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The influence of energy weighting on X-ray imaging quality

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Abstract

Recent developments in radiation imaging detectors offer a perspective towards energy sensitive X-ray pixel detectors. Such spectroscopic pixel detectors would provide energy information in addition to the spatial information. One way to use this additional energy information is to weight each photon by an energy dependent factor. Under stringent assumptions we can show how this energy weighting has to be performed to achieve maximum image quality. In order to visualise the impact of the weighting technique on the image quality we have simulated three radiologic cases with the EGS4-based Monte Carlo Roentgen Simulation ROSI (Nucl. Instr. and Meth. A 509 (2003) 151; www.pi4.physik.uni-erlangen.de/Giersch/ROSI).

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1. Introduction

A photon counting hybrid pixel detector is a very promising approach for X-ray imaging, as has been shown in several publications [1–3]. The technological progress will lead to even more sophisticated electronics within each pixel. One future possibility might be the integration of an analogue-to-digital converter within each pixel. This would allow not "only" to detect each single photon but also to measure its energy. The question arising hereby is what benefit can be achieved by such a spectroscopic pixel detector. One way to improve the image quality by using the energy information is to weight each photon by an energy dependent factor.

2. Definition of energy weighting

Let us consider a very basic X-ray imaging situation (see Fig. 1). We have a parallel photon flux with a spectrum $\Phi(E)$ and an object with two regions with different absorption coefficients.

If we read out the detector we will get an intensity curve in dependence of the pixel number looking typically like Fig. 2: The two different intensities correspond to the two regions with different absorption coefficients.

The signal covering the interesting information about the object is going to be defined as the

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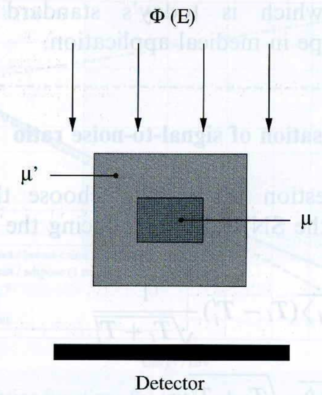

Fig. 1. A basic X-ray imaging situation.

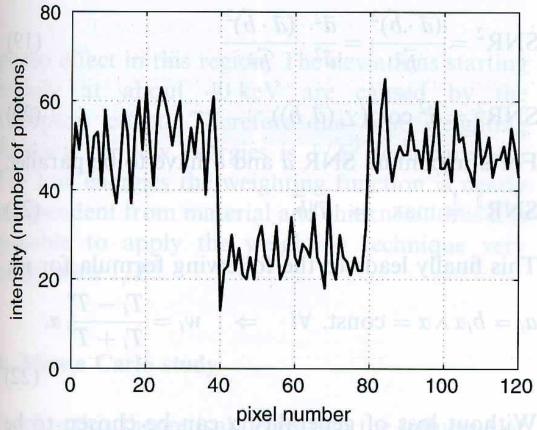

Fig. 2. Typical detector output in the region of interest for the imaging situation in Fig. 1.

difference of the expectation values of these intensities:

$$S = \langle I' \rangle - \langle I \rangle \tag{1}$$

The statistical noise can then found to be:

$$\sigma_S = \sqrt{\sigma_{I'}^2 + \sigma_I^2}. \tag{2}$$

If we assume an energy sensitive detector it is possible to observe the energy dependent absorption. In Fig. 3 the transmittance of PMMA and water, both 1 cm thick is shown. The material information as a function of energy is included in

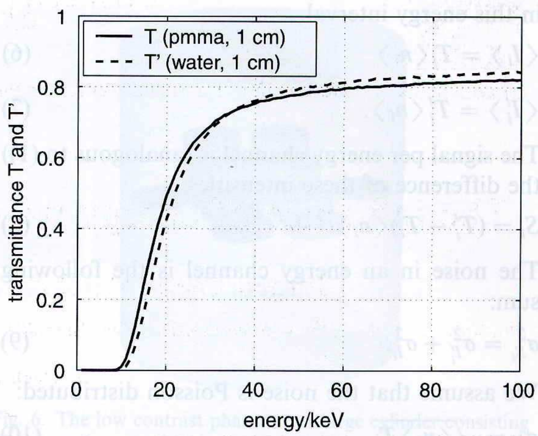

Fig. 3. Transmittances $T(E)$ of water and PMMA, each 1 cm thick.

