1

represent

Reg exp

Finite Automata $\Longleftrightarrow$ Regular grammars

check

build

- $\boxed{\text{Regular grammar}}$

→ right linear grammar $A \to aB \mid b$

→ Only S can produce ε the S not in r-side

- $\boxed{\text{Finite Automata}}$

5-tuple $(Q, \Sigma, \delta, q_0, F) = M$

Q - finite set of states

Σ - finite alphabet

δ - transition function $(\delta : Q \times \Sigma \to \mathcal{P}(Q))$

q_0 - initial state

$F \subseteq Q$ - set of final states

DFA (deterministic FA)

$|\delta(q, a)| \leq 1$

NFA (non deterministic FA)

$|\delta(q, a)| > 1$

$\boxed{\text{Property}}$ Any NFA has an equivalent DFA configuration (q, x)

state — unread sequence from input

final conf: (q_f, ε)

$\vdash$ move / transition $(q, ax) \vdash (p, x)$

$\overset{k}{\vdash}$ kmove , $\overset{+}{\vdash}$ + move $\exists k > 0$ a.î $C \overset{k}{\vdash} C'$

$\overset{*}{\vdash}$ move $C \overset{*}{\vdash} C' : \exists k \geq 0$ a.î $C \overset{k}{\vdash} C'$

! $\boxed{\text{Def}}$ Language accepted by FA $M = (Q, \Sigma, \delta, q_0, F)$ is $L(M) = \{ w \in \Sigma^* \mid (q_0, w) \overset{*}{\vdash} (q_f, \varepsilon), q_f \in F \}$

! $M_1 \sim M_2$ (FAs) $\Leftrightarrow L(M_1) = L(M_2)$

! $\varepsilon \in L(M) \Leftrightarrow q_0 \in F$ (initial state is also final)

REPRESENTING FAs

1. List of elem
2. Table
3. Graphical repres.

Ex 1

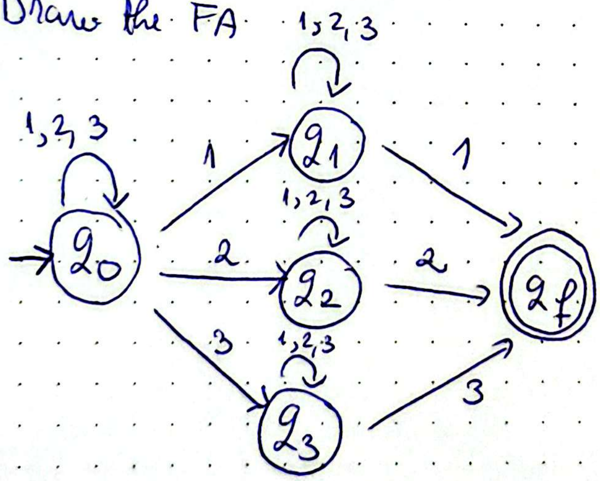
Given FA: $M = (Q, \Sigma, \delta, q_0, F)$

$Q = \{q_0, q_1, q_2, q_3, q_f\}$, $\Sigma = \{1, 2, 3\}$, $F = \{q_f\}$

δ	1	2	3
q_0	$\{q_0, q_1\}$	$\{q_0, q_2\}$	$\{q_0, q_3\}$
q_1	$\{q_1, q_f\}$	$\{q_1\}$	$\{q_1\}$
q_2	$\{q_2\}$	$\{q_2, q_f\}$	$\{q_2\}$
q_3	$\{q_3\}$	$\{q_3\}$	$\{q_3, q_f\}$
q_f	$\emptyset$	$\emptyset$	$\emptyset$

Prove that $w = 12321 \in L(M)$

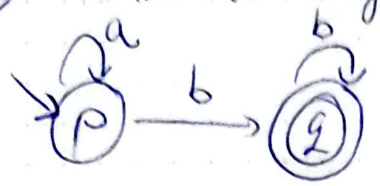
Draw the FA



$(q_0, 12321) \vdash (q_1, 2321) \overset{3}{\vdash} (q_1, 1) \vdash (q_f, \varepsilon)$

$\Rightarrow (q_0, w) \overset{*}{\vdash} (q_f, \varepsilon) \Rightarrow w \in L(M)$

E2) Find the language accepted by the FA below



Sol.

