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# Theoretical and experimental limits of triple photon energy absorptiometry in the measurement of bone mineral

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**Abstract.** Theoretical and experimental studies have been conducted to explore the possibilities of triple photon energy absorptiometry in the measurement of bone mineral content. The purpose of this technique is to correct the measured bone mineral density for fat and soft tissues. However, theoretical considerations lead us to doubt the precision and accuracy of such measurements. In a first approximation the absorption coefficient can be split into Compton and photoelectric energy-independent factors. A consequence of such a model is the impossibility of finding more than two independent mass attenuation coefficients for different energies. The existence of an energy-dependent third factor may justify the use of triple photon energy absorptiometry but experimental tests and numerical simulations have shown that its value is too low for triple photon energy absorptiometry to be considered as an adequate method for the measurement of bone mineral content.

## 1. Introduction

Single photon absorptiometry was first described by Cameron and Sorenson (1963) and was developed to determine the bone mineral content (BMC) in the diagnosis of osteoporosis. Based on the attenuation of a single photon beam of $^{125}I$, this technique required a constant thickness of soft tissue so that the measured bone had to be surrounded by a water bath. As a consequence, such a measurement was only possible with the appendicular skeleton.

Three years later, dual photon energy absorptiometry was described by Reed (1966) and then developed by a number of authors (Roos *et al* 1970, Mazess *et al* 1970). This method is based on measurements of the transmission of photons with two different energies through an environment made of bone and soft tissues. It has led to satisfactory measurements of BMC dedicated to the diagnosis of osteoporosis. These techniques would be a direct method of bone mass quantization if the human body could be regarded as a two-phase system made of bones and soft tissues. However, this is only a rough approximation and a third phase has to be taken into account because the mass attenuation coefficients of fat tissues at the relevant photoelectric peaks (44 and 100 keV for $^{153}Gd$) are different from those of bone mineral and soft tissues.

Classing fat tissues as soft tissues leads to a systematic error in the BMC. This error can be easily corrected by subtracting from the vertebral BMC, the BMC of a para-vertebral region (normally without mineral content). Even though most of the systems of dual photon absorptiometry are now using this simple correction of the BMC, it has been shown (Roos *et al* 1980) that this assesses a constant thickness of fat in the vertebral and para-vertebral regions. This hypothesis is far from always being satisfactorily verified.

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Recently, it has been suggested (Jonson *et al* 1988) that dual photon absorptiometry could be extended by using a third energy in order to solve directly the three-phase system that constitutes the human body. Those three energies are provided by one $^{153}$Gd source (44 and 100 keV) and one $^{241}$Am source (59.5 keV). However, such a technique seems to be contradictory to Alvarez' theory (Alvarez and Macovski 1976) according to which, for an energy between 30 and 200 keV, each mass attenuation coefficient can only be split into functions representing the Compton and photoelectric attenuations. This theory is the basis of dual energy tomodensitometry which has been developed by Avrin *et al* (1978) and Kalender *et al* (1987).

As a consequence, the purpose of this work is to study the experimental consequences of Alvarez' model in the measurement of BMC using triple photon energy absorptiometry, as far as the accuracy and the precision of the decomposition into bone mineral, fat and soft tissues are concerned.

## 2. Theoretical basis

### *2.1. Triple photon energy absorptiometry*

This technique is an attempt to eliminate the effect of soft tissues (S) and of fat tissues (F) in the measurement of the bone mineral content (B). This can be done by solving a three-equation Cramer system corresponding to the attenuation of three beams of different energies, $I_0^1$, $I_0^2$ and $I_0^3$:

$$\begin{aligned}
\ln (I_0^1/I^1) &= \mu_B^1 X_B + \mu_S^1 X_S + \mu_F^1 X_F \\
\ln (I_0^2/I^2) &= \mu_B^2 X_B + \mu_S^2 X_S + \mu_F^2 X_F \\
\ln (I_0^3/I^3) &= \mu_B^3 X_B + \mu_S^3 X_S + \mu_F^3 X_F.
\end{aligned} \tag{1}$$

