POLITECNICO MILANO 1863

Online Learning Application
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Pricing & Social Influence

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Introduction

Nowadays one big problem of e-commerces is to allocate the best price to its products so that, the seller can maximize its revenue.
The main issue is that increasing the price of a product leads to less people interested in that product, thus increasing the price is not necessarily beneficial to the seller. In contrast decreasing the price will increase the number of people interested in the product, but the revenue will be of course sub-optimal.
In order to maximize the revenue we can analyze the demand curve of a given product, which is a graphical representation of the relationship between the price p_i of a good or service i and the quantity demanded $q_i(p_i)$ for a given period of time, and find the price $\hat{p}$ such that:

$$\hat{p} = \arg\max_{p}(pq(p))$$

Unfortunately, in real world problems, the demand curve is not available, furthermore, we need to estimate this curve by interacting with the environment. One main problem of interacting with an unknown environment is that exploration costs a lot of money, so we want to find the best prices in the shortest amount of time to decrease the regret.
In order to do so, we can use reinforcement learning techniques such as Multi Armed Bandit (MAB) algorithms.

Practical example

In this project we want to study the case of a new e-commerce entering the market called ANS^2 that sells skateboarding clothes. More precisely, it is going to sell unisex t-shirts, hoodies, t-shirts, shoes and shirts.
For simplicity sake we can assume the website can sell an unlimited number of units without any storage cost whose goal is to minimize the cumulative regret while learning.
The web site of the vendor is structured as follows: in every webpage, a single product, called primary, is displayed together with its price. The user can add a number of units of this product to the cart. After the product has been added to the cart, two products, called secondary, are recommended. When displaying the secondary products, the price is hidden. Furthermore, the products are recommended in two slots, one above the other, thus providing more importance to the product displayed in the slot above. The website will propose only products that the customer has never seen before. If the user clicks on a secondary product, a new tab on the browser is opened and, in the loaded webpage, the clicked product is displayed as primary together with its price.

One main consideration we want to make is that the customer may buy different products during a visit, thus the price for a specific product may influence the total income generated by the customer.
For instance, let us assume that a customer lands on the webpage displaying a t-shirt: if the price is too high, the probability to buy that product is lower, but not only, also the probability to see the secondary products is lower, so it will decrease the probability that a customer visits and buys new products. In conclusion, when we choose the price for a specific product we have also to consider the indirect reward it will generate.

1 Step : Environment

The main aim of the environment is to simulate a real-world scenario. To simulate all the components we divide the model into various classes.

- Environment class: It is the wrapper that manages the environment and all its functions. There are two specializations of this class created for some specific use cases (which are EnvironmentContextual and EnvironmentNonStationary).
- Simulator class: It manages the simulation of the customers' interactions.
- Customer class: It contains all the information that defines a type of customer.

1.1 Parameters

The environment has a lot of parameters and each of them has a direct and significant impact on the behavior of the model. Some of the parameters are specifics of the environment:

- `customers_distribution` is a list of 4 floating values, that sum to 1. It indicates the probability of each type of customer appearing.
- `customer_per_day` is the average number of customers in a day.
- `variance_customers` is the standard deviation of the number of customers in a day.
- `products_graph` is the graph that indicates which is the primary and secondary product.
- `p_lambda` is the probability of observing each slot. The first value is 1, while the second is a number smaller than 1.
- `prices` indicates the reward for each product and each price level. Therefore, it is a 4 x 5 matrix.

Additionally, some parameters are specifics of the customer:

- `features` is the pair of binary features. In our case the first feature is associated to gender, whereas the second one to the age (for simplicity young/old)
- `alpha` is the probability distribution of starting from each product.
- `buy_distribution` is the probability of buying products given a selected price and type of product.
- `num_prods_distribution` is a matrix containing for each pair (product, price) the hyper parameter controlling the amount of units the customer is likely to buy for that specific product. In this case we have decided to model the customer's behaviour with a geometric distribution, thus the hyper parameters are the inverse of the means of the associated geometric distributions. The main idea behind the geometric distribution is that is a discrete distribution and is monotonically decreasing: higher is the number of products bought, lower will be the probability to buy that amount of units.
- `click_graph` is the probability of clicking the product given a selected price and type of product.

1.2 Learner interaction

The main aim of the learner is to minimize the cumulative regret by selecting the best price levels. Therefore, at the beginning of each day, the learner select the price levels (which is the super arm), and then at the end of the day it will obtain a report containing all the information about the customer activities, i.e. number of times bought, number of products seen, number of customers.

