## 2 Some basic knowledge about researching problem...

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Let's calculate smth with a given function: f(x, opa) = ln (1.000 + x · opa)
    Firstly, let's simplify this expression (if possible): f(x, opa) = ln (1.000 + x · opa)
3 Exploration of the expression as a function of multiple variables
Calculation value of function in the point BRITISH SCIENTISTS WERE SHOCKED, WHEN THEY COUNT IT!!! In the point M<sub>0</sub>(x<sub>0</sub>, opa<sub>0</sub>) = (1.000, 13.000) it's value = 2.63906
    Personally, I've always thought about first derivation of something like that function... Haven't you?
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Personally, I've always thought about first derivation of something like that function... Haven't you?
    But now, by using informatics and math skills I feel that I'm prepared enough to calculate it!
    1 step: Finding a derivation of opa
    When I was child, my father always told me: "Remember, son:
    (opa)' = \dots = [top secret] = \dots =
= 1.000
    2 step: Finding a derivation of x
    thanks to the results of my colleagues' scientific work, I know that:
    (x)' = \dots = [\text{top secret}] = \dots =
= 1.000
    3 step: Finding a derivation of x \cdot opa
    What if:
    (x \cdot opa)' = \dots = [\text{top secret}] = \dots =
    4 step: Finding a derivation of 1.000
   If someone asked me that in the middle of the night, I wouldn't hesitate to say:
    (1.000)' = \dots = [\text{top secret}] = \dots =
= 0.000
   5 step: Finding a derivation of 1.000 + x \cdot opa
   It's really easy to find:
    (1.000 + x \cdot opa)' = \dots = [\text{top secret}] = \dots =
= opa + x
    6 step: Finding a derivation of \ln (1.000 + x \cdot opa)
    My friends always beat me, because I didn't know that:
    (\ln(1.000 + x \cdot opa))' = \dots = [\text{top secret}] = \dots =
= \frac{1.000}{1.000 + x \cdot opa} \cdot (opa + x)
    Congratulations! The first derivation of the expression is:
   f'(x, opa) = \frac{1.000}{1.000 + x \cdot opa} \cdot (opa + x)
    In the point M_0(x_0, opa_0) = (1.000, 13.000) it's value = 1.00000
Finding the 3 derivation Let's find the 1 derivation of the expression:
    1 step: Finding a derivation of opa
    Sounds logical that it is the same as:
    (opa)' = \dots = [top secret] = \dots =
    2 step: Finding a derivation of x
    My roommate mumbled it in his sleep all night:
    (x)' = \dots = [\text{top secret}] = \dots =
    3 step: Finding a derivation of x \cdot opa
    What if:
    (x \cdot opa)' = \dots = [\text{top secret}] = \dots =
    4 step: Finding a derivation of 1.000
    While preparing for exams, I learned a lot of new things, for example:
    (1.000)' = \dots = [\text{top secret}] = \dots =
    5 step: Finding a derivation of 1.000 + x \cdot opa
    Sounds logical that it is the same as:
    (1.000 + x \cdot opa)' = \dots = [top secret] = \dots =
    6 step: Finding a derivation of \ln (1.000 + x \cdot opa)
    I was asked not to tell anyone that:
    (\ln (1.000 + x \cdot opa))' = \dots = [\text{top secret}] = \dots =
= \frac{\frac{1.000}{1.000 + x \cdot opa} \cdot (opa + x)}{\text{Let's find the 2 derivation of the expression:}}
    1 step: Finding a derivation of x
    Even my two-aged sister knows that:
    (x)' = \dots = [\text{top secret}] = \dots =
    2 step: Finding a derivation of opa
   I was asked not to tell anyone that:
    (opa)' = \dots = [top secret] = \dots =
    3 step: Finding a derivation of opa + x
    If someone asked me that in the middle of the night, I wouldn't hesitate to say:
    (opa + x)' = \dots = [top secret] = \dots =
    4 step: Finding a derivation of opa
    thanks to the results of my colleagues' scientific work, I know that:
    (opa)' = \dots = [top secret] = \dots =
    5 step: Finding a derivation of x
    My roommate mumbled it in his sleep all night:
    (x)' = \dots = [\text{top secret}] = \dots =
    6 step: Finding a derivation of x \cdot opa
    Even my two-aged sister knows that:
    (x \cdot opa)' = \dots = [top secret] = \dots =
    7 step: Finding a derivation of 1.000
    Man... Just look:
    (1.000)' = \dots = [\text{top secret}] = \dots =
= 0.000
    8 step: Finding a derivation of 1.000 + x \cdot opa
    For centuries, people have hunted for the secret knowledge that:
    (1.000 + x \cdot opa)' = \dots = [\text{top secret}] = \dots =
    9 step: Finding a derivation of 1.000
    I was asked not to tell anyone that:
    (1.000)' = \dots = [\text{top secret}] = \dots =
   10 step: Finding a derivation of \frac{1.000}{1.000+x\cdot opa}
For centuries, people have hunted for the secret knowledge that:
    \left(\frac{1.000}{1.000 + x \cdot opa}\right)' = \dots = [\text{top secret}] = \dots =
= \frac{(-1.000) \cdot (opa+x)}{(1.000+x \cdot opa)^{2.000}}
   11 step: Finding a derivation of \frac{1.000}{1.000+x \cdot opa} \cdot (opa + x) When I was child, my father always told me: "Remember, son:
    (\frac{1.000}{1.000+x \cdot opa} \cdot (opa + x))' = \dots = [\text{top secret}] = \dots = [\text{top secret}]
   \frac{(-1.000) \cdot (opa+x)}{(1.000+x \cdot opa)^{2.000}} \cdot (opa+x) + 2.000 \cdot \frac{1.000}{1.000+x \cdot opa}
    Let's find the 3 derivation of the expression:
    1 step: Finding a derivation of opa
    While preparing for exams, I learned a lot of new things, for example:
    (opa)' = \dots = [top secret] = \dots =
    2 step: Finding a derivation of x
    thanks to the results of my colleagues' scientific work, I know that:
    (x)' = \dots = [\text{top secret}] = \dots =
    3 step: Finding a derivation of x \cdot opa
    Sounds logical that it is the same as:
    (x \cdot opa)' = \dots = [\text{top secret}] = \dots =
    4 step: Finding a derivation of 1.000
    A true prince must know that:
    (1.000)' = \dots = [\text{top secret}] = \dots =
    5 step: Finding a derivation of 1.000 + x \cdot opa
    If someone asked me that in the middle of the night, I wouldn't hesitate to say:
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(1.000 + x \cdot opa)' = \dots = [\text{top secret}] = \dots =
= opa + x
    6 step: Finding a derivation of 1.000
    If someone asked me that in the middle of the night, I wouldn't hesitate to say:
    (1.000)' = \dots = [\text{top secret}] = \dots =
     7 step: Finding a derivation of \frac{1.000}{1.000+x\cdot o}
    My friends always beat me, because I didn't know that:
    \left(\frac{1.000}{1.000+x \cdot opa}\right)' = \dots = [\text{top secret}] = \dots = \frac{(-1.000) \cdot (opa+x)}{(-1.000) \cdot (opa+x)}
    (1.000+x\cdot opa)
    8 step: Finding a derivation of 2.000
    What if:
     (2.000)' = \dots = [\text{top secret}] = \dots =
= 0.000
    9 step: Finding a derivation of 2.000 \cdot \frac{1.000}{1.000 + x \cdot opa}
     Sounds logical that it is the same as:
