Seminar 9
LR(0)

PRINCIPLE

- current state
- current symbol
- prediction that uniquely determines → action to apply; → move to another state

LR(0) steps

1. Define items
2. Construct set of states
3. Construct table
4. Parse sequences based on moves between config.

Example:

$S' \to S$

$S \to aA$ (1)

$A \to bA$ (2)

$A \to c$ (3)

Items and states

$s_0 = closure(\{[S' \to .S]\}) = \{[S' \to .S], [S \to .aA]\}$

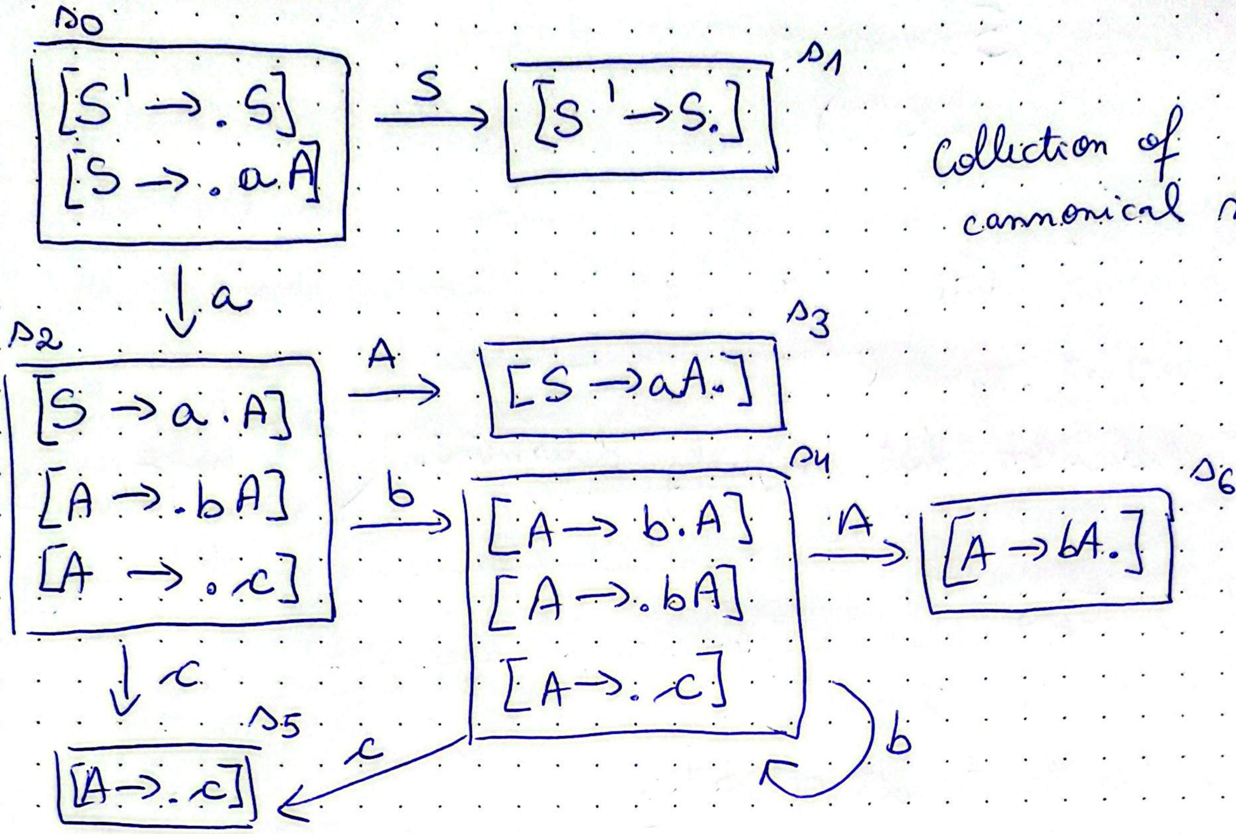
$s_1 = goto(s_0, S) = closure([S' \to S.]) = \{[S' \to S.]\}$

$s_2 = goto(s_0, a) = closure([S \to a.A]) =$

$= \{[S \to a.A], [A \to .bA], [A \to .c]\}$

and so on...

