Exercise 1.

Solution. Using the given model and fixing the clockwise orientation, we have $C_0 = \mathbb{Z}[V]$, $C_1 = \mathbb{Z}[a, b, c]$ and $C_2 = Z[S, T]$, where $\partial V = 0, \partial a = \partial b = \partial c = 0$ and $\partial S = a+b-c, \partial T = a - b + c$. Therefore

$$H_0 = \mathbb{Z}[V]/0 = \mathbb{Z},$$

$$H_2 = 0/0 = 0$$

(since $a + b - c$ and $a - b + c$ are $\mathbb{Z}$-linearly independent, $\partial : C_2 \to C_1$ must be injective) and finally

$$H_1 = \frac{\mathbb{Z}[a, b, c]}{Z[a + b - c, a - b + c]} = \frac{\mathbb{Z}[a + b - c, b - c, c]}{Z[a + b - c, 2(b - c)]} = \mathbb{Z}/2\mathbb{Z} \oplus \mathbb{Z}.$$

□

Exercise 2.

Solution. Using the given model and fixing the clockwise orientation, we have $C_0 = \mathbb{Z}[V, W]$, $C_1 = \mathbb{Z}[a, b, c]$ and $C_2 = Z[S, T]$, where $\partial V = \partial W = 0, \partial a = \partial b = V - W, \partial c = 0$ and $\partial S = a - b - c, \partial T = a - b + c$. Therefore

$$H_0 = \frac{\mathbb{Z}[V, W]}{\mathbb{Z}[V - W]} = \frac{\mathbb{Z}[V - W, W]}{\mathbb{Z}[V - W]} = \mathbb{Z},$$

$$H_2 = 0/0 = 0$$

(since $a - b - c$ and $a - b + c$ are $\mathbb{Z}$-linearly independent, $\partial : C_2 \to C_1$ must be injective) and finally

$$H_1 = \frac{\mathbb{Z}[a - b, c]}{Z[a - b - c, a - b + c]} = \frac{\mathbb{Z}[a - b - c, c]}{Z[a - b - c, 2c]} = \mathbb{Z}/2\mathbb{Z}.$$

□

Exercise 3.

Solution. Using exercise 6 from tutorial 1, we have

$$\begin{aligned}(X, x_0) \wedge (S^1, s_0) = X/\{x_0\} \wedge I/\partial I = X \times I/(X \times \partial I \cup \{x_0\} \times I) = \\ = X \times I/(X \times \{0\} \cup X \times \{1\} \cup \{x_0\} \times I) = \Sigma X,\end{aligned}$$

as needed. □