$L = \{a^n b^m \mid n \in \mathbb{N}, m \in \mathbb{N}^*\}$

? $L = L(M)$

$\boxed{1.\ L \subseteq L(M)}$ ($\leftarrow$ all sequences of that shape are accepted by M)

$\forall n \in \mathbb{N}, \forall m \in \mathbb{N}^* \quad a^n b^m \in L(M)$

Let $n \in \mathbb{N}, m \in \mathbb{N}^*$

$(p, a^n b^m) \overset{n}{\underset{ⓐ}{\vdash}} (p, b^m) \underset{ⓑ}{\vdash} (q, b^{m-1}) \overset{m-1}{\vdash} (q, \varepsilon)$

$\Rightarrow a^n b^m \in L(M)$

ⓐ $(p, a^n) \vdash (p, \varepsilon) \quad \forall n \in \mathbb{N}$

ⓑ $(q, b^k) \vdash (q, \varepsilon) \quad \forall k \in \mathbb{N}$

$(p, a^n) \vdash (p, \varepsilon)$ proof

$P(0)$: $(p, \varepsilon) \vdash (p, \varepsilon)$ $P(0)$ true

? $P(k)$ true $\Rightarrow P(k+1)$ true

$P(k)$ true $\Rightarrow (p, a^k) \overset{k}{\vdash} (p, \varepsilon)$

$(p, a^{k+1}) \vdash (p, a^k) \vdash (p, \varepsilon) \Rightarrow (p, a^{k+1}) \vdash (p, \varepsilon) \Rightarrow P(k+1)$ true

Same for (q, b^k)

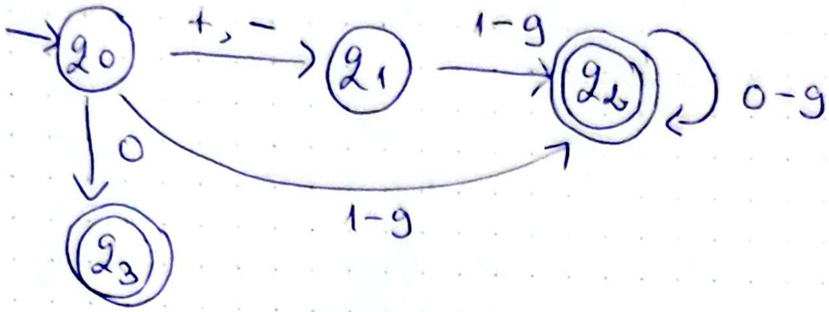
$\boxed{2.\ L(M) \subseteq L}$

Explaining that M accepts only sequences of $a^n b b^k$

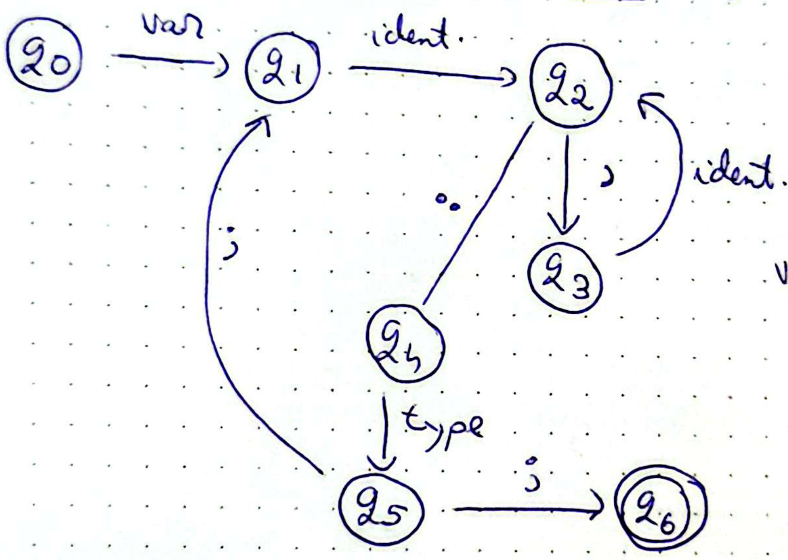
Cover all paths from initial state to final state

Ex3 Build the FAs that accepts the following languages

a) Integers numbers

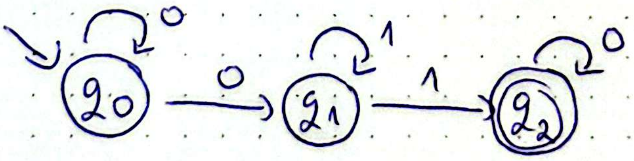


b) Variable declaration (Pascal, C)

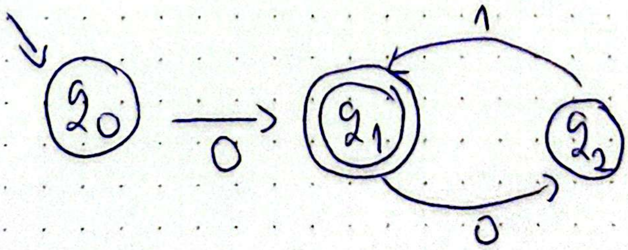


var a,b: string;

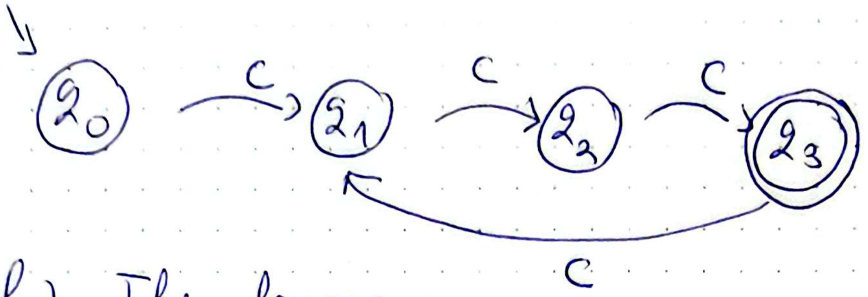
c) $L = \{ 0^n 1^m 0^q \mid n, m \in \mathbb{N}^*, q \in \mathbb{N} \}$