    (2.000 \cdot \frac{1.000}{1.000 + x \cdot opa})' = \dots = [\text{top secret}] = \dots =
= 2.000 \cdot \frac{(-1.000) \cdot (opa+x)}{(1.000 + x \cdot opa)^{2.000}}
     10 step: Finding a derivation of x
    thanks to the results of my colleagues' scientific work, I know that:
    (x)' = \dots = [\text{top secret}] = \dots =
    11 step: Finding a derivation of opa
    Even my two-aged sister knows that:
    (opa)' = \dots = [top secret] = \dots =
     12 step: Finding a derivation of opa + x
     While preparing for exams, I learned a lot of new things, for example:
     (opa + x)' = \dots = [top secret] = \dots =
    13 step: Finding a derivation of opa
     When I was child, my father always told me: "Remember, son:
     (opa)' = \dots = [top secret] = \dots =
    14 step: Finding a derivation of x
    It's really easy to find:
    (x)' = \dots = [\text{top secret}] = \dots =
    15 step: Finding a derivation of x \cdot opa
    For centuries, people have hunted for the secret knowledge that:
    (x \cdot opa)' = \dots = [\text{top secret}] = \dots =
= opa + x
    16 step: Finding a derivation of 1.000
    If someone asked me that in the middle of the night, I wouldn't hesitate to say:
    (1.000)' = \dots = [\text{top secret}] = \dots =
    17 step: Finding a derivation of 1.000 + x \cdot opa
    Sounds logical that it is the same as:
    (1.000 + x \cdot opa)' = \dots = [\text{top secret}] = \dots =
= opa + x
     18 step: Finding a derivation of (1.000 + x \cdot opa)^{2.000}
    I spend the hole of my life to find the answer and finally it's: ((1.000 + x \cdot opa)^{2.000})' = \dots = [\text{top secret}] = \dots =
= 2.000 \cdot (1.000 + x \cdot opa) \cdot (opa + x)
    19 step: Finding a derivation of x
     While preparing for exams, I learned a lot of new things, for example:
    (x)' = \dots = [\text{top secret}] = \dots =
    20 step: Finding a derivation of opa
    I spend the hole of my life to find the answer and finally it's:
    (opa)' = \dots = [top secret] = \dots =
     21 step: Finding a derivation of opa + x
     When I was child, my father always told me: "Remember, son:
     (opa + x)' = \dots = [top secret] = \dots =
     22 step: Finding a derivation of -1.000
    For centuries, people have hunted for the secret knowledge that:
     (-1.000)' = \dots = [\text{top secret}] = \dots =
     23 step: Finding a derivation of (-1.000) \cdot (opa + x)
     What if:
    ((-1.000) \cdot (opa + x))' = \dots = [top secret] = \dots =
= -2.000
    24 step: Finding a derivation of \frac{(-1.000) \cdot (opa+x)}{(1.000+x \cdot opa)^{2.000}}
     A true prince must know that:
     \left(\frac{(-1.000) \cdot (opa+x)}{(1.000+x \cdot opa)^{2.000}}\right)' = \dots = [\text{top secret}] = \dots = 
   \frac{(1.000+x \cdot opa)^{2.000}}{(-2.000) \cdot (1.000+x \cdot opa)^{2.000} - 2.000 \cdot (1.000+x \cdot opa) \cdot (opa+x) \cdot (-1.000) \cdot (opa+x)}{(2.000)^{2.000}}
    \frac{(-2.000) \cdot (1.000 + x \cdot opa)^{2.000} \cdot (1.000 + x \cdot opa)^{2.000}}{((1.000 + x \cdot opa)^{2.000})^{2.000}}
25 step: Finding a derivation of \frac{(-1.000) \cdot (opa + x)}{(1.000 + x \cdot opa)^{2.000}} \cdot (opa + x)
    Man... Just look:
     \left(\frac{(-1.000) \cdot (opa+x)}{(1.000+x \cdot opa)^{2.000}} \cdot (opa+x)\right)' = \dots = [\text{top secret}] = \dots = [\text{top secret}]