1.3 Customer interaction

The customer interaction works in the following manner:

1. Depending on the `alpha` distribution a starting point is randomly chosen.

2. The customer opens the page and she will buy one or more products with a probability depending on `buy_distribution`. If she does not buy the simulation stops, otherwise the number of items bought is sampled from a geometrical distribution.

3. Then, with a probability that depends on λ and the `click_graph`, she explores a different product. However, if she has already seen this product, she will not open that page.

From step 3 to step 6 the main assumption is that we have different classes of customers interact with the website, and each of them has a different behaviour. For each step, we know the distribution of the customers and some characteristics of them (for instance the mean of the number of items a customer buys for a specific product), but the website is not able to identify the customer is interacting with it, thus we are not able to estimate the unknown parameters for each customer but only an aggregated estimate.

1.4 Selection of the super arm

Since we have to take in consideration also the indirect income generated by other products that are bought after buying the first product, we have to find the combination of prices such that maximizes the overall income considering the indirect margins, thus we have to solve a combinatorial problem to select the best super arm to play.
In order to do so, we have to compute the believed expected reward $\mathbb{E}[r_a]$ for each superarm a and choose the arm with the highest expected reward. We compute the expected reward as follows:

$$\mathbb{E}[r_a] = \sum_{i \in \mathscr{C}} r_{a,i} w_i$$

where $\mathscr{C}$ is the set of indexes of customers, $r_{a,i}$ is the expected reward given the super arm a for customer i and w_i is the probability to see customer i.

1.4.1 Monte Carlo methods

A way to compute the expected reward for a given customer i is to use a Monte Carlo simulation. Monte Carlo methods, or Monte Carlo experiments, are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results. The underlying concept is to use randomness to solve problems that might be deterministic in principle.

Basically, we have 5 seeds (the products) and we run N simulations starting from each seed. A seed is a starting node in a graph, whose edges $e_{i,j}$ represent the probability of clicking product j given that product i has been bought, whereas each node has an activation threshold that coincides with the probability of buying that product.
The simulation works as follows:

1. Explore the simulation graph in a depth first search tree fashion. If a node has been already visited, it can not be reached anymore.

2. When selecting a node j, draw a sample x from a Bernoulli distribution $\mathscr{B}e(t_{i,p,j})$ where $t_{i,p,j}$ is the probability that customer i buys product j at price p. If $x < t_{i,p,j}$, activate node j, otherwise stop the branch.
   We can keep track the number of times a node j has been activated with the variable Λ_j.

3. When expanding an active node i towards node j, draw a sample x from a Bernoulli distribution $\mathscr{B}e(e_{i,j})$. If $x < e_{i,j}$ move towards node j, otherwise stop the branch.

Once a node is activated we draw a sample n from the distribution of number of items bought for that node (product). Every time a product is bought, we update the number of times that product has been bought:

$$n_{a,p} = n_{a,p} + n$$

where $n_{a,p}$ is the current number of units bought for product p given the super arm a.
Here the code for a single simulation:

```
def shopping_dfs(self, primary, displayed_primary, report, super_arm, c):
  displayed_primary[primary] = True
  report.seen(primary)
  if np.random.random() < c.get_probability_buy(primary, super_arm[primary]):
    amount = c.get_num_prods(primary, super_arm[primary]) #1
    report.bought(primary, amount)
    click_prob = [c.get_probability_click(primary, secondary) for secondary in
    ↪  self.products_graph[primary]]
    for secondary, edge_prob, lamb in zip(self.products_graph[primary],
    ↪  click_prob, lamb_SLOTS):
      if not displayed_primary[secondary] and np.random.random() < lamb *
      ↪  edge_prob:
        report.move(primary, secondary)
        self.shopping_dfs(secondary, displayed_primary, report, super_arm, c)
```

Finally we compute the expected reward as:

$$r_{a,i} = \frac{1}{N} \sum_{p \in \mathscr{P}} \alpha_{i,p} a_p n_{a,p}$$

where $\mathscr{P}$ is the set of indexes of the products, $\alpha_{i,p}$ is the probability for customer i to start from the seed p and a_p is the price selected for the product p. We can also compute the activation probability (probability that a specific item is bought) of each product as:

$$\pi_{p,i} = \frac{1}{N} \sum_{p \in \mathscr{P}} \alpha_{i,p} \Lambda_p$$

Thanks to the following theorem we have some theoretical guarantees on the accuracy of this method.

