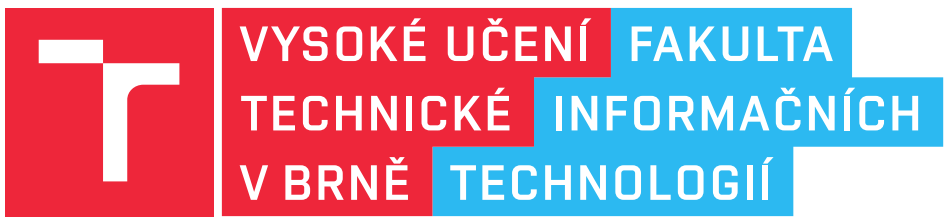
Topic 38 – Graph Radio Coloring Parallelization

Patrik Goldschmidt, Vladimír Dušek

Brno University of Technology, Faculty of Information Technology

Božetěchova 1/2. 612 66 Brno - Královo Pole, Czech Republic



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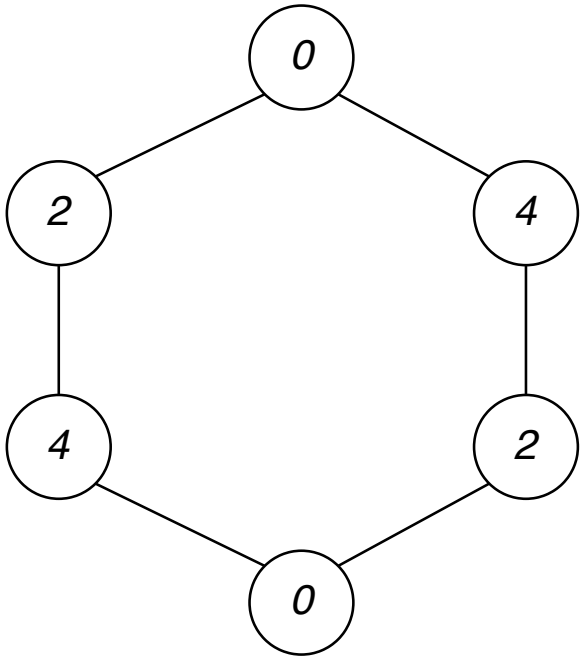
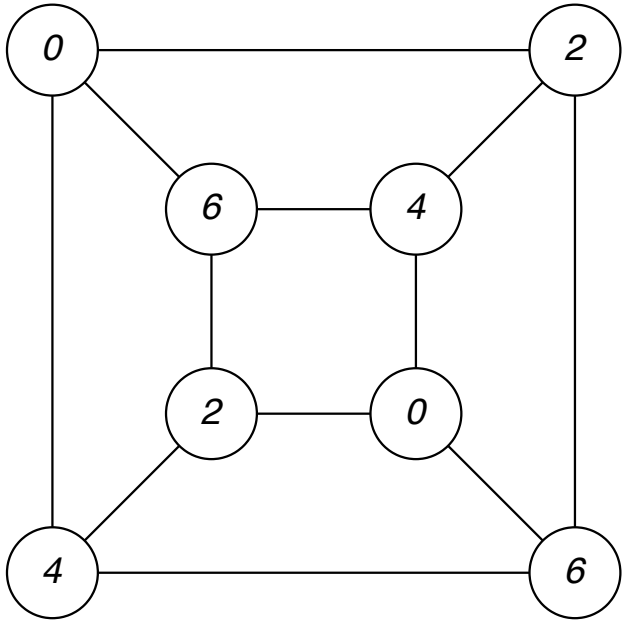


(a) 6-cycle graph radio coloring



(b) 3-dimensional cube graph radio coloring

Given an undirected graph $G = (V, E)$, the radiocoloring of G is a mapping $f : V \to N$, such that $\forall v, u \in V : |f(u) - f(v)| \geq k + 1 - d(u, v)$, where $d(u, v)$ is a distance between u and v in G.

```
# Step 1)
dist_matrix = floyd_warshall(adj_matrix)

C = Matrix(n, n)

# Step 2)
for i in (0 ... n):
    for j in (0 ... n):
        C(i)(j) = max(k+1 - dist_matrix(i)(j), 0)
    C(i)(i) = INF

labels = (0, INF, INF, ...)
