

Public Key Cryptography – Lab3

The Miller-Rabin Test

Input: $n \in \mathbb{N}, n \geq 3$ odd, $k \in \mathbb{N}^*$

Output: $P(n \text{ is prime})$ – the probability that n is prime

Algorithm miller_rabin(n, k)

```
probability :=  $1 - \frac{1}{4^k}$ 
if  $n = 2$  then
    return 1.0
# Step 0. Write  $n - 1 = 2^s t$ , where  $t$  is odd
 $s := 0$ 
while  $t$  is even do
     $s := s + 1$ 
     $t := \frac{n-1}{2^s}$ 

# Step 1: Choose (randomly)  $1 < a < n$ 
# Step 2: Compute the following sequence (modulo  $n$ ):  $a^t, a^{2t}, a^{2^2t}, \dots, a^{2^st}$ 
# Step 3: If either the first number in the sequence is 1 or one gets the value
1 and its previous number -1, then  $n$  is possible to be prime and we repeat the steps
1-3 at most  $k$  times
while  $k > 0$  do
     $a :=$  random number between from  $\{2, \dots, n-1\}$ 
    for  $i = \overline{1, s}$  do
         $e :=$  repeated squaring modular exponentiation( $a, 2^i t, n$ )
        if  $e = 1$  then
             $k := k - 1$ 
            goto Step 1
# Step 4: The algorithm stops and  $n$  is composite
    return 0.0
return probability
```

Input: $x \in \mathbb{N}, y \in \mathbb{N}, n \in \mathbb{N}$

Output: $x^y \pmod n$

Algorithm repeated squaring modular exponentiation(x, y, n)

```
result := 1
current :=  $x \bmod n$ 
while  $y > 0$  do
    if  $y$  is odd then
        result := (result * current) mod  $n$ 
    current := (current * current) mod  $n$ 
     $y := \lfloor \frac{y}{2} \rfloor$ 
return result
```