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The feasibility of triple-energy absorptiometry for the determination of bone mineral, Ca and P *in vivo*

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Abstract. The theoretical feasibility of triple-energy absorptiometry in general and the experimental conditions when using triple-energy absorptiometry for the determination of bone mineral, elemental calcium and phosphorus content *in vivo* have been investigated. A theoretical analysis of the decomposition of the mass attenuation coefficients is presented and discussed. The main obstacle to the effective use of triple-energy absorptiometry *in vivo* is the large number of pulses which must be detected to reduce the statistical fluctuations.

Keywords: triple-energy absorptiometry, photon absorptiometry, bone mineral, bone

1. Introduction

In body composition analyses a quantitative analysis of various tissue masses is desirable. Photon absorptiometry, neutron activation, ultrasound measurements and magnetic resonance tomography have been used for noninvasive measurements of body composition. In the case of photon absorptiometry, single- and dual-photon-energy techniques employing radionuclides are often used, and recently also single- and dual-energy x-ray absorptiometry using x-ray tubes. Success has also been reported using triple-photon absorptiometry (Jonson *et al* 1986).

In body composition analysis, the human body can, from an absorptiometric standpoint, be divided into a three-tissue-component system composed of bone mineral, lean soft tissue and adipose tissue (Tothill and Pye 1992).

Photon absorptiometric techniques are predominantly used for bone mineral content determination. These techniques are characterized by high precision while the absorbed radiation dose is low. However, when less than three photon energies are used, certain approximations must be made to solve the three-tissue-component system, if the total thickness of the attenuator is not known.

When using photon absorptiometry, combinations of three different materials other than bone mineral, lean soft tissue and adipose tissue could be of interest, for instance (i) bone mineral, lean soft tissue with adipose tissue and a metal prosthesis (Farrell and Webber 1992), (ii) calcium, phosphorus (the two major components of bone mineral) and lean soft tissue with adipose tissue.

Attempts have been made to analyse the above-mentioned three-component systems accurately using triple-photon absorptiometric methods. In the case of bone mineral content, soft tissue and adipose tissue, Jonson *et al* have reported success (1986, 1988).

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The use of three photon energies to determine the contents of three materials simultaneously is the subject of discussion. Several authors assume different approximations in the photon attenuation process in the clinical energy range (Alvarez and Macovski 1976, Lehmann *et al* 1981). Often, only photoelectric absorption and Compton scattering processes are taken into account when parametrization of attenuation coefficients in the clinical energy range is discussed. These approximations, within certain limits, will not cause substantial errors in the calculations. When it comes to triple-photon absorptiometry, however, these generalizations are not adequate since they have no basis in theory (Hawkes and Jackson 1980). Rayleigh scattering, photoelectric absorption and Compton scattering are the three dominating photon interaction processes which contribute to the photon attenuation in the clinical photon energy range ($h\nu < 1$ MeV) (Jackson and Hawkes 1981).

When using photon absorptiometry techniques to quantify the amount of bone mineral, it is the amount of hydroxyapatite in the skeleton that is measured. Calcium and phosphorus are two important constituents of hydroxyapatite. In healthy human skeletons the distributions of calcium and phosphorus follow mainly the hydroxyapatite concentration. The proportions of calcium and phosphorus in bone tissue vary only slightly with age (Kósa *et al* 1989). However, there are conditions which may influence the calcium to phosphorus ratio. It would therefore be desirable to analyse the amounts of calcium and phosphorus separately *in vivo*.

This paper discusses experimental and fundamental theoretical conditions determining the feasibility of triple-photon absorptiometry in the analysis of a three-component system in the clinical photon energy range. Concerning the experimental conditions, analyses are made for (i) bone mineral, lean soft tissue and adipose tissue, and (ii) calcium, phosphorus and lean soft tissue with adipose tissue as the three-component systems.

The aim of this theoretical study is to show that basic theoretical requirements for the feasibility of triple-photon absorptiometry exist and to discuss the experimental limitations of triple-photon absorptiometry *in vivo*.

2. Methods

Regarding photon attenuation, the human body can be generalized into a three-tissue-component system composed of bone mineral, soft tissue and adipose tissue. In order to use photon absorptiometry for an unequivocal determination of these three tissue layers, without any approximations, the applied transmission technique must make use of three different photon energies. A system of equations consisting of three equations, describing the exponential attenuation of the radiation beam, can be set up in the following form:

$$N_i = N_{0,i} \exp(-\mu_{x,i} m_x - \mu_{y,i} m_y - \mu_{z,i} m_z) \tag{1}$$

where $i = 1, 2, 3$, N_i is the number of counts in the energy interval i, $N_{0,i}$ the number of counts in the energy interval i without attenuation, μ the mass attenuation coefficient in $\mathrm{cm}^2\ \mathrm{g}^{-1}$, m the mass of attenuator per unit area in $\mathrm{g\ cm}^{-2}$ and x, y, and z the indices for the three attenuators, and in matrix form

$$\begin{pmatrix} \mu_{x,1} & \mu_{y,1} & \mu_{z,1} \\ \mu_{x,2} & \mu_{y,2} & \mu_{z,2} \\ \mu_{x,3} & \mu_{y,3} & \mu_{z,3} \end{pmatrix} \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} = \begin{pmatrix} \ln N_{0,1} - \ln N_1 \\ \ln N_{0,2} - \ln N_2 \\ \ln N_{0,3} - \ln N_3 \end{pmatrix}$$

$$(A) \qquad (B) \quad = \qquad (C).$$

We will then have for m_x, for instance,

$$m_x = \frac{a(\ln N_{0,1} - \ln N_1) + b(\ln N_{0,2} - \ln N_2) + c(\ln N_{0,3} - \ln N_3)}{a\mu_{x,1} + b\mu_{x,2} + c\mu_{x,3}} \tag{2}$$

with

$$a = \mu_{y,2}\mu_{z,3} - \mu_{z,2}\mu_{y,3}$$
$$b = \mu_{y,3}\mu_{z,1} - \mu_{z,3}\mu_{y,1}$$
$$c = \mu_{y,1}\mu_{z,2} - \mu_{z,1}\mu_{y,2}.$$

Similar expressions can be obtained for m_y and m_z.