Theorem With probability of at least $1 - \delta$, the estimated activation probability of every node is subject to an additive error of $\pm \epsilon n$ when the number of repetitions is:

$$R = O(\frac{1}{\epsilon^2} log(\mid S \mid) log(\frac{1}{\delta}))$$

where S is the number of seeds (5) and n is the number of nodes in the graph (still 5).

In our case, if we want to have an additive error of $\epsilon n = 0.1$ ($\epsilon = 0.02$) with a probability of 90 % ($\delta = 0.1$), we need to run $R = 1748$, thus the number of simulations for each product for each customer is $N = 350$. In conclusion, this method is stochastic and very noisy, and to obtain a decent estimate we need to run a massive number of simulations which makes the simulation process astonishingly slow.
Given the size of the parameters for this problem, we can afford to compute the exact value for the expected value for a given arm in a reasonable time with a different approach.

1.4.2 Dynamic programming approach

To overcome the limitation of the Monte Carlo simulation we develop a dynamic programming solution that returns the expected number of items bought. This method has a time complexity of $\Theta(2^N NM)$, where N is the number of items and M is the number of types of customers, therefore, this solution is only feasible because the number of items is quite small.

```python
def run_dp(self, super_arm):
  ans = 0
  for c, p in zip(self.customers, self.customers_distribution):
    @lru_cache(maxsize=None)
    def dp(primary, mask):
      mask |= 1 << primary
      ans = np.zeros(5)
      ans[primary] = 1 / c.num_prods_distributions[primary][super_arm[primary]]
      click_prob = [c.get_probability_click(primary, secondary) for secondary in
      ↪  self.products_graph[primary]]
```

```
    for secondary, edge_prob, lamb in zip(self.products_graph[primary],
    ↪  click_prob, lamb_SLOTS):
      if (mask & (1 << secondary)) == 0:
        ans += lamb * edge_prob * dp(secondary, mask)
    ans *= c.get_probability_buy(primary, super_arm[primary])
    return ans
  for primary, alpha in enumerate(c.get_distribution_alpha()):
    ans += p * alpha * dp(primary, 0)
return ans
```

2 Step : Optimization algorithm

Here we consider the case in wich all the parameters are known and the goal is to maximize the cumulative expected reward, following a greedy approach.
The algorithm works as follow:

1. set the prices of all the pruducts with the lowest one
2. collect the reward obtained by increasing, each time, the price of just one product of the original super arm
3. compare the five different configuration obtained with the first one. There could be two case
   (a) there is an increase of the reward, so select the one that gave the maximum reward (the highest increase) as the best one and repeat the algorithm from point 2
   (b) there is no increse (all the new configuration is worst than the previous one) and stop the algoritm
4. Return the actual best configuration.

