Лабораторная работа 7

In[690]:=
```
f = 11*x^2 - 2*x*y - 2*x*z + 2*y*z + 9*z^2 - 4*x + y + z;
```

In[691]:=
```
TraditionalForm[f]
```

Out[691]//TraditionalForm=

$$11 x^2 - 2 x y - 2 x z - 4 x + 2 y z + y + 9 z^2 + z$$

In[692]:=
```
A = {
{11, -1, -1},
{-1, 0, 1},
{-1, 1, 9}
};
```

In[693]:=
```
MatrixForm[A]
```

Out[693]//MatrixForm=

$$\begin{pmatrix} 11 & -1 & -1 \\ -1 & 0 & 1 \\ -1 & 1 & 9 \end{pmatrix}$$

Составляем характеристическое уравнение

In[694]:=
```
l =.
AE = A - IdentityMatrix[3] * l;
MatrixForm[AE]
```

Out[696]//MatrixForm=

$$\begin{pmatrix} 11 - l & -1 & -1 \\ -1 & -l & 1 \\ -1 & 1 & 9 - l \end{pmatrix}$$

In[697]:=
```
myCharPoly = Det[AE]
```

Out[697]= $-18 - 96 l + 20 l^2 - l^3$

In[698]:=
```
wolframCharPoly = CharacteristicPolynomial[A, l]
```

Out[698]= $-18 - 96 l + 20 l^2 - l^3$

Автоматическая проверка на равенство характеристических уравнений

In[699]:=
```
FullSimplify[myCharPoly == wolframCharPoly]
```

Out[699]= True

Ищем собственные значения

In[700]:=
```
sol = Solve[myCharPoly == 0, l];
```

In[701]:=
```
myEigenVals = l /. sol;
```

In[702]:=
```
% // N
```

Out[702]= {-0.18064, 8.61753, 11.5631}

Автоматическая проверка на равенство собственных значений

In[703]:=
```
wolframEigenVals = Eigenvalues[A];
Sort[wolframEigenVals] == Sort[myEigenVals]
```

Out[704]= True

In[705]:=
```
X = {x, y, z};
one = AE /. l → myEigenVals[[1]];
two = AE /. l → myEigenVals[[2]];
three = AE /. l → myEigenVals[[3]];
myOne = one.X
myTwo = two.X
myThree = three.X
```

Out[709]= $\{-y - z + x(11 - -0.181\ldots), -x + z - y\,(-0.181\ldots), -x + y + z(9 - -0.181\ldots)\}$

Out[710]= $\{-y - z + x(11 - 8.62\ldots), -x + z - y\,8.62\ldots, -x + y + z(9 - 8.62\ldots)\}$

Out[711]= $\{-y - z + x(11 - 11.6\ldots), -x + z - y\,11.6\ldots, -x + y + z(9 - 11.6\ldots)\}$

Ищем собственные вектора

In[712]:=
```
myEigenVec1 = Solve[myOne == 0 /. z → 1];
myEigenVec2 = Solve[myTwo == 0 /. z → 1];
myEigenVec3 = Solve[myThree == 0 /. z → 1];
myEigenVec1 = {x, y, 1} /. myEigenVec1[[1]]
myEigenVec2 = {x, y, 1} /. myEigenVec2[[1]]
myEigenVec3 = {x, y, 1} /. myEigenVec3[[1]]
```

Out[715]= $$\left\{-\frac{-1 - 9\,(-0.181\ldots) + (-0.181\ldots)^2}{1 + (-0.181\ldots)}, -\frac{8 - (-0.181\ldots)}{1 + (-0.181\ldots)}, 1\right\}$$

Out[716]= $$\left\{-\frac{-1 - 9\,(8.62\ldots) + (8.62\ldots)^2}{1 + 8.62\ldots}, -\frac{8 - 8.62\ldots}{1 + 8.62\ldots}, 1\right\}$$

Out[717]= $$\left\{-\frac{-1 - 9\,(11.6\ldots) + (11.6\ldots)^2}{1 + 11.6\ldots}, -\frac{8 - 11.6\ldots}{1 + 11.6\ldots}, 1\right\}$$

In[718]:=
```
wolframEigenSys = Eigensystem[A]
```

Out[718]= $\{\{11.6\ldots, 8.62\ldots, -0.181\ldots\}, \{\{-2.28\ldots, 0.284\ldots, 1\}, \{0.447\ldots, 0.0642\ldots, 1\}, \{-0.804\ldots, -9.98\ldots, 1\}\}\}$

