Лабораторная работа 7

In[113]:=
```
f = 11 * x^2 - 2 * x * y - 2 * x * z + 2 * y * z + 9 * z^2 - 4 * x + y + z;
```

In[114]:=
```
TraditionalForm[f]
```

Out[114]//TraditionalForm=

$$11\,x^2 - 2\,x\,y - 2\,x\,z - 4\,x + 2\,y\,z + y + 9\,z^2 + z$$

In[115]:=
```
A = {
{11, -1, -1},
{-1, 0, 1},
{-1, 1, 9}
};
```

In[116]:=
```
MatrixForm[A]
```

Out[116]//MatrixForm=

$$\begin{pmatrix} 11 & -1 & -1 \\ -1 & 0 & 1 \\ -1 & 1 & 9 \end{pmatrix}$$

Составляем характеристическое уравнение

In[117]:=
```
l =.
AE = A - IdentityMatrix[3] * l;
MatrixForm[AE]
```

Out[119]//MatrixForm=

$$\begin{pmatrix} 11 - l & -1 & -1 \\ -1 & -l & 1 \\ -1 & 1 & 9 - l \end{pmatrix}$$

In[120]:=
```
myCharPoly = Det[AE]
```

Out[120]= $-18 - 96\,l + 20\,l^2 - l^3$

In[121]:=
```
wolframCharPoly = CharacteristicPolynomial[A, l]
```

Out[121]= $-18 - 96\,l + 20\,l^2 - l^3$

Автоматическая проверка на равенство характеристических уравнений

In[122]:=
```
FullSimplify[myCharPoly == wolframCharPoly]
```

Out[122]= True

Ищем собственные значения

In[123]:=
```
sol = Solve[myCharPoly == 0, l];
```

In[124]:=
```
myEigenVals = l /. sol;
```

In[125]:=
```
% // N
```

Out[125]= {-0.18064, 8.61753, 11.5631}

Автоматическая проверка на равенство собственных значений

In[126]:=
```
wolframEigenVals = Eigenvalues[A];
Sort[wolframEigenVals] == Sort[myEigenVals]
```

Out[127]= True

In[128]:=
```
X = {x, y, z};
one = AE /. l → myEigenVals[[1]];
two = AE /. l → myEigenVals[[2]];
three = AE /. l → myEigenVals[[3]];
myOne = one.X
myTwo = two.X
myThree = three.X
```

Out[132]= $\{-y - z + x(11 - -0.181\ldots), -x + z - y\,(-0.181\ldots), -x + y + z(9 - -0.181\ldots)\}$

Out[133]= $\{-y - z + x(11 - 8.62\ldots), -x + z - y\,(8.62\ldots), -x + y + z(9 - 8.62\ldots)\}$

Out[134]= $\{-y - z + x(11 - 11.6\ldots), -x + z - y\,(11.6\ldots), -x + y + z(9 - 11.6\ldots)\}$

Ищем собственные вектора

In[135]:=
```
myEigenVec1 = Solve[myOne == 0 /. z → 1];
myEigenVec2 = Solve[myTwo == 0 /. z → 1];
myEigenVec3 = Solve[myThree == 0 /. z → 1];
myEigenVec1 = {x, y, 1} /. myEigenVec1[[1]]
myEigenVec2 = {x, y, 1} /. myEigenVec2[[1]]
myEigenVec3 = {x, y, 1} /. myEigenVec3[[1]]
```

Out[138]= $$\left\{-\frac{-1 - 9\,(-0.181\ldots) + (-0.181\ldots)^2}{1 + -0.181\ldots}, -\frac{8 - -0.181\ldots}{1 + -0.181\ldots}, 1\right\}$$

Out[139]= $$\left\{-\frac{-1 - 9\,(8.62\ldots) + (8.62\ldots)^2}{1 + 8.62\ldots}, -\frac{8 - 8.62\ldots}{1 + 8.62\ldots}, 1\right\}$$

Out[140]= $$\left\{-\frac{-1 - 9\,(11.6\ldots) + (11.6\ldots)^2}{1 + 11.6\ldots}, -\frac{8 - 11.6\ldots}{1 + 11.6\ldots}, 1\right\}$$

In[141]:=
```
wolframEigenSys = Eigensystem[A]
```

Out[141]= $\{\{11.6\ldots, 8.62\ldots, -0.181\ldots\}, \{\{-2.28\ldots, 0.284\ldots, 1\}, \{0.447\ldots, 0.0642\ldots, 1\}, \{-0.804\ldots, -9.98\ldots, 1\}\}\}$

In[142]:=
```
myEigenVals [[1]]
myEigenVals [[2]]
myEigenVals [[3]]
```

Out[142]= -0.181 …

Out[143]= 8.62 …

Out[144]= 11.6 …

Автоматическая проверка на равенство собственных векторов

In[145]:=
```
wolframEigenSys [[2, 3]] == N[myEigenVec1 ]
wolframEigenSys [[2, 2]] == N[myEigenVec2 ]
wolframEigenSys [[2, 1]] == N[myEigenVec3 ]
```

Out[145]= True

Out[146]= True

Out[147]= True

Составляем матрицу из нормированных собственных векторов

In[148]:=
```
S = {
Normalize [myEigenVec1 ],
Normalize [myEigenVec2 ],
Normalize [myEigenVec3 ]
};
N[S] // MatrixForm
```

Out[149]//MatrixForm=

$$\begin{pmatrix} -0.0798259 & -0.991846 & 0.0993417 \\ 0.407145 & 0.0585251 & 0.911487 \\ -0.909869 & 0.113207 & 0.399153 \end{pmatrix}$$