$I_0$ and $I$ are respectively the beam intensities in the air and after passage through the body. The coefficients, $\mu$, are the mass attenuation coefficients (cm$^2$ g$^{-1}$) of each phase whose surface densities are $X$ (g cm$^{-2}$). If the determinant of this system is different from zero, then

$$X_B = \begin{pmatrix} \ln (I_0^1/I^1)\mu_S^1\mu_F^1 \\ \ln (I_0^2/I^2)\mu_S^2\mu_F^2 \\ \ln (I_0^3/I^3)\mu_S^3\mu_F^3 \end{pmatrix} \begin{pmatrix} \mu_B^1\mu_S^1\mu_F^1 \\ \mu_B^2\mu_S^2\mu_F^2 \\ \mu_B^3\mu_S^3\mu_F^3 \end{pmatrix}^{-1}. \tag{2}$$

That is:

$$X_B = \ln (I_0^1/I^1) \begin{pmatrix} \mu_S^2\mu_F^2 \\ \mu_S^3\mu_F^3 \end{pmatrix} D^{-1} - \ln (I_0^2/I^2) \begin{pmatrix} \mu_S^1\mu_F^1 \\ \mu_S^3\mu_F^3 \end{pmatrix} D^{-1} + \ln (I_0^3/I^3) \begin{pmatrix} \mu_S^1\mu_F^1 \\ \mu_S^2\mu_F^2 \end{pmatrix} D^{-1}. \tag{3}$$

The surface densities of fat and soft tissues can be calculated in the same way.

As a consequence, before any kind of triple photon energy absorptiometry measurement, it is necessary to establish that the determinant of the system (1) is different from zero, i.e. to be sure that the mass attenuation coefficient (for bone, soft and fat tissues) at energy $E_1$ is not linearly dependent on the mass attenuation coefficient at energies $E_2$ and $E_3$.

*2.2. The mass attenuation coefficients*

According to Alvarez and Macovski (1976), at the energies available with $^{153}Gd$ and $^{241}Am$, each mass attenuation coefficient can be split up into two functions corresponding to Compton and photoelectric processes:

$$(\mu/\rho)(E) = a_C f_C(E) + a_{ph} f_{ph}(E) \tag{4}$$

where $a_C$ and $a_{ph}$ do not depend on the energy: they are characteristic of the tissue. $f_{ph}(E)$ and $f_C(E)$ depend only on the energy of the photon. Many formulae have been proposed for $f_{ph}(E)$ and $f_C(E)$. Among them, the most commonly used is the formula of McCullough (1975):

$$(\mu/\rho)_{MC}(E) = a_C f_{KN}(E) + a_{ph} E^{-3.2} \tag{5}$$

where $f_{KN}(E)$ is the Klein-Nishina function:

$$f_{KN}(E) = \frac{1+d}{d^2}\left[\left(\frac{2+2d}{1+2d}\right) - \frac{1}{d}\ln(1+2d)\right] + \frac{1}{2d}\ln(1+2d) - \frac{(1+3d)}{(1+2d)^2} \tag{6}$$

and $d = E/mc^2$.

In comparison with Hubbell's tables (Hubbell 1969), the accuracy of McCullough's formula is about 1% for water and fat tissues between 30 and 200 keV (Alvarez and Macovski 1976). These authors gave no data about hydroxyapatite.

In equation (5), the mass attenuation coefficients appear to be vectors of a two-dimensional space. The vectors $\boldsymbol{a}_C$ and $\boldsymbol{a}_{ph}$ span this space:

$$\begin{aligned}
\boldsymbol{\mu}_1/\boldsymbol{\rho} &= \begin{pmatrix} \mu/\rho_{1B} \\ \mu/\rho_{1S} \\ \mu/\rho_{1F} \end{pmatrix} = f_{KN}(E_1)\begin{pmatrix} a_{CB} \\ a_{CS} \\ a_{CF} \end{pmatrix} + E_1^{-3.2}\begin{pmatrix} a_{phB} \\ a_{phS} \\ a_{phF} \end{pmatrix} \\
\boldsymbol{\mu}_2/\boldsymbol{\rho} &= \begin{pmatrix} \mu/\rho_{2B} \\ \mu/\rho_{2S} \\ \mu/\rho_{2F} \end{pmatrix} = f_{KN}(E_2)\begin{pmatrix} a_{CB} \\ a_{CS} \\ a_{CF} \end{pmatrix} + E_2^{-3.2}\begin{pmatrix} a_{phB} \\ a_{phS} \\ a_{phF} \end{pmatrix} \\
\boldsymbol{\mu}_3/\boldsymbol{\rho} &= \begin{pmatrix} \mu/\rho_{3B} \\ \mu/\rho_{3S} \\ \mu/\rho_{3F} \end{pmatrix} = f_{KN}(E_3)\begin{pmatrix} a_{CB} \\ a_{CS} \\ a_{CF} \end{pmatrix} + E_3^{-3.2}\begin{pmatrix} a_{phB} \\ a_{phS} \\ a_{phF} \end{pmatrix}.
\end{aligned} \tag{7}$$

According to this theory, it is rigorously impossible to find three independent mass attenuation coefficients for three different energies: the third equation in (1) can only be a linear combination of the two first ones and the system is impossible to solve (the determinant is zero). Fortunately, McCullough's formula remains an approximation so that it is possible to hope that even if the determinant is very near zero, its value is big enough to allow the use of triple photon energy absorptiometry. It is then important to quantify the influence of the low value of this determinant on the accuracy and the precision of the measured BMC.

## 3. Material and method

In the first instance, McCullough's formula is applied to the evaluation of the hydroxyapatite ($Ca_{10}(PO_4)_6(OH)_2$) mass attenuation coefficient. For several energies, the calculated data are compared with Hubbell's experimental data. This is merely a numerical simulation. The error in the surface density can be found by partial differentiation of equation (3). If a Poisson law is assumed for the incident beam, in a first-order

**Table 1.** Experimental mass attenuation coefficients used in the determinants and variances evaluation.

| | | Mass attenuation coefficients ($cm^2\ g^{-1}$) | | | |
|---|---|---|---|---|---|
| | Energy (keV) | Smith *et al* (1983) | Hubbell (1969) | Jonson *et al* (1988) | This study |
| Soft tissue | 44 | 0.245 | 0.2496 | 0.2454 | 0.2428 |
| | 60 | 0.205 | 0.2063 | 0.2025 | 0.1991 |
| | 100 | 0.170 | 0.1707 | 0.1677 | 0.1699 |
| Fat | 44 | 0.216 | 0.2186 | 0.2163 | 0.2265 |
| | 60 | 0.195 | 0.1944 | 0.1940 | 0.1918 |
| | 100 | 0.170 | 0.1691 | 0.1674 | 0.1677 |
| Bone | 44 | 0.790 | 0.7890 | 1.0930 | 0.7770 |
| | 60 | 0.402 | 0.4015 | 0.5873 | 0.3908 |
| | 100 | 0.195 | 0.2013 | 0.2296 | 0.2058 |

approximation

$$\sigma_B^2 = \left(\frac{1}{I_0^1}+\frac{1}{I^1}\right)\begin{pmatrix}\mu_S^2\mu_F^2\\ \mu_S^3\mu_F^3\end{pmatrix}^2 D^{-2} + \left(\frac{1}{I_0^2}+\frac{1}{I^2}\right)\begin{pmatrix}\mu_S^1\mu_F^1\\ \mu_S^3\mu_F^3\end{pmatrix}^2 D^{-2} + \left(\frac{1}{I_0^3}+\frac{1}{I^3}\right)\begin{pmatrix}\mu_S^1\mu_F^1\\ \mu_S^2\mu_F^2\end{pmatrix}^2 D^{-2}. \quad (8)$$

The variances for soft tissues and fat, as well as the errors for dual photon energy absorptiometry, can be defined in the same way.

The variance depends on two parameters: (i) on the activity of the source; and (ii) on the various determinants which are linked to the mass attenuation coefficients.

As a consequence, for a given number of incident photons, the error is a function of those coefficients. From an experimental point of view, they are determined using the following apparatus.

(1) The radioactive sources $^{153}Gd$ (3.7 GBq) and $^{241}Am$ (1.85 GBq) are collimated by 100 mm of lead, with an aperture of 3 mm. The measurement time has been adjusted to provide an incident beam of $10^5$ photons.

