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## 1 Chapter 0 — Lab Solutions

*Vlad Prytula*

These solutions demonstrate how theory meets practice in reinforcement learning. Every solution weaves mathematical analysis with runnable code, following the Application Mode principle: **mathematics and code in constant dialogue**.

All outputs shown are actual results from running the code with the specified seeds.

## 1.1 Lab 0.1 — Tabular Boost Search (Toy World)

**Goal:** Reproduce the  $\geq 90\%$  of oracle guarantee using `scripts/ch00/toy_problem_solution.py`.

### 1.1.1 Solution

```
from scripts.ch00.toy_problem_solution import (
    TabularQLearning,
    discretize_action_space,
    run_learning_experiment,
    evaluate_policy,
    OraclePolicy,
)

# Configure experiment (parameters from exercises_labs.md)
actions = discretize_action_space(n_bins=5, a_min=-1.0, a_max=1.0)
print(f"Action space: {len(actions)} discrete templates (5x5 grid)")

agent = TabularQLearning(
    actions,
    epsilon_init=0.9,
    epsilon_decay=0.995,
    epsilon_min=0.05,
    learning_rate=0.15,
)

results = run_learning_experiment(
    agent,
    n_train=800,
    eval_interval=40,
    n_eval=120,
    seed=314,
)

# Compute oracle baseline
oracle = OraclePolicy(actions, n_eval=200, seed=314)
oracle_results = evaluate_policy(oracle, n_episodes=300, seed=314)
oracle_mean = oracle_results['mean_reward']

pct_oracle = 100 * results['final_mean'] / oracle_mean
print(f"Final mean reward: {results['final_mean']:.2f} (target >= 0.90 * oracle)")
print(f"Per-user reward: {results['final_per_user']}")
```

#### Actual Output:

```
Action space: 25 discrete templates (5x5 grid)
Oracle: Computing optimal actions via grid search...
price_hunter: w*=(0.5, 0.0), Q*=17.80
premium: w*=(-1.0, 1.0), Q*=19.02
```

bulk\_buyer: w\*=(0.5, 1.0), Q\*=12.88

Final mean reward: 16.13 (target  $\geq 0.90 \times \text{oracle}$ )

Per-user reward: {'price\_hunter': 14.62, 'premium': 22.65, 'bulk\_buyer': 11.02}

Oracle mean: 16.77

Percentage of oracle: 96.2%

Result: SUCCESS

### 1.1.2 Analysis

#### 1. We achieve 96.2% of oracle performance—well above the 90% target.

The Q-learning agent learns effective context-adaptive policies purely from interaction data.

#### 2. Per-User Performance Breakdown:

Segment	Q-Learning	Oracle	% of Optimal
price_hunter	14.62	17.80*	82.1%
premium	22.65	19.02*	119.0%**
bulk_buyer	11.02	12.88*	85.6%

\*Oracle Q-values are per-action estimates; actual oracle mean across users is 16.77.

**Why does premium exceed oracle estimates?** The stochasticity in user interactions means different random seeds produce different outcomes. The agent found an action that happened to perform well on the evaluation seed.

#### 3. Learned Policy vs. Oracle Policy:

User Type	Learned Action	Oracle Action
price_hunter	(0.5, 0.5)	(0.5, 0.0)
premium	(-1.0, 0.0)	(-1.0, 1.0)
bulk_buyer	(-0.5, 0.5)	(0.5, 1.0)

The learned actions differ from oracle because: 1. Q-table hasn't converged perfectly in 800 episodes 2. Stochastic rewards mean multiple actions have similar expected values 3. The  $5 \times 5$  grid may not include the truly optimal continuous action

**Key Insight:** Even without matching the oracle's exact actions, Q-learning achieves near-oracle reward. This demonstrates the robustness of value-based learning.

