwise
- query succeeds ("yes") if every call succeeds; fails otherwise ("no")

Unification

match a call in a query to the head of a clause

- easy for constants, e.g. sunny
- variables can be matched with other variables or constants, e.g. X matches sunny
- what about compound terms?
 e.g. father(person(X), john)

Prolog terms

- constant: starts with lower case letter sunny
- number: 5
- variable: starts with upper case letter Sunny
- compound term: function name(term 1, ..., term n) father(person(X), john)
 note: this has nothing to do with functions in mathematics!

Unification Rules

two terms unify (=) if and only if

- 1 two constants/numbers: if and only if they are the same
 - bill = bill
 - **7** = 7
 - bill \= 7
 - note: 'bill'= bill
 - note: '7'\= 7

Unification Rules

two terms unify (=) if and only if

- 2 a constant/number and a variable: always unify ⇒ variable is instantiated with constant/number
 - X = 7 X instantiated with 7
 - bill = X X instantiated with bill
- 3 two variables: always unify
 - \Rightarrow variables are considered the same, i.e. have same value
 - X = Y
 X and Y are the same: X = _154, Y = _154

 \Rightarrow instantiation of variables is sometimes called **variable binding**



Unification Rules

two terms unify (=) if and only if

- two compound terms: if and only if
 - 1 same function name
 - 2 same number of arguments
 - 3 all corresponding arguments unify
 - 4 variable instantiations are compatible
 - ⇒ variables are instantiated with unified constants/numbers
 - k(X,p) = k(Y,Mp) instantiation: X = Y, Mp = p
 - k(X,p) = k(f(1,p),Mp)
 instantiation: X = f(1,p), Mp = p
 - $\mathbf{k}(X,p) = \mathbf{k}(f(1,p),1)$
 - k(X,p) \= k(f(1,p),Mp,Y)
 - $\mathbf{k}(X,p,m(Y)) = k(t(Z), Z, X)$
 - $\mathbf{k}(X,p,t(Y)) = k(t(Z), Z, X)$



Unification - Try it yourself

Do these terms unify? If so, what is the instantiation of the variables?

- m(X,Y) and p(Y,X)
- mia(X) and 'mia'(f(a))
- \bullet t(X,Y) and t(Y,Z)
- p(1,Y,f(a)) and p(X,m,Z)
- k(X,m(Y)) and k(p,X)
- k(X,m(Y)) and k(m(5),X)
- s(X,Y) and s(Y,f(X))

- = Do two terms unify?
- == Are two terms identical?

$$X = Y, X == Y$$

$$\Rightarrow$$
 if term1 == term2 then term1 = term2

Note the difference to the is and the $=:= (=\=)$ predicates

■ X is
$$5+7$$
 X =\= $5+7$ X = $5+7$ X \== $5+7$

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

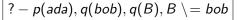
cl_5 : u(ada).

cl_6 : u(sam)
```

```
Query: p(ada), q(bob), q(B), B \setminus = bob
```

Solution:

```
cl_1: p(bob).
cl_2: p(X).
cl_3: q(X): -u(X), p(X).
cl_4: q(bob).
cl_5: u(ada).
cl_6: u(sam)
```



```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

Query: p(ada), q(bob), q(B), $B \setminus = bob$ Solution: $? - p(ada), q(bob), q(B), B \setminus = bob$

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

$$? - p(ada), q(bob), q(B), B \setminus = bob$$

$$cl_2$$
 (ada = $_{-}123$)

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

 $? - p(ada), q(bob), q(B), B \setminus = bob$ cl_2 (ada = .123) $? - q(bob), q(B), B \setminus = bob$

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

$$?-p(ada), q(bob), q(B), B \setminus = bob$$

$$cl_2$$

$$(ada = _123)$$

$$?-q(bob), q(B), B \setminus = bob$$

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

$$?-p(ada), q(bob), q(B), B \setminus = bob$$

$$cl_2 \\ (ada = .123)$$

$$?-q(bob), q(B), B \setminus = bob$$
Choice Point

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

$$?-p(ada), q(bob), q(B), B \setminus = bob$$
 cl_2
 $(ada = _123)$
 $?-q(bob), q(B), B \setminus = bob$
 cl_3
 $(bob = _436)$

Choice
Point

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

$$\begin{array}{c|c} ?-p(ada), q(bob), q(B), B \setminus = bob \\ \hline cl_2 \\ (ada = .123) \\ \hline ?-q(bob), q(B), B \setminus = bob \\ \hline cl_3 \\ (bob = .436) \\ \hline ?-u(bob), p(bob), q(B), B \setminus = bob \\ \hline \end{array}$$