Generally, the total mass attenuation coefficient for an element is a product of a material constant k and the total photon interaction cross-section per atom, σ^a_{Tot}

$$\frac{\mu_{\text{Tot}}}{\rho}(h\nu) = k\sigma^a_{\text{Tot}} = \frac{N_A}{A}\sigma^a_{\text{Tot}} \tag{3}$$

where N_A is Avogadro's number, A is the atomic mass and σ^a_{Tot} is the total cross-section per atom including the partial interaction processes: coherent or Rayleigh scattering, photoelectric absorption and incoherent or Compton scattering ($h\nu < 1$ MeV).

The total mass attenuation coefficient $(\mu_{\text{Tot}}/\rho)(h\nu)$ can also be expressed as a sum of the partial mass attenuation coefficients of the different photon interaction processes (McCullough 1975).

$$\frac{\mu_{\text{Tot}}}{\rho}(h\nu) = \frac{\mu_R}{\rho}(h\nu) + \frac{\mu_{\text{ph}}}{\rho}(h\nu) + \frac{\mu_C}{\rho}(h\nu). \tag{4}$$

Subsequently, σ^a_{Tot} can be expressed as

$$\sigma^a_{\text{Tot}} = \sigma^a_R + \sigma^a_{\text{ph}} + \sigma^a_C \tag{5}$$

where σ^a_R, σ^a_{ph} and σ^a_C are the cross-sections for Rayleigh scattering, photoelectric absorption and Compton scattering per atom, respectively.

Thus

$$\frac{\mu_{\text{Tot}}}{\rho}(h\nu) = \frac{N_A}{A}(\sigma^a_R + \sigma^a_{\text{ph}} + \sigma^a_C). \tag{6}$$

In principle, a further step can be made to decompose the total mass attenuation coefficient, as proposed by McCullough (1975).

$$\frac{\mu_{\text{Tot}}}{\rho}(h\nu) = \frac{ZN_A}{A}\sigma^e_{\text{Tot}} \tag{7}$$

where

$$\sigma^e_{\text{Tot}} = \sigma^e_R + \sigma^e_{\text{ph}} + \sigma^e_C \tag{8}$$

and Z is the atomic number.

The total electronic cross-section, σ^e_{Tot}, can thus be written as a sum of the individual photon interaction cross-sections per electron (McCullough 1975).

However, this decomposition ((7) and (8)) assumes that all the electrons in the atoms contribute to the different interaction processes with the same probability, which is not true, or that an average interaction probability is assumed for all the atomic electrons. Particularly for photoelectric absorption and Rayleigh scattering, the expression for the interaction probability per electron (σ^e_R and σ^e_{ph}) is not completely appropriate.

In the case of σ_R^e, the interaction probability for coherent scattering between a photon and an electron, the Thomson scattering cross-section may 'erroneously' be used. The Thomson cross-section is given by:

$$\sigma_{\mathrm{Th}} = 8\pi r_0^2/3 \tag{9}$$

where $r_0 = e^2/4\pi\varepsilon_0 m_e c^2$ (the classical electron radius, which has nothing to do with the actual size of the electron).

As can be seen from (9) the Thomson scattering cross-section, which is given for elastic scattering of a photon by a single free electron, is independent of the photon energy and is a universal constant (Heitler 1954).

When the photon wavelength is comparable to the size of atoms (the atomic diameter is $\sim$2–4 $\times$ 10^{-10} m) all the electrons in an atom of the attenuating medium, and not only single electrons, participate simultaneously in the scattering process (Anderson 1984). Thus, the interaction probability, σ, of coherent scattering may be expressed per atom σ_R^a. This is also emphasized by the fact that, due to conservation of momentum, Rayleigh scattering is an atomic process: it involves strongly bound atomic electrons (Jackson and Hawkes 1981, Greening 1977, Roy and Reed 1968). The Rayleigh scattering cross-section can be calculated by numerical integration of the Thomson scattering differential cross-section weighted by the square of the coherent scattering form factor $[F(x, Z)]^2$ where $x = (1/\lambda)\sin(\theta/2)$ is the momentum transfer variable, λ the incident photon wavelength and θ the scattering angle (Hubbell and Seltzer 1995, Hawkes and Jackson 1980, Ling *et al* 1982, Jackson and Hawkes 1981, Anderson 1984, Roy and Reed 1968). The coherent scattering form factor $F(x, Z)$ accounts for the scattering effects from the atomic electrons. Thus

$$\sigma_R^a = \int \mathrm{d}\sigma_{\mathrm{Th}}(\theta)[F(x, Z)]^2. \tag{10}$$

At small momentum transfer, $F(x, Z)$ approaches the atomic number Z. Thus the Rayleigh scattering cross-section is proportional to Z^2. It is difficult to derive an analytical expression for the Rayleigh scattering cross-section. However, in the nonrelativistic energy range, σ_R^a can be expressed analytically (Roy and Reed 1968). According to Roy and Reed σ_R^a can be written

$$\sigma_R^a = 4\pi e^4 Z^2 (m_e c^2)^{-2} (\lambda/b)^2 (0.8 - \pi\lambda/2b) \tag{11}$$

where

$$b = 5.9 \times 10^{-8} Z^{-1/3} \text{ cm}.$$

In the case of σ_{ph}^e, photoelectric absorption between photons and free electrons cannot take place due to the requirement for conservation of momentum. Therefore, the interacting electron in the photoelectric process must be bound. The law of conservation of momentum indicates that for photoelectric absorption the interaction probability is enhanced if the electron is more strongly bound. This is because the interaction probability for photoelectric absorption of the K-shell electrons is higher than that for the other atomic electrons. The photoelectric interaction occurs mostly (80–90%) with the K-shell electrons for most elements (Anderson 1984).

Due to the conservation of momentum, the photoelectron must be bound before the interaction and consequently the photoelectric process must be associated with a reaction between a photon and an atom. It follows that it is more correct to express the interaction probability, σ, of photoelectric absorption per atom σ_{ph}^a than per electron σ_{ph}^e.

The photoelectron liberated by the photoelectric process is not highly relativistic since in the clinical photon energy range the photon energies are below 1 MeV ($h\nu$ is not $\gg m_e c^2$).

In this energy range, the atomic cross-section derived by Bethe can be used for the photoelectric absorption probability (Lapp and Andrews 1972, Hawkes and Jackson 1980). Bethe's formula is (Lapp and Andrews 1972)

$$\sigma_{\mathrm{ph}}^{a} = (2 \times 10^{-8})\sigma_{\mathrm{Th}} S (Z - 0.3)^5 \left(\frac{m_e c^2}{h\nu} \right)^{7/2} \tag{12}$$

where σ_{Th} is the Thomson cross-section (9), S is a function of photon energy and atomic number and $h\nu$ the photon energy in keV.