---

## 1.2 Exercise 0.2 (from exercises\_labs.md) — Stress-Testing Reward Weights

**Goal:** Validate that oversized engagement weight  $\delta$  inflates rewards despite unchanged GMV.

### 1.2.1 Solution

```
from scripts.ch00.toy_problem_solution import (
    USER_TYPES, compute_reward, rank_products, simulate_user_interaction,
)

# Exact parameters from exercises_labs.md
alpha, beta, delta = 0.6, 0.3, 0.3 # delta intentionally oversized
user = USER_TYPES["price_hunter"]
ranking = rank_products(0.8, 0.1)
interaction = simulate_user_interaction(user, ranking, seed=7)
reward = compute_reward(interaction, alpha=alpha, beta=beta, gamma=0.0, delta=delta)
print(f"Reward with delta={delta:.1f}: {reward:.2f}")
```

#### Actual Output:

```
Interaction: {'clicks': [5], 'purchases': [], 'n_clicks': 1, 'n_purchases': 0, 'gmv': 0.0, 'cmv': 0.0}
Reward with delta=0.3: 0.30
```

This matches the expected output in `exercises_labs.md`.

### 1.2.2 Extended Analysis

Running 100 samples per user type with a “clickbait” ranking (high discount boost):

Standard weights ( $\alpha=0.6$ ,  $\beta=0.3$ ,  $\delta=0.1$ ):

Ratio  $\delta/\alpha = 0.167$

price_hunter	: R=20.01, GMV=28.30, CM2=9.60, Clicks=1.56
premium	: R=12.71, GMV=17.98, CM2=6.17, Clicks=0.76
bulk_buyer	: R=11.35, GMV=16.19, CM2=5.12, Clicks=1.01

Oversized delta ( $\alpha=0.6$ ,  $\beta=0.3$ ,  $\delta=0.3$ ):

Ratio  $\delta/\alpha = 0.500$  (5x above guideline!)

price_hunter	: R=20.32 (+1.6%), GMV=28.30, Clicks=1.56	<-- Same GMV, higher reward!
premium	: R=12.86 (+1.2%), GMV=17.98, Clicks=0.76	
bulk_buyer	: R=11.55 (+1.8%), GMV=16.19, Clicks=1.01	

**Key Observation:** With  $\delta/\alpha = 0.50$ , reward increases  $\sim 1.5\%$  while GMV stays constant. The agent could learn to prioritize clicks over conversions—a form of engagement gaming.

**Guideline:** Keep  $\delta/\alpha \leq 0.10$  to ensure GMV dominates the reward signal.

---

## 1.3 Exercise 0.1 — Reward Sensitivity Analysis

**Goal:** Compare learned policies under three reward configurations.

### 1.3.1 Solution

```
# Three configurations as specified in the exercise
configs = [
    ((1.0, 0.0, 0.0), "Pure GMV"),
```

```

    ((0.4, 0.5, 0.1), "Profit-focused"),
    ((0.5, 0.2, 0.3), "Engagement-heavy"),
]

# Run Q-learning for each configuration
for weights, label in configs:
    result = run_sensitivity_experiment(weights, label, n_train=1200, seed=42)
    print(f"{label} (alpha={weights[0]}, beta={weights[1]}, delta={weights[2]}):")
    print(f"  Learned policy: ...")

```

### Actual Output:

Pure GMV (alpha=1.0, beta=0.0, delta=0.0):

Final reward: 23.99

Final GMV: 23.99

Learned policy:

```

price_hunter    -> w_discount=+1.0, w_quality=+0.0
premium         -> w_discount=-1.0, w_quality=+1.0
bulk_buyer      -> w_discount=+0.5, w_quality=+1.0

```

Profit-focused (alpha=0.4, beta=0.5, delta=0.1):

Final reward: 14.03

Final GMV: 24.24

Learned policy:

```

price_hunter    -> w_discount=+0.5, w_quality=+0.0
premium         -> w_discount=-1.0, w_quality=+1.0
bulk_buyer      -> w_discount=+0.5, w_quality=+1.0

```

Engagement-heavy (alpha=0.5, beta=0.2, delta=0.3):

Final reward: 14.40

Final GMV: 24.48

Learned policy:

```

price_hunter    -> w_discount=+1.0, w_quality=-1.0
premium         -> w_discount=-1.0, w_quality=+1.0
bulk_buyer      -> w_discount=+0.5, w_quality=+1.0

```

### 1.3.2 Analysis

Does the policy change? Yes, for some user types:

Configuration	price_hunter	premium	bulk_buyer
Pure GMV	(+1.0, 0.0)	(-1.0, +1.0)	(+0.5, +1.0)
Profit-focused	(+0.5, 0.0)	(-1.0, +1.0)	(+0.5, +1.0)
Engagement-heavy	(+1.0, -1.0)	(-1.0, +1.0)	(+0.5, +1.0)

### Key Observations:

1. **premium users:** Policy is stable across all configurations at  $(-1.0, +1.0)$ —quality-heavy.

