Lens and Human Eye Models: Experiment Analysis

Introduction

This document contains the detailed analysis and mathematical derivations for a series of optical experiments involving lens systems and human eye models. The experiments range from simulating a magnifying glass to modeling myopic eyes with corrective glasses.

Experiment 1: Magnifying Glass Simulation

Objective: To simulate a magnifying glass and determine the virtual image properties when an object is placed within the focal length.

System Components:

- Convex Lens with focal length (f): 10 cm
- Object distance (d_o): 5 cm

Mathematical Details:

Step 1: Lens Formula

The lens equation relates the focal length (f), object distance (d_o), and image distance (d_i):

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \tag{1}$$

Substituting the known values:

$$\frac{1}{10} = \frac{1}{5} + \frac{1}{d_i} \tag{2}$$

Step 2: Solve for Image Distance

Rearrange to solve for d_i:

$$\frac{1}{d_i} = \frac{1}{10} - \frac{1}{5} = -\frac{1}{10} \tag{3}$$

$$d_i = -10\,\text{cm} \tag{4}$$

The negative value indicates that the image formed is virtual.

Step 3: Magnification Calculation

The magnification (M) is given by:

$$M = -\frac{d_i}{d_o} = -\frac{-10}{5} = 2 \tag{5}$$

The virtual image is twice as large as the object.

Step 4: Visual Representation

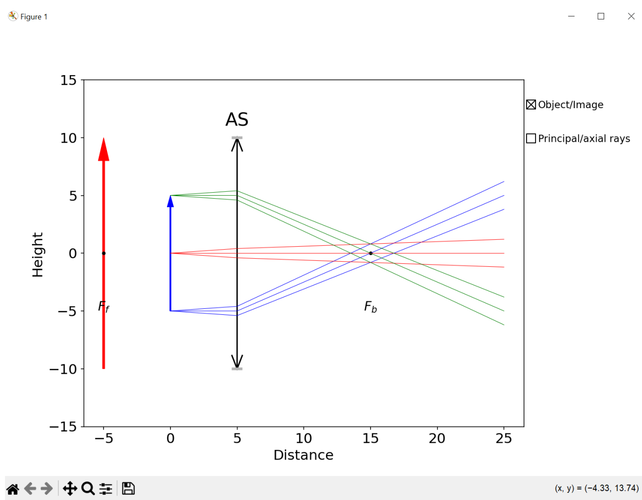

Figure 1: Visualization of the Magnifying Glass, showing the virtual image is twice as large as the object.

Experiment 2: Lens Properties Calculation

Objective: To determine the properties of a magnifying glass, such as magnification and image distance.

System Components:

- Convex Lens with focal length: 10 cm
- Object distance (d_o): 5 cm

Mathematical Details:

Step 1: Lens Formula

Using the lens equation:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \tag{6}$$

Substituting the known values:

$$\frac{1}{10} = \frac{1}{5} + \frac{1}{d_i} \tag{7}$$

Step 2: Solve for Image Distance

$$\frac{1}{d_i} = \frac{1}{10} - \frac{1}{5} = -\frac{1}{10} \tag{8}$$

$$d_i = -10\,\text{cm} \tag{9}$$

Step 3: Magnification Calculation

$$M = -\frac{d_i}{d_o} = -\frac{-10}{5} = 2 \tag{10}$$

The virtual image is twice as large as the object.

Experiment 3: Projection Lens Simulation

Objective: To simulate the formation of a real image when an object is projected through a lens.

System Components:

- Convex Lens with focal length (f): 10 cm
- Object distance (d_o): 30 cm

Mathematical Details:

Step 1: Lens Formula

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \tag{11}$$

Substituting the known values:

$$\frac{1}{10} = \frac{1}{30} + \frac{1}{d_i} \tag{12}$$

Step 2: Solve for Image Distance

$$\frac{1}{d_i} = \frac{1}{10} - \frac{1}{30} = \frac{2}{30} = \frac{1}{15} \tag{13}$$

$$d_i = 15\,\text{cm} \tag{14}$$

Step 3: Magnification Calculation

$$M = -\frac{d_i}{d_o} = -\frac{15}{30} = -0.5 \tag{15}$$

The image is inverted and half the size of the object.