(2) The detection is performed by a germanium detector cooled with liquid nitrogen and connected to a multichannel analyser. The protection against noise is realized by 100 mm of lead with a 3 mm diameter collimation. The source-detector distance is 1500 mm.

(3) The standards are made of 4.18 g $cm^{-2}$ of pure water, 4.74 g $cm^{-2}$ of trioleate glycerol (simulating fat tissues) and 1.218 g $cm^{-2}$ of hydroxyapatite. The variances of the calculated mass attenuation coefficients are below 1%. These coefficients are reported (table 1) with the results of Smith *et al* (1983) and Jonson *et al* (1988). They are compared with the values for hydroxyapatite calculated from Hubbell's tables (Hubbell 1969). For each of these series of coefficients, the variance using dual or triple photon absorptiometry has been calculated using equation (8) and by the simulation of a passage through 1 g of hydroxyapatite, 10 g of water and 5 g of fat. This allows the research of a set of three optimal energies.

## 4. Results

### *4.1. Hydroxyapatite mass attenuation coefficient and McCullough's formula*

Compared with Hubbell's tables (Hubbell 1969), mass attenuation coefficients derived from McCullough's formula are higher, especially for low energies. The differences

range from 27% at 30 keV to 0.7% at 200 keV. They are respectively 17, 12 and 3.7% at 44, 60 and 100 keV. These errors are mainly due to calcium and phosphorus. However, as the mass of bone is relatively small (compared with the mass of water and fat tissues), the total error introduced by the Compton-photoelectric decomposition is below 4% in the worst case (44 keV), for energies between 40 and 100 keV and for an environment made of 15 g cm$^{-2}$ of soft tissues and 1 g cm$^{-2}$ of hydroxyapatite.

### *4.2. The variance effect*

All the calculations have been made with an incident beam similar to those used in dual photon energy absorptiometry, that is $10^5$ photons. After passage through 1 g cm$^{-2}$ of bone, 10 g cm$^{-2}$ of water and 5 g cm$^{-2}$ of fat tissues, $10^3$, $3\times10^3$ and $6\times10^3$ photons were counted at 44, 60 and 100 keV respectively. Table 2 gives the determinant and the variance for each series of data when dual photon energy absorptiometry ($^{153}$Gd) is used. These results are rather independent of the choice of the mass attenuation coefficients: the bone mass variance ranges from 0.0024 g cm$^{-2}$ with the coefficients of Jonson *et al* to 0.0048 g cm$^{-2}$ with Hubbell's and our coefficients. The accuracy on the BMC is approximately 0.5%. The soft tissue variance is slightly bigger.

**Table 2.** Determinants and variances for several sets of mass attenuation coefficients using dual and triple photon energy absorptiometry.

| | | Smith *et al* (1983) | Hubbell (1969) | Jonson *et al* (1988) | This study |
|---|---|---|---|---|---|
| Dual | Determinant | 0.074 | 0.084 | 0.127 | 0.079 |
| | Bone variance | 0.0045 | 0.0048 | 0.0024 | 0.0048 |
| | Soft tissue variance | 0.0189 | 0.0191 | 0.0156 | 0.0197 |
| Triple | Determinant | $7.19\times10^{-5}$ | $6.75\times10^{-5}$ | $4.57\times10^{-5}$ | $5.3\times10^{-5}$ |
| | Bone variance | 2.4 | 2.63 | 0.0615 | 0.82 |
| | Soft tissue variance | 714 | 867 | 51.4 | 1134 |
| | Fat variance | 699 | 775 | 46 | 1091 |

As for triple photon energy absorptiometry, table 2 also shows that the variance ranges from 0.062 g cm$^{-2}$ (Jonson *et al* 1988) to 2.63 g cm$^{-2}$ (Hubbell 1969). As a consequence, the accuracy on the BMC is 10 times worse than the accuracy achieved with dual photon energy absorptiometry when using the coefficients of Jonson *et al*. This accuracy becomes unacceptable with Hubbell's coefficients and would require in this case a drastic increase in the counting time. Our coefficients show a variance of 0.82 g cm$^{-2}$ for bone tissues. Those for water and fat tissues range from 46 to 1091 g cm$^{-2}$ depending on the mass attenuation coefficients. The variance on the BMC resulting from an increase of 1% in Johnson's coefficients has been evaluated. For the BMC measured using triple photon energy absorptiometry, this number ranges from 0.051 to 0.95 g cm$^{-2}$ depending on the increased coefficient. On the other hand, dual photon energy absorptiometry provides a much more stable variance ($0.0023 < \sigma^2 < 0.0028$) g cm$^{-2}$.

