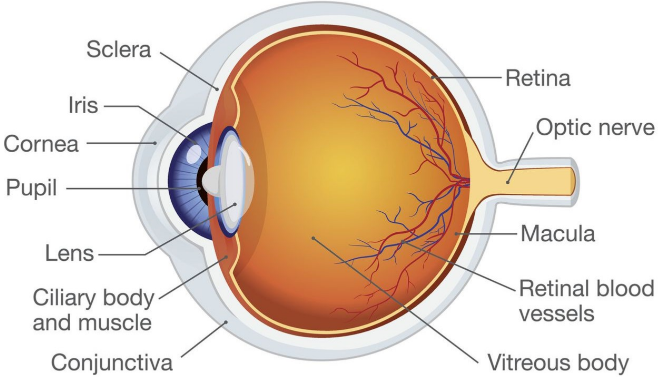
Camera model

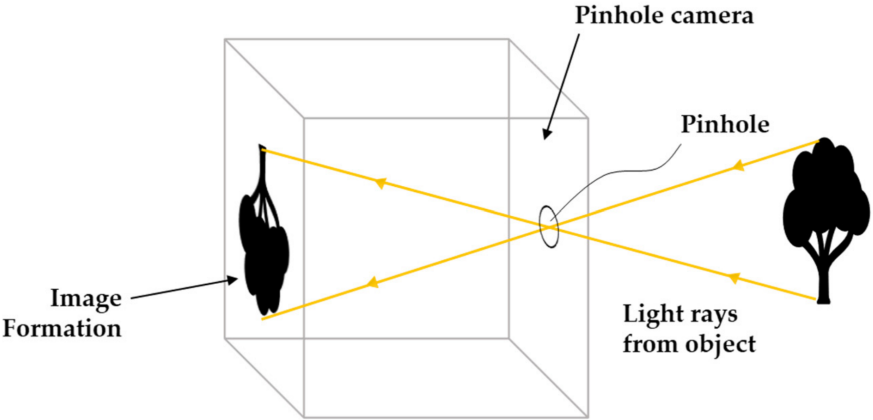
Laboratory of robotics and control systems
ITT PROJECT MANAGEMENT SERVICES L.L.C

Human eye anatomy



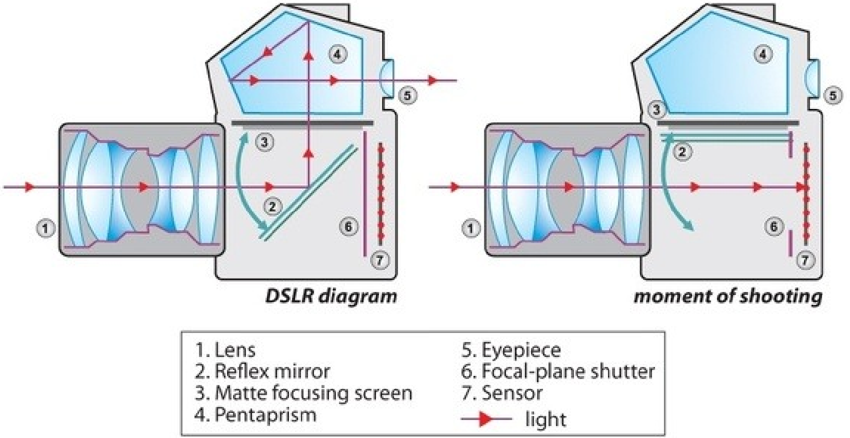
https://www.thoughtco.com/how-the-human-eye-works-4155646

Pinhole camera



https://sciprofiles.com/publication/view/690913ad90efc2401d603c26af6d7107

Digital camera scheme

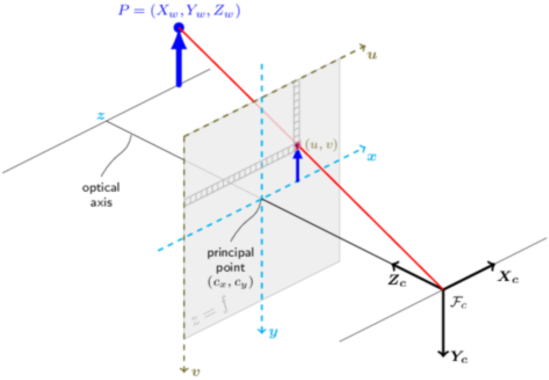


https://photodrugs.wordpress.com/2010/10/20/dslr-principle-scheme-of-work/

Overview

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} = [R|t] \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \frac{f_x X_c}{Z_c} + c_x \\ \frac{f_y X_c}{Z_c} + c_y \end{bmatrix}$$