This makes sense: premium users convert well on quality products regardless of reward weighting.

2. **price\_hunter**: Shows the most variation:

- Pure GMV:  $(+1.0, 0.0)$  — maximize discount, ignore quality
- Profit-focused:  $(+0.5, 0.0)$  — moderate discount (high-margin products)
- Engagement-heavy:  $(+1.0, -1.0)$  — extreme discount, actively penalize quality (clickbait risk!)

3. **bulk\_buyer**: Stable at  $(+0.5, +1.0)$ —balanced approach works across configurations.

**The engagement-heavy case shows clickbait risk:** For price\_hunter, the policy shifts to  $(+1.0, -1.0)$ , actively demoting quality products. This maximizes clicks but may hurt long-term user satisfaction.

---

## 1.4 Exercise 0.2 — Action Geometry and the Cold Start Problem

**Goal:** Understand how exploration strategy effectiveness depends on policy quality.

This exercise teaches a fundamental insight through a structured investigation. We start with a hypothesis, test it empirically, discover it's wrong, diagnose why, and then design an experiment that validates when the original intuition *does* hold.

---

### 1.4.1 Part A — The Hypothesis

Intuition suggests that **local exploration**—small perturbations around our current best action—should be more efficient than **uniform random sampling**. After all, once we find a good region of action space, why waste samples exploring far away?

*# Two exploration strategies:*

*# - Uniform: When exploring (with prob epsilon), sample ANY action uniformly*

*# - Local: When exploring (with prob epsilon), sample NEIGHBORS of current best (+/-1 grid c*

**Hypothesis:** Local exploration converges faster because it exploits structure near good solutions (gradient descent intuition).

Let's test this.

---

### 1.4.2 Part B — Cold Start Experiment

We train both agents from scratch ( $Q=0$  everywhere) for 500 episodes.

```
from scripts.ch00.lab_solutions import exercise_0_2_action_geometry
```

```
results = exercise_0_2_action_geometry(  
    n_episodes_cold=500,  
    n_episodes_warmup=200,  
    n_episodes_refine=300,
```

```

    n_runs=5,
    seed=42,
)

```

### Actual Output (Cold Start):

Starting BOTH agents from random initialization (Q=0 everywhere).  
 Training each for 500 episodes...

Results (averaged over 5 runs):

Strategy	Final Reward	Std
-----	-----	-----
Uniform	15.66	18.48
Local	10.33	15.43

Winner: Uniform (by 34.0%)

**\*\* SURPRISE!** Uniform exploration wins decisively.  
 Our hypothesis was WRONG. But why?

### 1.4.3 Part C — Diagnosis: The Cold Start Problem

#### Why does local exploration fail from cold start?

The problem is **initialization**. With Q=0 everywhere: - **Uniform agent**: Explores the ENTIRE action space randomly - **Local agent**: Starts at action index 0 (the corner:  $w = (-1, -1)$ ) and only explores NEIGHBORS of that corner!

The local agent is doing “**local refinement of garbage**”—there’s no good region nearby to refine. It’s stuck in a bad neighborhood.

#### Action Coverage After 200 Episodes:

```

Uniform explored 18/25 actions (72%)
Local explored   4/25 actions (16%)

```

Local agent never discovered the optimal region!

This is the **COLD START PROBLEM**: > Local exploration assumes you’re already in a good basin. > From random initialization, you’re not.

### 1.4.4 Part D — Warm Start Experiment

If local exploration fails from cold start, when SHOULD it work?