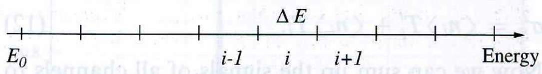

Fig. 4. Energy sampling inside the detector.

the difference between those curves and their characteristic progression.

The energy sensitive detector is assumed to have N energy channels with a bin size of ΔE (see Fig. 4) for every pixel. Therefore each detected photon increases a counter in the appropriate channel.

$$E = E_0 + i\Delta E. \tag{3}$$

The running index numbering the energy bins is i. In an energy bin we will obtain an expectation value of the number of photons n_i and the transmittance T_i for this pixel:

$$\langle n_i \rangle = \int_{E_0+i\Delta E}^{E_0+(i+1)\Delta E} \Phi(E')\,\mathrm{d}E' \tag{4}$$

$$T_i = \frac{\int_{E_0+i\Delta E}^{E_0+(i+1)\Delta E} \Phi(E')T(E')\,\mathrm{d}E'}{\langle n_i \rangle}. \tag{5}$$

With this assumption the detector provides an energy dependent intensity. The intensity per energy channel can then be found to be the product of the expectation value of the number of photons before the object and the transmittance in this energy interval.

$$\langle I_i \rangle = T_i \langle n_i \rangle \tag{6}$$

$$\langle I'_i \rangle = T'_i \langle n_i \rangle. \tag{7}$$

The signal per energy channel is (analogous to (1)) the difference of these intensities:

$$S_i = (T'_i - T_i)\langle n_i \rangle. \tag{8}$$

The noise in an energy channel is the following sum:

$$\sigma^2_{S_i} = \sigma^2_{I'_i} + \sigma^2_{I_i}. \tag{9}$$

We assume that the noise is Poisson distributed:

$$\sigma_{I_i} = \sqrt{\langle n_i \rangle T_i} \tag{10}$$

$$\sigma_{I'_i} = \sqrt{\langle n_i \rangle T'_i}. \tag{11}$$

Consequently we get the following expression:

$$\sigma^2_{S_i} = \langle n_i \rangle T'_i + \langle n_i \rangle T_i. \tag{12}$$

Now we can sum up the signals of all channels to get the total signal. Energy weighting means that we multiply the signal of each channel with an individual weighting factor w_i before summing:

$$\tilde{S} = \sum_i S_i w_i. \tag{13}$$

Those weighting factors take into account that some signals can be of higher importance for the image quality than others. If we construct the signal in this way we get the expression of the noise stringently by error propagation:

$$\sigma^2_{\tilde{S}} = \sum_i w_i^2 \sigma^2_{S_i}. \tag{14}$$

Finally we get the following term for the SNR, as one of the most important properties for the image quality:

$$\mathrm{SNR} = \frac{\tilde{S}}{\sigma_{\tilde{S}}} \tag{15}$$

$$\mathrm{SNR}^2 = \frac{(\sum_i \langle n_i \rangle (T_i - T'_i) w_i)^2}{\sum_i \langle n_i \rangle (T_i + T'_i) w_i^2}. \tag{16}$$

It should be mentioned that this expression also includes two special cases: If we choose w_i constant we get a counting detector and if w_i is proportional to the photon energy, we will get an integrating detector, which is today's standard imaging detector type in medical application.

3. Maximisation of signal-to-noise ratio

The question is, how to choose the w_i to maximise the SNR? By introducing the vectors $\vec{a}$ and $\vec{b}$:

$$a_i := \sqrt{\langle n_i \rangle}(T_i - T'_i)\frac{1}{\sqrt{T_i + T'_i}} \tag{17}$$

$$b_i := \sqrt{\langle n_i \rangle}\sqrt{T_i + T'_i}\, w_i \tag{18}$$

the expression for SNR can easily be transformed into a scalar product of these vectors:

$$\mathrm{SNR}^2 = \frac{(\vec{a}\cdot\vec{b})^2}{\vec{b}^2} = \frac{\vec{a}^2}{\vec{a}^2}\frac{(\vec{a}\cdot\vec{b})^2}{\vec{b}^2} \tag{19}$$

$$\mathrm{SNR}^2 = \vec{a}^2 \cos^2(\angle(\vec{a},\vec{b})). \tag{20}$$

For a maximum SNR $\vec{a}$ and $\vec{b}$ have to be parallel:

$$\mathrm{SNR}^2 \overset{!}{=} \max \Leftrightarrow \vec{a} \| \vec{b}. \tag{21}$$

This finally leads to the following formula for w_i:

$$a_i = b_i \alpha \wedge \alpha = \text{const.}\ \forall i \quad \Rightarrow \quad w_i = \frac{T_i - T'_i}{T_i + T'_i}\alpha. \tag{22}$$

Without loss of generality α can be chosen to be equal to 1. The weighting factor is the difference of the transmittances divided by the sum of the transmittances.

