Proceedings of the Institute of Radio Engineers
Volume 26, Number 3 *March, 1938*

A THEORY OF NOISE FOR ELECTRON MULTIPLIERS*

By
W. Shockley and J. R. Pierce
(Bell Telephone Laboratories, Inc., New York City)

Summary—*The noise in secondary-emission electron multipliers is considered from a theoretical viewpoint. The noise properties of a stage are correlated with its secondary-emission properties: the mean value m and mean-square deviation* δ^2 *of the number of secondaries per primary. If* $\overline{I_{p\Delta f}^2}$ *and* $\overline{I_{s\Delta f}^2}$ *denote the mean-square noise current lying in the frequency band* Δf *in the primary- and secondary-electron currents, then* $\overline{I_{s\Delta f}^2} = m^2\ \overline{I_{p\Delta f}^2} + \delta^2 2e\overline{I_p}\Delta f$ *where* $\overline{I}_p$ *is primary direct current. This result is applied to many-stage multipliers. For n similar stages* $\overline{I_{s\Delta f}^2} = \overline{M^2 I^2_{p\Delta f}} + \delta^2[M(M-1)/m(m-1)]2e\,\overline{I_p}\Delta f$ *where* $M = m^n$ *is the over-all gain of the multiplier.*

The problem of noise in secondary-emission electron multipliers is essentially that of evaluating the fluctuation in the output current in terms of the fluctuation in the input current and in the manner in which the secondaries are produced, which may be ascribed to the properties of the secondary emitting surface or surfaces involved.

There are two kinds of electron multipliers, single-stage multipliers and multistage multipliers. Single-stage multipliers have only one multiplying surface. The electrons striking it form the input and the secondary electrons leaving it form the output. Multistage multipliers have several multiplying surfaces, arranged so that the output of the first surface forms the input of the second, and so on, the output of the last surface forming the output of the multiplier as a whole. In the treatment of the noise in multipliers it is desirable to obtain expressions for noise in multistage multipliers as well as single-stage multipliers.

Work previously published has not rigorously distinguished between output noise caused by noise in the input and that due to the manner in which secondary electrons are produced, which may be ascribed to the properties of the secondary emitting surfaces. Moreover, results have been borrowed with more or less justification from previous work on shot effect.

It is the purpose of this paper to derive expressions for noise in secondary-emission multipliers, of both the single-stage and the multistage varieties, using only the most elementary assumptions and borrowing methods only, and not results, from previous work in other fields.

* Decimal classification: R138. Original manuscript received by the Institute, July 8, 1937.

In discussing noise it is first necessary to define what we mean by noise, both in the electron stream which forms the input and in the output circuit of the multiplier.

It has been found most convenient to express all results in terms of currents, and to represent these currents expanded as a Fourier series over a period T which may be allowed to increase without limit without invalidating the final results.[1] The electron current arriving at the surface of a multiplier plate may thus be represented by

$$I_p = \sum_{n=-\infty}^{n=+\infty} a_n e^{in\omega t}, \qquad \omega = \frac{2\pi}{T} \cdot \tag{1}$$

This series will have a direct-current component or average value denoted by $\overline{I_p}$ equal to the constant term of the series, and an alternating-current component which we shall regard as noise.

The series may be equally well written in the more familiar but somewhat more awkward form

$$I_p = \sum_{0}^{\infty} 2 \mid a_n \mid \cos n(\omega t + \phi_n) . \tag{2}$$

It is seen that the amplitude of the component of frequency $n\omega$ is $2|a_n|$, and the mean-square value of this component is

$$\overline{I_{n\omega}{}^2} = 2 \mid a_n \mid^2 .$$

By adding such terms[2] we may obtain the total mean-square value of the noise-current components in the frequency range $\Delta\omega$ from $n_1\omega$ to $n_2\omega$, which we may speak of as that between ω_1 and ω_2. This noise current will be

$$\overline{I_{\Delta\omega}{}^2} = \sum_{n_1}^{n_2} 2 \mid a_n \mid^2 = \overline{2 \mid a_n \mid^2} T\Delta f \tag{3}$$

where $T\Delta f = n_2 - n_1$ and $\overline{|a_n|^2}$ is the average value of $|a_n|^2$ over the range.