Collection of cannonical states



LR(0) TABLE

Actions
- shift: if $[A \to \alpha . \beta] \in s_i$
- reduce if $[A \to \beta .] \in s_i$ & $A \neq S'$
- accept if $[S' \to S.] \in s_i$
- error otherwise

STATE	ACTION	GOTO a	b	c	S	A
0	shift	2			1	
1	acc					
2	shift		4	5		3
3	reduce 1					
4	shift		4	5		6
5	reduce 3					
6	reduce 2					

PARSING

$w = abbc$

α: working stack

β input stack

initial config: $(\$ s_0, w\$, \varepsilon)$

final config: $(\$ s_{acc}, \$, \pi)$

output (production number sequence)

α	β	π
$\$0$	$abbc\$$	ε
$\$0a2$	$bbc\$$	ε
$\$0a2b4$	$bc\$$	ε
$\$0a2b4b4$	$c\$$	ε
$\$0a2b4b4c5^{(1)}$	$\$$	ε
$\$0a2b4\underline{b4A6}^{(2)}$	$\$$	3
$\$0a2\underline{b4A6}$	$\$$	2, 3
$\$0\underline{a2A3}$	$\$$	2, 2, 3
$\$0S1$	$\$$	1, 2, 2, 3
$\$$ acc	$\$$	

(1) reduce with production 3 $\Rightarrow$ replace c5 with left hand side of production 3 which is A

After A write 6 because s4 with A will shift to s6

$$\$0a2b4b4\underbrace{\cancel{c5}}_{A} \rightarrow \$0a2b4b4A6$$

$4 \text{ with } A \Rightarrow 6$

(2) reduce with production 2 $(A \rightarrow bA)$

so replace b4A6 with A, then 4 with A $\Rightarrow$ 6

$$\$0a2b4\underbrace{b4A\cancel{6}}_{A \rightarrow bA} \rightarrow \$0a2b4A6$$

$4 \text{ with } A \Rightarrow 6$

$w_2 = baba$

$w_3 = abcc$

α	β	Π
\$0	baba\$	ε
\$0 err	baba\$	ε

error because there is no move in LR(0) for row 0 column b

$\Downarrow$

$w_2 \notin G$

\$0	abcc\$	ε
\$0a2	bcc\$	ε
\$0a2b4	cc\$	ε
\$0a2b4c5	c\$	
reduce		
\$0a2b4A6	c\$	3
reduce		
\$0a2A3	c\$	2, 3
reduce		
\$0S1	c\$	2, 3
\$ accept	c\$	2, 3

because is not empty and we reached accept it means the word is not accepted

$w_3 \notin G$

Ex2

$S' \to S$

$S \to AA$ (1)

$A \to aA^{(2)} \mid b^{(3)}$

Canonical collection

$s_0 = closure(\{[S' \to .S]\}) = \{[S' \to .S], [S \to .AA], [A \to .aA],$
$[A \to .b]\}$

$s_1 = goto(s_0, S) = closure(\{[S' \to S.]\}) = \{[S' \to S.]\}$

$s_2 = goto(s_0, A) = closure(\{[S \to A.A]\}) = \{[S \to A.A],$
$[A \to .aA], [A \to .b]\}$

$s_3 = goto(s_0, a) = closure(\{[A \to a.A]\}) = \{[A \to a.A],$
$[A \to .aA], [A \to .b]\}$

$s_4 = goto(s_0, b) = closure(\{[A \to b.]\}) = \{[A \to b.]\}$

$goto(s_1, X) = \emptyset, \forall X \in N \cup \Sigma$

$goto(s_2, A) = closure(\{[S \to AA.]\}) = \{[S \to AA.]\} = s_5$

$goto(s_2, a) = closure(\{[A \to a.A]\}) = \{[A \to a.A],$
$[A \to .aA], [A \to .b]\} = s_3$