**Answer:** After we’ve found a good region via global exploration!

**Experiment Design:** 1. Train with UNIFORM for 200 episodes (find good region) 2. Then continue training with each strategy for 300 more episodes 3. Compare which strategy refines better from this warm start

### Actual Output (Warm Start):

Results (averaged over 5 runs, after 200 warmup episodes):

Strategy	Final Reward	Std
-----	-----	-----
Uniform	14.04	16.76
Local	14.58	17.10

Winner: Local (by 3.7%)

Local exploration is now COMPETITIVE (or wins)!

Once we're in a good basin, local refinement works.

**Key Observation:** The gap between uniform and local *reverses* when starting from a warm policy. From cold start, uniform wins by 34%. From warm start, local actually *wins* by 3.7%! Local exploration works—and even excels—*once you're already in a good region*.

---

### 1.4.5 Part E — Synthesis: Adaptive Exploration

The key insight: **EXPLORATION STRATEGY SHOULD ADAPT TO POLICY MATURITY.**

Training Phase	Recommended Strategy	Rationale
Early (cold)	Uniform/global	Find good regions across action space
Late (warm)	Local/refined	Exploit structure within good regions

**This is exactly what sophisticated algorithms implement:**

Algorithm	Mechanism	Effect
<b>SAC</b>	Entropy bonus $\alpha \cdot H(\pi)$	Encourages broad exploration; decays naturally as policy sharpens
<b>PPO</b>	Decaying entropy coefficient	High entropy early (explore) $\rightarrow$ low late (exploit)
$\epsilon$ -greedy	$\epsilon$ decays ( $0.9 \rightarrow 0.05$ )	Global early, local late
<b>Boltzmann</b>	Temperature $\tau$ decays	High $\tau$ = uniform, low $\tau$ = local around best

### Connection to Theory:

The cold start problem explains why **optimistic initialization** (starting with high Q-values) helps—it forces global exploration before settling into local refinement. Starting with  $Q = \infty$  everywhere means the agent must try everything before any action looks “best.”



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### 1.4.6 Summary Table

Experiment	Uniform	Local	Winner	Gap
Cold start	15.66	10.33	Uniform	34.0%
Warm start	14.04	14.58	<b>Local</b>	3.7%

**The same exploration strategy can WIN or LOSE depending on whether the policy is cold (random) or warm (trained).** In fact, the winner *reverses*: uniform dominates cold start, but local wins after warm-up!

This is not a bug—it’s a fundamental insight about RL exploration.

---

### 1.4.7 Practical Guideline

When designing exploration strategies, ask:

“Is my policy already in a good region?”

- **If no** → Use global/uniform exploration first
- **If yes** → Local refinement is efficient

This principle applies beyond toy examples. In production RL: - **Curriculum learning** starts with easier tasks (warm start for harder ones) - **Transfer learning** initializes from pre-trained policies (warm start) - **Reward shaping** guides early exploration toward good regions

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## 1.5 Exercise 0.3 — Regret Analysis

**Goal:** Track cumulative regret and understand what it tells us about learning.

### 1.5.1 Background: What Is Regret?

**Cumulative regret** measures total performance loss compared to an oracle:

$$\text{Regret}(T) = \sum_{t=1}^T (R_t^* - R_t)$$

where  $R_t^*$  is the oracle’s reward and  $R_t$  is the agent’s reward at episode  $t$ .

**Sublinear regret** means  $\text{Regret}(T) = o(T)$ , i.e., average regret per episode vanishes:

$$\frac{\text{Regret}(T)}{T} \rightarrow 0 \quad \text{as } T \rightarrow \infty$$

This confirms the agent is *learning*—eventually performing as well as the oracle.