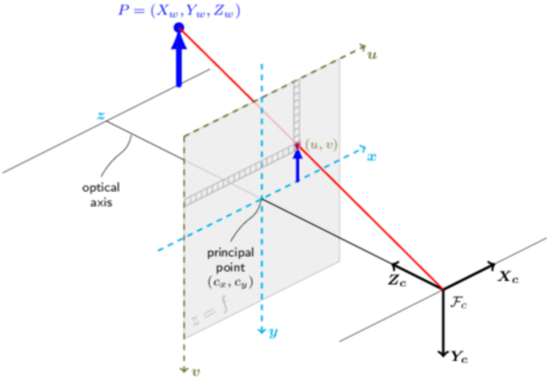
Pinhole camera model

https://docs.opencv.org/4.x/

Intrinsic parameters

$$\begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix}$$



Pinhole camera model

https://docs.opencv.org/4.x/

Rotation-translation matrix

$$P_c = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} P_w$$

$$\begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} r_{1_1} & r_{1_2} & r_{1_3} & t_x \\ r_{2_1} & r_{2_2} & r_{2_3} & t_y \\ r_{3_1} & r_{3_2} & r_{3_3} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

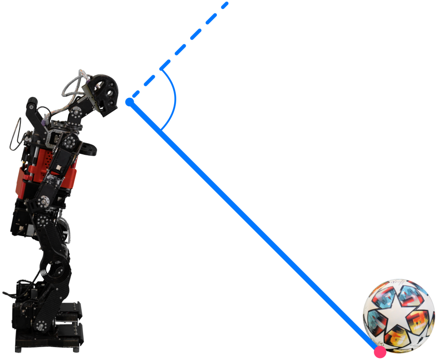
$$Z_c \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = [R|t] \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} = \begin{bmatrix} r_{1_1} & r_{1_2} & r_{1_3} & t_x \\ r_{2_1} & r_{2_2} & r_{2_3} & t_y \\ r_{3_1} & r_{3_2} & r_{3_3} & t_z \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

Complete distortionless model

Normalized coordinates

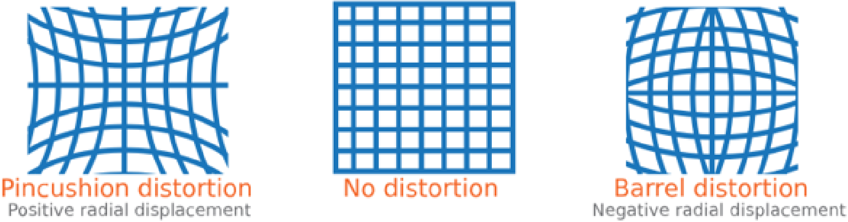
$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \frac{f_x X_c}{Z_c} + c_x \\ \frac{f_y X_c}{Z_c} + c_y \end{bmatrix}$$

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} = [R|t] \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$



Radial distortion

$$x_{distorted} = x(1 + k_1 r^2 + k_2 r^4 + k_3 r^6)$$
$$y_{distorted} = y(1 + k_1 r^2 + k_2 r^4 + k_3 r^6)$$

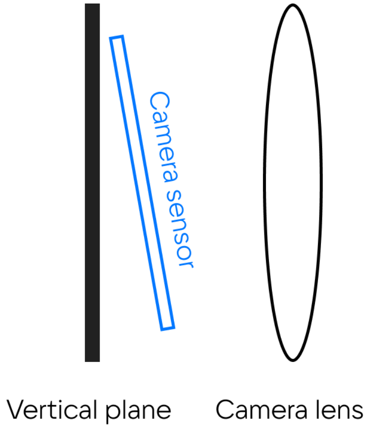


https://docs.opencv.org/4.x/

Tangential distortion

$$x_{distorted} = x + [2 \cdot p_1 \cdot x \cdot y + p_2 \cdot (r^2 + 2 \cdot x^2)]$$

$$y_{distorted} = y + [p_1 \cdot (r^2 + 2 \cdot y^2) + 2 \cdot p_2 \cdot x \cdot y]$$