2.1 Results

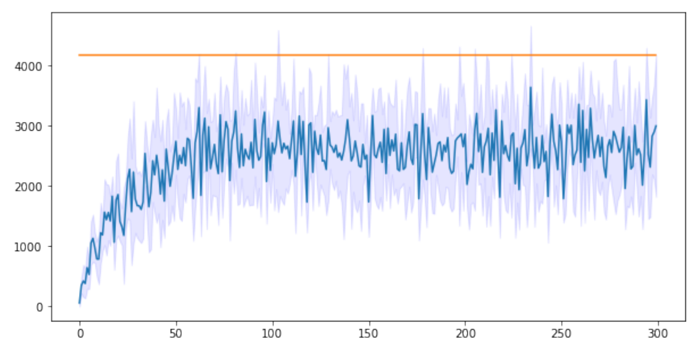


Figure 1: Reward

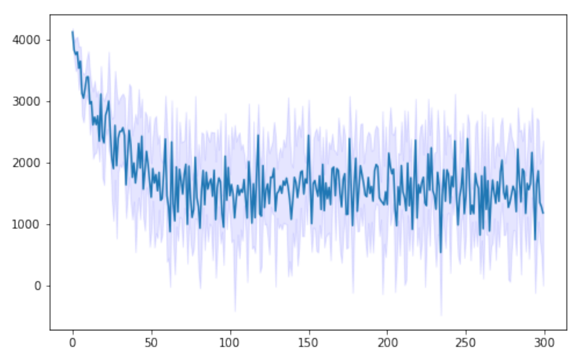


Figure 2: Regret

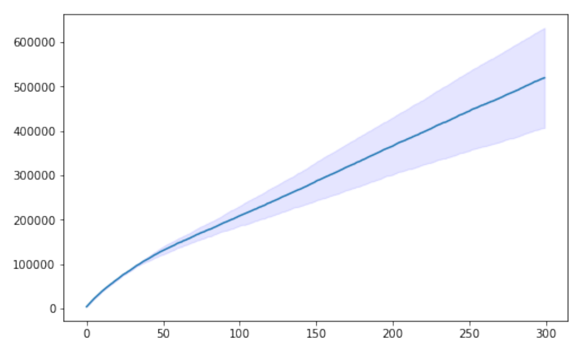


Figure 3: Cumulative regret

3 Step : Optimization with uncertain conversion rates

3.1 Results

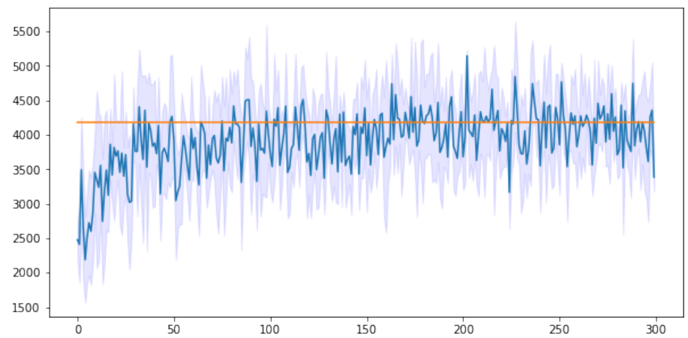


Figure 4: UCB Reward

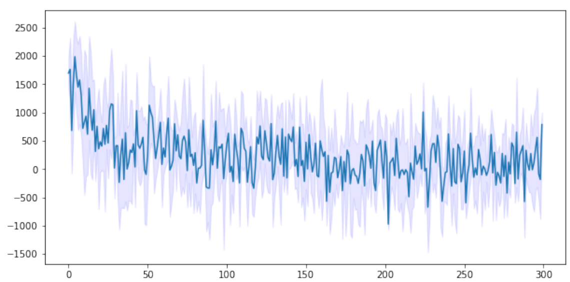


Figure 5: UCB Regret

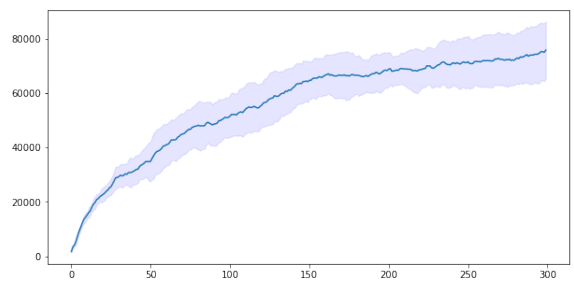
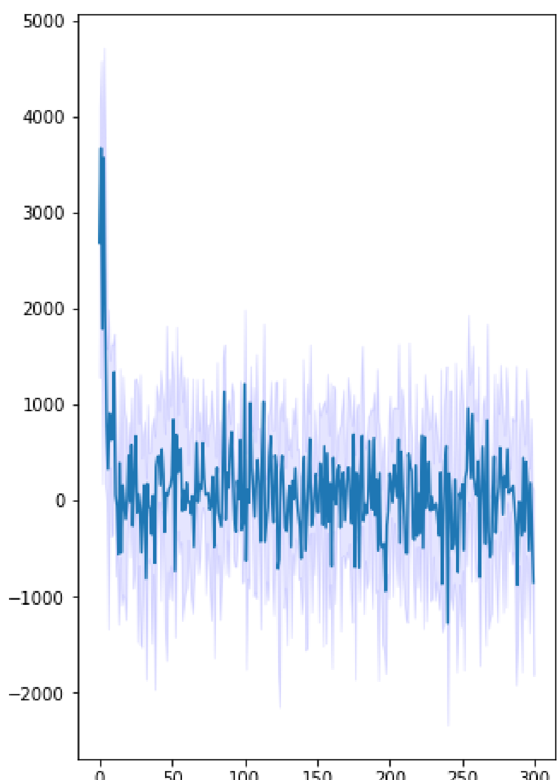