### *4.3. Optimizing the choice of the three energies*

Using Hubbell's tables and under the same experimental conditions, the BMC variance has been evaluated for each set of two or three energies ranging from 30 to 500 keV.

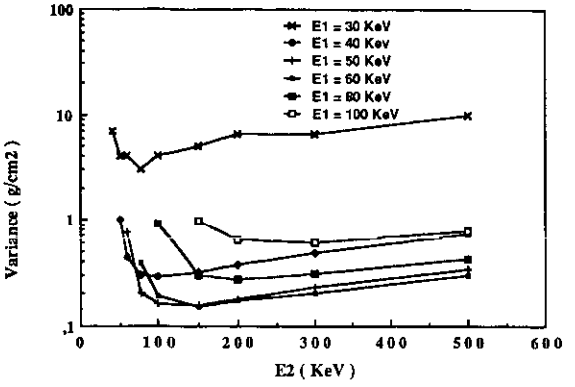


**Figure 1.** Dual photon energy absorptiometry: the BMC variance as a function of the second energy, $E_2$, for several values of the first energy, $E_1$.

When using dual photon energy absorptiometry and for all sets of energies the variance ranges from 0.003 to 0.22 g cm$^{-2}$. The corresponding curves (Watt 1975) are a way of choosing the radioactive source (figure 1). The same study has been done for triple photon energy absorptiometry (figure 2). The results range from 0.65 to more than 1000 g cm$^{-2}$, with large variations when changing one energy among the three.

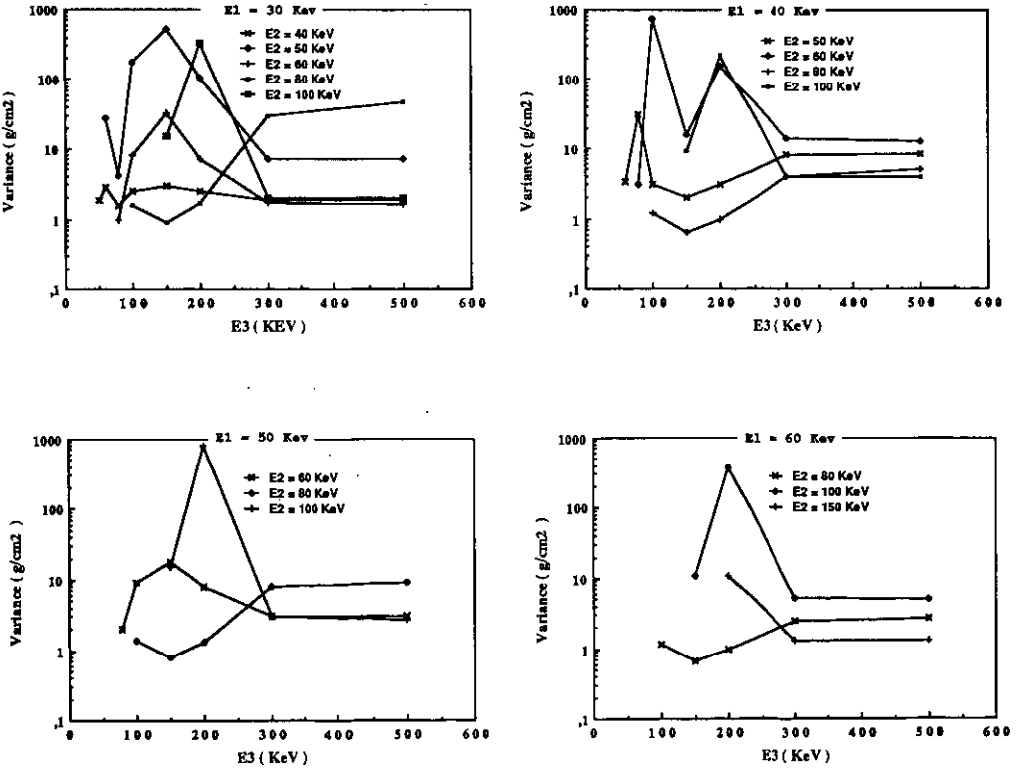


**Figure 2.** Triple photon energy absorptiometry: the BMC variance as a function of the third energy, $E_3$, for several values of the first and second energies, $E_1$ and $E_2$.

