Seminar 5

→ what is a right regular grammar? Recap

$\boxed{RG \to FA}$

$RG = (N, \Sigma, P, S)$ $\quad$ $FA = (Q, \Sigma, \delta, q_0, F)$

Rules:

$Q = N \cup \{K\}$ $\quad$ $K \leftarrow$ a final state we introduce

$q_0 = S$

$$F = \begin{cases} \{K\}, & S \to \varepsilon \notin P \\ \{S, K\}, & S \to \varepsilon \in P \end{cases}$$

$$\delta(A, a) = \{B \mid A \to aB \in P\} \cup K$$

where $K = \begin{cases} \{K\} & A \to a \in P \\ \emptyset & \text{otherwise} \end{cases}$

- Ex1

Given $G = (\{S, A, B, C\}, \{a, b, c\}, P, S)$

$$P = \begin{aligned} S &\to aB \mid bA \\ A &\to cB \\ C &\to aS \mid c \\ B &\to b \end{aligned}$$

Find the FA that accepts $L(G)$ and draw the graph representation.

Solution

$M = (\{S, A, B, C, K\}, \{a, b, c\}, \delta, S, \{K\})$

$\delta(S, a) = \{B\}$

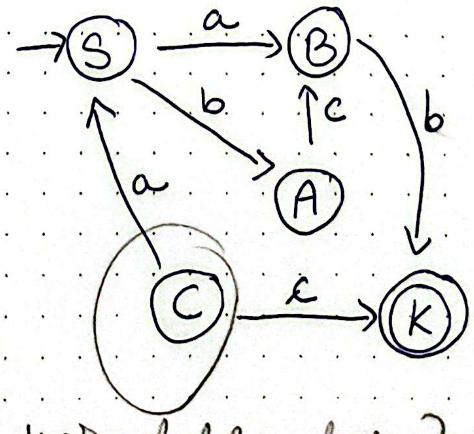
$\delta(S, b) = \{A\}$

$\delta(A, c) = \{B\}$

$\delta(C, a) = \{S\}$

$\delta(C, c) = \{K\}$

$\delta(B, b) = \{K\}$



will see in the next seminar

unreachable state? eliminate?

• Ex2

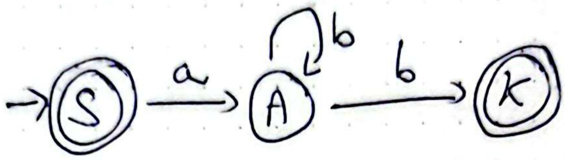
Given $G = (\{S, A\}, \{a, b\}, P, S)$

$P: \; S \to aA \mid \varepsilon$
$\quad\; A \to bA \mid b \mid a$ Find the FA

Solution

$M = (\{S, A, K\}, \{a, b\}, \delta, S, \{S, K\})$

$\delta(S, a) = \{A\}$
$\delta(A, b) = \{A\}$
$\delta(A, b) = \{K\}$
$\delta(A, a) = \{K\}$

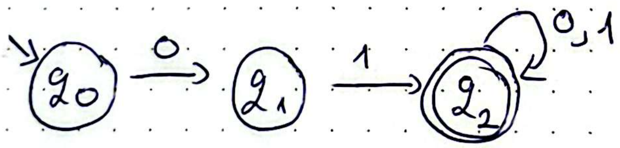


$\boxed{AF \to GR}$

$N = Q \qquad S = q_0$

Σ the same $\quad P = \{A \to aB \mid \delta(A, a) \ni B\} \cup \{A \to a \mid \delta(A, a) \ni B, B \in F\}$

• Ex3 Given



Find GR

Sol

$N = (\{q_0, q_1, q_2\}, \Sigma = \{0, 1\}, P, q_0)$

$P: \; q_0 \to 0q_1$
$\quad\; q_1 \to 1q_2 \mid 1$ because q_2 final state
$\quad\; q_2 \to 0q_2 \mid 0$
$\quad\; q_2 \to 1q_2 \mid 1$

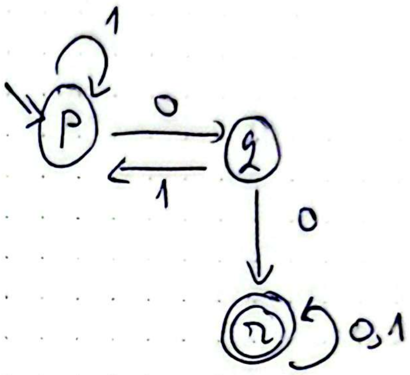
• Ex4

Given $M = (Q, \Sigma, \delta, q_0, F)$ where $Q = \{p, q, r\}$
$\Sigma = \{0, 1\}$, $q_0 = p$, $F = \{r\}$ and