Step 4: Visual Representation

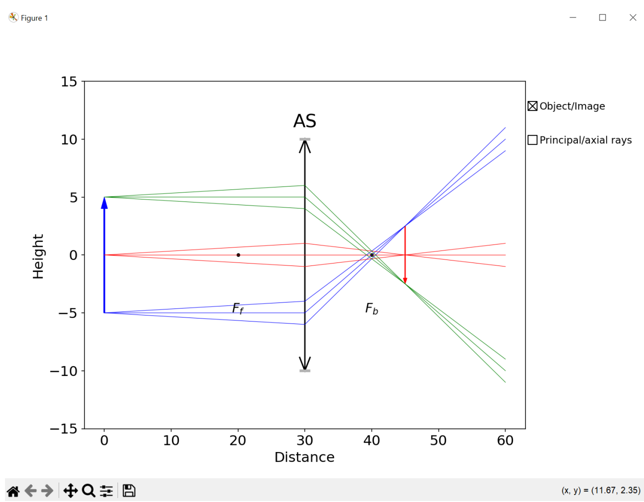

Figure 2: Visualization of the Projection Lens, showing the image is inverted and half the size of the object.

Experiment 4: Concave Mirror Simulation

Objective: To simulate a concave mirror using a lens and determine the properties of the resulting image.

System Components:

- Concave Mirror with focal length (f): 10 cm
- Object distance (d_o): 30 cm

Mathematical Details:

Step 1: Mirror Formula

The mirror equation is similar to the lens formula:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \tag{16}$$

Substituting the known values:

$$\frac{1}{10} = \frac{1}{30} + \frac{1}{d_i} \tag{17}$$

Step 2: Solve for Image Distance

$$\frac{1}{d_i} = \frac{1}{10} - \frac{1}{30} = \frac{2}{30} = \frac{1}{15} \tag{18}$$

$$d_i = 15\,\text{cm} \tag{19}$$

Step 3: Magnification Calculation

$$M = -\frac{d_i}{d_o} = -\frac{15}{30} = -0.5 \tag{20}$$

The image is inverted and half the size of the object.

Step 4: Visual Representation

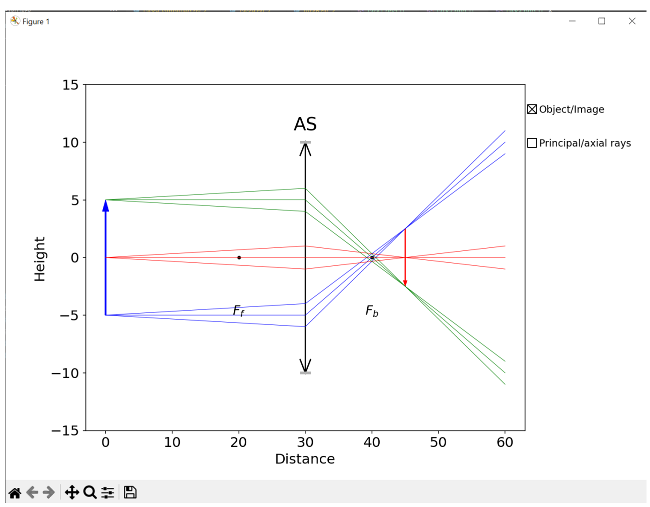

Figure 3: Visualization of the Concave Mirror, showing the image is inverted and half the size of the object.

Experiment 5: Lens Camera Simulation

Objective: To simulate a camera lens system with an aperture acting as a field stop and evaluate the system utilization.

System Components:

- Focal Length: 50 cm

- Object Distance: 100 cm
- Aperture Diameter: 10 units
- Lens Diameter: 30 units

Mathematical Details:

Step 1: System Utilization Calculation

The system utilization is calculated as the ratio of the limiting aperture diameter to the maximum possible lens diameter:

$$\text{Utilization} = \left(\frac{\text{Limiting Diameter}}{\text{Maximum Possible Diameter}}\right) \times 100 = \left(\frac{10}{30}\right) \times 100 = 33.33\% \tag{21}$$

Only 33.33% of the available light is utilized due to the limiting aperture.

Step 2: Visual Representation

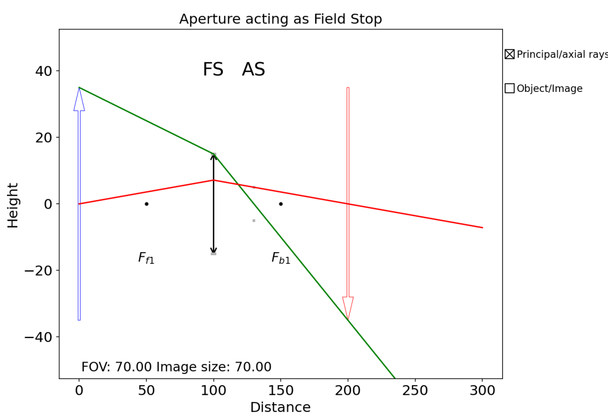

Figure 4: Visualization of the Camera Lens, showing that only 33.33% of the available light is utilized due to the limiting aperture.