Figure 6: UCB Cumulative regret



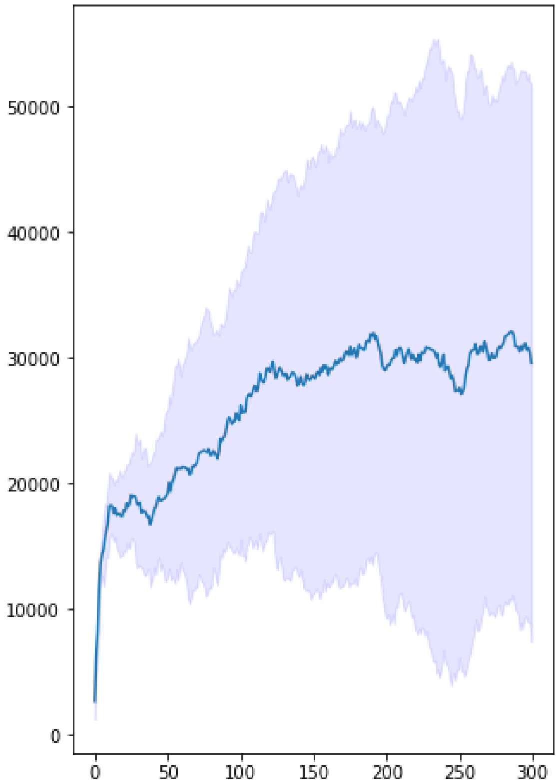


Figure 9: TS Cumulative regret

4 Step : Optimization with uncertain conversion rates, α ratios, and number of items sold per product

4.1 Results

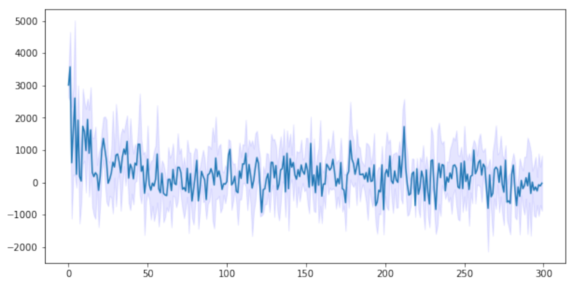


Figure 11: UCB Regret

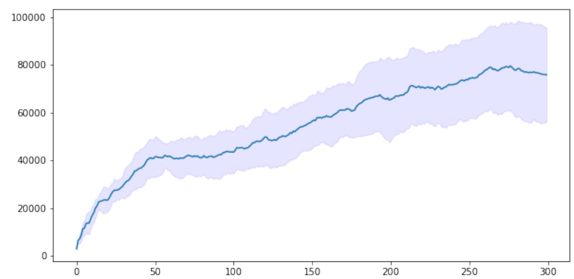


Figure 12: UCB Cumulative regret