$goto(s_2, b) = closure(\{[A \to b.]\}) = s_4$

$goto(s_3, A) = closure(\{[A \to aA.]\}) = \{[A \to aA.]\} = s_6$

$goto(s_3, a) = closure(\{[A \to a.A]\}) = s_3$

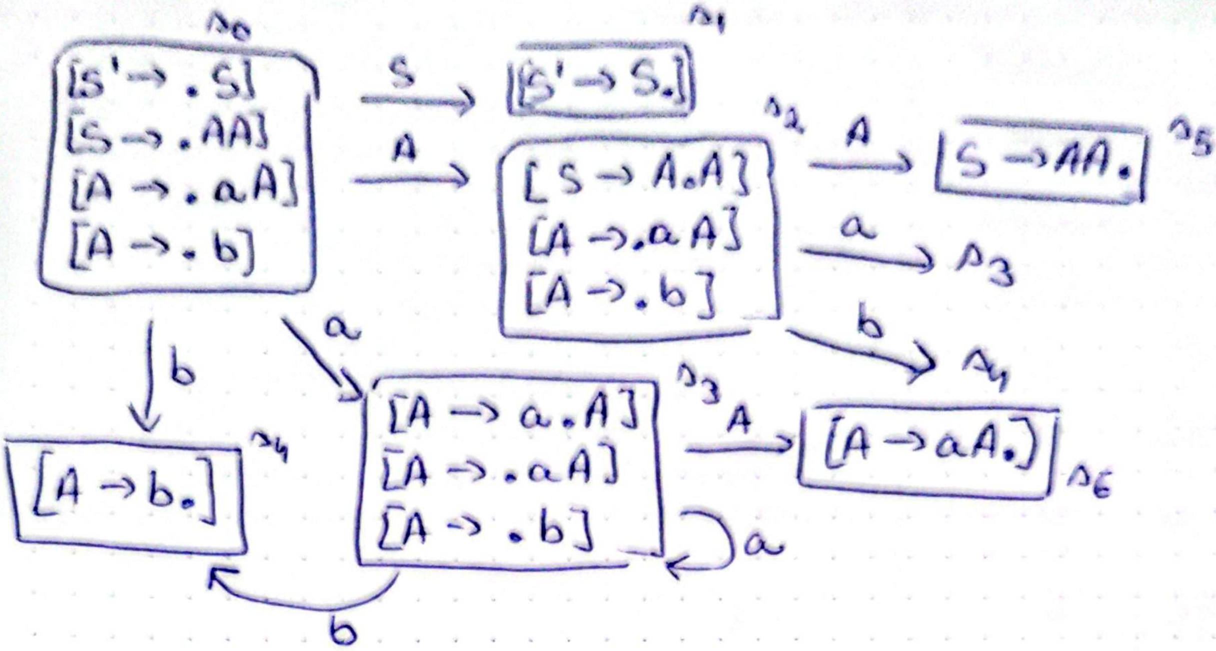
$goto(s_3, b) = closure(\{[A \to b.]\}) = s_4$

$goto(s_4, X) = \emptyset, \forall X \in N \cup \Sigma$

$goto(s_5, X) = \emptyset, \forall X \in N \cup \Sigma$

$goto(s_6, X) = \emptyset, \forall X \in N \cup \Sigma$

Visual representation



LR(0) table

State	Action	goto a	goto b	goto S	goto A
0	shift	3	4	1	2
1	accept				
2	shift	3	4		5
3	shift	3	4		6
4	reduce 3				
5	reduce 1				
6	reduce 2				

No conflicts in the table ⇒ $G \in LR(0)$

$w_1 = baab$

$w_2 = abba$

$w_3 = babb$

REBEL DOT

α	β	π
$0	baab$	ε
$0b4 (reduce)	aab$	ε
$0A2	aab$	3
$0A2a3	ab$	3
$0A2a3a3	b$	3
$0A2a3a3b4	$	3
$0A2a3a3A6	$	3,3
$0A2a3A6	$	2,3,3
$0A2A5	$	2,2,3,3
$0S1	$	1,2,3,3
$ accept	$	1,2,3,3