δ	0	1
p	q	p
q	r	p
r	r	r

1. Build the GR (equivalent right linear grammar)
2. Draw the FA

Solution



$G = \{N, \Sigma, P, S\}$
$\Sigma = \{0, 1\}$
$N = \{p, q, r\}$
$P:$ $p \to 0q \mid 1p$
$q \to 0r \mid 0 \mid 1p$
$r \to 0r \mid 0 \mid 1r \mid 1$
$S = p$

• Ex5

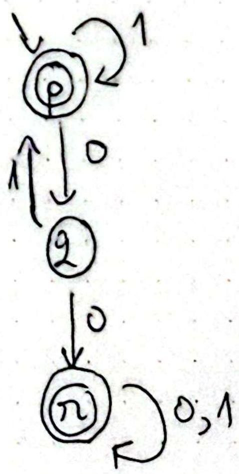
Given $M = (Q, \Sigma, \delta, q_0, F)$ where $Q = \{p, q, r\}$, $q_0 = p$
$F = \{p, r\}$, $\Sigma = \{0, 1\}$

δ	0	1
p	q	p
q	r	p
r	r	r

Build the GR
Draw the FA

because p final and initial state

Solution



$G = \{N, \Sigma, P, S\}$ $\quad P:$ $p \to 0q \mid 1p \mid 1 \mid \varepsilon$
$\Sigma = \{0, 1\}$ $\quad q \to 0r \mid 1p \mid 1 \mid 0$
$N = \{p, q, r\}$ $\quad r \to 0r \mid 1r \mid 0 \mid 1$

Regular sets

<u>Rules</u>

- $\emptyset$, $\{\varepsilon\}$, $\{a\}$ are regular sets over Σ
- if P, Q regular sets $\Rightarrow P \cup Q$, $P \cap Q$, PQ, P^* are regular sets over Σ
- any regular set is obtained with above rules applied in a recursive manner (a finite amount of time)

Regular expressions

<u>Rules</u>

Regular sets can be represented by regular expressions

$$\text{expr} \begin{cases} \emptyset \\ \varepsilon \\ a \end{cases} \xrightarrow{\text{represent}} \begin{cases} \{\emptyset\} \\ \{\varepsilon\} \quad \text{sets} \\ \{a\} \quad \forall a \in \Sigma \end{cases}$$

- if p, q are regular expr. representing regular sets P and Q, then

$$p+q \Leftrightarrow P \cup Q \quad (p|q)$$
$$pq \Leftrightarrow PQ$$
$$p^* \Leftrightarrow P^*$$

Any reg. expr. is buid applying the above rules

<u>Obs</u>

$$p^+ = pp^*$$

priority: $*$, concatenation, $+$ $\longrightarrow$

$a\varepsilon = \varepsilon a = a$

$\emptyset a = a\emptyset = a$

$a^* = a + a^*$

$(a^*)^* = a^*$

$a + a = a$

$a + \emptyset = a$

$\emptyset^* = \varepsilon$

REBEL DOT LEADING THE LINE

Ex 5:

Find the set representation of the following regular expressions

a) $(0+10^*)$
b) (0^*10^*)
c) $(0+\varepsilon)(1+\varepsilon)$
d) $(a+b)^*$
e) $(11)^*$
f) $(aa)^*(bb)^*b$

Solution

a) $\{0, 1, 10, 100, 10000 \dots\}$
b) $\{01, 1, 10, 010, 001000, \dots\}$
$\{0^n 1 0^m, n, m \in N\}$
c) $\{0, 1, 01, \varepsilon\}$
d) $\{\varepsilon, a, b, ab, aa, aaa \dots, ba, bb, \dots\}$
e) $\{\varepsilon, 11, 1111, \dots\} \Leftrightarrow \{\varepsilon, 1^{2n} \mid n \in N^*\}$

f) $\{b, aab, bbb, aaabbb \dots\}$
$\{a^{2n} b^{2m} b \mid n, m \in N\}$

! Regular sets are right linear languages (a lang generated by a right linear grammar)

Lema 1: $\emptyset, \{\varepsilon\}, \{a\}\ \forall a \in \Sigma$ are righ. linear lang

Lema 2: If L_1, L_2 righ-lin-lang $\Rightarrow L_1 \cup L_2, L_1 L_2, L_1^*$ are righ-lin-lang