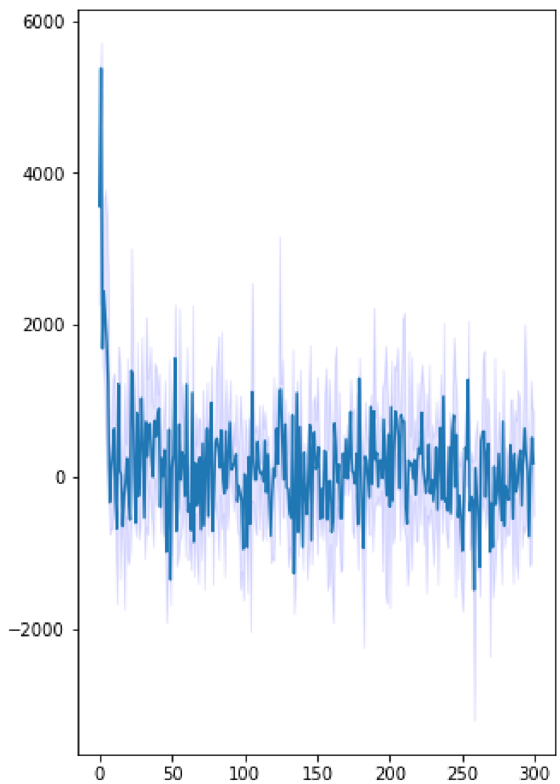


Figure 14: TS Regret

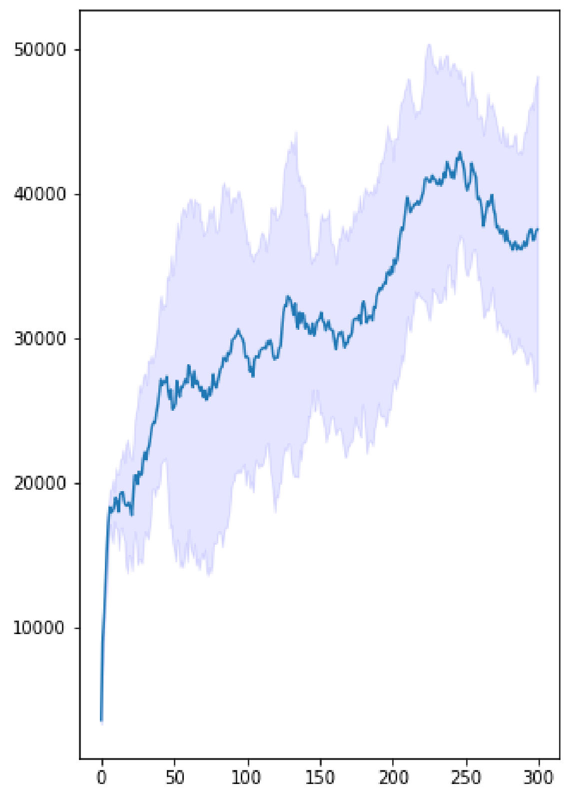


Figure 15: TS Cumulative regret

5 Step : Optimization with uncertain graph weights

5.1 Results

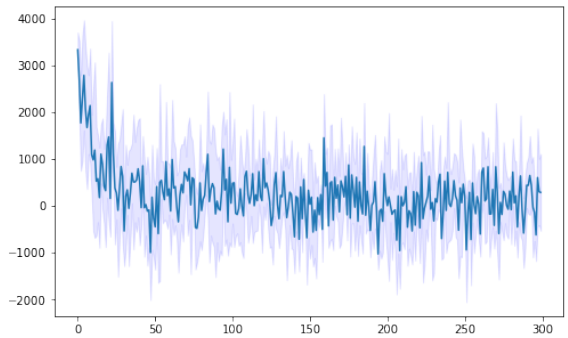


Figure 17: UCB Regret

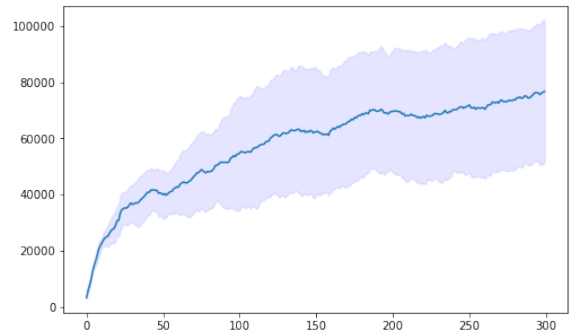


Figure 18: UCB Cumulative regret

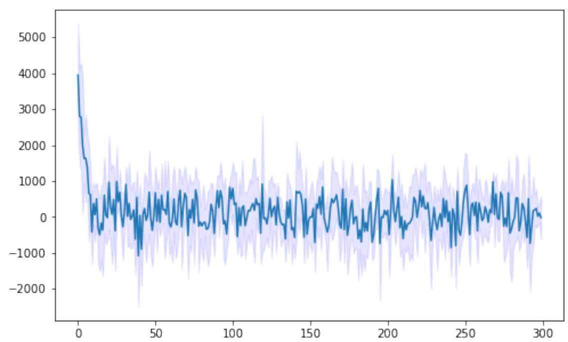