=> $w_1 \in G$

α	β	π
$0	abba$	ε
$0a3	bba$	ε
$0a3b4	ba$	ε
$0a3A6	ba$	3
$0A2	ba$	2,3
$0A2b4	a$	2,3
$0A2A5	a$	3,2,3
$0S1	a$	3,2,3
$ accept	a$	3,2,3

not empty => $w_2 \notin G$

Ex 2

$S' \to S$
$S \to AA$ (1)
$A \to aA^{2} \mid b^{3}$

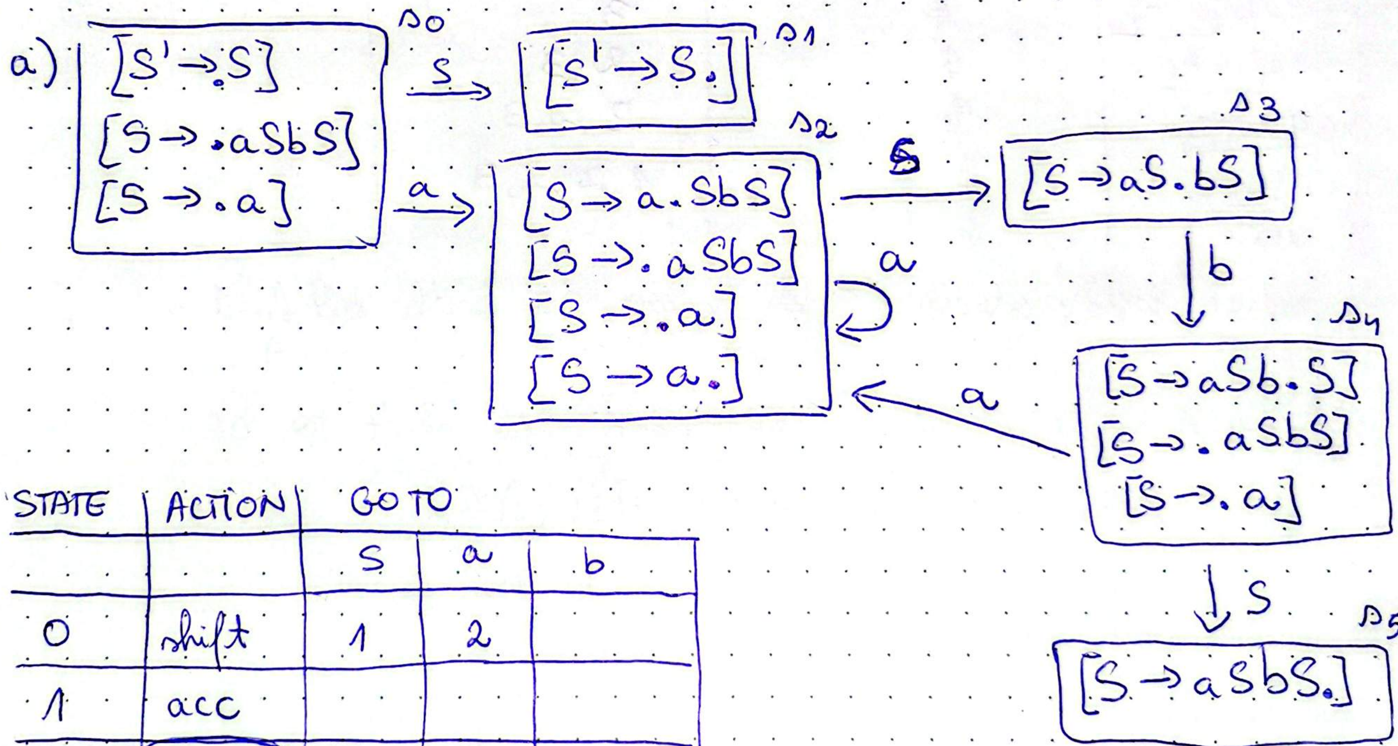
Ex 3

Are the following grammars LR(0)?

a) $S' \to S$
$S \to aSbS$ (1)
$S \to a$ (2)

b) HW
$S' \to S$
$S \to Ax$ (1)
$S \to By$ (2)
$A \to a$ (3)
$B \to a$ (4)

a)



STATE	ACTION	GOTO		
		S	a	b
0	shift	1	2	
1	acc			
2	reduce 2 / shift			

conflict shift-reduce ⇒ grammar $\notin$ LR(0)