last = 0
```

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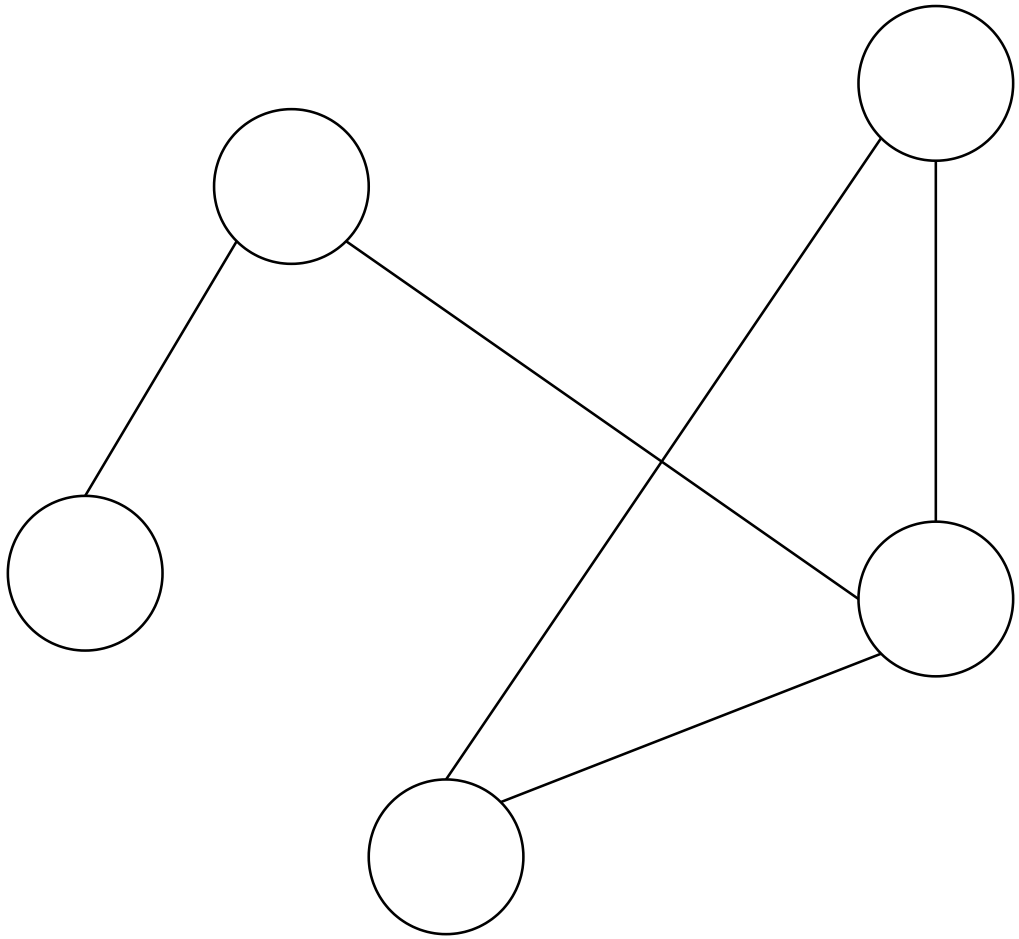
# Step 3)
repeat(n - 1):
    min_label = INF
    for j in (0 ... n):
        if min_label > C(last)(j):
            min_label = C(last)(j)