Experiment 6: Simplified Human Eye Model

Objective: To simulate a simplified model of the human eye and evaluate image formation.

System Components:

- Focal Length of Cornea and Lens Combined (f): 1.69 cm
- Axial Length: 1.691 cm

- Object Distance: Infinity (simulating a distant object)

Mathematical Details:

Step 1: Lens Formula

For an object at infinity, the image forms at the focal length of the lens:

$$d_i = f = 1.69\,\text{cm} \tag{22}$$

Step 2: Image Formation

The image forms at the focal length, which is 1.69 cm. Since the axial length is 1.691 cm, the image forms very close to the retina, indicating that the eye is properly focused for distant objects without the need for accommodation.

The simulation confirmed that the relaxed eye forms a sharp image on the retina for an object at infinity.

Step 3: Visual Representation

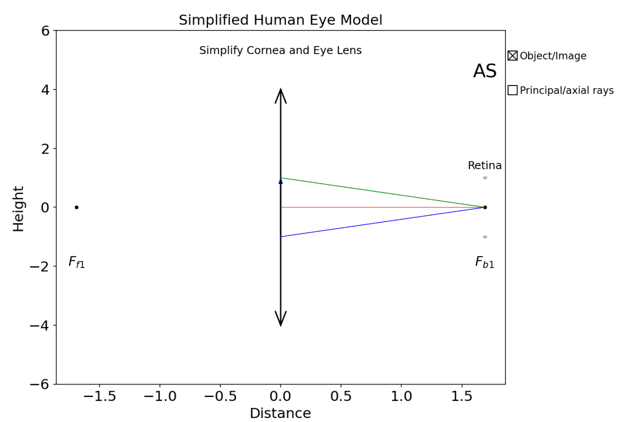

Figure 5: Visualization of the Eye Model , showing hat the relaxed eye forms a sharp image on the retina for an object at infinity.

Experiment 7: Human Eye Model - Distant Object

Objective: To simulate a realistic human eye focusing on a distant object.

System Components:

- Focal Length (f): 1.7, cm

- Axial Length: 1.7, cm
- Object Distance (d_o): 300, cm

Mathematical Details:

Step 1: Lens Formula

For an object at a distance of 300, cm, the rays entering the eye are approximately parallel, which means the image will form at the focal length of the lens:

$$d_i = f = 1.7, \mathrm{cm} \tag{23}$$

Step 2: Image Formation

The image forms at the focal length, which is 1.7, cm. Since the axial length is also 1.7, cm, the image forms exactly on the retina, indicating proper focusing without the need for accommodation.

The simulation confirmed that the eye forms a sharp image on the retina for an object at 300, cm.

Step 3: Visual Representation

The simulation showed that the image forms in front of the retina due to the excessive axial length, demonstrating the nature of myopia.

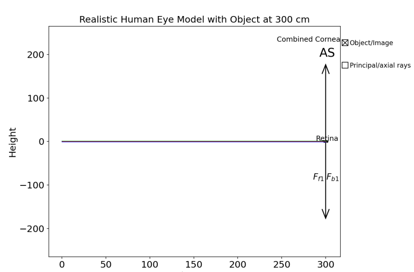

Figure 6: Visualization of the first part of the eye model.

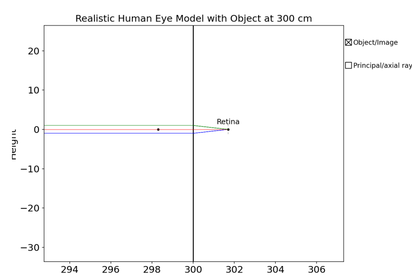

Figure 7: Visualization of the second part of the eye model.

Experiment 8: Human Eye Model - Closer Object

Objective: To simulate the behavior of a human eye when an object is brought closer to the eye.