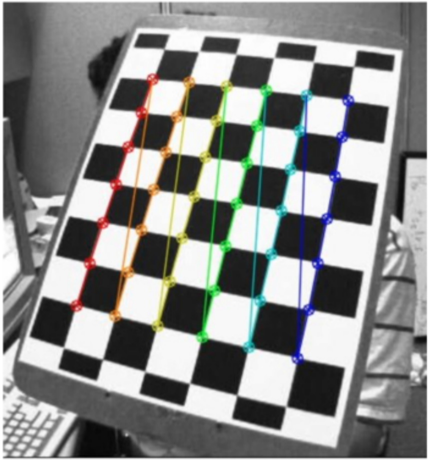
Lens and sensor are not parallel

General case

$$\begin{bmatrix} x'' \\ y'' \end{bmatrix} = \begin{bmatrix} x' \frac{1+k_1 r^2 + k_2 r^4 + k_3 r^6}{1+k_4 r^2 + k_5 r^4 + k_6 r^6} + 2p_1 x' y' + p_2(r^2 + 2x'^2) + s_1 r^2 + s_2 r^4 \\ y' \frac{1+k_1 r^2 + k_2 r^4 + k_3 r^6}{1+k_4 r^2 + k_5 r^4 + k_6 r^6} + p_1(r^2 + 2y'^2) + 2p_2 x' y' + s_3 r^2 + s_4 r^4 \end{bmatrix}$$

Camera calibration

- Minimum 10 images
- Enough to use approximately 40



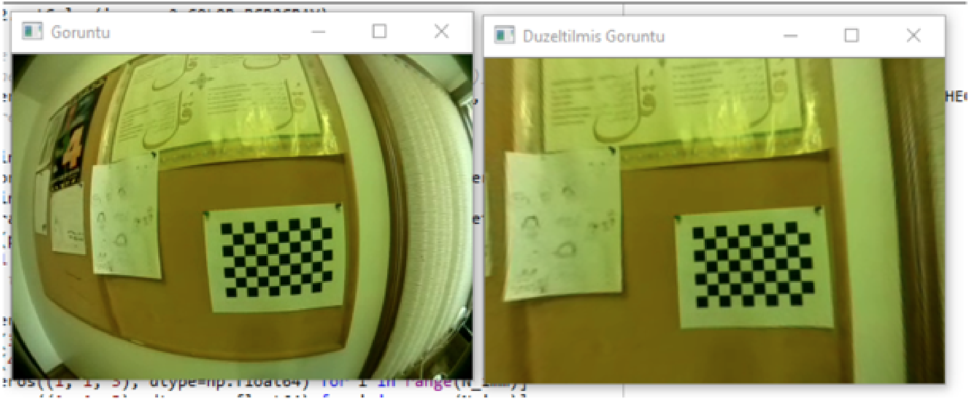
https://docs.opencv.org/4.x/

Camera calibration

$$A = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x'' \\ y'' \end{bmatrix} = \begin{bmatrix} x' \frac{1+k_1 r^2+k_2 r^4+k_3 r^6}{1+k_4 r^2+k_5 r^4+k_6 r^6} + 2p_1 x' y' + p_2(r^2 + 2x'^2) + s_1 r^2 + s_2 r^4 \\ y' \frac{1+k_1 r^2+k_2 r^4+k_3 r^6}{1+k_4 r^2+k_5 r^4+k_6 r^6} + p_1(r^2 + 2y'^2) + 2p_2 x' y' + s_3 r^2 + s_4 r^4 \end{bmatrix}$$