**Table 3.** Relative deviation of the mass attenuation coefficient between the theoretical formula of McCullough (1975) and Hubbell's tables (Hubbell 1969) for soft tissues (water), fat (trioleate glycerol) and bone (hydroxyapatite).

| Energy (keV) | Soft tissue | | | Fat | | |
|---|---|---|---|---|---|---|
| | Hubbell | McCullough | Deviation (%) | Hubbell | McCullough | Deviation (%) |
| 30 | 0.3756 | 0.3331 | −10.3 | 0.2915 | 0.2658 | −8.8 |
| 40 | 0.2669 | 0.2467 | −7.6 | 0.2313 | 0.2193 | −5.2 |
| 50 | 0.2248 | 0.2139 | −4.8 | 0.2065 | 0.1999 | −3.2 |
| 60 | 0.2063 | 0.1982 | −3.9 | 0.1944 | 0.1899 | −2.8 |
| 80 | 0.1837 | 0.1789 | −2.6 | 0.1788 | 0.1751 | −2.1 |
| 100 | 0.1707 | 0.1671 | −2.1 | 0.1674 | 0.1650 | −1.4 |
| 150 | 0.1512 | 0.1492 | −1.3 | 0.1491 | 0.1481 | −0.7 |

| | Hydroxyapatite | | |
|---|---|---|---|
| | Hubbell | McCullough | Deviation (%) |
| 30 | 2.050 | 2.472 | 20.6 |
| 40 | 0.9636 | 1.087 | 12.8 |
| 50 | 0.5741 | 0.6156 | 7.2 |
| 60 | 0.4016 | 0.4137 | 3.0 |
| 80 | 0.2578 | 0.2546 | −1.2 |
| 100 | 0.2013 | 0.1964 | −2.3 |
| 150 | 0.1509 | 0.1464 | −3.0 |

## 5. Discussion

The study of the variance shows differential behaviour of the system, depending on the number of unknown factors. With two unknowns, whatever the mass attenuation coefficients are, as the determinant is different enough from zero, the errors are acceptable. Moreover, those errors are relatively stable for the various sets of energies. As a consequence, the diagnosis of osteoporosis and a long term study of the loss of bone mineral content is possible, even if a new calibration of the densitometer happens during this time. On the other hand, the choice of three unknowns leads to a determinant which is very close to zero ($10^{-4}$–$10^{5}$).

The main parameter of the variance is, therefore, no longer the counting time, but the choice of those parameters. Unlike the other authors, Jonson *et al* (1988) benefited from a favourable experimental situation in this choice. In any case the observed error is at least 30 times bigger than the error defined under the same conditions for dual photon energy absorptiometry. The counting time therefore has to be increased in the same ratio. Moreover, a single change of 1% in one of the mass attenuation coefficients can have a drastic effect on the accuracy of the results.

The poor behaviour of the system can be explained using the Compton–photoelectric decomposition of the mass attenuation coefficient. However, according to Hawkes (1980) it is impossible to split rigorously the mass attenuation coefficient into energy-independent functions. Then, equation (4) can be written

$$(\mu/\rho)(E) = a_C f_C(E) + a_{ph} f_{ph}(E) + \varepsilon(E, Z) \tag{9}$$

where $\varepsilon(E, Z)$ is an energy- and tissue-dependent correction function. For instance,

$\varepsilon(E, Z)$ can be evaluated by comparison between McCullough's formula and Hubbell's results (table 3). As $\varepsilon(E, Z)$ is slightly different from zero, the system of three unknowns can be solved, but the small value of this correction leads to unacceptable errors in the determination of the BMC. For our environment, McCullough's formula used for energies ranging from 40 to 100 keV leads to an error which is below 4% for a layer consisting of 1 g $cm^{-2}$ hydroxyapatite, 10 g $cm^{-2}$ soft tissues and 5 g $cm^{-2}$ fat. Therefore, the better results of Jonson *et al* can be explained by the choice of the bone standard: in his experiments, the bone standard was richer in calcium and phosphorus (95.2% Ca and P) than hydroxyapatite (57.4% Ca and P). This leads to a bigger value of $\varepsilon(E, Z)$ and smaller errors.