For the interaction probability of incoherent scattering between a photon and an electron, σ_C^e, the Klein–Nishina cross-section can be used. The Klein–Nishina cross-section is given by

$$\sigma_{\mathrm{K-N}} = 2\pi r_0^2 \left\{ \frac{1+\alpha}{\alpha^2} \left[\frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{\alpha} \right] + \frac{\ln(1+2\alpha)}{2\alpha} - \frac{(1+3\alpha)}{(1+2\alpha)^2} \right\} \tag{13}$$

where $\alpha = h\nu / m_e c^2$. Note that the Klein–Nishina cross-section is independent of the atomic number Z, and is merely dependent on the incident photon energy. The Klein–Nishina formula describes the interaction probability of incoherent scattering between a photon with energy $h\nu$ and a free electron. In other words, the binding energy of the interacting atomic electron is not included in the Klein–Nishina cross-section. The heaviest element of importance in human tissues, concerning photon attenuation properties, is calcium. The binding energy of the K-shell electrons in a calcium atom is of the order of 4 keV. When the incident photon energy is substantially higher than the binding energy of the atomic electrons, the Klein–Nishina formula may be used to calculate the incoherent scattering cross-section. However, when the binding energies of the atomic electrons can no longer be ignored, the Klein–Nishina formula must be corrected for the electron binding. This correction is made by using the incoherent scattering function $S(x, Z)$, which represents the probability that an atom receives energy and will be excited or ionized when an incident photon gives momentum to an atomic electron (Hubbell and Seltzer 1995, Hawkes and Jackson 1980, Jackson and Hawkes 1981, Anderson 1984, Roy and Reed 1968). Thus, σ_C^a is given by

$$\sigma_C^a = \int \mathrm{d}\sigma_{\mathrm{K-N}}(\theta) S(x, Z). \tag{14}$$

At high incident photon energies the incoherent scattering function approaches the atomic number Z, and σ_C^a is thus proportional to Z.

The partial cross-sections σ_R^a, σ_{ph}^a and σ_C^a for the three photon interaction processes, i.e., (10), (12) and (14), can be used in (6).

For mixtures and compounds, the mass attenuation coefficient can be obtained from the coefficients of the constituent elements, according to the simple relation (Hubbell and Seltzer 1995, Greening 1977, McCullough 1975, Anderson 1984)

$$\frac{\mu_{\mathrm{Tot}}}{\rho}(h\nu) = \sum_i w_i \left(\frac{\mu_{\mathrm{Tot}}(h\nu)}{\rho} \right)_i \tag{15}$$

where w_i is the fraction by weight of the ith atomic element and $(\mu_{\mathrm{Tot}}(h\nu)/\rho)_i$ is its mass attenuation coefficient.

For the three tissue layers (m_x, m_y and m_z) the accompanying mass attenuation coefficients can be expressed by combining (6) and (15).

The mass attenuation coefficients in the matrix (A) can thus be written in the decomposed form. The partial mass attenuation coefficients are independent of one another (Greening 1977) and it can be shown that none of the three rows in the matrix (A) is a linear combination of the others, and, consequently, the determinant of the matrix (A) differs from zero, i.e. triple-photon absorptiometry is theoretically feasible. A three-tissue-layer system, i.e., an equation system with three unknowns, can be simultaneously analysed.

Among others, Lehmann *et al* (1981) and White (1977) have proposed that the partial mass attenuation coefficients can be parametrized approximately into a material-dependent constant and an energy-dependent function. Considering the three photon interaction processes, Rayleigh scattering, photoelectric absorption and Compton scattering, the approximate parametrization can be performed in the following way.

By combining (6), (11), (12) and (13), and using the relation in (7) for σ_C^a ($\sigma_C^a = Z\sigma_{\mathrm{K-N}}$) we obtain

$$\begin{aligned}\frac{\mu_{\mathrm{Tot}}}{\rho}(h\nu) = \frac{N_A}{A}(\sigma_R^a + \sigma_{\mathrm{ph}}^a + \sigma_C^a) = \frac{N_A}{A}(4\pi e^4 Z^2 (m_e c^2)^{-2}(\lambda/b)^2(0.8 - \pi\lambda/2b)) \\ + \frac{N_A}{A}\left((2\times 10^{-8})\sigma_{\mathrm{Th}} S (Z-0.3)^5 \left(\frac{m_e c^2}{h\nu}\right)^{7/2}\right) \\ + \frac{ZN_A}{A}\left(2\pi r_0^2 \left\{\frac{1+\alpha}{\alpha^2}\left[\frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{\alpha}\right]\right.\right. \\ \left.\left. + \frac{\ln(1+2\alpha)}{2\alpha} - \frac{(1+3\alpha)}{(1+2\alpha)^2}\right\}\right). \qquad (16)\end{aligned}$$

In (16) the Z parameter for the Rayleigh and photoelectric processes can be taken outside the parentheses and the expression for the total mass attenuation coefficient can be rewritten as

$$\frac{\mu_{\mathrm{tot}}}{\rho}(h\nu) = \frac{Z^{2-3}N_A}{A} f_R(h\nu) + \frac{Z^5 N_A}{A} f_{\mathrm{ph}}(h\nu) + \frac{ZN_A}{A} f_C(h\nu) \qquad (17)$$

where $f_R(h\nu)$ and $f_{\mathrm{ph}}(h\nu)$ are the cross-sections for Rayleigh scattering and photoelectric absorption, 'without' the Z parameter, respectively, and $f_C(h\nu)$ is the Klein–Nishina cross-section. Note that, for Rayleigh and Compton scattering, the same Z dependence is obtained as in (10) and (14), respectively.