Undistortion



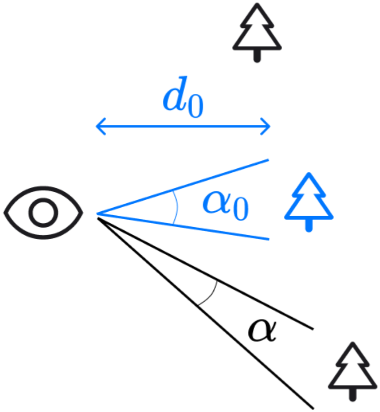
https://docs.opencv.org/4.x/

Object of known size

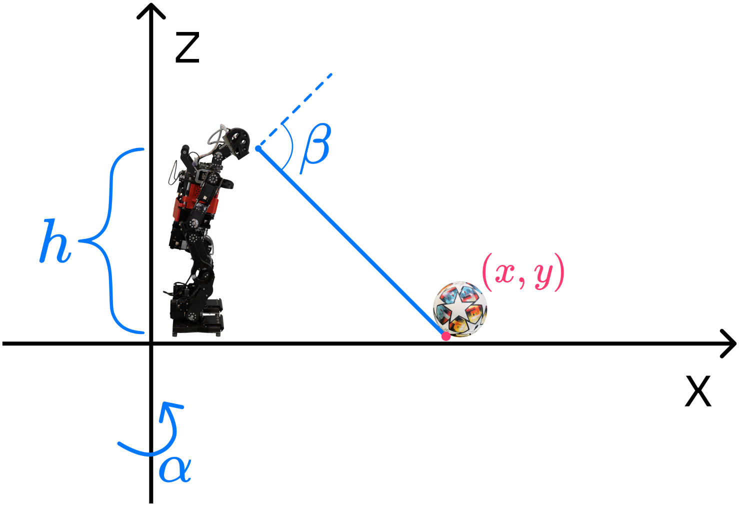
$$\sin\alpha \approx \alpha$$

$$h = 2 \cdot d \cdot \sin(\frac{\alpha}{2}) \approx 2 \cdot d \cdot \frac{\alpha}{2} = d_0\alpha_0 = d\alpha$$

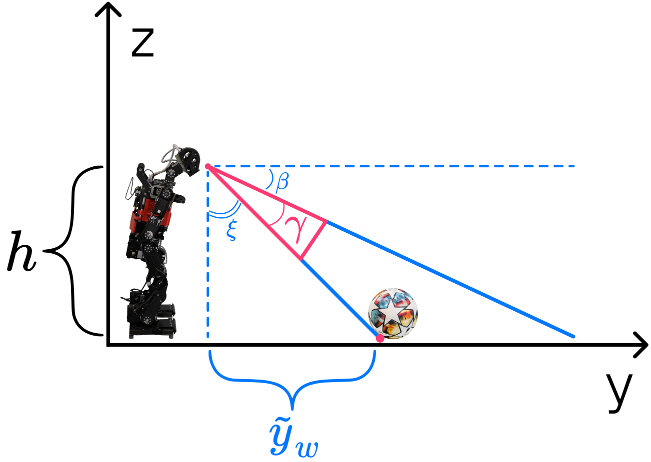
$$d = \frac{d_0}{\alpha_0} = \frac{h}{\alpha}$$



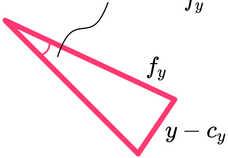
pic2r problem



pic2r y coordinate



$$\gamma = \arctan \frac{y - c_y}{f_y}$$

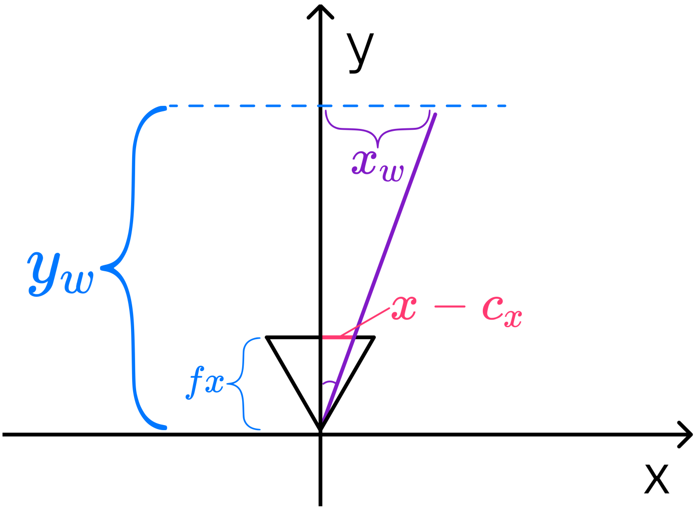


$$\xi = \frac{\pi}{2} - \arctan \frac{y - c_y}{f_y} - \beta$$

$$\tilde{y}_w = h \cdot \tan \xi =$$

$$= h \cdot \cot\left(\arctan \frac{y - c_y}{f_y} + \beta\right)$$

pic2r x coordinate



pic2r rotation

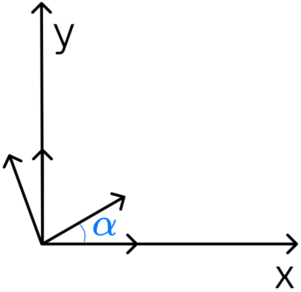
$$\overline{e_x} \to \cos\alpha\,\overline{e_x} + \sin\alpha\,\overline{e_y}$$

$$\overline{e_y} \to -\sin\overline{e_x} + \cos\alpha\,\overline{e_y}$$

$$R_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$R_\alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}; R_\alpha \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\sin\alpha \\ \cos\alpha \end{bmatrix}$$

$$\begin{bmatrix} x_w \\ y_w \end{bmatrix} = R_\alpha \begin{bmatrix} \tilde{x}_w \\ \tilde{y}_w \end{bmatrix}$$



Thank you for your time

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