It is of some importance to note that the current dealt with above is not a current flowing in a conducting circuit, but a current of electrons arriving at a multiplier plate, which is considered to consist of instantaneous pulses coinciding with the arrival of electrons at the surface of the plate. The same formal expressions can of course be used for currents in a circuit connecting plates between which electrons flow. This current flows only during the transit of electrons be-

[1] If desired, the phenomenon may be regarded as periodic over a period T.
[2] Such cross terms as $(4|a_{n_1}|\ |a_{n_2}|) \cos (\omega n_1 t + \phi_1) \cos (\omega n_2 + \phi_2)$ vanish in the time average.

tween the plates. It is shown in Appendix I that if the transit time is sufficiently small the expression obtained for the electron current leaving or arriving at one of two plates between which the electrons flow and the expression of the circuit current are identical.[3] Henceforward, such currents will not be distinguished except by special note.

In the Appendix the mean-square noise output current in frequency range Δf is evaluated for a generalized device, representing the multiplier, which has the following properties: 1. The device is linear in the sense that there is no interference between the effects of different input electrons. 2. Each electron entering the input has a probability $p(g)$ of producing g electrons at the output. 3. The output electrons appear simultaneously at a time t_0 after the input electron enters the device. ($p(g)$ and t_0 are the same for all input electrons.) The mean value of g, $\bar{g}$, represents the average multiplication of the device. The expression arrived at for the noise in the output secondary current is

$$\overline{I_{s\Delta f}{}^2} = \bar{g}^2\overline{I_{p\Delta f}{}^2} + 2e\overline{I_p}(\overline{g^2} - \bar{g}^2)\Delta f. \tag{4}$$

Here $\overline{I_p}$ is the average or direct-current component of the primary or input current, $\overline{g^2}$ is the mean-square value of g, the number of electrons leaving due to the arrival of an individual electron, and Δf is the width in cycles of the frequency band considered.

The expression is immediately applicable to single-stage multipliers, since according to our assumptions the arrival of an individual electron at a multiplier plate results in the emission of some number g of secondary electrons, which form the output. Here we shall interpret (4) in terms of the operation of a single-stage multiplier.

The expression shows a mean-square output noise current consisting of two numerically additive components. The first is the mean-square input noise current multiplied by the square of the mean multiplication. This is the component of output we would expect for any input amplified by the ratio $\bar{g} = \overline{I_s}/\overline{I_p}$.

The second component is a noise introduced because an individual primary electron may not produce exactly $\bar{g}$ secondaries, but 0, 1, 2, etc., up to a very large number. It can be seen that in case each primary did produce exactly $\bar{g}$ secondaries, $\overline{g^2}$ would be equal to $\bar{g}^2$ and the second term would be zero.

For a reasonable frequency range the expression is, as has been explained, valid for either the electron current leaving the multiplier

[3] This current is to be considered as applied to the electrodes by an external current generator.

plate or for the current in a circuit connecting an anode which collects the electrons with the multiplier plate. Assuming such a circuit had a constant resistance R for all frequencies, this second term would predict an infinite dissipation of power in the circuit due to noise current, since the power dissipation in any frequency band Δf wide is finite and the range of possible frequencies is infinite. This infinite power dissipation is avoided because of the impossibility of having a pure resistance and because (4) is valid for current in the circuit only when the transit time of an electron from plate to collector is small in comparison with $1/f$. An examination of the derivation in the Appendix will show that as the transit time is indefinitely increased, increased phase changes during transit can reduce the circuit current of a given frequency to the vanishing point although the corresponding electron current leaving the multiplier plate remains unchanged.

For convenience it seems desirable to rewrite (4) when applied to the input and output of an individual multiplier plate in terms of new quantities defined below:

$m = \bar{g}$, average number of secondaries produced by one primary

$\delta^2 = \overline{g^2} - \bar{g}^2$, mean-square deviation of this number

$b = \dfrac{\delta^2}{m^2}$, relative mean-square deviation.

