Exercise 1.

Solution. Let X be the CW-complex $S_1^{2017} \vee S_2^{2017} \vee S_1^{2018} \cup e_2^{2018} \cup e^{2019}$ together with the constant attaching maps $S_1^{2017} \to e^0$, $S_2^{2017} \to e^0$ and $S^{2018} \to e^0$, an arbitrary attaching map $\partial D_2^{2018} = S^{2017} \to S_2^{2017}$ of degree 6 and an arbitrary attaching map $\partial D^{2019} = S^{2018} \to S_1^{2018}$ of degree 4 (we already know such maps do exist). Then the homology groups of X are the same as the homology groups of the chain complex

$$\cdots \to 0 \to 0 \to \underbrace{\mathbb{Z}}_{\dim 2019} \xrightarrow{\cdot\begin{pmatrix}4 & 0\end{pmatrix}} \underbrace{\mathbb{Z} \oplus \mathbb{Z}}_{\dim 2018} \xrightarrow{\cdot\begin{pmatrix}0 & 0\\ 0 & 6\end{pmatrix}} \underbrace{\mathbb{Z} \oplus \mathbb{Z}}_{\dim 2017} \to 0 \to 0 \to \cdots \to \underbrace{0}_{\dim 1} \to \underbrace{\mathbb{Z}}_{\dim 0} \to 0.$$

Therefore (since the kernel of $\cdot \begin{pmatrix}0 & 0\\ 0 & 6\end{pmatrix}$ is $\mathbb{Z} \oplus 0$ and $\cdot \begin{pmatrix}4 & 0\end{pmatrix}$ is injective) we have

$$\begin{aligned}
H_0(X) &= \mathbb{Z}/0 = \mathbb{Z},\\
H_{2017}(X) &= (\mathbb{Z} \oplus \mathbb{Z})/(0 \oplus 6\mathbb{Z}) = \mathbb{Z} \oplus \mathbb{Z}/6,\\
H_{2018}(X) &= (\mathbb{Z} \oplus 0)/(4\mathbb{Z} \oplus 0) = \mathbb{Z}/4
\end{aligned}$$

and all the other homology groups are trivial, as required. □