Figure 20: TS Regret

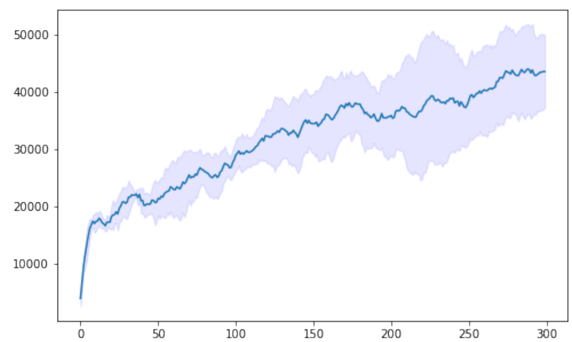


Figure 21: TS Cumulative regret

6 Step : Non-stationary demand curve

6.1 Results

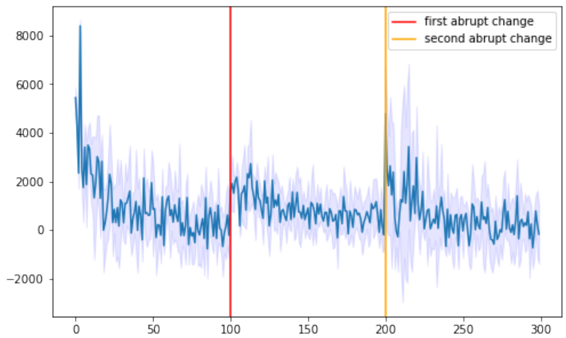


Figure 23: Change detection UCB Regret

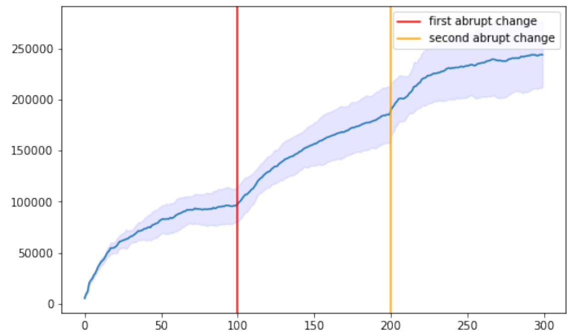


Figure 24: Change detection UCB Cumulative regret

7 Step : Context generation

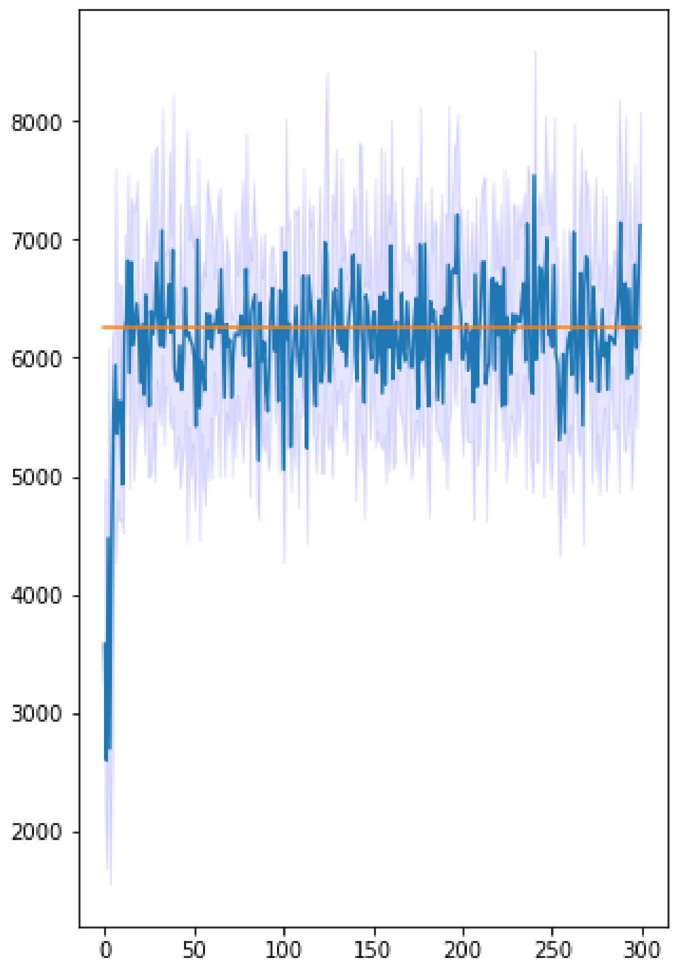