This means that in case of a small difference of high transmittances the weighting factor w_i will be small. This is intuitive, because we would get two similar but high intensities, which leads to a small but noisy signal. Thus w_i will be low, if the uncertainty of the signal is high.

In order to get an idea how the weighting function looks like, Fig. 5 shows the weighting factors for two cases: breast-tissue to breast-calcification and breast-tissue to adipose in the energy range from 10 to 100 keV.

Up to 40 keV the weighting function is approximately parallel to the straight line of $1/E^3$. This accordance comes from the domination of the

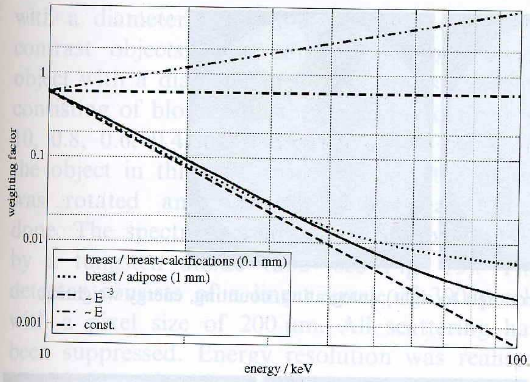

Fig. 5. Weighting functions for two material combinations and the plot for $1/E^3$. For comparison also the plot for an integrating detector ($\sim E$) and a counting detector (const) is shown.

Fig. 6. The low contrast phantom: A large cylinder consisting of breast tissue including five cylinders consisting of adipose and five cylinders consisting of water.

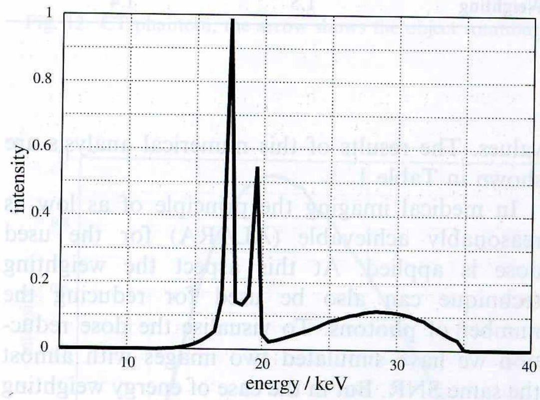

Fig. 7. Molybdenium spectrum, 35 kV, filtering: 1 mm Be, 30 μm Mo, 2 mm Al.

photo effect in this region. The deviations starting mainly at about 40 keV are caused by the Compton effect. Therefore the best weighting function for low energies is $1/E^3$ (compare [4]). For low energies the weighting function is nearly independent from material and thickness, thus it is possible to apply the weighting technique very easily with $1/E^3$.

4. Monte Carlo study

After this theoretical derivation the influence of this weighting technique on the image quality will be shown in three radiologic cases. For these calculations the simulation tool ROSI has been used [5,6]. ROSI is EGS4 based, object-oriented and capable of parallel computing. All material compositions have been taken from the ICRU report 46 [7]. The tube spectra have been generated with the "boone-algorithm" [8,9].

4.1. *Case 1: Low contrast*

In the first case we defined a cylinder consisting of breast tissue with a height of 30 mm including five adipose and five water cylinders with different thicknesses: 0.5, 1, 2, 4 and 6 mm (see Fig. 6). We have used a standard mammographic spectrum (see Fig. 7). The detector consists of 200 × 200 pixels with a pixel size of 150 μm. It is an ideal one with 100% quantum efficiency and 100% scatter suppression. The energy resolution was realised by 1 keV wide bins with no energy blurring between them.