Equation (17) is in accordance with the finding of White (1977) concerning the decomposition of the mass attenuation coefficients. Using the same terminology as White (1977), $f_R(h\nu)$, $f_{\mathrm{ph}}(h\nu)$ and $f_C(h\nu)$ are the energy-dependent 'constants' at the energy ($h\nu$) and $Z^{2-3}N_A/A$, Z^5N_A/A and ZN_A/A are the material-dependent constants of the partial mass attenuation coefficients. Thus, the total mass attenuation coefficient is decomposed into an approximate parameter form (Lehmann *et al* 1981, White 1977) and can be written

$$\frac{\mu_{\mathrm{Tot}}}{\rho}(h\nu) = g_R f_R(h\nu) + g_{\mathrm{ph}} f_{\mathrm{ph}}(h\nu) + g_C f_C(h\nu) \qquad (18)$$

where

$$g_R = \frac{Z^{2-3}N_A}{A} \qquad g_{\mathrm{ph}} = \frac{Z^5 N_A}{A} \qquad g_C = \frac{ZN_A}{A}.$$

Note that the purpose of this step-by-step analysis of the decomposition of the mass attenuation coefficient is to show how this approximate parametrization can be performed, rather than using the expressions for numerical calculations of the mass attenuation coefficients, which are well tabulated by, for instance, Berger and Hubbell (1987).

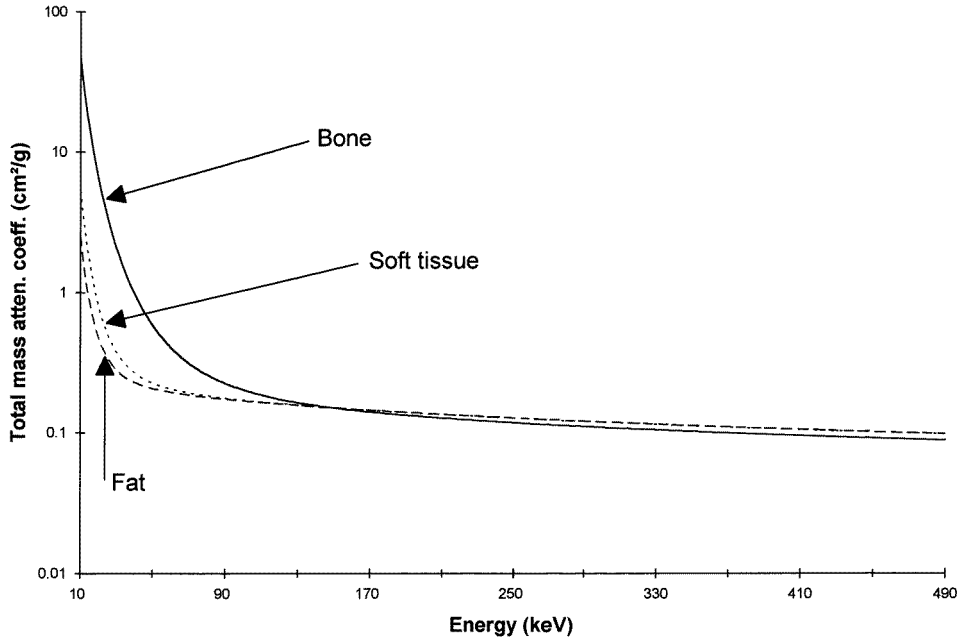

Figure 1. Total mass attenuation coefficients for bone (hydroxyapatite), soft tissue (water) and fat (triolein).

As has been stated above, it should be borne in mind that the mass attenuation coefficient expressed in the parametrized form of (17) is an approximation since the function S in (12) and both the coherent scattering form factor F (in (10)) and the incoherent scattering function S (in (14)) are dependent on the atomic number Z (Hawkes and Jackson 1980, Jackson and Hawkes 1981). Also the Klein–Nishina formula is valid only when the binding energy of the atomic electrons is negligible compared with the incident photon energy, and all the atomic electrons can be treated as if they were free in the interaction process, which is, however, not the case in the clinical photon energy range.

The mass attenuation coefficients for each of the three tissue layers can, in combination with (15), also be expressed in the approximate parametrized form of (18). Thus,

$$\mu_x(h\nu) = g_{C,x} f_C(h\nu) + g_{\mathrm{ph},x} f_{\mathrm{ph}}(h\nu) + g_{R,x} f_R(h\nu)$$
$$\mu_y(h\nu) = g_{C,y} f_C(h\nu) + g_{\mathrm{ph},y} f_{\mathrm{ph}}(h\nu) + g_{R,y} f_R(h\nu)$$
$$\mu_z(h\nu) = g_{C,z} f_C(h\nu) + g_{\mathrm{ph},z} f_{\mathrm{ph}}(h\nu) + g_{R,z} f_R(h\nu).$$

When the mass attenuation coefficients are expressed in the parametrized form (18), it can also be shown that the determinant of the matrix $(A) \neq$ zero and thus triple-photon energy absorptiometry is feasible. However, the non-zero condition in this case is a mathematical, or algebraic, effect. When the coefficients in a quadratic $(n \times n)$ matrix are expressed according to (18) (i.e. as a sum of products in the form $g_i f_i$, where $i = 1, 2, 3, \ldots$, the g_i and f_i values are constants (>0) and they differ from each other as indicated by (18) and figures 2 and 4, respectively), it can be shown that the determinant of the $(n \times n)$ matrix will only differ from zero when the number of products of the individual coefficients is at least the size (n) of the matrix. When the number of products is less than n there is linear combination in the matrix and in consequence the determinant is zero. The partial mass