Equation (4) then becomes

$$\overline{I_{s\Delta f}{}^2} = m^2\overline{I_{p\Delta f}{}^2} + 2e\overline{I_s}mb\Delta f \tag{5}$$

where $\overline{I_s} = m\overline{I_p}$ is the direct-current secondary current. We may here note that except for the factor (mb), the second term is just the mean-square noise current to be expected for shot effect in the current $\overline{I_s}$. Thus the plate of a multipler multiplies the input noise to it like a signal and adds to this a noise equal to mb times the shot noise corresponding to the output current.

As (5) is valid for any single multiplier stage whose noise input is defined, we may easily derive the expression for a multistage multiplier by considering that the secondary electron current from one multiplier plate forms the primary electron current of the next. Carrying this treatment out for an n-stage multiplier, we obtain

$$\overline{I_{n\Delta f}{}^2} = M^2\overline{I_{1\Delta f}{}^2} + 2e\overline{I_n}M\left(b_1 + \frac{b_2}{m_1} + \cdots + \frac{b_n}{m_1 m_2 \cdots m_{n-1}}\right)\Delta f. \tag{6}$$

Here $\overline{I_n{}^2{}_{\Delta f}}$ is the noise current in the output of the nth stage, $\overline{I_1}$ is the average primary current, b_r and m_r refer to the rth stage, and $M = (m_1 m_2 \cdots m_n)$, the over-all multiplication.

The first term is, of course, merely the noise in the primary electron stream as amplified by the multiplier.

It is easy to see the genesis of each component of the second term. According to (5), each stage introduces a noise proportional to the average current leaving that stage and to the product mb for that stage. This noise will be multiplied by the subsequent action of the multiplier.

The rth component in term 2 of (6) multiplied by the common factor, is

$$2e\overline{I_n}M\frac{b_r}{m_1m_2\cdots m_{r-1}}\Delta f.$$

This may be rewritten

$$[2e\overline{I_1}m_1m_2\cdots m_r\Delta f][m_rb_r][m_{r+1}\cdots m_n]^2.$$

The quantity in the first brackets is the shot noise corresponding to the output from the rth plate. $[m_rb_r]$ is the modifying factor discussed above. The quantity in the third brackets is the multiplication for all subsequent stages.

If m and b are the same for all stages, the result given in (8) may be summed to give

$$\overline{I_{n\Delta f}^2} = M^2\overline{I_{1\Delta f}^2} + 2e\overline{I_n}M\frac{\left(1-\dfrac{1}{M}\right)}{\left(1-\dfrac{1}{m}\right)}b\Delta f. \tag{7}$$

It is seen that for a given multiplication M, the most important quantity in this expression is b. As has been explained, if each primary electron produced exactly m secondaries, b would be equal to zero, and the multiplier would in itself introduce no noise, but would only amplify that present in the input.

We may rewrite (6) in terms of a relative mean-square deviation for the multiplier as a whole

$$\overline{I_{n\Delta f}^2} = M^2\overline{I_{1\Delta f}^2} + 2e\overline{I_1}M^2B\Delta f \tag{8}$$

where, according to our formula,

$$B = \left(b_1 + \frac{b_2}{m_1} + \frac{b_3}{m_1m_2} + \cdots + \frac{b_n}{m_1\cdots m_{n-1}}\right). \tag{9}$$

This gives an expression for multistage multipliers analogous to (5) for single-stage multipliers. In this form the result might have been obtained immediately, since in a multistage multiplier a primary electron in the input results in the practically simultaneous production of some number, g, of electrons in the output, and hence (4) is applicable. The only difficulty lies in arriving at the value of B, the relative mean-square deviation for the multiplier as a whole, in terms of the relative mean-square deviation b for a single stage. A direct evaluation of B in terms of its components is discussed in Appendix II.