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

$$\begin{array}{c|c} ?-p(ada), q(bob), q(B), B \setminus = bob \\ \hline & cl_2 \\ (ada = .123) \\ \hline ?-q(bob), q(B), B \setminus = bob \\ \hline & cl_3 \\ (bob = .436) \\ \hline ?-u(bob), p(bob), q(B), B \setminus = bob \\ \hline \end{array}$$

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

Query: p(ada), q(bob), q(B), $B \setminus = bob$ Solution:

$$\begin{array}{c|c} ?-p(ada), q(bob), q(B), B \setminus = bob \\ \hline cl_2 \\ (ada = _123) \\ \hline ?-q(bob), q(B), B \setminus = bob \\ \hline cl_3 \\ (bob = _436) \\ \hline ?-u(bob), p(bob), q(B), B \setminus = bob \\ \hline \text{no applicable} \\ \end{array}$$

clause

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

```
Query: p(ada), q(bob), q(B), B \setminus = bob Solution:
```

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

$$\begin{array}{c|c} ?-p(ada), q(bob), q(B), B \setminus = bob \\ \hline cl_2 \\ (ada = _123) \\ \hline ?-q(bob), q(B), B \setminus = bob \\ \hline cl_3 \\ (bob = _436) \\ \hline ?-u(bob), p(bob), q(B), B \setminus = bob \\ \hline no applicable \\ clause \\ \hline \end{array}$$

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

Query: p(ada), q(bob), q(B), $B \setminus = bob$ Solution:

$$\begin{array}{c|c} ?-p(ada), q(bob), q(B), B \setminus = bob \\ \hline cl_2 \\ (ada = .123) \\ \hline ?-q(bob), q(B), B \setminus = bob \\ \hline cl_3 \\ (bob = .436) \\ \hline ?-u(bob), p(bob), q(B), B \setminus = bob \\ \hline no applicable \\ clause \\ \end{array}$$

fail

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

$$?-p(ada), q(bob), q(B), B \setminus = bob$$

$$cl_2 \\ (ada = _123)$$

$$?-q(bob), q(B), B \setminus = bob$$

$$cl_3$$

$$fail$$
Choice
Point

```
cl_1 : p(bob).

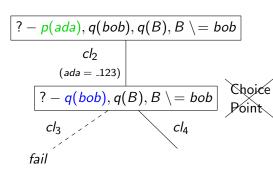
cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```



```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

$$?-p(ada), q(bob), q(B), B \setminus = bob$$
 cl_2
 $(ada = .123)$
 $?-q(bob), q(B), B \setminus = bob$
 cl_3
 cl_4

fail

 $?-q(B), B \setminus = bob$

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

$$?-p(ada), q(bob), q(B), B \setminus = bob$$
 cl_2
 $(ada = .123)$
 $?-q(bob), q(B), B \setminus = bob$
 cl_3
 cl_4

fail

 $?-q(B), B \setminus = bob$

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

$$?-p(ada), q(bob), q(B), B \setminus = bob$$
 cl_2
 $(ada = .123)$
 $?-q(bob), q(B), B \setminus = bob$
 cl_3
 cl_4
 $fail$
 $?-q(B), B \setminus = bob$

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

$$?-p(ada), q(bob), q(B), B \setminus = bob$$
 cl_2
 $(ada = .123)$
 $?-q(bob), q(B), B \setminus = bob$
 cl_3
 cl_4
 cl_4
 cl_4
 cl_4
 cl_4

```
cl_1 : p(bob).

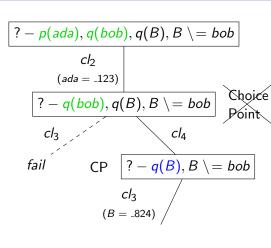
cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```



```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

$$(P, p(ada), q(bob), q(B), B = bob)$$

$$cl_2$$

$$(ada = .123)$$

$$Point$$

$$cl_3$$

$$cl_4$$

$$fail$$

$$CP$$

$$Point$$

$$cl_3$$

$$(B = .824)$$

$$Point$$

$$cl_3$$

$$(B = .824)$$

$$Point$$

$$cl_3$$

$$(B = .824)$$

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

?
$$-p(ada), q(bob), q(B), B \setminus = bob$$
 cl_2
 $(ada = .123)$
? $-q(bob), q(B), B \setminus = bob$
 cl_3
 cl_4

fail

 CP
? $-q(B), B \setminus = bob$
 cl_3
 $(B = .824)$
? $-u(.824), p(.824), .824 \setminus = bob$