In[719]:=
```
myEigenVals [[1]]
myEigenVals [[2]]
myEigenVals [[3]]
```

Out[719]= -0.181 …

Out[720]= 8.62 …

Out[721]= 11.6 …

Автоматическая проверка на равенство собственных векторов

In[722]:=
```
wolframEigenSys [[2, 3]] == N[myEigenVec1 ]
wolframEigenSys [[2, 2]] == N[myEigenVec2 ]
wolframEigenSys [[2, 1]] == N[myEigenVec3 ]
```

Out[722]= True

Out[723]= True

Out[724]= True

Составляем матрицу из нормированных собственных векторов

In[725]:=
```
S = {
Normalize [myEigenVec1 ],
Normalize [myEigenVec2 ],
Normalize [myEigenVec3 ]
};
N[S] // MatrixForm
```

Out[726]//MatrixForm=

$$\begin{pmatrix} -0.0798259 & -0.991846 & 0.0993417 \\ 0.407145 & 0.0585251 & 0.911487 \\ -0.909869 & 0.113207 & 0.399153 \end{pmatrix}$$

Составляем каноническое уравнение

In[727]:=
```
a = {1, 1, 1};
a1 = Transpose[S].a;
N[a1] // MatrixForm
p = myEigenVals[[1]] * x1^2 + myEigenVals[[2]] * y1^2 +
   myEigenVals[[3]] * z^2 + 2 * a1[[1]] * x1 + 2 * a1[[2]] * y1 + 2 * a1[[3]] * z1 - 10;
p = FullSimplify[p];
N[p] // TraditionalForm
p /. {x1 → x, y1 → y, z1 → z}
```

Out[729]//MatrixForm=

$$\begin{pmatrix} -0.58255 \\ -0.820114 \\ 1.40998 \end{pmatrix}$$

Out[732]//TraditionalForm=

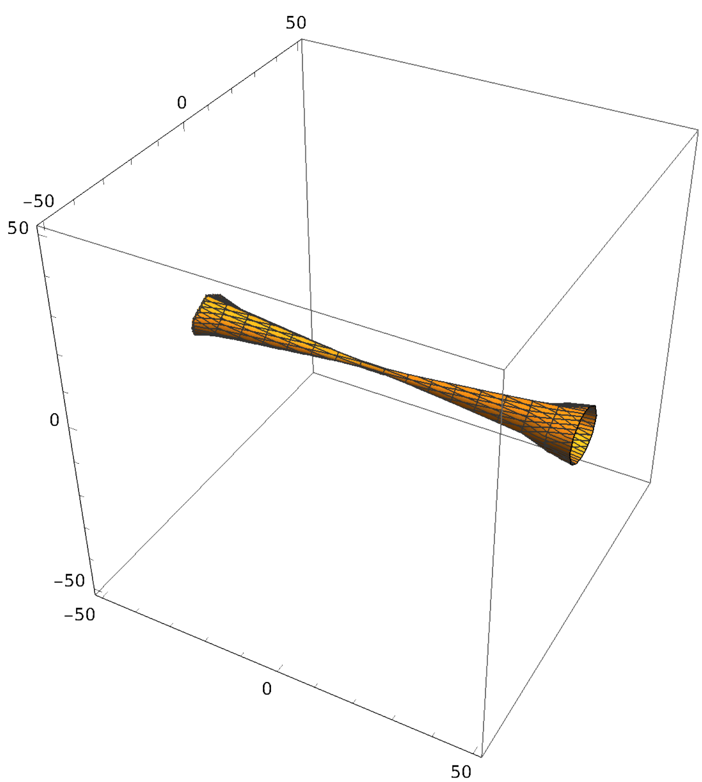
$$8.61753\,(\mathrm{y1}^2 - 1.\,\mathrm{x1}^2) + 11.5631\,(z^2 - 1.\,\mathrm{x1}^2) + 20.\,\mathrm{x1}^2 - 1.1651\,\mathrm{x1} - 1.64023\,\mathrm{y1} + 2.81996\,\mathrm{z1} - 10.$$

Out[733]= $-10 + 20\,x^2 + (-x^2 + y^2)$ 8.62 … $+ (-x^2 + z^2)$ 11.6 … + x -1.17 … + z 2.82 … + y -1.64 …

Полученная фигура

In[744]:=
```
x =.; y =.; z =.;
fk[a_, b_, c_] := p /. {x1 → a, y1 → b, z1 → c}
ContourPlot3D[fk[x, y, z] == 0, {x, -50, 50}, {y, -50, 50}, {z, -50, 50}]
```

Out[746]=



Исходная фигура

In[739]:=

```
fnk[a_, b_, c_] := f
ContourPlot3D[fnk[x, y, z] == 0, {x, -50, 50}, {y, -50, 50}, {z, -50, 50}]
```

Out[740]=