            p = j
    for j in (0 ... n):
        C(p)(j) += min_label
    for j in (0 ... n):
        if C(p)(j) < C(last)(j):
            C(p)(j) = C(last)(j)
    labels(p) = min_label
    last = p
```

- Largest-degree first
- Algorithm
  - Sort the vertices in descending order by their degree.
  - Compute distance 2 binary matrix.
  - Initialize a n x $2n-1$ binary matrix "forbidden" ($O(n^2)$).
  - Find the smallest non-conflicting color, assign it and update forbidden matrix.

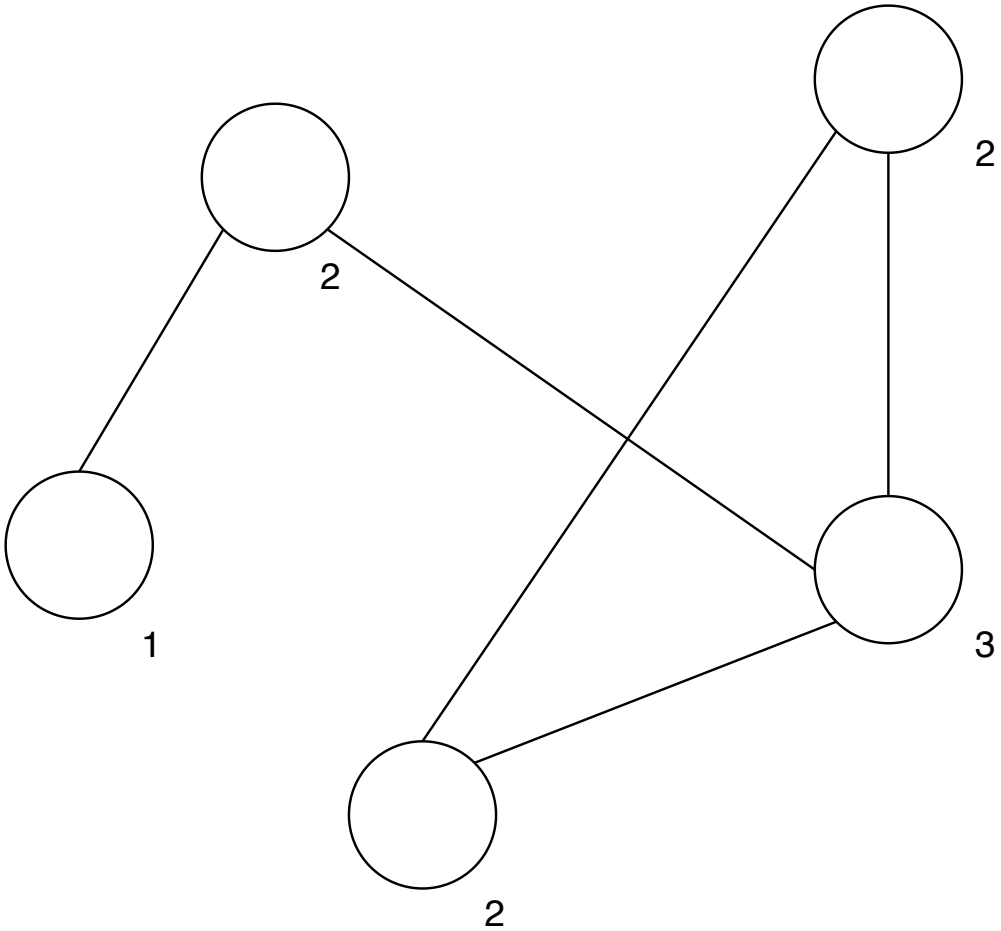
$$\mathbf{DTIME} \approx O(n^2) + O(n \log n) + O(n^3) + O(n^3) \approx O(n^3)$$

$$\mathbf{DTIME}_n \approx O(n) + O(\log n) + O(n^2) + O(n \log n) \approx O(n^2)$$

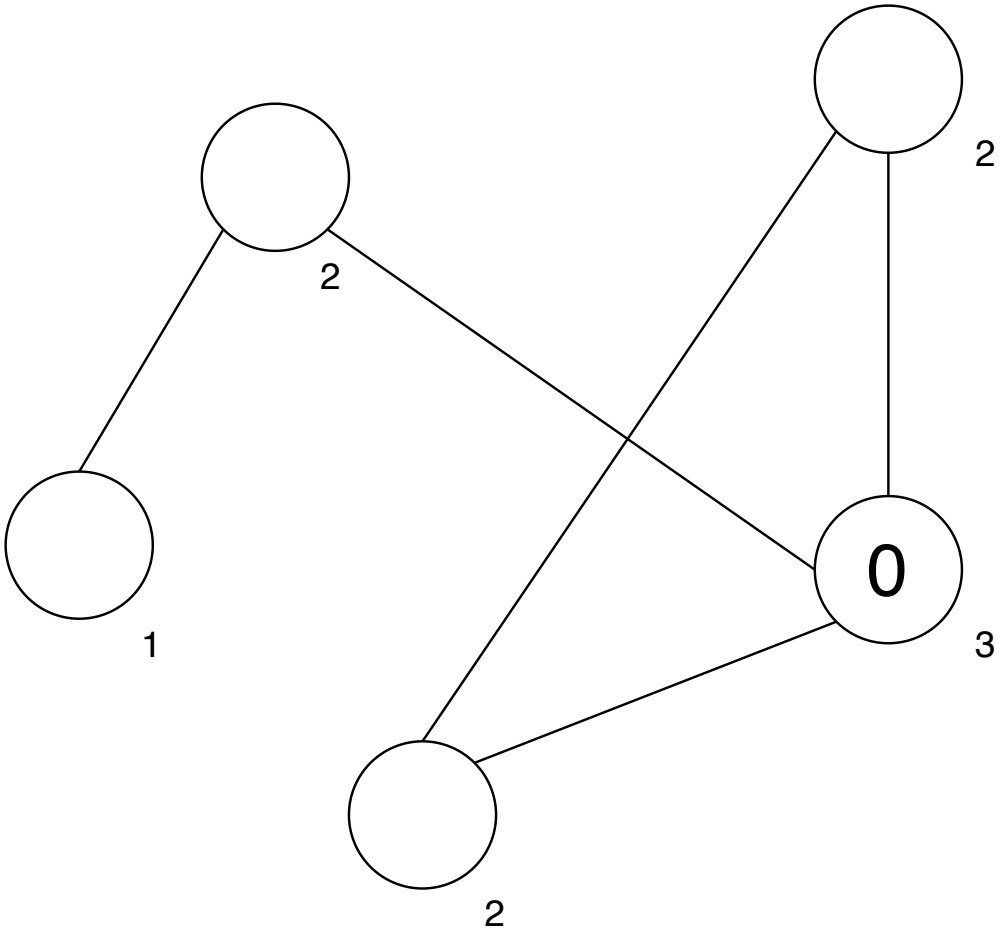