### 1.5.2 Solution

```
# Cumulative regret:  $\text{Regret}(T) = \sum_{t=1}^T (R*_t - R_t)$ 
# where  $R*_t$  is oracle reward and  $R_t$  is agent reward

# Run 2000 episodes with geometric decay:  $\text{eps}_t = 0.9 * 0.998^t$ 
```

#### Actual Output:

=== Exercise 0.3: Regret Analysis ===

Oracle policy computed:

```
price_hunter: w*=(1.0, -0.5), Q*=17.46
premium: w*=(-0.5, 1.0), Q*=19.22
bulk_buyer: w*=(0.5, 1.0), Q*=12.04
```

Regret Analysis Summary:

```
Total episodes: 2000
Final cumulative regret: 5680.0
Average regret per episode: 2.840
Regret growth slowing? True (avg regret: 6.161 early -> 2.840 late)
```

Empirical curve fitting (for illustration only):

```
sqrt(T) model:  $\text{Regret}(T) \sim 127.0 * \sqrt{T}$ 
Power model:  $\text{Regret}(T) \sim 53.9 * T^{0.62}$ 
```

WARNING: Don't conflate empirical fits with asymptotic bounds!

The exponent  $\alpha=0.62$  describes the learning TRANSIENT,  
not a fundamental asymptotic rate.

Theoretical expectations (for reference):

```
Constant epsilon-greedy:  $\Theta(T)$  -- LINEAR regret (explores forever)
Geometric decay epsilon:  $O(1)$  -- BOUNDED regret (sum of  $\text{eps}_t$  converges)
UCB:  $O(\sqrt{KT \log T})$ 
```

Our schedule ( $\text{eps}=0.998^t$ ) is geometric -> regret should plateau eventually.  
The  $T^{0.62}$  fit captures the transient, not the asymptote.

### 1.5.3 Interpretation

**Is regret sublinear?** Yes. The average regret per episode (2.84) is well below the oracle's mean reward (~16), and inspection of the regret curve shows growth slowing over time.

### 1.5.4 Theory-Practice Gap: Why Curve Fitting Is Misleading

**Caution:** Fitting a power law to 2000 points and claiming “regret scales as  $O(T^{0.62})$ ” conflates two different things:

Concept	What It Means
<b>Empirical fit</b>	$\text{Regret} \approx c \cdot T^\alpha$ for <i>observed</i> data
<b>Asymptotic bound</b>	$\lim_{T \rightarrow \infty} \text{Regret}(T)/T^\alpha < \infty$

The empirical exponent  $\alpha = 0.62$  could drift as  $T \rightarrow \infty$ . With finite samples, you can fit almost any functional form.

### 1.5.5 What Should We Actually Expect?

Our implementation uses **geometric decay**:  $\varepsilon_t = 0.9 \cdot 0.998^t$ .

This is *summable*:

$$\sum_{t=0}^{\infty} \varepsilon_t = \frac{0.9}{1 - 0.998} = 450$$

Since total exploration is bounded, **regret should plateau** as  $T \rightarrow \infty$ —not grow as any power of  $T$ !

Exploration Schedule	Asymptotic Regret	Notes
Constant $\varepsilon$	$\Theta(T)$	Linear! Never stops exploring randomly
$\varepsilon = c/t$	$O(\log T)$ or $O(\sqrt{T})$	Depends on problem structure
$\varepsilon = \varepsilon_0 \cdot \lambda^t$ (geometric)	$O(1)$ — bounded!	Total exploration is finite
UCB	$O(\sqrt{KT \log T})$	Optimal for stochastic bandits

**Common misconception:** “Constant  $\varepsilon$ -greedy gives  $O(T^{2/3})$ .” This is **wrong**. Constant  $\varepsilon$  gives *linear* regret  $\Omega(\varepsilon T)$  because you explore forever. The  $O(T^{2/3})$  bound requires *decaying*  $\varepsilon$  or Explore-Then-Commit.

### 1.5.6 What the Empirical Fit Actually Shows

The  $T^{0.62}$  fit over 2000 episodes captures the **transient learning phase**:

1. **Early** ( $t < 200$ ): High  $\varepsilon \rightarrow$  lots of exploration  $\rightarrow$  high per-episode regret
2. **Middle** ( $200 < t < 1000$ ): Q-values converging  $\rightarrow$  regret growth slows
3. **Late** ( $t > 1000$ ):  $\varepsilon \approx 0.9 \cdot 0.998^{1000} \approx 0.12 \rightarrow$  mostly exploitation

The power-law fit interpolates this transition but doesn’t reflect any fundamental asymptotic rate.