Discussion of Previous Work and Experimental Results

Most writers treating noise in secondary-emission devices have assumed for b the value given by Poisson's formula. The assumptions underlying this formula are as follows: When an electron strikes the plate any number of electrons from zero to N (where N is very large) may be emitted. The probability that any particular one out of N be emitted is m/N. When N is allowed to go to infinity while m is held constant, the Poisson distribution is obtained. For this the probability that g electrons be emitted is

$$p(g) = \frac{m^g e^{-m}}{g!}\cdot$$

Using this distribution one easily finds that

$$\begin{aligned} \bar{g} &= m \\ \overline{g^2} &= m^2 + m \\ b &= 1/m \text{ or } bm = 1\,. \end{aligned}$$

If this distribution is assumed, and if the noise in the input is considered to be that of shot effect, as is the case in currents arising from photoelectric emission, and from thermionic emission in the absence of space charge, then we obtain

$$\overline{I_{p\Delta f}{}^2} = 2e\overline{I_p}\Delta f\,.$$

For this case (7) reduces to

$$\overline{I_{n\Delta}{}^2} = 2eM^2\overline{I_p}\Delta f + 2e\overline{I_n}M\left[\frac{1-\dfrac{1}{M}}{1-\dfrac{1}{m}}\right]\frac{1}{m}\,\Delta f = 2e\overline{I_n}\left(\frac{Mm-1}{m-1}\right)\Delta f. \quad (10)$$

This is just the expression obtained by Zworykin, Morton, and Malter[4] by assuming

[4] V. K. Zworykin, G. A. Morton, L. Malter, "The secondary emission multiplier—a new electronic device," Proc. I.R.E., vol. 24, pp. 351–375; March, (1936).

"1. Shot noise from an emitter is multiplied by the subsequent stages in the same way in which an ordinary signal is multiplied.

"2. Secondary emission from a target is subject to shot noise such that

$$i_n^2 = 2eI\Delta f."$$

Here I is the output current from the plate and Δf is the frequency band.

These assumptions amount to assuming the results of the present investigation, with a value of $b=1/m$. The experimental data obtained by these writers check the theory to within a few per cent; however, the test is not as rigorous as might appear from their tables. The total output noise they obtained by experiment and calculation is largely due to amplified shot effect in the primary current. Here the fact that bm differs from unity would affect this total noise little, although it would affect the noise introduced by the multiplier considerably. Table I shows a compilation of Zworykin, Morton and Malter's results and a calculation of bm from the data given. In their work

$$K'/K = M\left[\frac{1-\dfrac{1}{M}}{1-\dfrac{1}{m}}\,\frac{1}{m}+1\right].$$

Thus their K'/K is equal to our $M(B+1)$. Their computed value of MB, denoted by MB', thus lacks a factor mb which appears in the measured value MB. Hence $MB/MB'=mb$.

TABLE I

Values Computed from the Data of Zworykin, Morton, and Malter

No. of Stages	M Total Multiplication	K'/K Meas. $=M(1+B)$	K'/K Comp. $=M(1+B')$	MB	MB'	$\dfrac{MB}{MB'}=bm$	m*
3	60	77	80	17	20	0.85	3.91
3	28	40	41	12	13	0.92	3.04
3	6.8	12.1	12.3	5.3	5.5	0.96	1.90
2	29.5	36.2	36.0	6.7	6.5	1.03	5.45
1	6	7.2	7.0	1.2	1.0	1.20	6.0

* In the absence of contradictory information, we assume m is the same for all stages.

It is seen that the critical quantity affecting noise introduced by multiplication bm which should be unity according to Zworykin, Morton, and Malter's assumptions, differs from unity by as much as 20 per cent.

M. Ziegler has attacked the problem of secondary-emission noise both theoretically and experimentally.[5] His theory agrees with ours but he considers only the case of shot effect in the primary current and one stage. His values of mb differ greatly from unity. Table II shows values in the present notation computed from Ziegler's data, the secondary emitting surface consisting of activated BaO and SrO.

TABLE II
VALUES COMPUTED FROM ZIEGLER'S DATA

Volts	50	100	200	300	400	500	600	700	800
m	1	1.6	2.6	3.4	3.9	4.3	4.6	4.9	5.2
b	0.88	0.69	0.59	0.60	0.64	0.72	0.82	0.92	1.0
$1/m$	1.	0.62	0.38	0.29	0.26	0.23	0.22	0.20	0.19
bm	0.88	1.1	1.5	2.0	2.5	3.1	3.8	4.5	5.2

It is seen that at low voltages the value of b is nearly $1/m$, as it is for a Poisson distribution but that for higher voltages the values deviate, showing a considerably greater spread than would be predicted on the basis of a Poisson distribution.