After simulating 10^9 photons we got the results shown in Fig. 8. The first image is the integrating case and is defined to be the reference. For the counting image a slight enhancement can be seen, the weighting technique results in a clear enhancement.

Different regions of interest inside the images have been evaluated to obtain the mean and standard deviation for the calculation of the SNR

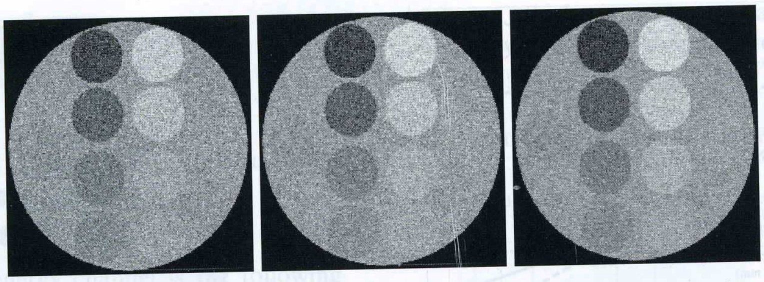

Fig. 8. Results from low contrast simulation done with ROSI (from left to right): integrating, counting, energy weighting.

Table 1
SNR enhancement for low contrast breast phantom

SNR enhancement	Water/breast	Adipose/breast
Integrating	1.0	1.0
Counting	1.2	1.2
Weighting	1.5	1.4

values. The results of this numerical analysis are shown in Table 1.

In medical imaging the principle of as low as reasonably achievable (ALARA) for the used dose is applied. At this aspect the weighting technique can also be used for reducing the number of photons. To visualise the dose reduction we have simulated two images with almost the same SNR. But in the case of energy weighting we need only a dose which is 2.5 times smaller (see Fig. 9)!

4.2. Case 2: Breast-calcification

The second investigated case concerns the problem of small objects with high contrast like calcifications in breast tissue. We defined a cylinder consisting of 30 mm thick breast tissue, including 35 breast calcifications with different thicknesses and diameters (see Fig. 10 and Table 2). The X-ray setup has been the same as in case 1.

After simulating 8×10^8 photons we got the images shown in Fig. 11. The SNR is enhanced by 1.1 in the counting case and 1.2 in the weighting case (see Table 3).

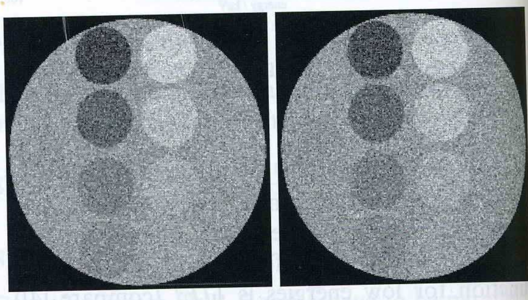

Fig. 9. Dose reduction: right image was simulated with a dose 2.5 times smaller.

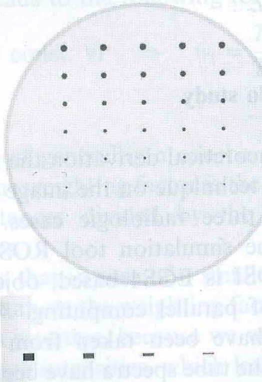

Fig. 10. Breast-calcification phantom.

4.3. Case 3: Computed tomography for mammography

Finally we investigated a computed tomography (CT) case (see Fig. 12). We defined a phantom with a diameter of 56 mm including eight low contrast objects: One adipose and one water object with a diameter of 20 mm and six objects consisting of blood with the following diameters: 10, 0.8, 0.6, 0.4, 0.2 mm. The X-rays penetrate the object in this case from the side, the object was rotated and 180 projections have been done. The spectrum was assumed to be emitted by a tungsten anode tube (see Fig. 13). The detector consists of a linear array of 300 pixels with a pixel size of 200 μm. All scattering has been suppressed. Energy resolution was realised by 3 keV wide bins with no energy blurring between them.

After simulating 10^9 photons we have done a Shepp–Logan reconstruction [10] based on the three sinograms (integrated, counted, weighted) and get the images and values shown in Fig. 14 and Table 4. In this case again the weighting technique results in a SNR enhancement.

Note the beam hardening artefact in the image based on the weighted data (reconstructed values are smaller in the central region of the image). This is due to the stronger weighting of lower photon energies which are more affected by this effect. Beam hardening is known and there exist already methods to suppress the artefacts. This was not investigated but will be done in future.