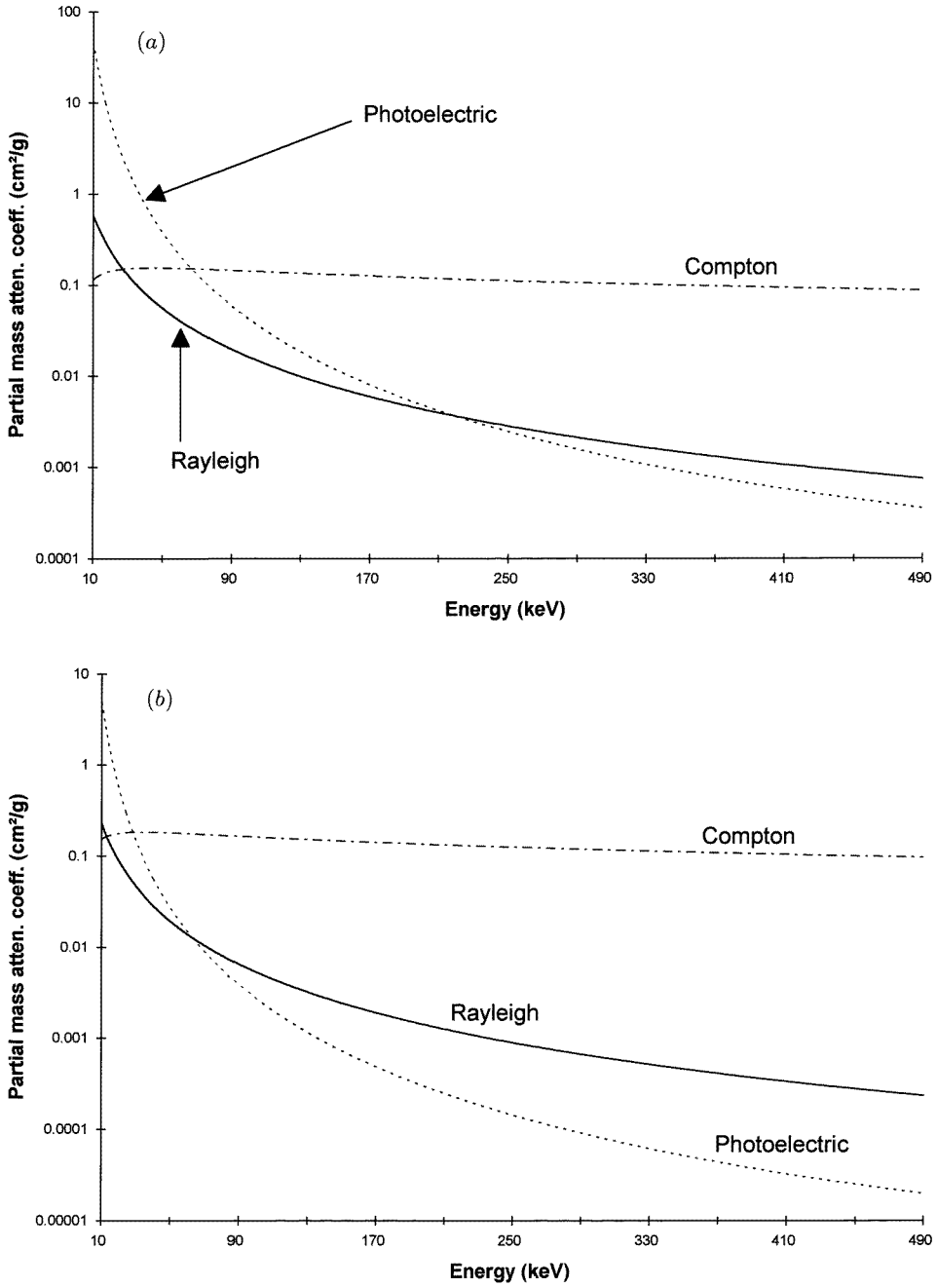

Figure 2. Partial mass attenuation coefficients: Rayleigh scattering, photoelectric absorption and Compton scattering for bone (*a*), soft tissue (*b*) and fat (*c*).

attenuation coefficients cannot be rigorously split into a material-dependent constant and an energy-dependent function (Hawkes and Jackson 1980, Jackson and Hawkes 1981, Kotzki *et al* 1991). This indicates that the number of photon interaction processes may not limit

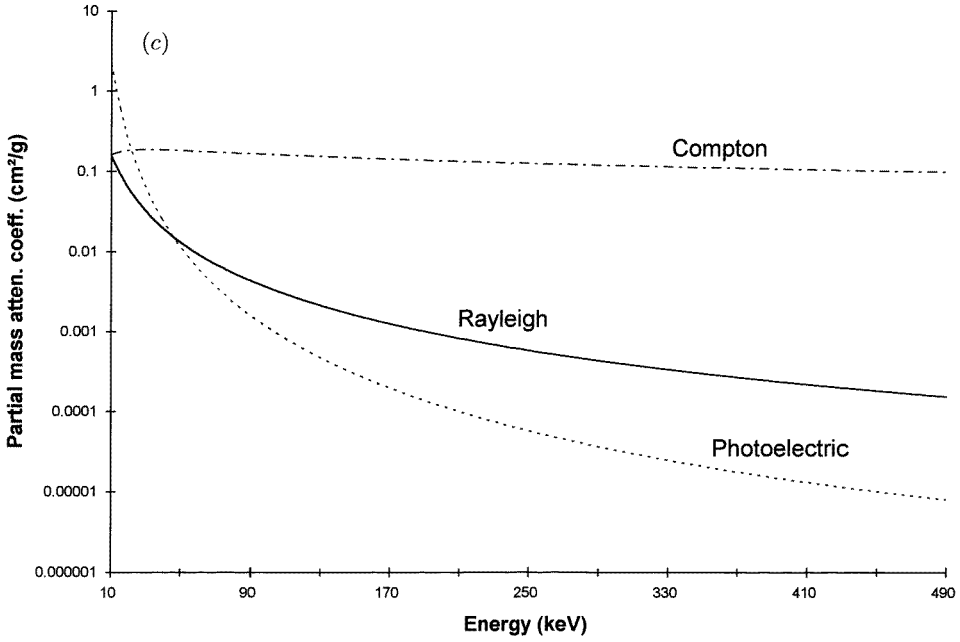

Figure 2. (Continued)

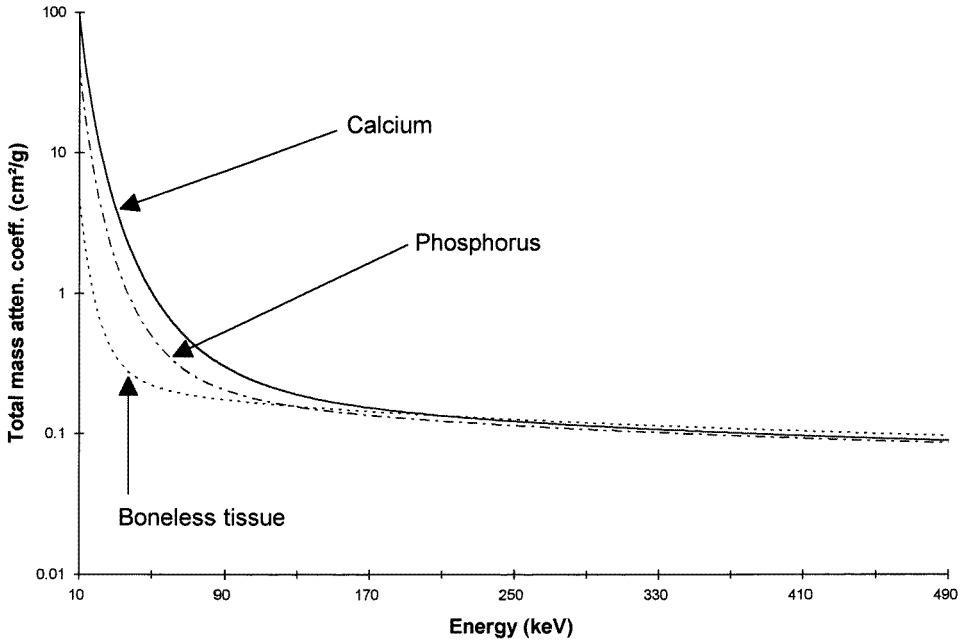

Figure 3. Total mass attenuation coefficients for Ca, P and boneless tissue.

the number of different materials which can be simultaneously quantified using the same number of photon energies as the number of unknown materials.