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

$$\begin{array}{c|c} ?-p(ada), q(bob), q(B), B \setminus = bob \\ \hline cl_2 \\ (ada = .123) \\ \hline ?-q(bob), q(B), B \setminus = bob \\ \hline cl_3 \\ (B = .824) \\ \hline \\ CP \hline ?-u(.824), p(.824), .824 \setminus = bob \\ \hline \end{array}$$

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

CP
$$?-q(B), B = bob$$

$$cl_3 \\ (B = .824)$$

$$?-u(.824), p(.824), .824 = bob$$

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

CP
$$?-q(B), B \setminus = bob$$
 cl_3
 $(B = .824)$

CP $?-u(.824), p(.824), .824 \setminus = bob$
 cl_5
 $(.824 = ada)$

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

CP
$$\boxed{?-q(B), B \setminus = bob}$$

$$cl_3$$

$$(B = .824)$$
CP $\boxed{?-u(.824), p(.824), .824 \setminus = bob}$

$$cl_5$$

$$(.824 = ada)$$

$$\boxed{?-p(ada), ada \setminus = bob}$$

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

Query: p(ada), q(bob), q(B), $B \setminus = bob$ Solution:

(ada = .581)

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

Query: p(ada), q(bob), q(B), $B \setminus = bob$ Solution:

CP
$$?-q(B), B = bob$$
 cl_3
 $(B = .824)$

CP $?-u(.824), p(.824), .824 = bob$
 cl_5
 $(.824 = ada)$
 $?-p(ada), ada = bob$
 cl_2
 $(ada = .581)$

 $|?-ada \rangle = bob$

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

Query: p(ada), q(bob), q(B), $B \setminus = bob$ Solution:

 $? - ada \setminus = bob$

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

Query: p(ada), q(bob), q(B), $B \setminus = bob$ Solution:

CP
$$? - q(B), B \setminus = bob$$
 cl_3
 $(B = .824)$

CP $? - u(.824), p(.824), .824 \setminus = bob$
 cl_5
 $(.824 = ada)$
 $? - p(ada), ada \setminus = bob$
 cl_2
 $(ada = .581)$

 $? - ada \setminus = bob$

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

Query: p(ada), q(bob), q(B), $B \setminus = bob$ Solution:

 $|?-ada \rangle = bob$

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

CP
$$? - q(B), B \setminus = bob$$
 cl_3
 $(B = .824)$

CP $? - u(.824), p(.824), .824 \setminus = bob$
 cl_5
 $(.824 = ada)$
 $? - p(ada), ada \setminus = bob$
 cl_2
 $(ada = .581)$
 $? - ada \setminus = bob$

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

CP
$$?-q(B), B \setminus = bob$$
 cl_3
 $(B = .824)$

CP $?-u(.824), p(.824), .824 \setminus = bob$
 cl_5
 $(.824 = ada)$
 $?-p(ada), ada \setminus = bob$
 cl_2
 $(ada = .581)$
 $?-ada \setminus = bob$

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

CP
$$?-q(B), B \setminus = bob$$
 cl_3
 $(B = .824)$

CP $?-u(.824), p(.824), .824 \setminus = bob$
 cl_5
 $(.824 = ada)$
 $?-p(ada), ada \setminus = bob$
 cl_2
 $(ada = .581)$
 $?-ada \setminus = bob$

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

CP
$$?-q(B), B \setminus = bob$$
 cl_3
 $(B = .824)$

CP $?-u(.824), p(.824), .824 \setminus = bob$
 cl_5
 $(.824 = ada)$
 $?-p(ada), ada \setminus = bob$
 cl_2
 $(ada = .581)$
 $?-ada \setminus = bob$

```
cl_1 : p(bob).

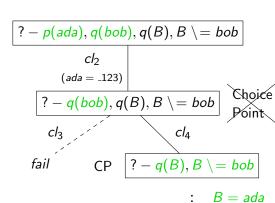
cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```



```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

$$?-p(ada), q(bob), q(B), B \setminus = bob$$
 cl_2
 $(ada = .123)$
 $?-q(bob), q(B), B \setminus = bob$
 cl_3
 cl_4
 $fail$
 CP
 $?-q(B), B \setminus = bob$
 $B = ada$

```
cl_1 : p(bob).

cl_2 : p(X).

cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

$$?-p(ada), q(bob), q(B), B \setminus = bob$$
 cl_2
 $(ada = .123)$
 $?-q(bob), q(B), B \setminus = bob$
 cl_3
 cl_4
 $fail$
 CP
 $?-q(B), B \setminus = bob$
 $B = ada$

```
cl_1 : p(bob).

cl_2 : p(X).