Составляем каноническое уравнение

In[172]:=
```
(*fk1*)
a = {-2, 0.5, 0.5};
a1 = S.a;
N[a1] // MatrixForm
fk1 = myEigenVals[[1]] * x1^2 + myEigenVals[[2]] * y1^2 +
   myEigenVals[[3]] * z1^2 + 2 * a1[[1]] * x1 + 2 * a1[[2]] * y1 + 2 * a1[[3]] * z1;
fk1 = FullSimplify[fk1];
fk1 /. {x1 → x, y1 → y, z1 → z} // TraditionalForm
```

Out[174]//MatrixForm=

$$\begin{pmatrix} -0.286601 \\ -0.329284 \\ 2.07592 \end{pmatrix}$$

Out[177]//TraditionalForm=

$$(-0.18064\ x - 0.573201)\,x + y\,(8.61753\ y - 0.658567) + z\,(11.5631\ z + 4.15183)$$

In[178]:=
```
(* Дополняем члены до полного квадрата - fk2 *)
a0 = -myEigenVals[[1]] * (a1[[1]] / myEigenVals[[1]])^2 - myEigenVals[[2]] *
    (a1[[2]] / myEigenVals[[2]])^2 - myEigenVals[[3]] * (a[[3]] / myEigenVals[[3]])^2;
fk2 = myEigenVals[[1]] * (x1 + a1[[1]] / myEigenVals[[1]])^2 +
  myEigenVals[[2]] * (y1 + a1[[2]] / myEigenVals[[2]])^2 +
  myEigenVals[[3]] * (z1 + a1[[3]] / myEigenVals[[3]])^2 + a0
TraditionalForm[
 fk2]
```

Out[179]=

$$0.420512 + (1.58658 + x1)^2\ \boxed{-0.181\ldots} + (-0.0382109 + y1)^2\ \boxed{8.62\ldots} + (0.179529 + z1)^2\ \boxed{11.6\ldots}$$

Out[180]//TraditionalForm=

$$(x1 + 1.58658)^2\ \boxed{-0.181\ldots} + (z1 + 0.179529)^2\ \boxed{11.6\ldots} + (y1 - 0.0382109)^2\ \boxed{8.62\ldots} + 0.420512$$

Замена переменных:

x2 = x1 + 1.58658

y2 = y1 - 0.0382109

z2 = z1 + 0.179529

In[181]:=
```
(* fk3 *)
fk3 = myEigenVals[[1]] * x^2 + myEigenVals[[2]] * y^2 + myEigenVals[[3]] * z^2 + a0;
TraditionalForm[fk3 == 0]
```

Out[182]//TraditionalForm=

$$x^2\ \boxed{-0.181\ldots} + z^2\ \boxed{11.6\ldots} + y^2\ \boxed{8.62\ldots} + 0.420512 = 0$$

In[161]:=
```
(* fk4 *)
fk4 = myEigenVals[[1]] / a0 * x^2 +
   myEigenVals[[2]] / a0 * y^2 + myEigenVals[[3]] / a0 * z^2 + 1;
TraditionalForm[fk4 == 0]
```

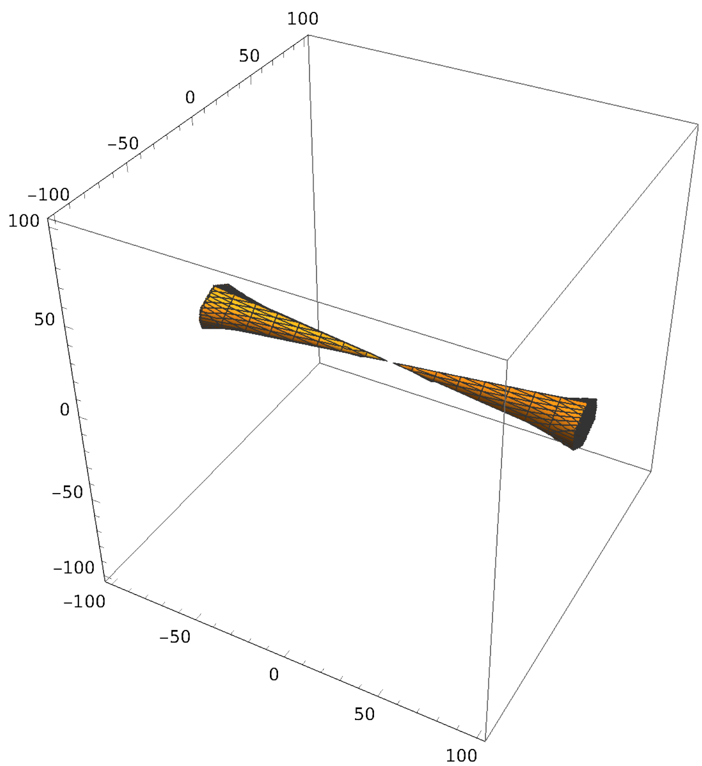
Out[162]//TraditionalForm=

$$-0.429573\ x^2 + 20.4929\ y^2 + 27.4977\ z^2 + 1 = 0$$

Полученная фигура

In[187]:=
```
x =.; y =.; z =.;
value = 100;
fk[a_, b_, c_] := fk4 /. {x2 → a, y2 → b, z2 → c}
ContourPlot3D[fk[x, y, z] == 0,
 {x, -value, value}, {y, -value, value}, {z, -value, value}]
```

Out[190]=



Исходная фигура

In[167]:=
```
fnk[a_, b_, c_] := f
ContourPlot3D[fnk[x, y, z] == 0,
 {x, -value, value}, {y, -value, value}, {z, -value, value}]
```

Out[168]=