Construction

- $L_1 \cup L_2 = L_3$

$\boxed{G_3}$: $N_3 = N_1 \cup N_2 \cup \{S_3\}$

$P_3 = P_1 \cup P_2 \cup \{S_3 \to \alpha_1 \mid S_1 \to \alpha_1 \in P_1\} \cup$
$\{S_3 \to \alpha_2 \mid S_2 \to \alpha_2 \in P_2\}$

we add S_3 which is $S_1 \cup S_2$ as prod

$G_1 = (N_1, \Sigma_1, P_1, S_1)$
$N_1 = \{S_1, A_1, B_1\}$
$\Sigma_1 = \{a, b\}$
P_1: $S_1 \to aA_1 \mid bB_1$
$A_1 \to aA_1 \mid a$
$B_1 \to bB_1 \mid b$

$G_2 = (N_2, \Sigma_2, P_2, S_2)$
$N_2 = \{S_2, A_2\}$
$\Sigma_2 = \{c, d\}$
P_2: $S_2 \to cA_2 \mid dS_2$
$A_2 \to c$

$N_3 = N_1 \cup N_2 \cup \{S_3\}$

$\Sigma = \{a, b, c, d\}$

P_3:

$S_3 \to \underbrace{aA_1 | bB_1}_{S_1} | \underbrace{cA_2 | dS_2}_{S_2}$

$A_1 \to aA_1 | a$, $B_1 \to bB_1 | b$, $S_2 \to cA_2 | dS_2$, $A_2 \to c$

- $L_4 = L_1 L_2$

$\boxed{G_4}$ $N_4 = N_1 \cup N_2$, $S_4 = S_1$

P_4: $\{A \to aB \mid \text{if } A \to aB \in P_1\} \cup$

$\{A \to aS_2 \mid \text{if } A \to a \in P_1\} \cup P_2$

$N_4 = \{S_1, A_1, B_1, S_2, A_2\}$ $\Sigma = \{a, b, c, d\}$

P_4: $S_1 \to aA_1 | bB_1$

$A_1 \to aA_1 | aS_2$ s-au adăugat

$B_1 \to bB_1 | bS_2$

$S_2 \to cA_2 | dS_2$

$A_2 \to c$

$L_5 = L_1^*$ $\boxed{G_5}$

$N_5 = N_1 \cup \{S_5\}$

$P_5 = P_1 \cup \{S_5 \to \varepsilon\} \cup \{S_5 \to \alpha_1 \mid \text{if } S_1 \to \alpha_1 \in P_1\} \cup$

$\{A \to \alpha S_5 \mid \text{if } A \to \alpha \in P_1\}$

ex.

P_1:

$S_1 \to aA_1 | bB_1$

$A_1 \to aA_1 | a$

$B_1 \to bB_1 | b$

$N_5 = \{S_1, A_1, B_1, S_5\}$

$G_5 = \{N_5, \Sigma_1, P_5, S_5\}$

P_5: $S_5 \to \varepsilon | aA_1 | bB_1$

$A_1 \to aA_1 | aS_5$

$B_1 \to bB_1 | bS_5$

RG → RE

System of equation

nonterminals → indeterminants
terminals → coefficients
equation for A → all productions of A
$X = aX + b$ has solution $X = a^*b$
→ we want to solve for S
regular expression = solution for S

Ex: 6

$G = (N, \Sigma, P, S)$
$N = \{S, A, B\}$
$\Sigma = \{a, b\}$
P:
$S \to aA \mid bB$
$A \to aA \mid a$
$B \to bB \mid b$

Find RE ?

Solution

$$\begin{cases} S = aA + bB \\ A = aA + a \\ \underbrace{B = bB + b} \end{cases} \Rightarrow A = a^*a = a^+$$

$$\Downarrow$$

$$B = b^*b = b^+$$

$$\Rightarrow S = aa^+ + bb^+$$

Ex 7

Give the RE for :

$G = (\{S, A, B\}, \{a, b\}, P, S)$
P: $S \to aA$
$A \to aA \mid bB \mid b$
$B \to bB \mid b$

Sol

$$\begin{cases} S = aA \\ A = aA + \overbrace{bB + b}^{b^+} \\ \underbrace{B = bB + b} \end{cases}$$

$$\Downarrow$$

$$B = b^*b = b^+$$

$$\Rightarrow A = aA + bb^+ + b$$

$$A = aA + b^+$$

$$\Downarrow$$

$$A = a^*b^+$$

$$S = aa^*b^+ = a^+b^+$$