The significant variations in the hydroxyapatite, fat and soft tissue variances can be easily explained. According to equation (8), the variances are proportional to the minor determinants of the system which are bigger for soft tissues and fat than for hydroxyapatite (table 4). The reason is that the hydroxyapatite mass attenuation coefficients are noticeably different from the mass attenuation coefficients for fat and soft tissues.

**Table 4.** Variance calculations for soft tissue, fat and hydroxyapatite: minor determinants of the system.

| Energy (keV) | Minor determinant | | |
|---|---|---|---|
| | Soft tissue | Fat | Hydroxyapatite |
| 44–100 | 0.1154 | 0.1113 | 0.0052 |
| 44–60 | 0.0937 | 0.0907 | 0.0042 |
| 60–100 | 0.0284 | 0.0274 | 0.0013 |

In conclusion, it appears that a methodology based on the resolution of a three-phase system is not suited to the determination of bone mineral content because triple photon energy absorptiometry requires the solution of a linear system of equations whose determinant is too close to zero to provide acceptable errors when changing slightly the choice of the mass attenuation coefficients.

**Résumé**

Une étude tant théorique qu'expérimentale a été menée dans le but d'évaluer les performances des techniques d'absorptiométrie tri-photonique quant à l'estimation du contenu minéral osseux (CMO). Cette technique utilise trois faisceaux d'energies différentes de manière à tenir compte de l'influence des tissus mous et adipeux lors de la mesure du contenu minéral osseux. Cependant, des considérations théoriques nous ont conduit à douter de la justesse et de la reproductibilité de telles mesures. En première approximation, un coefficient d'absorption peut en effet être décomposé en la somme de deux facteurs indépendants de l'énergie et représentant les processi Compton et photoélectrique. Il est dès lors impossible de choisir plus de deux énergies différentes sans obtenir des coefficients d'absorption linéairement dépendants. En fait, l'existence d'un troisième facteur lié à l'énergie et intervenant dans l'expression de ces coefficients rend théoriquement possible les techniques d'absorptiométrie tri-photonique mais des tests expérimentaux ainsi que des simulations numériques nous ont montré que l'influence relative de ce facteur était trop faible pour justifier une utilisation pertinente de l'absorptiométrie tri-photonique.

## Zusammenfassung

Theoretische und experimentelle Grenzen der Drei-Photonen-Energieabsorptiometrie bei der Messung des Knochenmineralgehalts.

Theoretische und experimentelle Studien wurden durchgeführt, um die Möglichkeiten der Drei-Photonen-Energieabsorpiometrie bei der Bestimmung des Mineralgehaltes von Knochen zu untersuchen. Zweck dieses Verfahrens ist es, die gemessene Knochenmineraldichte für Fett und Weichgewebe zu korrigieren. Theoretische Betrachtungen führen jedoch zu Zweifeln an der Genauigkeit solcher Messungen. In erster Näherung kann der Absorptionskoeffizient in einem Compton- und einen photoelektrischen Energieunabhängigen Anteil gespalten werden. Als Konsequenz daraus ergibt sich, daß nicht mehr als zwei unabhängige Massenschwächungskoeffizienten für verschiedene Energien gefunden werden können. Die Existenz eines nicht Energie-abhängigen dritten Faktors könnte die Anwendung der Drei-Photonen-Energieabsorptiometrie rechtfertigen, aber experimentelle Tests und numerische Simulationen haben gezeigt, daß dieser Wert zu niedrig ist, als daß die Drei-Photonen-Energieabsorptiometrie als adäquate Methode zur Messung des Knochenmineralgehalts betrachtet werden könnte.

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