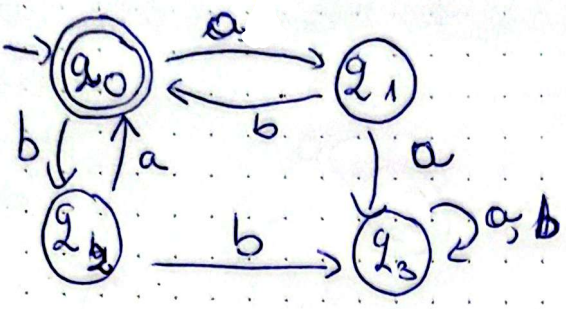
Seminar 6

FA ⇒ RE

Ex 1



Find the RG

Solution

q_1 initial

$$\begin{cases} q_0 = q_1 b + q_2 a + \varepsilon \\ q_1 = q_0 a \\ q_2 = q_0 b \\ q_3 = q_1 a + q_2 b + q_3(a+b) \end{cases}$$

We solve for q_0 because q_0 is final

$q_0 = q_1 b + q_2 a + \varepsilon$

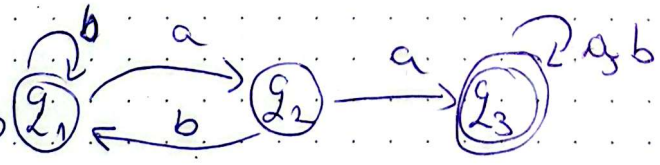
$q_0 = q_0 ab + q_0 ba + \varepsilon$

$q_0 = q_0 (ab + ba) + \varepsilon$

$X \quad X \quad a \quad b \qquad (X = Xa + b \Rightarrow X = ba^*)$

$q_0 = \varepsilon (ab+ba)^* = (ab+ba)^*$

Ex 3



$$\begin{cases} q_1 = q_1 b + q_2 b + \varepsilon \\ q_2 = q_1 a \\ q_3 = q_2 a + q_3 a + q_3 b \end{cases}$$

$q_1 = q_1 b + q_1 ab + \varepsilon$

$q_1 = q_1 (b + ab) + \varepsilon$

$q_1 = \varepsilon (b+ab)^* = (b+ab)^*$

$q_2 = q_1 \cdot a$

$q_2 = (b+ab)^* a$

$q_3 = q_2 a + q_3 a + q_3 b$

$q_3 = q_2 a + q_3 (a+b)$

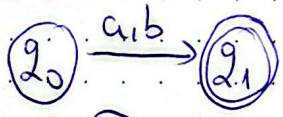
$q_3 = (b+ab)^* aa + q_3 (a+b)$

$q_3 = (b+ab)^* aa (a+b)^*$

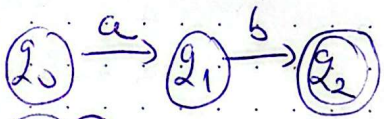
RE → FA

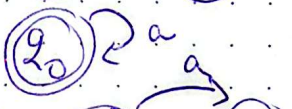
(1) Basic expressions

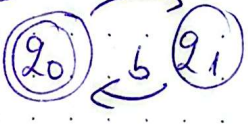
1. a: 

2. a+b: 

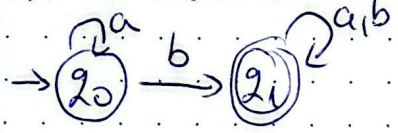
3. $(a+b)^*$ 

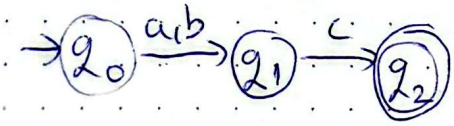
4. ab: 