### 1.5.7 The Honest Conclusion

**What we can say:** - Regret growth slows over time  $\rightarrow$  the agent is learning - Average regret per episode decreases  $\rightarrow$  converging toward oracle performance - With geometric decay, regret will eventually plateau (bounded total regret)

**What we should NOT say:** - “Regret scales as  $O(T^{0.62})$ ” — this conflates empirical fits with asymptotic bounds - Comparisons to UCB/theoretical bounds without matching assumptions

**The key insight:** Sublinear regret growth confirms learning. The specific exponent from curve-fitting is an artifact of the learning transient, not a fundamental property.

---

## 1.6 Exercise 0.4 — Constrained Q-Learning with CM2 Floor

**Goal:** Add profitability constraint  $\mathbb{E}[\text{CM2} \mid x, a] \geq \tau$  and study the GMV–CM2 tradeoff.

### 1.6.1 Solution

```
class ConstrainedQLearning:
    """Q-learning with CM2 floor constraint.

    Maintains separate estimates for  $Q(x,a)$  (reward) and  $\text{CM2}(x,a)$  (margin).
    Filters actions based on estimated CM2 feasibility.
    """
    def get_feasible_actions(self, user_name):
        return [a for a in range(n_actions) if self.CM2[(user_name, a)] >= self.tau]
```

**Actual Output:**

=== Exercise 0.4: Constrained Q-Learning ===

Pareto Frontier (GMV vs CM2):

```
-----
tau= 0.0: GMV=23.73, CM2= 7.98, Violations= 0%
tau= 2.0: GMV=23.16, CM2= 8.02, Violations= 50%
tau= 4.0: GMV=22.50, CM2= 7.94, Violations= 50%
tau= 6.0: GMV=21.41, CM2= 6.49, Violations= 72%
tau= 8.0: GMV=23.38, CM2= 8.19, Violations= 58%
tau=10.0: GMV=23.03, CM2= 7.23, Violations= 72%
tau=12.0: GMV=20.17, CM2= 6.93, Violations= 66%
```

Analysis:

```
Unconstrained GMV: 23.73
Unconstrained CM2: 7.98
Best CM2 at tau=8.0: CM2=8.19, GMV=23.38
```

### 1.6.2 Theory-Practice Gap: Per-Episode Constraints Are Hard!

The results don't show a clean Pareto frontier. Why?

1. **High CM2 variance:** CM2 is 0 when no purchase occurs (common!), and can be 30+ when a high-margin product sells. Per-episode CM2 is extremely noisy.
2. **Constraint satisfaction is probabilistic:** Even if  $\mathbb{E}[\text{CM2} \mid x, a] \geq \tau$ , individual episodes often violate the constraint due to variance.
3. **Optimistic initialization:** We initialize CM2 estimates at 10.0 (optimistic). As estimates converge to true values, many actions become infeasible, leading to policy instability.

### Better Approaches (Chapter 3, Section 3.6):

1. **Lagrangian relaxation:** Instead of hard constraints, penalize violations:

$$\max_{\pi} \mathbb{E}[R] - \lambda(\tau - \mathbb{E}[\text{CM2}])$$

2. **Chance constraints:** Require  $P(\text{CM2} \geq \tau) \geq 1 - \delta_c$  instead of expected value.
3. **Batch constraints:** Aggregate over episodes/users, not per-episode.

**Key Insight:** Single-episode CMDP constraints with high-variance outcomes require sophisticated handling. The simple primal feasibility approach shown here is educational but not production-ready.

## 1.7 Exercise 0.5 — Bandit-Bellman Bridge (Conceptual)

**Goal:** Show that contextual bandit Q-learning is the  $\gamma = 0$  case of MDP Q-learning.

### 1.7.1 Solution

**The Bellman optimality equation:**

$$V^*(s) = \max_a \left\{ R(s, a) + \gamma \sum_{s'} P(s' \mid s, a) V^*(s') \right\}$$

**Setting  $\gamma = 0$ :**

$$V^*(s) = \max_a R(s, a)$$

The future value term vanishes! The Q-function becomes:

$$Q^*(s, a) = R(s, a) = \mathbb{E}[\text{reward} \mid s, a]$$

This is exactly what our bandit Q-table estimates.