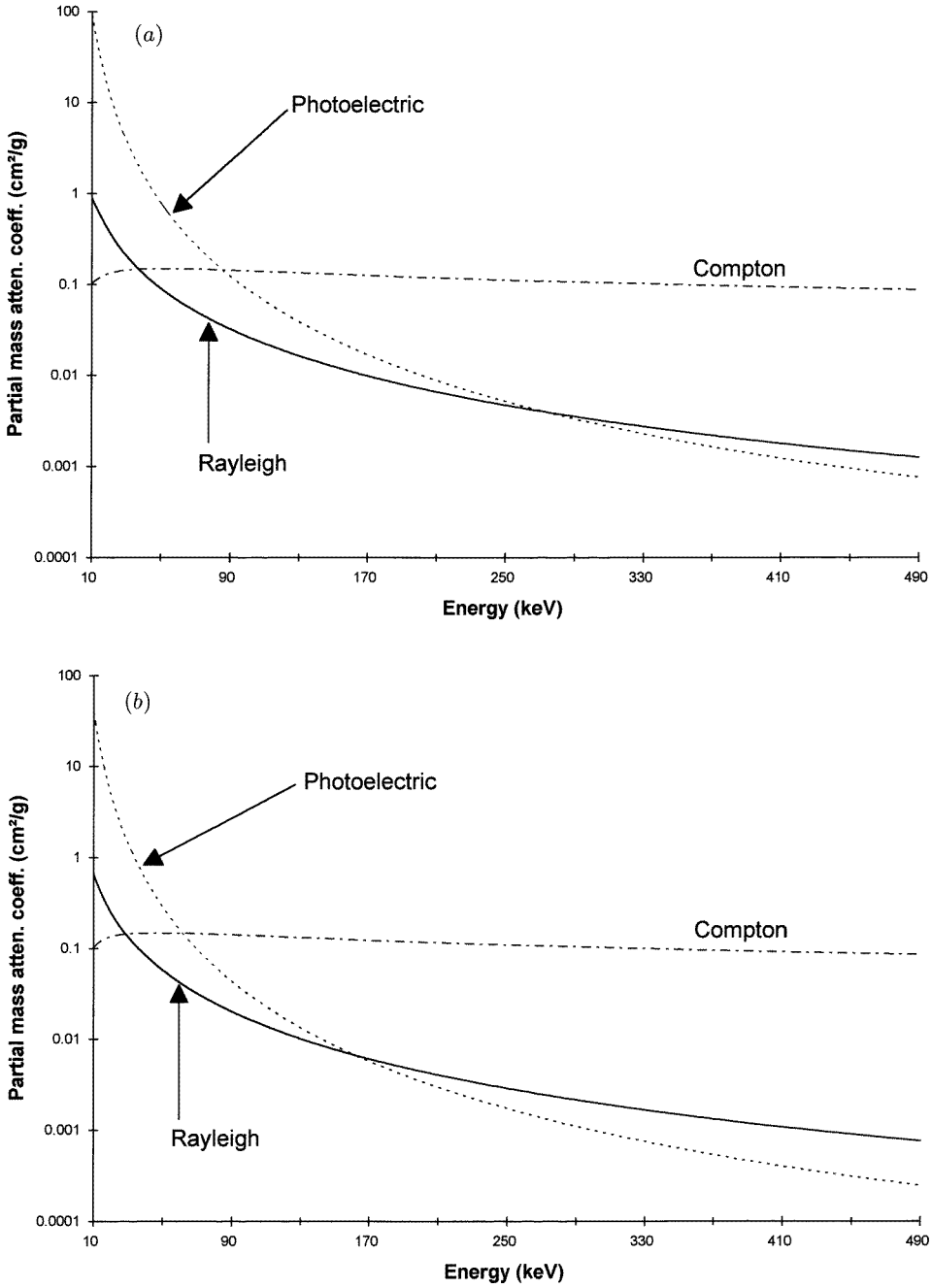

Figure 4. Partial mass attenuation coefficients: Rayleigh scattering, photoelectric absorption and Compton scattering for Ca (*a*), P (*b*) and boneless tissue (*c*).

Regarding the experimental conditions, there are several standard methods of solving the three-equation system, containing attenuation coefficients and the logarithm of the counted pulses (i.e. $\ln N_0 - \ln N$), arising from the attenuation equation system. If the matrix (A) is ill