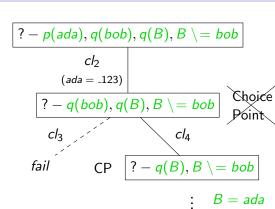
cl_3 : q(X) : -u(X), p(X).

cl_4 : q(bob).

cl_5 : u(ada).

cl_6 : u(sam)
```

$$B = ada$$

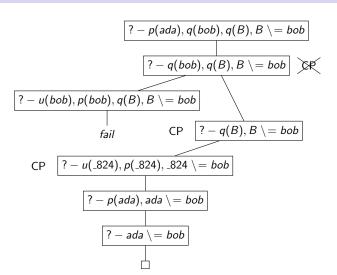


The whole search tree

 $cl_1 : p(bob).$ $cl_2 : p(X).$ $cl_3 : q(X) :$ -u(X), p(X). $cl_4 : q(bob).$ $cl_5 : u(ada).$ $cl_6 : u(sam)$

Query: p(ada), q(bob), $q(B), B \setminus = bob$ Solution:

B = ada



Are there any other solutions to the query?

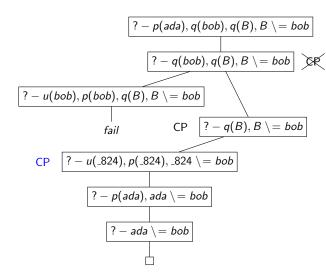
Prolog: Are there any choice points left that might lead to further solutions?

starting from the last choice point

 $cl_1 : p(bob).$ $cl_2 : p(X).$ $cl_3 : q(X) :$ -u(X), p(X). $cl_4 : q(bob).$ $cl_5 : u(ada).$ $cl_6 : u(sam)$

Query: p(ada), q(bob), $q(B), B \setminus = bob$ Solution:

 $\mathbf{B} = \mathbf{ada}$

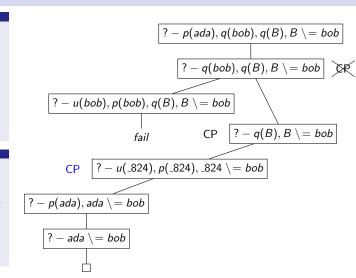


 $cl_1: p(bob).$ $cl_2: p(X).$ $cl_3: q(X):$ -u(X), p(X). $cl_4: q(bob).$ cl_5 : u(ada). $cl_6: u(sam)$

Query: p(ada), q(bob), $q(B), B \setminus = bob$

Solution:

B = ada

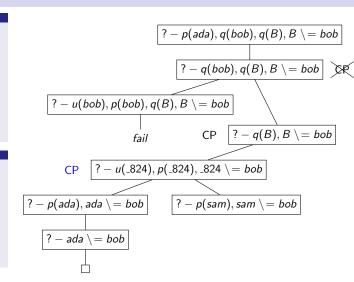


 $cl_1 : p(bob).$ $cl_2 : p(X).$ $cl_3 : q(X) :$ -u(X), p(X). $cl_4 : q(bob).$ $cl_5 : u(ada).$ $cl_6 : u(sam)$

Query: $p(ada), q(bob), q(B), B \setminus = bob$

Solution:

 $\mathbf{B} = \mathbf{ada}$

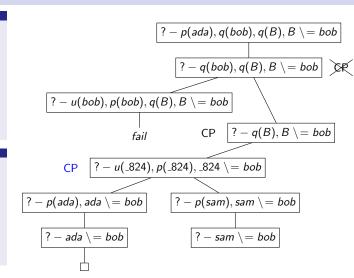


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Query: p(ada), q(bob), $q(B), B \setminus = bob$

Solution:

B = ada

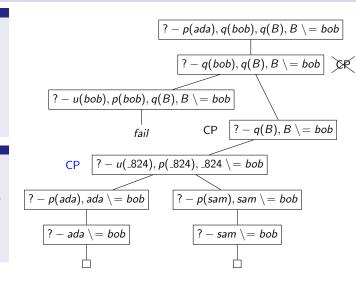


 $cl_1 : p(bob).$ $cl_2 : p(X).$ $cl_3 : q(X) :$ -u(X), p(X). $cl_4 : q(bob).$ $cl_5 : u(ada).$ $cl_6 : u(sam)$