Figure 7: TS Reward

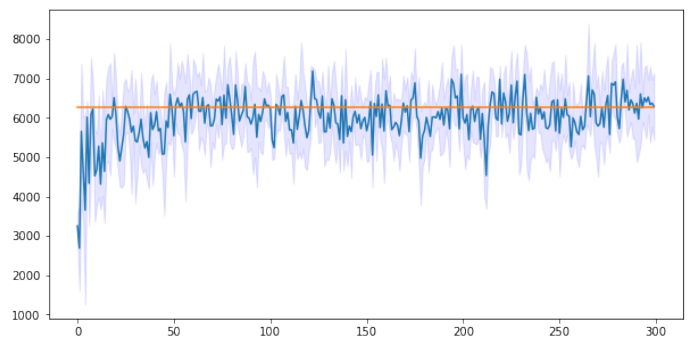


Figure 10: UCB Reward

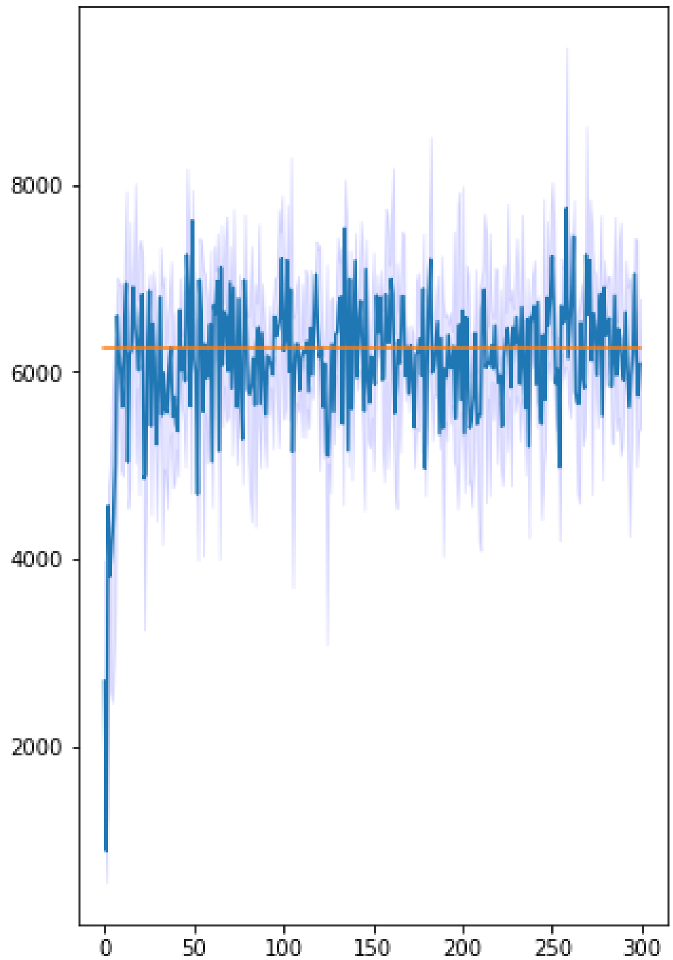


Figure 13: TS Reward

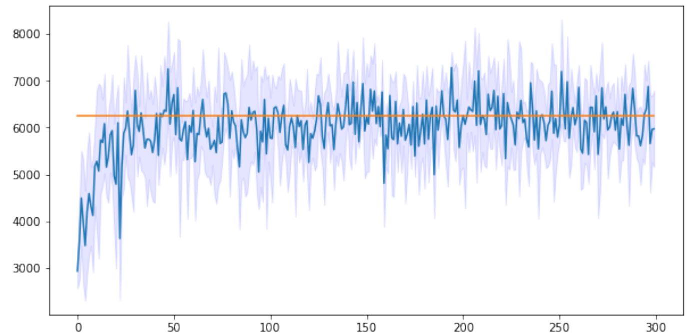


Figure 16: UCB Reward

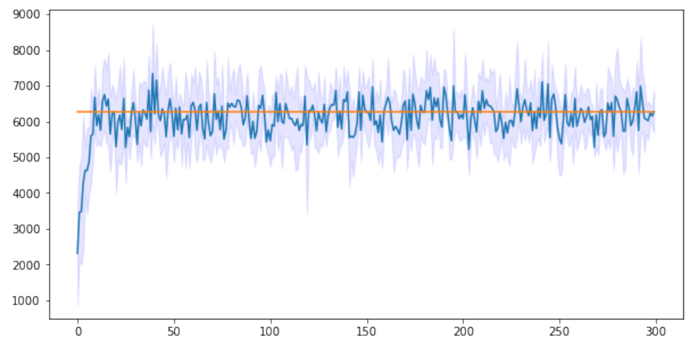


Figure 19: TS Reward

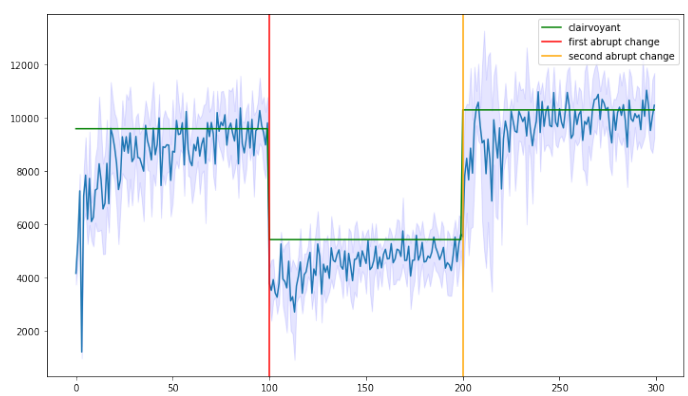


Figure 22: Change detection UCB Reward