Theoretical work has also been carried out by Campbell.[6] His results are essentially in agreement with Ziegler's.

Experiments have been carried out by L. J. Hayner and B. Kurrelmeyer.[7] Their results show definitely that mb is not equal to unity. They are, however, able to correlate a large amount of data by assuming that the primary electrons may be divided into three fractions, x, y, and z. The fraction x is taken to represent reflected electrons, y "buried electrons" which produce no secondaries, z electrons which produce true secondary electrons—not reflected. For the fraction z they assume that $m_z b_z = 1$. These assumptions enable them to calculate the value of m_z from noise measurements of secondary emission and they have investigated the effect of primary voltage upon this basis.

CONCLUDING REMARKS

There are, of course, limitations in the analysis as presented. There are two effects which have been neglected and should be mentioned. First, it is evident that electrons of different energies are not equivalent and an electron of higher energy produces more secondaries on the average than one of low energy. If the distribution in energy of electrons emitted by a given plate is independent of the number emitted, this spread of energy can be combined with the properties of

[5] M. Ziegler, *Physica*, vol. 1, p. 1; January, (1936), and vol. 3, p. 307; May, (1936). For comparison with Ziegler's notation we have $J = m$, $n\delta = M^2\ (b+1)$.

[6] N. R. Campbell, *Proc. Camb. Phil. Soc.*, vol. 15, pp. 117, 310, and 513; October, (1909–1910); *Phil. Mag.*, vol. 50, p. 81, July, (1925).

[7] *Physics*, vol. 6, p. 323; (1935); *Phys. Rev.*, vol. 52, p. 952; November, (1937). The writers are indebted to Professor Kurrelmeyer for several valuable discussions upon secondary emission.

the next plate and the whole analysis carried through as before. Under these conditions the quantities b and m for a plate are to be determined when electrons of varying energy strike it. If, however, the average energy per electron is low when a large number are emitted and high when a few are emitted, then essential modification will be needed. It seems unlikely that corrections along these lines will be necessary; the energies of the secondaries will probably be small compared to the voltage between stages. For such conditions, all secondaries can be regarded as having the same energy.

Second, we have not considered the possibility of electrons missing a stage entirely and perhaps striking a later stage with several times normal energy. This can quite probably be worked out along the lines used in Appendix II but since there appeared to be no immediate need for the results, it has not been attacked.

Appendix I

Our problem is to expand the output current from the device described in the text in terms of its characteristics and the input current. Suppose that N electrons arrive at the device during the time interval T to constitute an input current I_p. Let the rth of these arrive at instant t_r and cause the simultaneous appearance of g_r electrons at the output. (Transit time down the multiplier will be considered later.) These may be considered as being collected by an anode which is connected to the device by an external circuit. Thus an electron current flows to the device, an electron current flows from the device to the anode, and a current flows in the circuit connecting the device and the anode. Either of these latter two currents may be in some cases considered as the output current of the multiplier.

Let us denote the pulse of output current due to the rth primary electron as I_r, and expand it in the form of a Fourier series, over a period T. (The final results are independent of the length of T as long as T is large compared with the reciprocal of the lowest frequency of interest.)

$$I_r = \sum_{n=-\infty}^{+\infty} b_{rn} e^{in\omega t}, \qquad \omega = 2\pi/T.$$

Then

$$b_{rn} = \frac{1}{T}\int_0^T I_r e^{-in\omega t} dt. \tag{A1}$$

Either of two cases may be considered.

If we are interested in the electron current leaving the device or arriving at the anode, we know that this is zero except for a pulse at the instant of departure or arrival of the electrons. Thus we may consider the exponential factor as constant during the integration and take it outside the integral.

If we are interested in the circuit current, we know that this flows only during the transit of the electron from the device to the anode. If the transit angle of the electron is large compared to π, the exponential factor will so alternate during the transit time as greatly to reduce the integral. However, for audio noise frequencies at least the transit angle is very small and we may regard the exponential as constant during the period the current is flowing, and take it out of the integral.