Query: $p(ada), q(bob), q(B), B \setminus = bob$

Solution:

B = ada

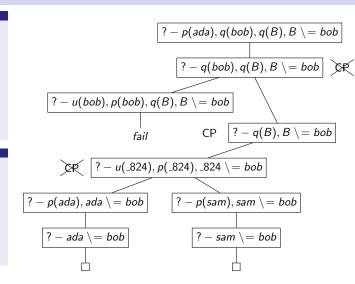


 $cl_1 : p(bob).$ $cl_2 : p(X).$ $cl_3 : q(X) :$ -u(X), p(X). $cl_4 : q(bob).$ $cl_5 : u(ada).$ $cl_6 : u(sam)$

Query: $p(ada), q(bob), q(B), B \setminus = bob$

Solution:

 $\mathbf{B} = \mathbf{ada}$

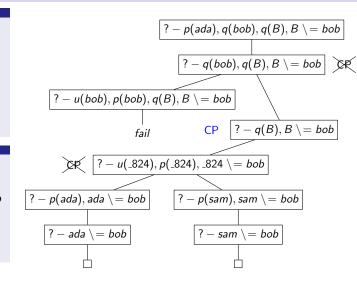


 $cl_1 : p(bob).$ $cl_2 : p(X).$ $cl_3 : q(X) :$ -u(X), p(X). $cl_4 : q(bob).$ $cl_5 : u(ada).$ $cl_6 : u(sam)$

Query: $p(ada), q(bob), q(B), B \setminus = bob$

Solution:

B = ada



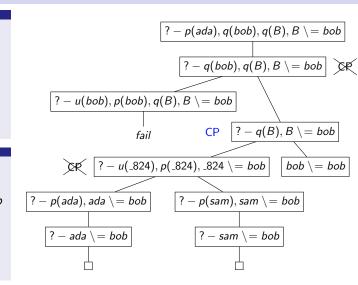
 $cl_1 : p(bob).$ $cl_2 : p(X).$ $cl_3 : q(X) :$ -u(X), p(X). $cl_4 : q(bob).$ $cl_5 : u(ada).$ $cl_6 : u(sam)$

Query:

p(ada), q(bob), $q(B), B \setminus = bob$

Solution:

 $\mathbf{B} = \mathbf{ada}$

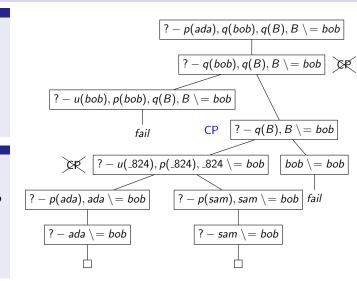


 $cl_1 : p(bob).$ $cl_2 : p(X).$ $cl_3 : q(X) :$ -u(X), p(X). $cl_4 : q(bob).$ $cl_5 : u(ada).$ $cl_6 : u(sam)$

Query: $p(ada), q(bob), q(B), B \setminus = bob$

Solution:

 $\mathbf{B} = \mathbf{ada}$



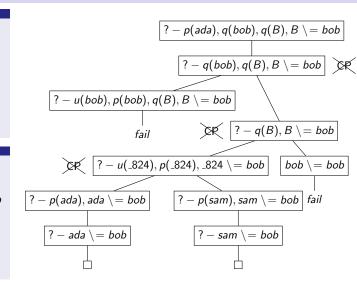
 $cl_1: p(bob).$ $cl_2: p(X).$ $cl_3: q(X):$ -u(X), p(X). $cl_4: q(bob).$ $cl_5: u(ada).$ $cl_6: u(sam)$

Query:

p(ada), q(bob), $q(B), B \setminus = bob$

Solution:

 $\mathbf{B} = \mathbf{ada}$



 $cl_1: p(bob).$ $cl_2: p(X).$ $cl_3: q(X):$ -u(X), p(X). $cl_4: q(bob).$ cl_5 : u(ada). $cl_6: u(sam)$

Query: p(ada), q(bob), $q(B), B \setminus = bob$

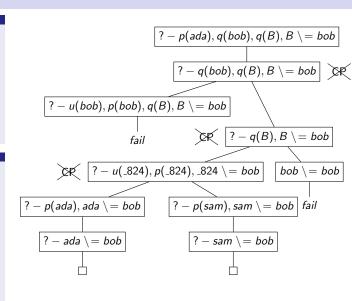
Solution:

B = ada

B = sam

No more

solutions



trace

To see how this looks in Prolog:

```
trace.
:
notrace.
```

Summary – How does Prolog generate answers?

A general query Q

?- C1 ..., Cn

- 1 try to prove first call C1
 - by unifying it with next matching head of a clause (top to bottom)
- 2 if unification fails go back to last choice point and retry
- \blacksquare else substitute C_1 in Q with conditions in body of the clause
- 4 repeat 1.) and 2.) until
 - no goals left in query solution found⇒ return all variable bindings
 - goals left in query & no choice points left to retry unification query fails/cannot be proven
 - \Rightarrow return no

Summary – How does Prolog generate answers?



Remember: order matters

- Order of goals in a query first one first
- Order of clauses in a program top one first
- Order of conditions in a clause first one first

What is recursion?

Most powerful technique for programming in Prolog! (in particular for working with lists – more on that later)

 \Rightarrow If you understand recursion, you can program in Prolog

The **idea** of recursion:

- similar to while/for loop: repeat a procedure until some lower/upper bound is reached
- in contrast to while/for loop: a whole function calls itself repeatedly
- in Prolog: a predicate calls itself
 ⇒ one of the conditions in a clause refers to the same predicate as the head of this clause

A recursive predicate

```
 \begin{split} my\_predicate\_name(X,Y) :- \\ check\_first(X), \\ do\_second(Y,Z), \\ my\_predicate\_name(X,Z). \end{split}
```

Ancestor Example

```
is_ancestor_of(Ancestor, Person) :-
human(Person),
human(Ancestor),
is_parent_of(Parent, Person),
is_ancestor_of(Ancestor, Parent).
```

The Ancestor Example corrected

Ancestor Example

```
is_ancestor_of(Parent, Person) :-
    is_parent_of(Parent, Person).

is_ancestor_of(Ancestor, Person) :-
    is_parent_of(Parent, Person),
    human(Person),
    is_ancestor_of(Ancestor, Parent),
    human(Ancestor).
```

Base case & recursive definition

Don't forget the **base case**

⇒ Prolog won't find the correct solutions or even loop forever

Defining a recursive predicate

- **1 base case** most basic case (most basic arguments) which is not recursive; terminates the recursion
- recursive definition one of the conditions is the predicate itself

Order matters – especially for recursion

```
Natural Number Example
Tail1:
                           NoTail1:
natural no(0).
                           natural_no(0).
natural_no(X) :-
                           natural_no(X) :-
   Y is X-1,
                               natural_no(Y),
   natural no(Y).
                               Y is X-1.
Tail2:
                           NoTail2:
natural_no(0).
                           natural_no(0).
natural_no(X) :-
                           natural_no(X) :-
   X is Y+1,
                               natural_no(Y),
   natural no(Y).
                               X is Y+1.
```

Order matters – especially for recursion

| | test | generate | reversed, test 0 |
|---------|------|----------|------------------|
| Tail1 | 1 | Х | loop |
| Tail2 | X | X | no |
| NoTail1 | Х | X | loop |
| NoTail2 | 1 | ✓ | loop |

- base case (usually) before recursive definition
- think about whether you want to test or generate (or both)
- tail recursion is more efficient!
 - ⇒ but it's not always possible to use it

Prolog versus Logic

The previous examples have the same **declarative** (**logical**) **meaning** but a different **procedural meaning** $\Rightarrow \text{ different behaviour}$

```
p :- p
```

- declarative meaning: "If p holds then p holds"
- in logic: $p \rightarrow p$ ⇒ a tautology – it's always true
- procedural meaning: "To prove p you must prove p" ⇒ ?- p loops forever

So: Prolog is not a full logic programming language! (not fully declarative)



What you should know now

- How does Prolog generate answers?
 - What is unification and how does it work?
 - Why does the order of Prolog clauses matter?
 - What is a choice point?
 - How can trace be used for debugging?
- How does recursion work in Prolog?
 - Why is the base case important?
 - What is tail recursion?
 - What is the difference between the declarative and the procedural meaning of a recursive program?

Useful reading/resources

Introductory Book:

- "Learn Prolog Now!" Blackburn, Bos, Striegnitz
 - ⇒ A free online version is also available

Prolog Manual

- HTML: https://sicstus.sics.se/sicstus/docs/ latest4/html/sicstus.html/
- PDF: https://sicstus.sics.se/sicstus/docs/ latest4/pdf/sicstus.pdf