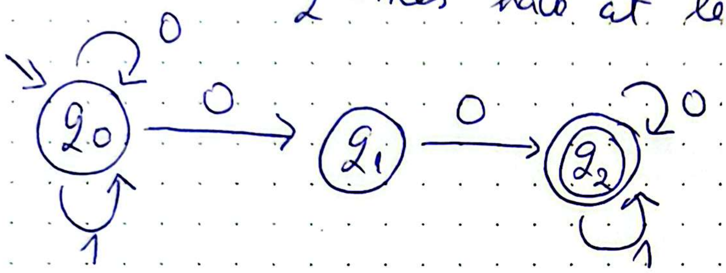
d) $L = \{ 0(01)^n \mid n \in \mathbb{N} \}$



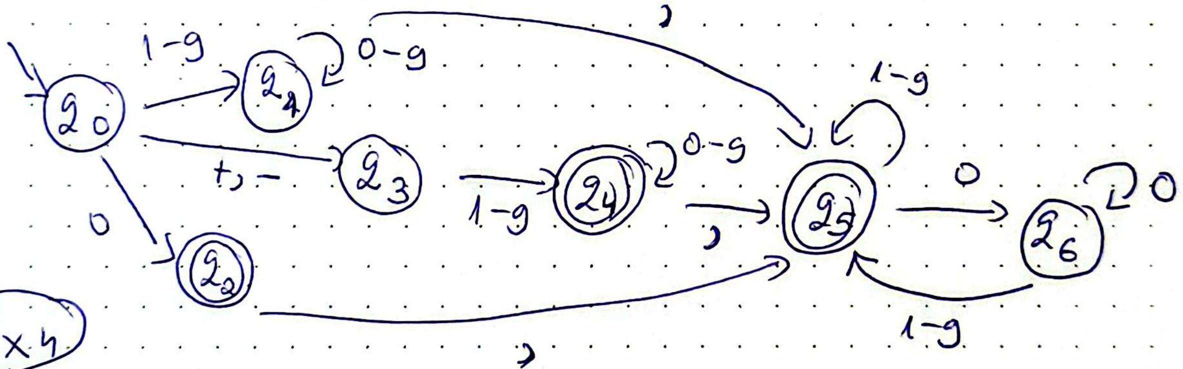
e) $L = \{c^{3n} \mid n \in \mathbb{N}^*\}$



f) The language over $\Sigma = \{0, 1\}$ with property that all sequences have at least two consecutive 0's

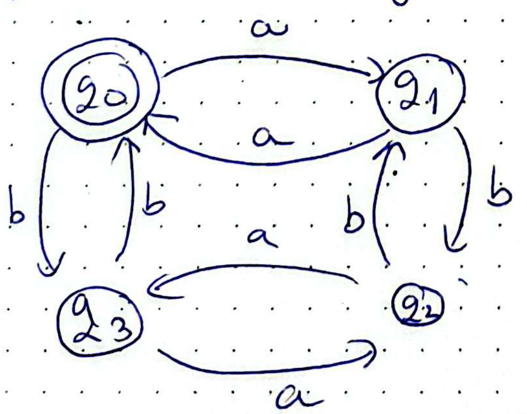


g) real numbers



Ex.4

h) The set of strings with ~~?~~ even numbers of a and b

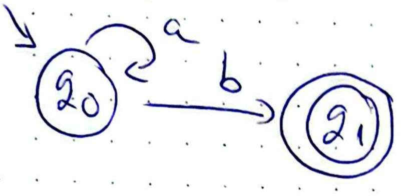


+ table

Ex 5. For the following regular grammars describe their generated language. Give the corresponding FA

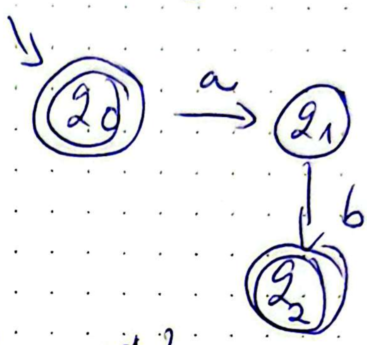
a) $A \to aA$
$A \to b$

$\mathcal{L} = \{a^n b, n \in \mathbb{N}\}$



b) $S \to \varepsilon$
$S \to aA$
$A \to b$

$L = \{\varepsilon, ab\}$



c) $S \to \varepsilon$
$S \to aA$
$A \to bA$
$A \to c$

$L = \{ab^n c \mid n \in \mathbb{N}^*\}$