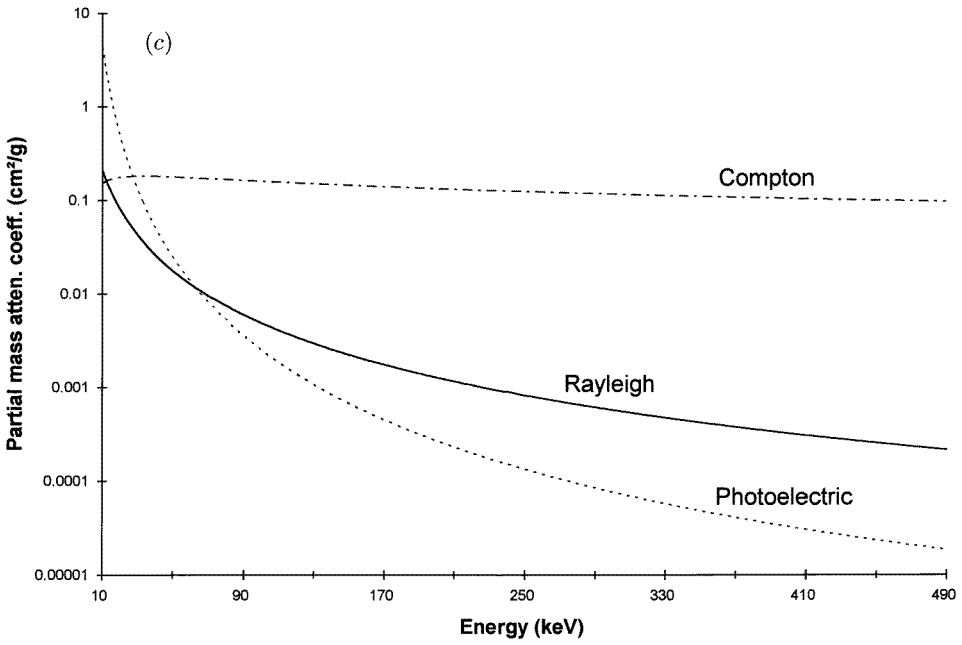

Figure 4. (Continued)

conditioned, this will give rise to a sensitive inverted matrix, and small perturbations in the input data will introduce large errors into the solution. Accurate experimental determination of the attenuation coefficients is therefore necessary and the number of pulses recorded must be high enough to reduce the statistical fluctuations.

In an experimental situation, the following factors are of prime importance: the amounts of the different components in the attenuator, their mutual attenuation properties and the number of counts detected. In most absorptiometry measurements, bone mineral content is the parameter of interest. Comparison between mass attenuation coefficients for bone, soft tissue and fat shows the largest difference for low photon energies (see figure 1). However, from about 140 keV the mass attenuation coefficients for hydroxyapatite, water and fat are almost equal. The Rayleigh scattering component and photoelectric absorption component decrease rapidly with increasing photon energy, due to the decreasing probabilities of these photon interaction processes, for all three above-mentioned materials (see figure 2). (Note that all the figures presented in this article have been constructed from the theoretical data of Berger and Hubbell (1987).)

Simultaneous quantitative *in vivo* analysis of the amount of calcium and phosphorus in the skeleton may be of interest. In this case, a tissue combination composed of calcium, phosphorus and soft tissue with adipose tissue could be used as a three-component system. The soft tissue and the adipose tissue should be combined into one single tissue since, for the four-component system (Ca, P, soft tissue and adipose tissue), the corresponding coefficient matrix is generally more ill conditioned than the matrix for the three-component system. The mixture of the soft and adipose tissue is composed of 10.3% hydrogen, 23.6% carbon, 2.7% nitrogen, 62.7% oxygen, 0.14% sodium, 0.21% sulphur, 0.12% chlorine and 0.21% potassium, by weight, according to ICRP 23 (ICRP 1975), and is the major elemental composition of human boneless tissue.

The mass attenuation coefficients for Ca, P and soft tissue with adipose tissue can be seen in figure 3. The largest differences can be seen in the low-photon-energy region.

The scattering components (Rayleigh and Compton) and the photoelectric absorption component of the mass attenuation coefficients exhibit similar energy dependences for the combination of Ca, P and boneless tissue as for bone, lean soft tissue and adipose tissue. This is shown in figure 4.

One prerequisite for an *in vivo* method is that the measurement time is reasonably short for patient comfort and compliance. For good Poisson counting statistics the technique must be associated with photon-generating devices or radionuclides which can produce high count rates in a well collimated beam, and also fast-counting electronics, which can handle the high count rates involved.

To obtain reasonably good precision, it is necessary to detect a large number of photons. For a three-component system, the variance in, for instance, m_x can be written

$$V(m_x) = 1/D^2[a^2(1/N_{0,1} + 1/N_1) + b^2(1/N_{0,2} + 1/N_2) + c^2(1/N_{0,3} + 1/N_3)] \qquad (19)$$

where D is the denominator of (2) and a, b and c are as in (2).

Equation (19) shows that the variance, V, is strongly dependent on the inverted values of N_0 and N. Thus, improvements in precision could be made with increased values of N_0 and N.

The selection of the photon energies is also of importance since the mass attenuation coefficients have different values for different photon energies, particularly in the low-photon-energy region for all of the above-discussed tissues (figures 1 and 3). Thus, the choice of photon energies also indirectly influences the variance, since both D (the determinant of matrix (A)) and a, b and c (in (2) and (19)) contain mass attenuation coefficients. However, this will not alter the variance from measurement to measurement, if the photon energies are the same, since the mass attenuation coefficients are constants.

For a measurement situation concerning a three-component system composed of bone, lean soft tissue and adipose tissue with the thicknesses (which are representative for normal patient values and counts)

$$m_b = 1.0 \text{ g cm}^{-2} \qquad m_s = 15 \text{ g cm}^{-2} \qquad m_f = 5 \text{ g cm}^{-2}$$
$$N_{0,1} = 5 \times 10^8 \qquad N_{0,2} = 4 \times 10^8 \qquad N_{0,3} = 2 \times 10^8$$

where b, s and f are indices for bone mineral, lean soft tissue and fat, respectively, for the photon energies 44 keV, 59.5 keV and 100 keV (Jonson *et al* 1988) we will have a precision (expressed as the coefficient of variation) of about 5% for the bone mineral values. This is somewhat lower than the short-term *in vitro* precision of 2.1% reported by Szücs *et al* using a continuous x-ray spectrum as a photon source for triple-photon absorptiometry (Szücs *et al* 1993).

When the three-component system is composed of Ca, P and soft/adipose (s/a) tissue, with the thicknesses $m_{\text{Ca}} = 0.4$ g cm^{-2}, $m_{\text{P}} = 0.185$ g cm^{-2} and $m_{s/a} = 20$ g cm^{-2} (representative for normal patient values), and we require the same precision for Ca and P as for the bone, lean soft tissue and adipose tissue combination, that is about 5%, we need the following primary counts for the three photon energies:

$$N_{0,1} = 5 \times 10^{11} \qquad N_{0,2} = 4 \times 10^{11} \qquad N_{0,3} = 2 \times 10^{11}.$$

This shows that the number of pulses detected must be increased by a factor of 1000 to obtain about the same precision *in vivo* as for the bone, lean soft tissue and adipose tissue combination obtained by Jonson *et al* (1988).

3. Discussion

In body composition analyses using photon absorptiometry techniques, a three-component system consisting of bone mineral, lean soft tissue and adipose tissue is often analysed.