   \frac{(-2.000) \cdot (1.000 + x \cdot opa)^2 \cdot 000}{((1.000 + x \cdot opa)^2 \cdot 000} - 2.000 \cdot (1.000 + x \cdot opa) \cdot (opa + x) \cdot (-1.000) \cdot (opa + x)}{((1.000 + x \cdot opa)^2 \cdot 000)^2 \cdot 000} \cdot (opa + x) + 2.000 \cdot \frac{(-1.000) \cdot (opa + x)}{(1.000 + x \cdot opa)^2 \cdot 000}}{(1.000 + x \cdot opa)^2 \cdot 000}
    26 step: Finding a derivation of \frac{(-1.000)\cdot(opa+x)}{(1.000+x\cdot opa)^{2.000}}\cdot(opa+x) + 2.000\cdot\frac{1.000}{1.000+x\cdot opa}
    I was asked not to tell anyone that:
     \left(\frac{(-1.000)\cdot(opa+x)}{(1.000+x\cdot opa)^{2.000}}\cdot(opa+x)+2.000\cdot\frac{1.000}{1.000+x\cdot opa}\right)'=\dots=[\text{top secret}]=\dots=
= \frac{(-2.000) \cdot (1.000 + x \cdot opa)^{2.000} - 2.000 \cdot (1.000 + x \cdot opa) \cdot (opa + x) \cdot (-1.000) \cdot (opa + x)}{((1.000 + x \cdot opa)^{2.000})^{2.000}} \cdot (opa + x) + 2.000 \cdot \frac{(-1.000) \cdot (opa + x)}{(1.000 + x \cdot opa)^{2.000}} + 2.000 \cdot \frac{(-1.000) \cdot (opa + x)}{(1.000 + x \cdot opa)^{2.000}}
     Finally... The 3 derivation of the expression:
     f^{(3)}(\mathbf{x}, \text{opa}) = \frac{(-2.000) \cdot (1.000 + x \cdot opa)^{2.000} - 2.000 \cdot (1.000 + x \cdot opa) \cdot (opa + x) \cdot (-1.000) \cdot (opa + x)}{((1.000 + x \cdot opa)^{2.000})^{2.000}} \cdot (opa + x) + 2.000 \cdot \frac{(-1.000) \cdot (opa + x)}{(1.000 + x \cdot opa)^{2.000}} + 2.000 \cdot \frac{(-1.000) \cdot (opa + x)}{(1.000 + x \cdot opa)^{2.000}}
     BRITISH SCIENTISTS WERE SHOCKED AGAIN, WHEN THEY COUNT THE 3 DERIVATION OF THIS EXPRESSION!!!
    In the point M_0(x_0, opa_0) = (1.000, 13.000) it's value = 1.57143
Finding partical derivations Partial derivation of the expression on the variable x:
     \frac{\partial f}{\partial x} = 13.000 \cdot \frac{1.000}{1.000 + 13.000 \cdot x}
     In the point M_0(x_0, opa_0) = (1.000, 13.000) it's value = 0.92857 !!!
    Partial derivation of the expression on the variable opa:
     \frac{\partial f}{\partial opa} = \frac{1.000}{1.000 + opa}
     In the point M_0(x_0, opa_0) = (1.000, 13.000) it's value = 0.07143 !!!
Finding full derivation Full derivation:
     \sqrt{\left(13.000 \cdot \frac{1.000}{1.000 + 13.000 \cdot x}\right)^{2.000} + \left(\frac{1.000}{1.000 + opa}\right)^{2.000}}
    In the point M_0(x_0, opa_0) = (1.000, 13.000) it's value = 0.93131!!!
4 Exploration the expression as a function of the first variable
Now let's consider the expression as a function of x variable: f(x) = \ln(1.000 + 13.000 \cdot x)
Decomposing on Macloren's formula Maklorens formula for x \to x_0 = 1.000:
    f(x) = 2.639 + 0.929 \cdot (x - 1.000) + (-0.431) \cdot (x - 1.000)^{2.000} + 0.267 \cdot (x - 1.000)^{3.000} + (-0.186) \cdot (x - 1.000)^{4.000} + o((x - 1.000)^{4.000})
Graphics Graph f(x) = \ln(1.000 + 13.000 \cdot x) on the diapasone x \in [-10:10]:
                                                                  (\log((1.000)+((13.000)*(x))))
          4.5
            4
          3.5
            3
          2.5
            2
          1.5
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0.5 <del>-</del>

-5

0

5

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## Equations in the point Tangent equation in the point $x_0 = 1.000$ : $f(\mathbf{x}) = 0.929 \cdot (x - 1.000) + 2.639$ Normal equation in the point $x_0 = 1.000$ : $f(\mathbf{x}) = (-1.077) \cdot (x - 1.000) + 2.639$ Their graphs in $\delta = 1.00000$ coverage of the point $x_0 = 1.000000$