Thus, ordinarily, for either the case of the electron current or the circuit current we may say

$$b_{rn} = \frac{e^{-in\omega t_r}}{T} \int I_r dt = \frac{e g_r}{T} e^{-in\omega t_r}.$$

(over transit time)

The total current I due to all the electrons is given by[8]

$$I = \sum_{n=-\infty}^{+\infty} b_n e^{in\omega t} = \sum_{n=-\infty}^{+\infty} \sum_{r=1}^{N} r\, b_{rn} e^{in\omega t}. \tag{A2}$$

The mean-square noise current in Δf is given by (3)

$$\overline{I_{s\Delta f}}^2 = 2\overline{|b_n|^2} T\Delta f. \tag{A3}$$

In this

$$\begin{aligned} |b_n|^2 &= \frac{e^2}{T^2}\left[\sum_{r=1}^{N} g_r e^{-in\omega t_r}\right]\left[\sum_{s=1}^{N} g_s e^{+in\omega t_s}\right] \\ &= \frac{e^2}{T^2}\left[\sum_{r=1}^{N} g_r^2 + \sum_{r=1}^{N}\sum_{s=1}^{N} g_r g_2 e^{in\omega(t_s - t_r)}\right]. \end{aligned} \tag{A4}$$

with $r \neq s$

We are interested in the value of $|b_n|^2$ which corresponds to typical

[8] Some doubts as to the rigor of dealing with expressions like (A2) and (A3) may reasonably arise at this point. Equation (A3) is obviously in a form designed for calculating power measured by some device due to fluctuations in the electron stream and it is somewhat unphysical to make power calculations upon the electron stream without reference to the measuring device. However it may be shown by a rigorous but rather tedious analysis that if the current defined by (A2) is applied to the calculation of power in any practical circuit and the averaging over the allowed transit instants and multiplications carried out correctly the result will be the same as would be obtained by calculating with the average value of (A3).

This result, that expressions like (A3) can be used for calculations of noise power, has been arrived at previously for the case of shot noise ($I^2\Delta f = 2eI\Delta f$) by T. C. Fry, *Jour. Frank. Inst.*, vol. 199, p. 203; February, (1925), who uses a very different method of attack.

The authors are indebted to Dr. Fry for showing them that the results of their analysis can be equally well obtained by using his method.

multiplication and transit instants. Hence we should average over allowed multiplications and transit instants. Consider the g_r first. Let the probability of obtaining value x for g_r be $p(x)$. Then the average of g_r is $\Sigma x\ p(x) \equiv \bar{g}$. The average of $g_r{}^2 = \Sigma x^2\ p(x) = \overline{g^2}$. Every electron has the same values for these two quantities. Hence averaging over the multiplications we get

$$\begin{aligned} |\, b_n\,|^2 &= \frac{e^2}{T^2}\left[N\overline{g^2} + \bar{g}^2 \sum_{\substack{r=1\\ r\neq s}}^{N} \sum_{s=1}^{N} e^{in\omega(t_s - t_r)} \right] \\ &= \frac{e^2}{T^2}\left[N(\overline{g^2} - \bar{g}^2) + \bar{g}^2 \sum_{r=1}^{N} \sum_{s=1}^{N} e^{in\omega(t_s - t_r)} \right]. \end{aligned} \tag{A5}$$

Before examining this further, consider the noise in the primary current. It is determined exactly as are the b_n's save with $g_r = 1$. From (A4) we get

$$|\, a_n\,|^2 = \frac{e^2}{T^2}\left[\sum_{r=1}^{N} \sum_{s=1}^{N} e^{in\omega(t_s - t_r)} \right]. \tag{A6}$$

The second term of (A5) is merely $\overline{g^2}$ times this.