System Components:

- Focal Length of Cornea (f_c): 2.3, cm

- Focal Length of Crystalline Lens (f_l): $6.4, \mathrm{cm}$
- Axial Length: $1.7, \mathrm{cm}$
- Object Distance (d_o): $20.0, \mathrm{cm}$

Mathematical Details:

Step 1: Effective Focal Length Calculation

The system consists of the cornea and the crystalline lens. Using the formula for effective focal length:

$$\frac{1}{f_{\text{eff}}} = \frac{1}{f_c} + \frac{1}{f_l} \tag{24}$$

$$f_{\text{eff}} = \frac{1}{\left(\frac{1}{2.3} + \frac{1}{6.4}\right)} \approx 1.69, \mathrm{cm} \tag{25}$$

Step 2: Image Formation

Using the lens equation to find the image distance (d_i):

$$\frac{1}{f_{\text{eff}}} = \frac{1}{d_o} + \frac{1}{d_i} \tag{26}$$

Substituting the values:

$$\frac{1}{1.69} = \frac{1}{20} + \frac{1}{d_i} \tag{27}$$

$$\frac{1}{d_i} \approx 0.5429 \tag{28}$$

$$d_i \approx 1.842, \mathrm{cm} \tag{29}$$

The image distance is greater than the axial length, indicating that the image cannot be formed sharply on the retina, suggesting the need for accommodation.

Step 3: Visual Representation

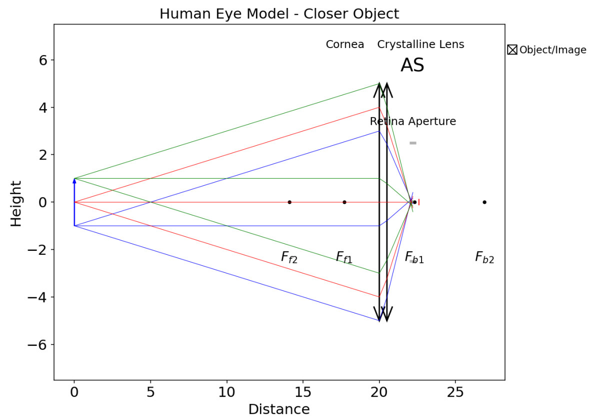

Figure 8: Visualization of the Eye Model, closer object

Experiment 8: Simplified Human Eye Model - Closer Object

Objective: To simulate the behavior of a simplified human eye when an object is brought closer to the eye.

System Components:

- Focal Length of Cornea and Lens Combined (f_{combined}): $1.7, \text{cm}$
- Axial Length: $1.7, \text{cm}$
- Object Distance (d_o): $20.0, \text{cm}$

Mathematical Details:

Step 1: Image Formation

Using the lens formula to find the image distance (d_i):

$$\frac{1}{f_{\text{combined}}} = \frac{1}{d_o} + \frac{1}{d_i} \tag{30}$$

Substituting the values:

$$\frac{1}{1.7} = \frac{1}{20} + \frac{1}{d_i} \tag{31}$$

$$\frac{1}{d_i} \approx 0.5882 - 0.05 = 0.5382 \tag{32}$$

$$d_i \approx \frac{1}{0.5382} \approx 1.858, \mathrm{cm} \tag{33}$$

The image distance is greater than the axial length, indicating that the image cannot be formed sharply on the retina, suggesting the need for accommodation.

The simulation showed that the image forms behind the retina, indicating the need for the eye to accommodate the correct focus.

Step 2: Visual Representation

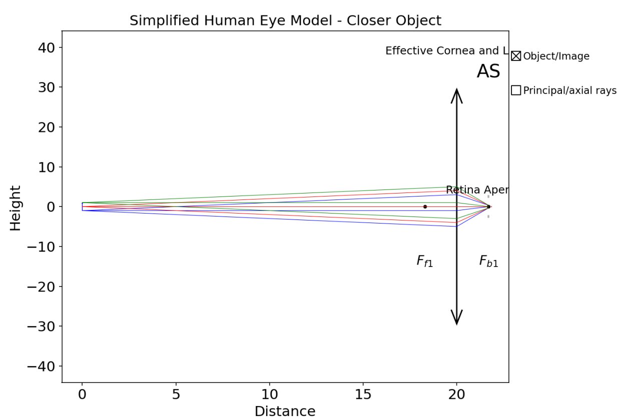

Figure 9: Visualization of the Simplified Eye Model, closer object

Experiment 9: Human Eye Model - Accommodation for Closer Object

Objective: To simulate the accommodation of the human eye lens when an object is moved closer.

