Laborator 2

Tema 1

a) $x[n] = e^{j\omega_0 n}$, $\forall n \in \overline{0, N-1}$

Arătați că TF a sinusoidei are următoarea expresie

$$X(\omega) = \frac{e^{-j(\omega-\omega_0)N/2}}{e^{-j(\omega-\omega_0)/2}} \cdot \frac{\sin\frac{(\omega-\omega_0)N}{2}}{\sin\frac{(\omega-\omega_0)}{2}}, \quad \forall \omega \in \mathbb{R} \quad \text{③}$$

Sol: Transformata Fourier pt x este

$$TF = X(\omega) = \sum_{n=0}^{N-1} x[n] \cdot e^{-j\omega n} = \sum_{n=0}^{N-1} e^{j\omega_0 n} \cdot e^{-j\omega n}$$

$$= \sum_{n=0}^{N-1} e^{-j(\omega n - \omega_0 n)} = \sum_{n=0}^{N-1} e^{-jn(\omega - \omega_0)} \quad \text{①}$$

Pentru simplitate $\omega - \omega_0 \overset{not}{=} \Omega$

$$TF = \sum_{\substack{n=0 \\ \text{①}}}^{N-1} e^{-jn\Omega} = \frac{e^{j\Omega N} - 1}{e^{j\Omega N} - e^{j\Omega(N-1)}} =$$

$$= \frac{e^{j\Omega(N-1)}}{e^{j\Omega(N-1)}} \cdot \frac{e^{j\Omega} - 1}{e^{j\Omega} - e^{j\Omega(N-1)}} = \frac{e^{j\Omega(N-1)}\left[e^{j\Omega} - e^{-j\Omega(N-1)}\right]}{e^{j\Omega(N-1)}\left[e^{j\Omega} - 1\right]}$$

$$= \frac{e^{j\Omega}\left(1 - e^{-j\Omega N}\right)}{e^{j\Omega}\left(1 - e^{-j\Omega}\right)} \Rightarrow TF = \frac{1 - e^{-j\Omega N}}{1 - e^{-j\Omega}} \quad \text{②}$$

$$\sin\frac{\Omega N}{2} = \frac{e^{j\frac{\Omega N}{2}} - e^{-j\frac{\Omega N}{2}}}{2j}$$

$$\sin\frac{\Omega}{2} = \frac{e^{j\frac{\Omega}{2}} - e^{-j\frac{\Omega}{2}}}{2j} \qquad \Bigg| \Rightarrow$$

$$\Rightarrow \frac{\sin\frac{\Omega N}{2}}{\sin\frac{\Omega}{2}} = \frac{e^{j\frac{\Omega N}{2}} - e^{-j\frac{\Omega N}{2}}}{e^{j\frac{\Omega}{2}} - e^{-j\frac{\Omega}{2}}}$$

$$\Rightarrow X(\omega) = \frac{e^{-j\frac{\Omega N}{2}} \cdot \left(e^{j\frac{\Omega N}{2}} - e^{-j\frac{\Omega N}{2}}\right)}{e^{-j\frac{\Omega}{2}} \left(e^{j\frac{\Omega}{2}} - e^{-j\frac{\Omega}{2}}\right)} =$$

③

$$= \frac{e^{-j\frac{\Omega N}{2}} \cdot e^{j\frac{\Omega N}{2}} - e^{-j\frac{\Omega N}{2}} \cdot e^{-j\frac{\Omega N}{2}}}{e^{-j\frac{\Omega}{2}} \cdot e^{j\frac{\Omega}{2}} - e^{-j\frac{\Omega}{2}} \cdot e^{-j\frac{\Omega}{2}}} =$$

$$= \frac{1 - e^{-j\left(\frac{\Omega N}{2} + \frac{\Omega N}{2}\right)}}{1 - e^{-j\left(\frac{\Omega}{2} + \frac{\Omega}{2}\right)}} \qquad \Rightarrow TF = \frac{1 - e^{-j\Omega N}}{1 - e^{-j\Omega}} \quad ②$$

Cum "1" = "2" și "3" = "2" ⇒ "1" = "3"

⇒ S-a arătat că mărimile are TF cerută