Parallel algorithm - functionality



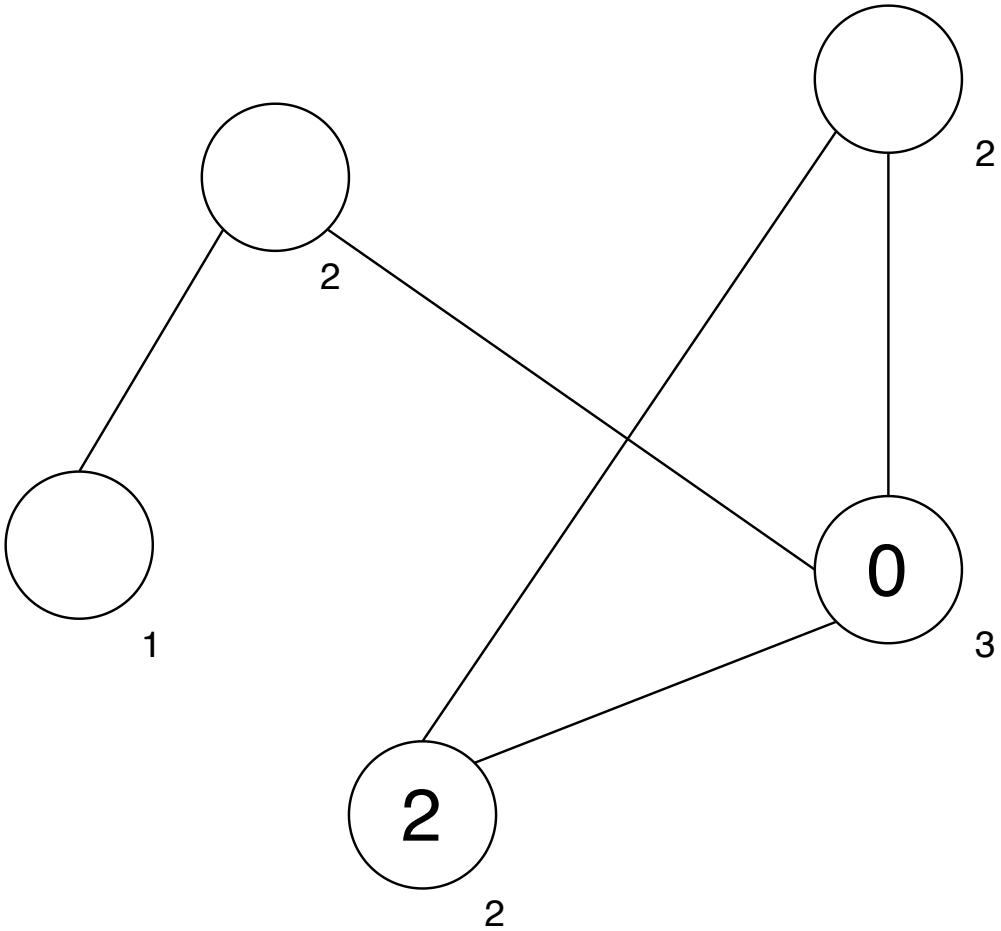
Parallel algorithm - functionality



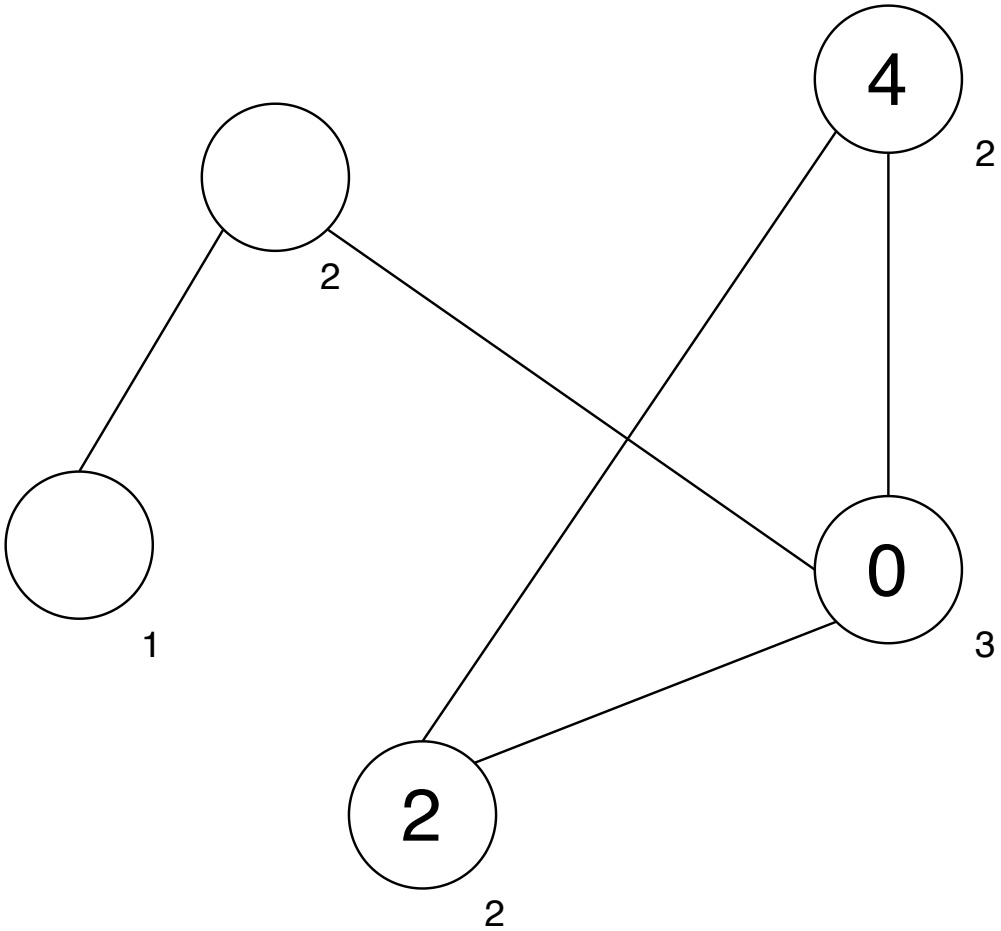
Parallel algorithm - functionality



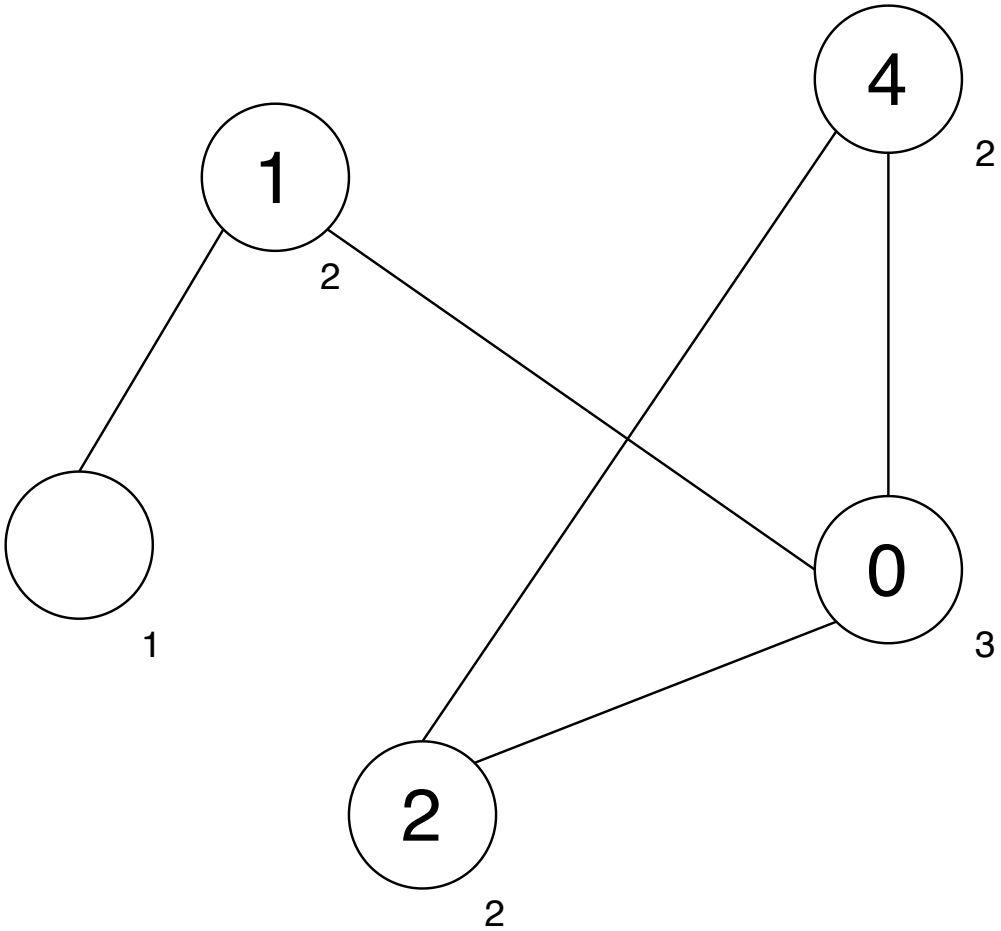