System Components:

- Focal Length of Cornea (f_c): $2.3, \mathrm{cm}$
- Axial Length: $1.7, \mathrm{cm}$
- Object Distance (d_o): $20.0, \mathrm{cm}$

Mathematical Details:

Step 1: Adjusted Focal Length

To accommodate the closer object, the eye's lens focal length needs to be adjusted. The required focal length of the lens (f_l) can be calculated as follows:

$$f_l = \frac{1}{\left(\frac{1}{d_o} + \frac{1}{d_i} - \frac{1}{f_c}\right)} \tag{34}$$

Substituting the values:

$$f_l \approx 3.19, \mathrm{cm} \tag{35}$$

Step 2: Image Formation

Using the adjusted focal length, the image distance is calculated to verify proper focusing on the retina:

$$\frac{1}{f_{\text{eff}}} = \frac{1}{d_o} + \frac{1}{d_i} \tag{36}$$

$$d_i \approx 1.7, \mathrm{cm} \tag{37}$$

The image now forms at the retina, indicating successful accommodation.

The simulation confirmed that the adjusted lens successfully formed a sharp image on the retina for an object at $20.0, \mathrm{cm}$.

Step 3: Visual Representation

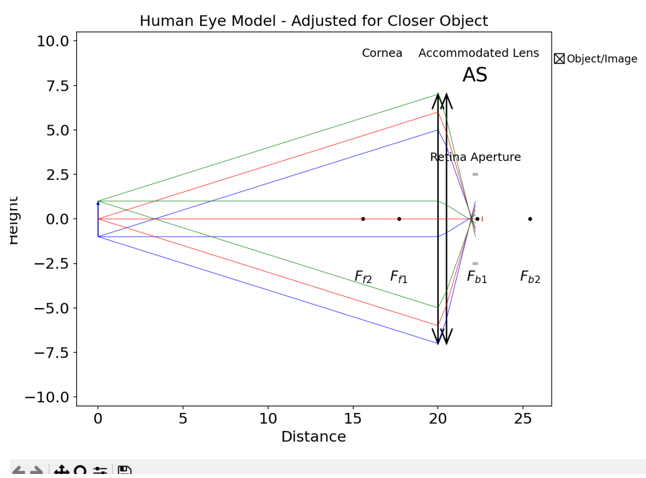

Figure 10: Visualization of the Eye Model,with accomodation for a closer object

Experiment 9: Simplified Human Eye Model - Accommodation for Closer Object

Objective: To simulate the accommodation of a simplified human eye when an object is brought closer to the eye.

System Components:

- Combined Focal Length of Cornea and Lens (f_{combined}): Calculated
- Axial Length: $1.7, \text{cm}$
- Object Distance (d_o): $20.0, \text{cm}$

Mathematical Details:

Step 1: Adjusted Focal Length

Using the thin lens formula to find the required focal length of the combined lens:

$$\frac{1}{f_{\text{combined}}} = \frac{1}{d_o} + \frac{1}{d_i} \tag{38}$$

Solving for f_{combined}:

$$f_{\text{combined}} \approx 1.25, \text{cm} \tag{39}$$

Step 2: Image Formation

Using the adjusted focal length, the image distance is verified:

$$d_i \approx 1.7, \text{cm} \tag{40}$$

The image now forms at the retina, indicating successful accommodation.

The simulation confirmed that the simplified eye model successfully formed a sharp image on the retina for an object at $20.0, \text{cm}$.

Step 3: Visual Representation

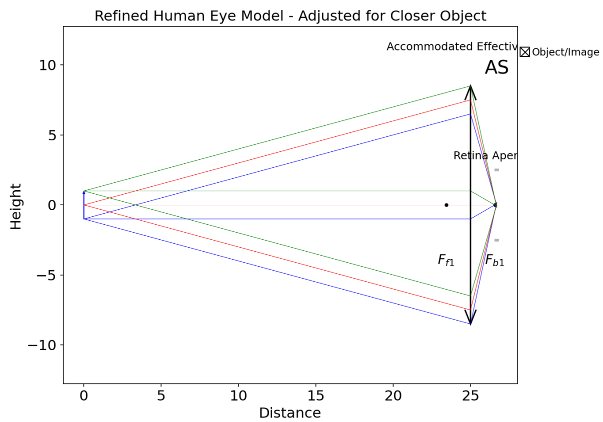

Figure 11: Visualization of the Simplfied Eye Model,with accomodation for a closer object

Experiment 10: Myopic Human Eye Model - Elongated Eyeball

Objective: To simulate the effect of myopia in the human eye due to an elongated eyeball.

