Integral Calculation and Visualization

AI Assistant

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Problem Statement

Calculate the definite integral of the function $f(x) = x^3 + 5x$ on the interval $[-4, 5]$ and provide a graphical representation of the function with the integrated area highlighted.

Integral Calculation

We need to calculate the definite integral:

$$\int_{-4}^{5} (x^3 + 5x)dx$$

First, find the indefinite integral (antiderivative) of $f(x) = x^3 + 5x$:

$$F(x) = \int (x^3 + 5x)dx = \frac{x^{3+1}}{3+1} + 5\frac{x^{1+1}}{1+1} + C = \frac{x^4}{4} + \frac{5x^2}{2} + C$$

Now, evaluate the definite integral using the Fundamental Theorem of Calculus:

$$\int_{-4}^{5} (x^3 + 5x)dx = \left[\frac{x^4}{4} + \frac{5x^2}{2}\right]_{-4}^{5}$$

$$= \left(\frac{5^4}{4} + \frac{5(5^2)}{2}\right) - \left(\frac{(-4)^4}{4} + \frac{5(-4)^2}{2}\right)$$

$$= \left(\frac{625}{4} + \frac{5(25)}{2}\right) - \left(64 + \frac{80}{2}\right)$$

$$= \left(\frac{625}{4} + \frac{125}{2}\right) - (64 + 40)$$

$$= \left(\frac{625}{4} + \frac{250}{4}\right) - (64 + 40)$$

$$= \frac{875}{4} - 104$$

$$= \frac{875}{4} - \frac{416}{4}$$

$$= \frac{459}{4}$$

$$= 114.75$$

The value of the definite integral is **114.75**.

Python Script for Plotting

The following Python script was used to generate the graph of the function and highlight the area under the curve.

Listing 1: plot_function.py

```
import numpy as np
import matplotlib.pyplot as plt

def f(x):
    return x**3 + 5*x

x = np.linspace(-6, 7, 500)
y = f(x)

plt.figure(figsize=(10, 6))
plt.plot(x, y, label='$f(x) = x^3 + 5x$', color='blue')

# Fill the area under the curve between -4 and 5
x_fill = np.linspace(-4, 5, 500)
y_fill = f(x_fill)
plt.fill_between(x_fill, y_fill, color='lightblue', alpha=0.5, label='Area of Integration')

plt.axvline(x=-4, color='gray', linestyle='--', linewidth=0.8)
plt.axvline(x=5, color='gray', linestyle='--', linewidth=0.8)
plt.axhline(y=0, color='black', linewidth=0.5)
plt.axvline(x=0, color='black', linewidth=0.5)

plt.title('Graph of $f(x) = x^3 + 5x$ and Area of Integration')
plt.xlabel('x')
plt.ylabel('f(x)')
plt.grid(True)
plt.legend()
plt.tight_layout()
plt.savefig('function_plot.png')
print("Plot saved as function_plot.png")
```

Graphical Representation

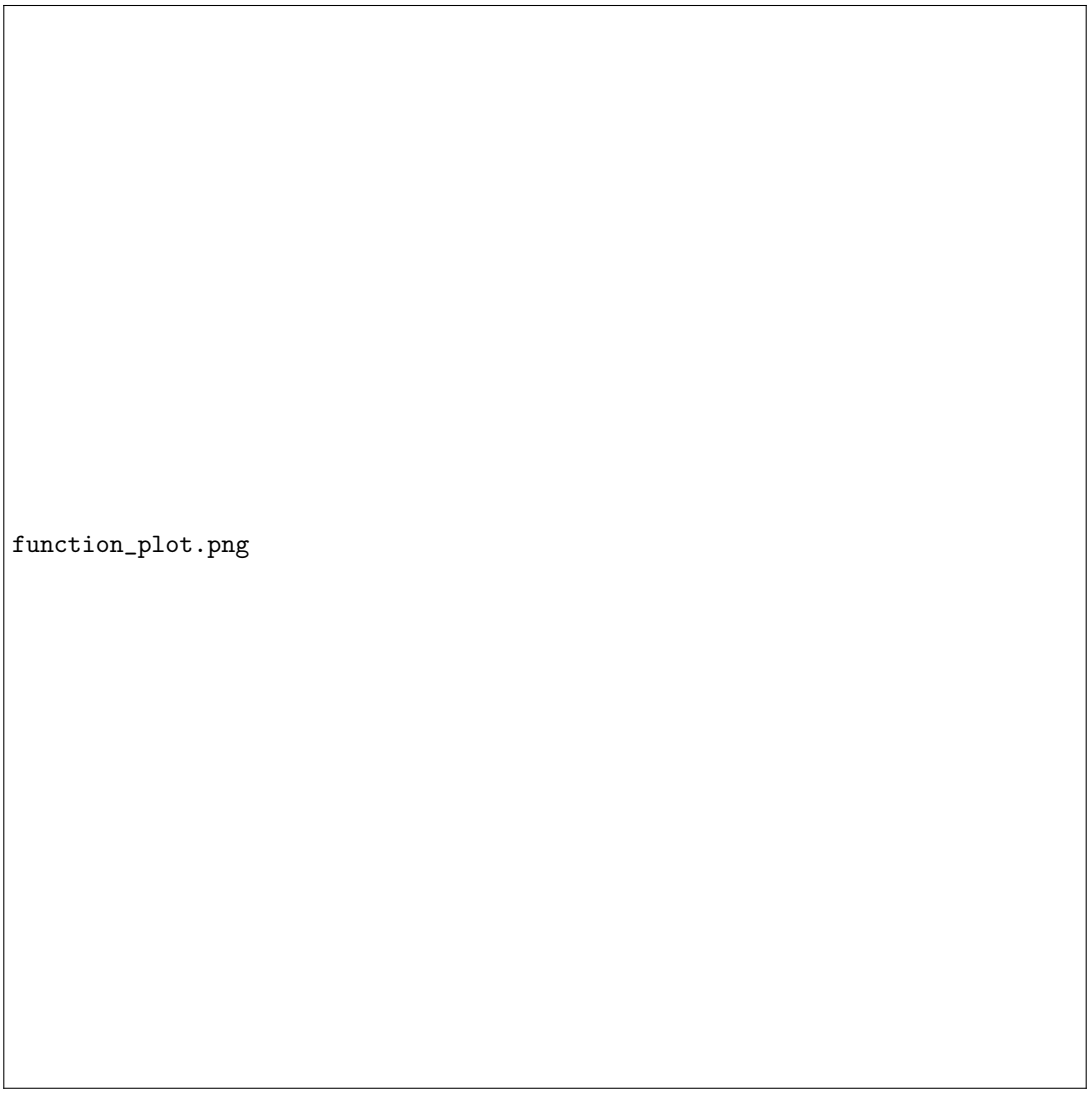

Figure 1: Graph of $f(x) = x^3 + 5x$ with the integrated area from -4 to 5 highlighted.