Parallel algorithm - functionality



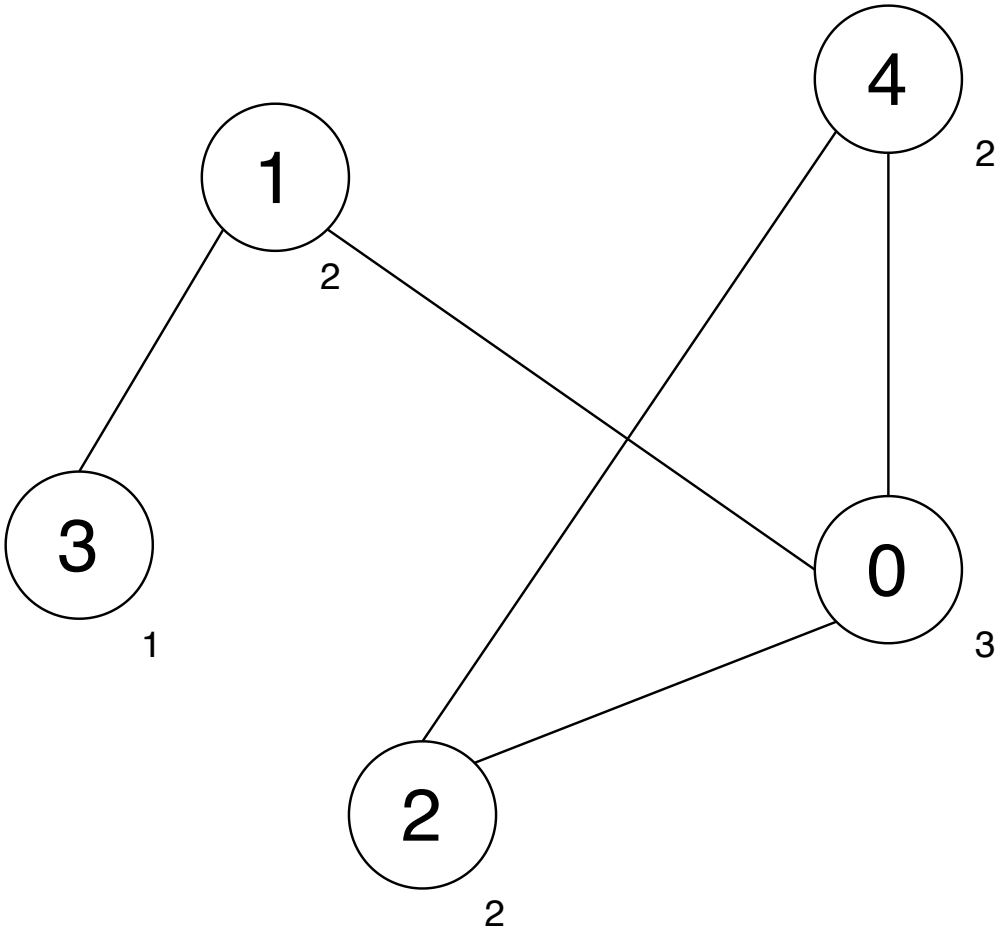
Parallel algorithm - functionality



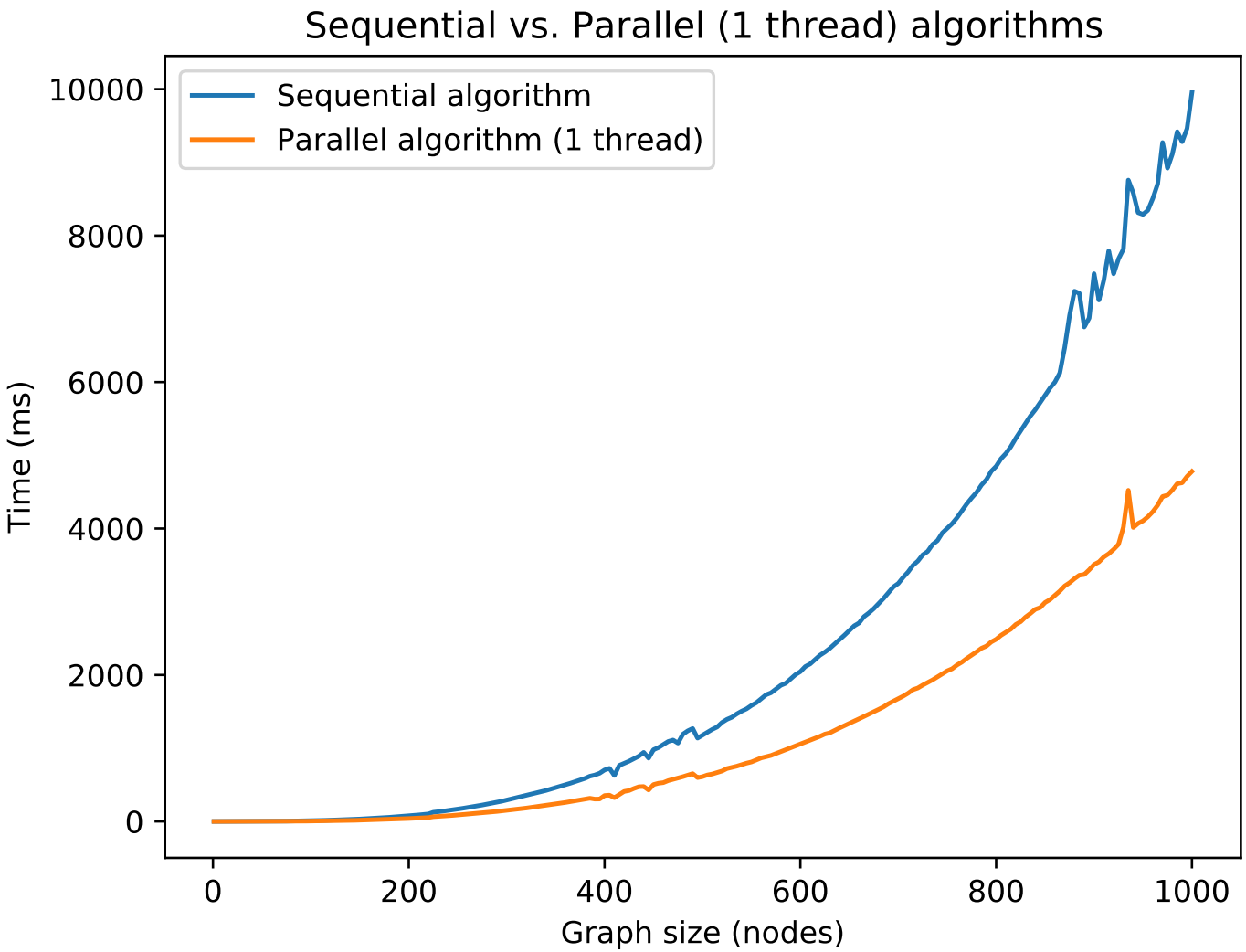
Parallel algorithm - functionality



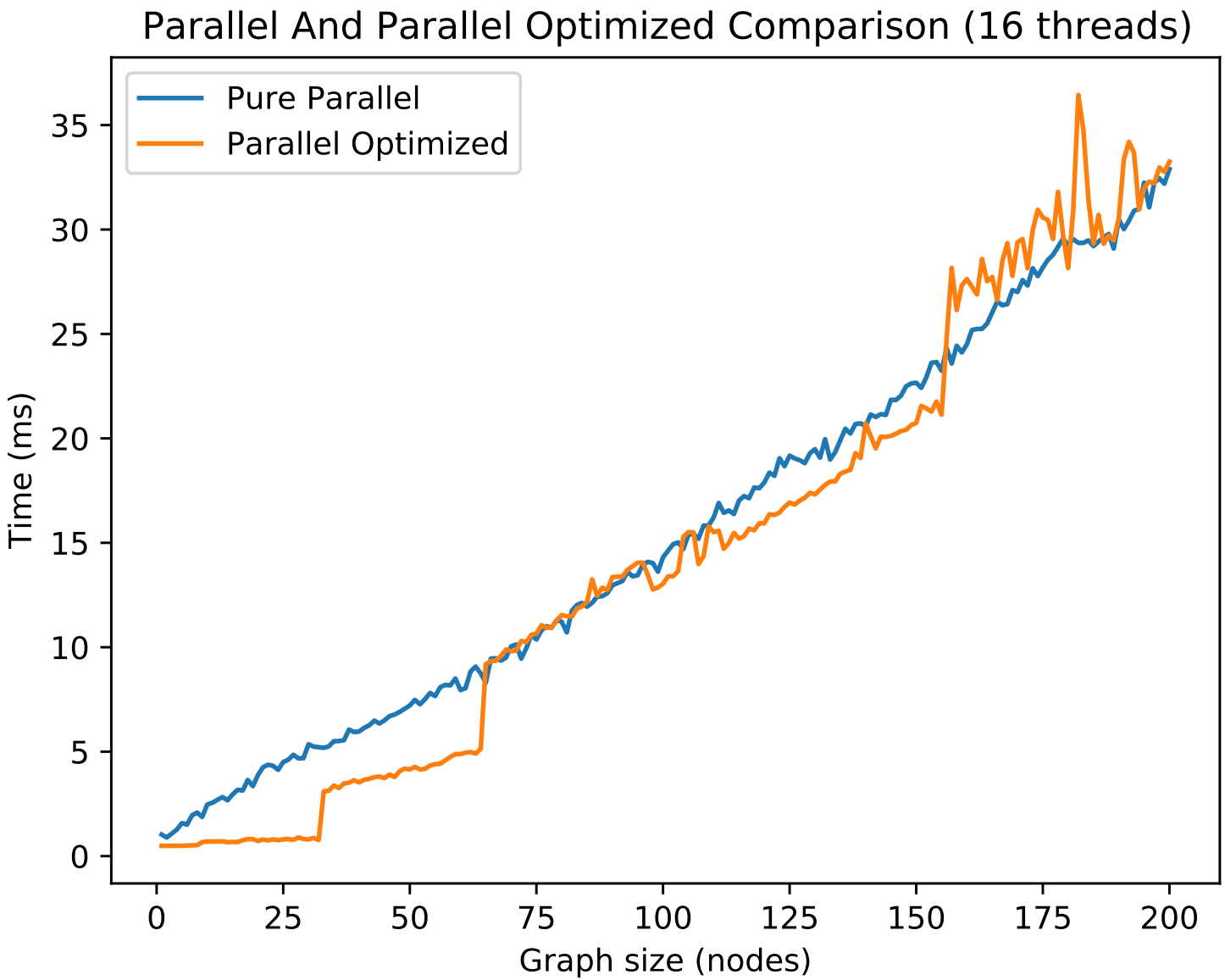