Hence the mean-square noise current in the secondaries is

$$\begin{aligned} \overline{I_{s\Delta f}{}^2} &= 2\,|\, b_n\,|^2 T\Delta f = 2\left(\frac{Ne}{T}\right) e(\overline{g^2} - \bar{g}^2)\Delta f + \bar{g}^2 2\,|\, a_n\,|^2 T\Delta f \\ &= 2e\overline{I_p}(\overline{g^2} - \bar{g}^2)\Delta f + \bar{g}^2 \overline{I_{p\Delta f}{}^2}. \end{aligned} \tag{A7}$$

We can check our method by setting $g = 1$ so that the multiplier has no effect at all on the current and then averaging over the t's independently. This corresponds to the case of random emission such as occurs in the shot effect. Averaging over the t's in (A6) we see that only the terms with $r = s$ contribute anything, and $\overline{|a_n|^2}$ becomes $e^2N/T^2 = e\ \overline{I_p}/T$. Inserting this in (A7) we obtain $\overline{I_{s\Delta f}{}^2} = 2e\overline{I_{p\Delta} f}$, the well-known equation of the shot effect.

It is to be noted that for (A7) we have needed to make no assumptions about the precise nature of the primary noise. No matter what form it takes it appears multiplied by $\bar{g}^2$ in the output noise. Also the first term, noise due to multiplication, is independent of the nature of the input noise and depends only on direct input current and the multiplying process.

The formula is thus applicable to a multiplier as a whole or to any individual stage or group of stages.

We should also point out that the formula is applicable even if the transit time down a whole multiplier is large provided that it is the same for all electrons. Under these circumstances we shall have to distinguish between two transit instants, t_r the transit instant of primary, and $t_r' = t_r + t_0$, the transit instant of the secondaries in the output circuit. The addition term t_0 merely changes the phase of b_n without altering its magnitude and does not affect the value of the noise current. It must, however, remain true that the transit angle between the plates in the input or output circuits be small.

Appendix II

We shall show here that the formula for the over-all properties M and B can be obtained by statistics rather than by noise calculations. Suppose the multiplier consists of n plates which have values m_1, δ_1^2, $b_1; m_2, \delta_2^2, b_2; \cdots; m_n, \delta_n^2, b_n$, for their characteristics. We wish to find the mean value $\bar{g}$, and mean-square value $\overline{g^2}$ of g, the number of secondaries leaving the nth plate due to the incidence of one electron on the first plate. Comparing (4) and (8) of the text we see that $M = \bar{g}$ and $B = (\overline{g^2} - \bar{g}^2)/M^2$.

Suppose one electron strikes the first plate. Then x_1 secondaries will be produced, where according to the definitions of m_1 and δ_1^2, we have

$$\overline{x_1} = m_1$$
$$\Delta_1^2 \equiv \overline{x_1^2} - \overline{x_1}^2 = \delta_1^2.$$

Let us denote the multiplication of plate 2 by y_2 when there are x_1 incident electrons, the number of secondaries being $x_1 y_2$. Then the values of $\overline{y_2}$ and $\overline{y_2^2} - \overline{y_2}^2$, which are m_2 and δ_2^2 for one incident electron, are (according to well-known statistical theorems) m_2 and δ_2/x_1, respectively. Hence, averaging first over y_2, since it depends on x_1, and then over x_1, we find

$$\begin{aligned}
\overline{x_2} &= \overline{x_1 y_2} = \overline{x_1 m_2} = m_1 m_2 \\
\Delta_2^2 &= \overline{(x_1 y_2)^2} - (m_1 m_2)^2 = \overline{(x_1^2 - m_1^2) m_2^2} + \overline{x_1^2 (y_2^2 - m_2^2)} \\
&= \overline{(x_1^2 - m_1^2) m_2^2} + \overline{x_1^2 \delta_2^2 / x_1} \\
&= \Delta_1^2 m_2^2 + \overline{x_1} \delta_2^2.
\end{aligned}$$

This result is obviously in the form of a recursion formula which can be applied successively up to the nth plate. Noting that by definition, $g = x_n$, we obtain

$$\begin{aligned}
M &= \bar{g} = \overline{x_n} = m_1 m_2 \cdots m_n \\
M^2 B &= \overline{g^2} - \bar{g}^2 = \Delta_n^2 = \delta_1^2 (m_2 m_3 \cdots m_n)^2 + m_1 \delta_2^2 (m_3 \cdots m_n)^2 \\
&\quad + m_1 m_2 \delta_3^2 (m_4 \cdots m_n^2) + \cdots + (m_1 \cdots m_{n-1}) \delta_n^2.
\end{aligned}$$

Recalling that $\delta^2/m^2 = b$, we see that these lead to (9).