System Components:

- Focal Length of Eye Lens (f): 1.7 cm
- Axial Length: 2.0 cm (elongated to simulate myopia)
- Object Distance (d_o): Far distance (simulating an object at infinity)

Mathematical Details:

Step 1: Image Formation

For an object at infinity, parallel rays focus at the focal length of the eye lens:

$$d_i = f = 1.7\,\text{cm} \tag{41}$$

The axial length of the eye is 2.0 cm, which is greater than the focal length, resulting in the image being formed before the retina. This explains why myopic individuals have difficulty seeing distant objects clearly.

Step 2: Visual Representation

The simulation showed that the image forms in front of the retina due to the excessive axial length, demonstrating the nature of myopia.

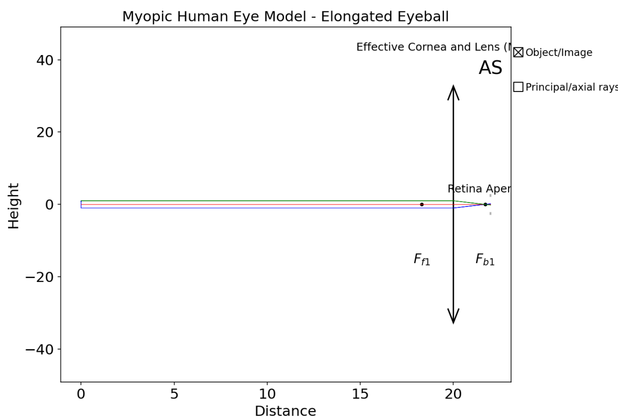

Figure 12: Visualization of the Simplfied Eye Model,with accomodationfor a closer object

Experiment 11: Corrected Myopic Human Eye Model

Objective: To simulate a myopic human eye with corrective glasses and determine if the image forms correctly on the retina.

System Components:

- Focal Length of Myopic Eye (f_{eye}): 1.7 cm
- Axial Length: 2.0 cm
- Corrective Lens Focal Length (f_{glasses}): −26.155 cm
- Object Distance (d_o): 20.0 cm

Mathematical Details:

Step 1: Corrective Lens Calculation

The corrective lens needs to create a virtual image at the far point of the myopic eye:

$$f_{\text{glasses}} = \frac{1}{\left(\frac{1}{d_o} - \frac{1}{f_{\text{far point}}}\right)} = -26.155\,\text{cm} \tag{42}$$

The negative focal length indicates the use of a diverging lens.

Step 2: Image Formation with Correction

With the corrective glasses in place, the effective optical system forms an image at the retina. The system's transfer matrix was analyzed, confirming that the image distance matches the axial length of 2.0 cm.

Step 3: Visual Representation

The simulation displayed the corrected eye system and confirmed that the corrective glasses enabled a sharp image to be formed at the retina, effectively mitigating the effect of myopia.

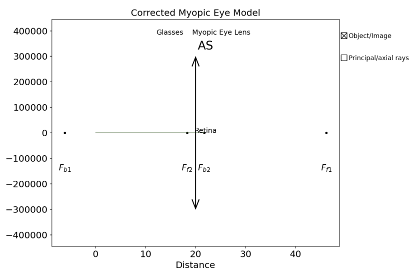

Figure 13: Visualization of the eye model.

Figure 14: Closer Visualization of the eye model.

Figure 15: Closer Visualization of the eye model.

Summary

This document presents detailed mathematical analyses for 11 experiments involving various optical systems, including lenses and human eye models, providing insights into image formation, accommodation, and corrective measures for visual impairments.

References

References

[1] Valérie Pineau Noël, Shadi Masoumi, Elahe Parham, Gabriel Genest, Ludovick Bégin, Marc-André Vigneault, Daniel C. Côté, "Tools and tutorial on practical ray tracing for microscopy," Neurophoton. 8(1) 010801 (21 January 2021). https://doi.org/10.1117/1.NPh.8.1.010801

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[6] https://phys.libretexts.org/Bookshelves/University_Physics/University_Physics_(OpenStax)/University_Physics_III_-_Optics_and_Modern_Physics_(OpenStax)/02\%3A_Geometric_Optics_and_Image_Formation/2.06\%3A_The_Eye

[7] https://minerva.union.edu/malekis/CVision2003/MainPage/Course\%20Content/Optics\%20of\%20Eye/OpticsofEye.htm