### 1.7.2 Numerical Verification

```
def bandit_update(Q, r, alpha):
    """Bandit: Q <- (1-alpha)Q + alpha*r"""
    return (1 - alpha) * Q + alpha * r
```

```
def mdp_update(Q, r, Q_next_max, alpha, gamma):
    """MDP:  $Q \leftarrow Q + \alpha[r + \gamma \max(Q') - Q]$ """
    td_target = r + gamma * Q_next_max
    return Q + alpha * (td_target - Q)
```

### Actual Output:

Test 1:

```
Initial Q: 5.0, Reward: 7.0, alpha: 0.1
Bandit update: 5.200000
MDP update (gamma=0): 5.200000
Difference: 0.00e+00
PASSED
```

Test 2:

```
Initial Q: 0.0, Reward: 10.0, alpha: 0.5
Bandit update: 5.000000
MDP update (gamma=0): 5.000000
Difference: 0.00e+00
PASSED
```

Test 3:

```
Initial Q: -3.0, Reward: 2.0, alpha: 0.2
Bandit update: -2.000000
MDP update (gamma=0): -2.000000
Difference: 4.44e-16 <-- Floating point precision
PASSED
```

Test 4:

```
Initial Q: 100.0, Reward: 50.0, alpha: 0.01
Bandit update: 99.500000
MDP update (gamma=0): 99.500000
Difference: 0.00e+00
PASSED
```

Verified: Bandit Q-update = MDP Q-update with gamma=0

### 1.7.3 Implications

Property	Contextual Bandit	Full MDP
Horizon	1 step	$T$ steps (or infinite)
State transitions	None	$s \rightarrow s'$ via $P(s'   s, a)$
Update target	$r$	$r + \gamma \max_{a'} Q(s', a')$
Convergence	Stochastic approximation	Bellman contraction

**For Chapter 11 (multi-episode search):** Today's ranking affects tomorrow's return probability. This requires  $\gamma > 0$  and the full Bellman machinery.

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## 1.8 Summary: Theory–Practice Insights

These labs revealed important insights about RL in practice:

Exercise	Key Discovery	Lesson
Lab 0.1	96.2% of oracle achieved	Q-learning works for small discrete action spaces
Ex 0.1	Policy varies with reward weights	Engagement-heavy configs risk clickbait
<b>Ex 0.2</b>	<b>Cold start problem discovered</b>	<b>Exploration strategy must match policy maturity</b>
Ex 0.3	Regret growth slows over time	Sublinear regret confirms learning; don't conflate empirical fits with $O(\cdot)$ bounds
Ex 0.4	No clean Pareto frontier	Per-episode constraints need Lagrangian methods
Ex 0.5	Bandit = MDP with $\gamma = 0$	Unified view of bandits and MDPs

### Key Lessons:

1. **Q-learning works well** for small discrete action spaces with clear structure
2. **Exploration strategy depends on context:**
  - Cold start  $\rightarrow$  uniform/global exploration
  - Warm start  $\rightarrow$  local refinement is competitive
  - This explains why  $\varepsilon$ -greedy decays, SAC uses entropy, etc.
3. **Per-episode constraints** with high-variance outcomes need careful handling (Lagrangian methods)
4. **Bandits are  $\gamma = 0$  MDPs**—understanding this connection is foundational for Chapter 11

**The Cold Start Problem (Ex 0.2) is the pedagogical highlight:** We started with a hypothesis (local exploration is more efficient), discovered it was wrong, diagnosed why (cold start), and then validated when the intuition *does* hold (warm start). This is honest empiricism in action.

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## 1.9 Running the Code

All solutions are in `scripts/ch00/lab_solutions.py`:

```
# Run all exercises
python scripts/ch00/lab_solutions.py --all

# Run specific exercise
python scripts/ch00/lab_solutions.py --exercise lab0.1
python scripts/ch00/lab_solutions.py --exercise 0.3

# Interactive menu
python scripts/ch00/lab_solutions.py
```

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*End of Lab Solutions*