Classification of the human body into three uniformly distributed types of tissue is a gross simplification since the composition of the tissues and their attenuation properties may vary slightly at different parts in the human body, and there may be individual variations in tissue composition. However, the mass attenuation coefficients are constants and the effects of differences due to location and individuals appear as uncertainties in the results and are common for all photon absorptiometric methods.

In simultaneous analysis of tissues using photon absorptiometry the number of tissue classes is the subject of discussion (Lehmann *et al* 1981, Kotzki *et al* 1991, Sutcliffe 1996). It is often proposed that the limitation of this number is set by the number of interaction processes in the photon attenuation. It has been shown in our study, and discussed by, among others, Jackson and Hawkes (1981) and Hawkes and Jackson (1980), that the partial mass attenuation coefficients cannot be rigorously decomposed into a material-Z-dependent constant and an energy-dependent function. Thus, the number of interaction processes may not restrict the number of tissue classes, and consequently may not be a basic requirement for the feasibility of triple-photon absorptiometry. A basic requirement for the feasibility of triple-photon absorptiometry is, however, that the determinant of the coefficient matrix differs from zero and only then does the quadratic linear equation system have, according to definition, a unique solution. Numerical calculation of the determinant to test the non-zero condition for feasibility of triple-photon absorptiometry is not valid since the mass attenuation coefficients (theoretical) are 'approximate' and are presented with a finite number of digits. This means that the theoretical models (expressions) used for the calculation of the partial mass attenuation coefficients (cross-sections) are associated with approximations. The calculations are performed by numerical analysis and are connected with certain accuracy errors (Hubbell and Seltzer 1995, Scofield 1973).

In the clinical photon energy range ($<$1 MeV) three independent photon interaction processes dominate: Rayleigh scattering, photoelectric absorption and Compton scattering. The importance of Rayleigh scattering is often neglected or questioned. However, its importance can be seen from figure 2, where the contribution to the total photon attenuation is equal for Rayleigh scattering and photoelectric absorption at about 60 keV for water and at 45 keV for fat, and the Rayleigh/photoelectric ratio is greater than unity above these energy levels (Jackson and Hawkes 1981). Although the partial mass attenuation coefficients for Rayleigh scattering and photoelectric interaction are lower than for Compton scattering (for instance in water above about 35 keV), Rayleigh scattering and photoelectric absorption take part in the attenuation process (McCullough 1975).

Also, when the approximate parametrization (16)–(18) is considered, it can be shown that the determinant (denominator of (2)) of the coefficient matrix differs from zero. This allows three tissue classes to be simultaneously quantified since we have an equation system composed of three unknown tissue thicknesses and three different photon interaction processes. In this case, the presence of the independent interaction processes makes it possible to avoid linear combinations in the matrix (A). Thus, it is proven that it is theoretically possible to analyse a three-component model with the aid of triple-photon energy absorptiometry. However, as has been pointed out above, the decomposition (16–18) is very approximate and simplified. In an experimental set-up, the measurement geometry is also of importance since the Rayleigh-scattered photons are scattered in the forward

direction. To avoid Rayleigh-scattered photons entering the detector, narrow-beam geometry and a relatively large scatterer-to-detector distance are required.

Regarding the Rayleigh-to-Compton technique for bone mineral content determination, the importance of Rayleigh scattering has been emphasized (Puumalainen *et al* 1976, Ling *et al* 1982).

The large number of detected pulses required makes it difficult to construct a practical triple-photon absorptiometer for the determination of bone mineral content in humans. Difficulties arise from the fast-pulse-counting devices required and in the selection of the photon source, since the measurement time should be kept to a minimum. Jonson *et al* (1986) have, however, solved these problems by using a technique where a complete x-ray spectrum is analysed.

The only way to improve the variance is to increase the number of transmitted photons, as shown in the present study. However, this is a practical or experimental task and the theoretical conditions for feasibility of triple-photon absorptiometry are independent of this.

The calculations of the variances for bone mineral, lean soft tissue and adipose tissue composition compared with Ca, P and bone-free mass simply illustrate an elemental difficulty which arises when changing the composition, and consequently the attenuation coefficients.

For the Ca and P measurement situation, bone-free mass is the dominating attenuator of the human body, and the amounts of calcium and phosphorus are very low. According to ICRP 23 the Ca content (of the total-body Ca), for soft and adipose tissue is 1.4% and 0.034%, respectively. The P content (of the total-body P), for soft and adipose tissue is 10.3 and 0.3%, respectively. In other words, only the extraskeletal phosphorus is appreciable compared with the amount of calcium and phosphorus in the skeleton. However, at appropriate measurement sites, such as the fingers, these percentages, particularly for P, are strongly reduced. The measured volumes are small and the Ca and P contents in soft tissues in the radiation beam will be negligible.

The calculated number of 10^{11} photons required for a precision of 5% (expressed as the coefficient of variation) in Ca and P measurements is a theoretical finding. Currently available detector systems cannot handle the photon count rates which are required to obtain 10^{11} photons during a reasonably short measurement time, at a stable level. The aim of the calculation of 10^{11} photons is to show the number of photons needed to obtain an adequate precision for Ca and P measurements *in vivo* using triple-photon absorptiometry.

Due to the high number of photons required in combination with the fact that the amounts of calcium and phosphorus in the skeleton are small in comparison with the total bone-free mass, it will be difficult to achieve accurate values of the amounts of calcium and phosphorus using triple-photon absorptiometry *in vivo*. This difficulty may be somewhat reduced if peripheral skeletal parts are chosen as measurement sites (fingers) where the proportion of bone-free mass is small.

This study has shown the difficulties expected in the determination of calcium and phosphorus *in vivo* using triple-photon absorptiometry. However, bone mineral content in combination with lean soft tissue and adipose tissue can be determined using triple-photon absorptiometry (Jonson *et al* 1986, 1988, Szücs *et al* 1993).

To summarize, the feasibility of triple-photon absorptiometry may be separated into a theoretical and an experimental part. It is theoretically possible to use triple-photon absorptiometry for the analysis of a three-component system. The experimental conditions, such as the similarities of the mass attenuation coefficients and the high number of detected photons necessary, set the limits for the effective use of triple-photon absorptiometry. These

experimental conditions have been discussed in our study and by, among others, Kotzki *et al* (1991), Tothill and Pye (1992) and Farrell and Webber (1990).

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