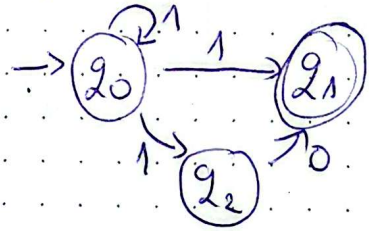
5. a^* 

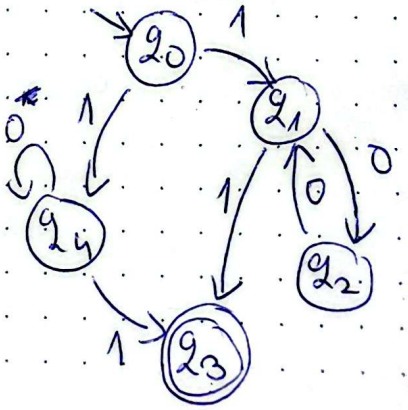
6. $(ab)^*$ 

(2) Other expressions

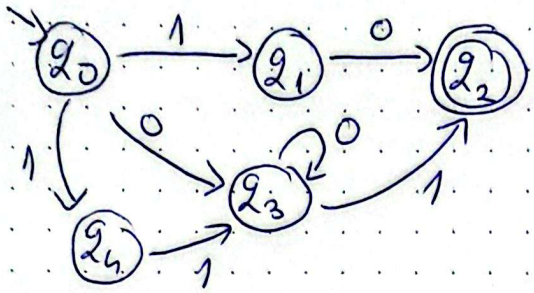
a) $a^* b (a+b)^*$ 

b) $(a+b)c$ 

c) $0^* 1 + 10$ 

d) $1(00)^* 1 + 10^* 1$ 

e) $10+(0+11)0^*1$



RE → RG

1. Do RE → FA
2. FA to RG

SIMPLIFY

Remove epsilon/null productions

(1) Eliminate epsilon productions from the following context free gramar

$S \to ABA$
$A \to 0A \mid \varepsilon$
$B \to 1B \mid \varepsilon$

(2) $S \to AB$
$A \to aAA \mid \varepsilon$
$B \to bBB \mid \varepsilon$

$\Rightarrow$

$S \to AB \mid A\varepsilon \mid \varepsilon B \mid \varepsilon$
$A \to a\varepsilon A \mid aA\varepsilon \mid a\varepsilon\varepsilon \mid aAA$
$B \to bBB \mid b\varepsilon B \mid bB\varepsilon \mid b\varepsilon\varepsilon$

$S \to AB \mid A \mid B \mid \varepsilon$
$A \to aA \mid a \mid aAA$
$B \to bBB \mid bB \mid b$

1. Identify null/ε prod: $A \to \varepsilon$, $B \to \varepsilon$
2. Find all prod where A, B appears in the right side and replace them one by one with ε

$S \to ABA \mid \varepsilon BA \mid A\varepsilon B \mid AB\varepsilon \mid A\varepsilon A \mid \varepsilon\varepsilon A \mid A\varepsilon\varepsilon \mid \varepsilon$

$\Rightarrow S \to ABA \mid BA \mid AB \mid B \mid AA \mid A \mid \varepsilon$

3. Do for the rest

$A \to 0A \mid 0\varepsilon \Rightarrow A \to 0A \mid 0$
$B \to 1B \mid 1\varepsilon \quad\quad B \to 1B \mid 1$

Remove useless symbols

useless (unreachable symbol)

I → cannot be derived from start symbol
(they are not in the left hand of any prod)

II → it did not produced a string of terminals (unproductive symbol)

ex: S → AB | a
A → b

B is not in left side

Solution: remove those productions

S → a
A → b

ex: S → AbbB | bbA (II)
S → bbB | bb
A → a
B → BB

! for RG is not the case !
! for CFG, it can be

Sol:
S → bbA
S → bb
A → a

Eliminate unit productions

$S \to aA \mid bB \mid C$

$A \to aS \mid aa$

$B \to b \mid A$

$C \to ab$

$S \to C$, $B \to A$ unit prod.

Eliminate $S \to C$

find C in rh side

$C \to ab$ eliminate and combine

$S \to C$ => $S \to ab$

Eliminate $B \to A$

find A in rh side

$A \to aS \mid aa$

=> $B \to aS \mid aa$

$S \to aA \mid bB \mid ab$

$A \to aS \mid aa$

$B \to b \mid aS \mid aa$

$C \to ab$

Order

0 → remove 1: ε-productions

2: UNIT (singulare) symbols

3: USELESS symbols (unreachable or unproductive)