Parallel algorithm - functionality



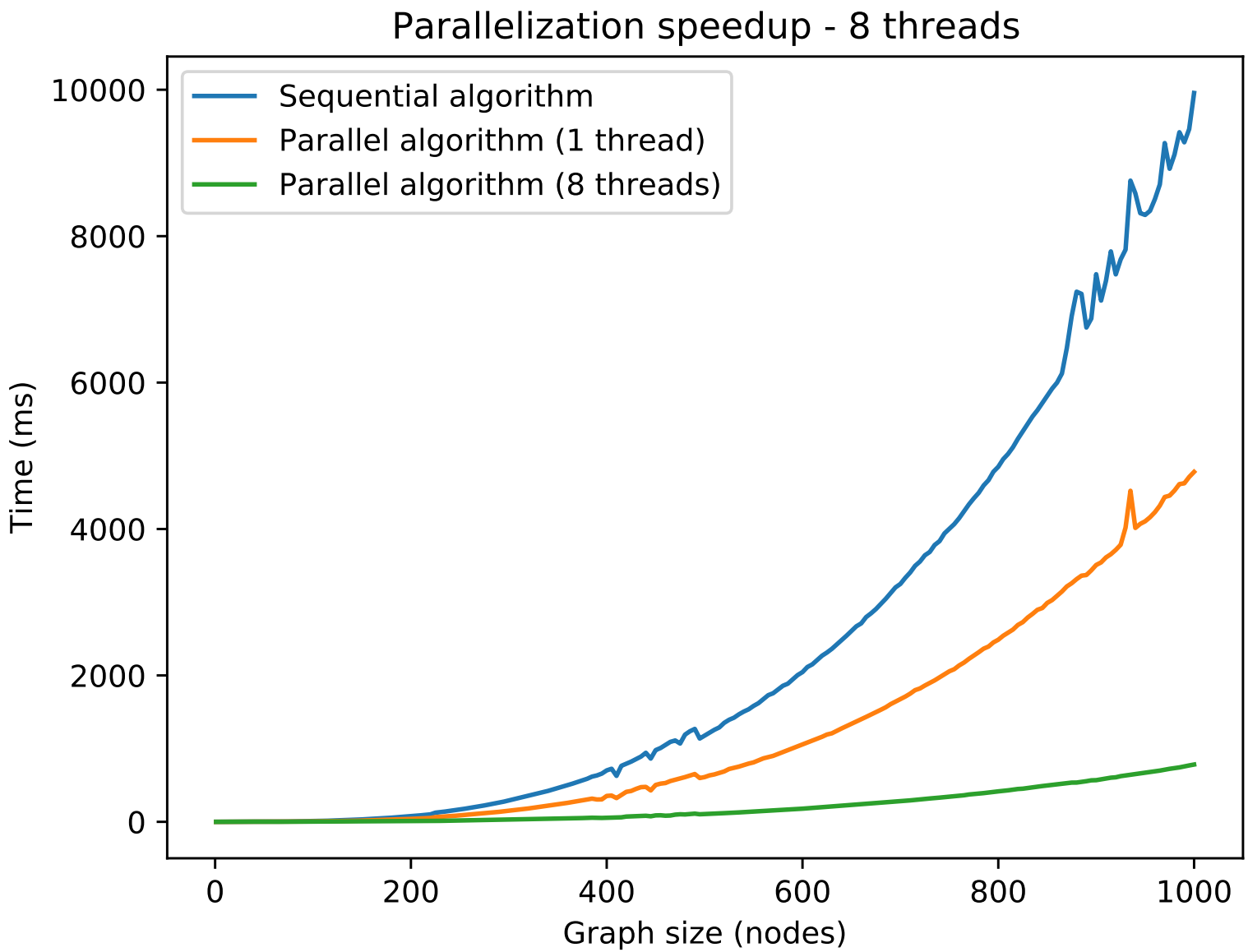
Evaluation – 1-threaded Algorithms Run



Evaluation – "Optimized" Parallel Variant



Evaluation – Sequential vs. Parallel



Thank you for your attention