Task 3
HW4
Optimization

Solve the following problem

$$\min_{w\in\mathbb{R}^n} \sum_{i=1}^{m} \alpha_i(\langle x_i, w\rangle - b_i)^2,$$ where $\alpha_i > 0$ are some constants

1. Feasible set $w\in\mathbb{R}^n$ - is convex.
2. Objective function $f(w)$ is a quadratic function (and differentiable) $\Rightarrow$ $f(w)$ - convex function.

1. and 2. $\Rightarrow$ We have unconstrained convex problem $\Rightarrow$
$\Rightarrow$ $\nabla f_w = 0$ is necessary and sufficient condition for global minimum.

$$f(w) = \sum_{i=1}^{m} \alpha_i(\langle x_i, w\rangle - b_i)^2$$

def
$$A_{m\times m} = \begin{bmatrix} \alpha_1 & & 0 \\ & \alpha_2 & \\ 0 & & \ddots \; \alpha_m \end{bmatrix}$$

$$\boxed{b^T A X w = \langle Ab, Xw\rangle = \langle Xw, Ab\rangle = w^T X^T A b}$$

$X_{m\times n}$ - matrix composed of x_i
b - vector composed of b_i

$$f(w) = A(Xw-b)^2 = (Xw-b)^T A(Xw-b) = w^T X^T A X w - w^T X^T A b -$$
$$- b^T A X w + b^T A b = w^T X^T A X w - 2w^T X^T A b + b^T A b$$
$$\nabla f_w(w) = 2X^T A X w - 2X^T A b = 2X^T A(Xw-b) = 0$$
$$X^T A X w = X^T A b$$

$$\boxed{w^* = (X^T A X)^{-1} X^T A b}$$

where:
$$A_{m\times n} = \begin{bmatrix} \alpha_1 & & 0 \\ & \alpha_2 & \\ 0 & & \ddots \; \alpha_m \end{bmatrix}$$

$X_{m\times n}$ - matix composed of x_i

b - vector composed of b_i