Table 2
SNR enhancement for breast calcifications

Thickness/mm	0.5, 0.35, 0.2, 0.1, 0.05
Diameter/mm	0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1, 0.05

Table 3
SNR enhancement for high contrast breast phantom

SNR enhancement	Breast/calcifications
Integrating	1.0
Counting	1.1
Weighting	1.2

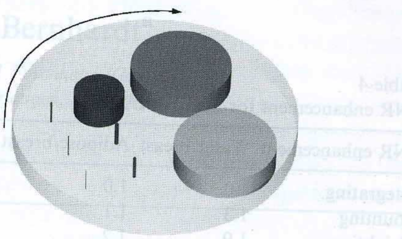

Fig. 12. CT phantom, the arrow shows the object rotations.

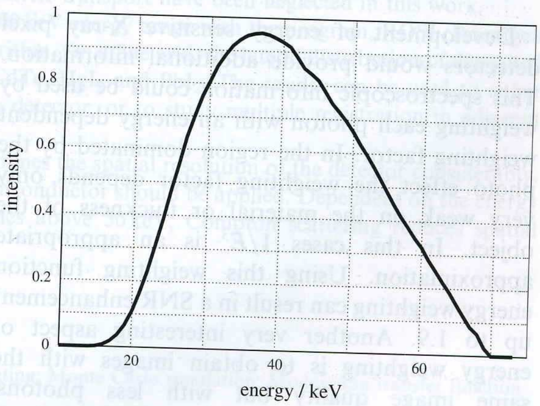

Fig. 13. Tungsten spectrum, 70 kV, filtering: 2 mm Al.

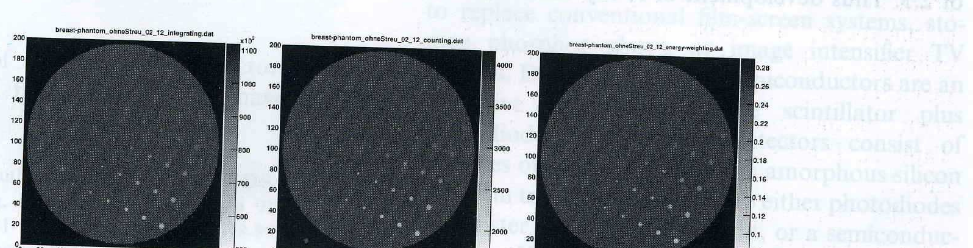

Fig. 11. Results from the breast-calcification simulation done with ROSI (from left to right): integrating, counting, energy weighting.

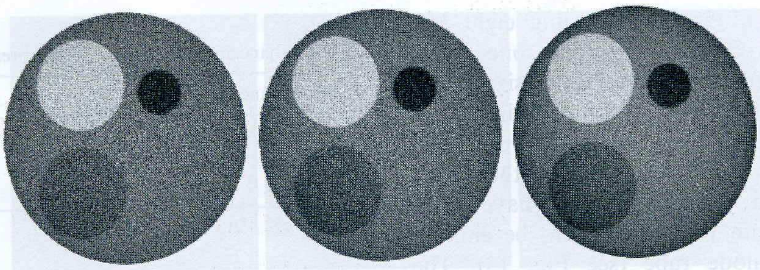

Fig. 14. Results from mamma CT simulation done with ROSI (from left to right): integrating, counting, energy weighting.

Table 4
SNR enhancement for mamma-CT

SNR enhancement	Water/breast	Adipose/breast	Blood/breast
Integrating	1.0	1.0	1.0
Counting	1.3	1.1	1.2
Weighting	1.9	1.2	1.4

5. Conclusion

Development of energy sensitive X-ray pixel detectors would provide additional information. This spectroscopic information could be used by weighting each photon with an energy dependent weighting factor. In the region dominated by the photo effect the weighting factor depends only very weak on the material or thickness of the object. In this cases $1/E^3$ is an appropriate approximation. Using this weighting function energy weighting can result in a SNR enhancement up to 1.9. Another very interesting aspect of energy weighting is to obtain images with the same image quality but with less photons. Energy weighting can reduce the dose by a factor of 2.5. Thus development of X-ray detectors with energy resolution is worth its effort and very promising.

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