A Graph Radio k-Coloring Algorithm

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Abstract. For a positive integer k, a radio k-coloring of a simple connected graph $G = (V(G), E(G))$ is a mapping $f\colon V(G) \to \{0, 1, 2, \ldots\}$ such that $|f(u) - f(v)| \geqslant k + 1 - d(u, v)$ for each pair of distinct vertices u and v of G, where $d(u, v)$ is the distance between u and v in G. The *span* of a radio k-coloring f, $rc_k(f)$, is the maximum integer assigned by it to some vertex of G. The *radio k-chromatic number*, $rc_k(G)$ of G is $\min\{rc_k(f)\}$, where the minimum is taken over all radio k-colorings f of G. If k is the diameter of G, then $rc_k(G)$ is known as the radio number of G. In this paper, we give an algorithm to find an upper bound of $rc_k(G)$. We also give an algorithm that implement the result in [16,17] for lower bound of $rc_k(G)$. We check that for cycle C_n, upper and lower bound obtained from these algorithms coincide with the exact value of radio number, when n is an even integer with $4 \leqslant n \leqslant 400$. Also applying these algorithms we get the exact value of the radio number of several circulant graphs.

Keywords: Channel assignment, Radio k-coloring, Radio k-chromatic number, Span.

1 Introduction

A number of graph coloring problems have their roots in a communication problem known as the channel assignment problem. The channel assignment problem is the problem of assigning channels (non-negative integers) to the stations in an optimal way such as the interference is avoided, see Hale [3]. The interference is closely related to the location of the stations. Radio k-coloring of graphs is a variation of this channel assignment problem. For a positive integer k, a *radio k-coloring* f of a simple connected graph G is an assignment of non-negative integers to the vertices of G such that for every two distinct vertices u and v of G, $|f(u) - f(v)| \geqslant k + 1 - d(u, v)$. The *span* of a radio k-coloring f, $rc_k(f)$, is the maximum integer assigned by f to some vertex of G. The *radio k-chromatic number*, $rc_k(G)$ of G is $\min\{rc_k(f)\}$, where the minimum is taken over all radio k-colorings f of G. If k is the diameter of G, then $rc_k(G)$ is known as the radio number and is denoted by $rn(G)$.

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The problem of finding radio k-chromatic number of a graph is of great interest for its widespread applications to channel assignment problem. So far, radio k-chromatic number is known for very limited families of graphs and specific values of k. Radio number of C_n and P_n [12], C_n^2[11], P_n^2[10], Q_n [4] and complete m-ary trees [13] are determined. Ortiz et al. [15] have studied the radio number of generalized prism graphs and have computed the exact value for some particular cases. A lower bound for radio number of any tree have been given in [9] by Liu. In [16,17], Saha et al. have given a lower bound for $rc_k(G)$ of an arbitrary graph G.
The objective of this paper is to provide an algorithm to find an upper bound of $rc_k(G)$ of an arbitrary graph G and implementation of the results in [16,17] for a lower bound of the same.

2 Algorithm to Find a Lower Bound of $rc_k(G)$

In this section, we give the Algorithm 1 to find a lower bound of $rc_k(G)$.

Algorithm 1. Finding a lower bound of $rc_k(G)$

Data: Positive integer k and G be an n-vertex graph.
Result: Lower bound of $rc_k(G)$.
Using Floyed-Warshall's algorithm compute the distance matrix $D[n][n]$ $=(d[i][j])_{n\times n}$, where $d[i][j]$ represents the distance between the vertex i and j of G.

```
for l = 0 to n − 1 do
    for i = 0 to n − 1 do
        for j = 0 to n − 1 do
            s[l][i][j] = d[l][j] + a[i][j] + a[j][i]
            if b ⩽ s[l][i][j] and i ∉ {l, j} and j ≠ l then
                b = s[l][i][j]
            end
        end
    end
end
if k = diam(G) and n is an even integer then
    Lower bound = ⌈(3(k+1) − b)/2⌉ ((n−2)/2) + 1
end
else
    Lower bound = ⌈(3(k+1) − b)/2⌉ ⌊(n−2)/2⌋
end
Print Lower bound.
```

3 Radio k-Coloring Algorithm

In this section, we developed an algorithm that gives a radio coloring of an arbitrary graph G and hence finds an upper bound of $rc_k(G)$. The time complexity of this algorithm is $O(n^3)$.

Algorithm 2. Finding a radio k-coloring of a graph

```
input  : G be an n-vertex simple connected graph and k be a positive integer.
output: A radio k-coloring of G.
begin
    Compute the adjacency matrix a[n][n] of G.
    Using Floyed-Warshall's algorithm compute the distance matrix
    D[n][n] = (d[i][j])_{n×n}, where d[i][j] represents the distance between the
    vertex i and j of G.
    Initialization : Choose a vertex r, c be a two dimensional matrix
    for i = 0 to n − 1 do
        for j = 0 to n − 1 do
            if k + 1 ⩾ d[i][j] then
                c[i][j] = k + 1 − d[i][j]
            else
                c[i][j] = 0
            c[j][i] = c[i][j]
        c[i][i] = ∞, a large number
    l = r                                  /* Use r as the initialization vertex  */
    print : l and its color is zero        /* Use zero is the color of r  */
    for i = 1 to n − 1 do
        min = ∞, a large number
        for j = 0 to n − 1 do
            if min ⩾ c[l][j] then
                min = c[l][j]
                p = j
        for j = 0 to n − 1 do
            c[p][j] = c[p][j] + min
        for j = 0 to n − 1 do
            if c[p][j] < c[l][j] then
                c[p][j]=c[l][j]
        print : p and min      /* Use min is the color of the vertex p  */
        l = p
```

Example 1. In this example, we explain the intermediate stages of Algorithm 2 considering G as the graph in Fig. 1.

Let $k = 4$. Here D is the distance matrix and C_0 is the k-labeling matrix. Let R_j^p be the j^{th}-row of a matrix C_p. Here we consider $r = x_1$ is the initial vertex.

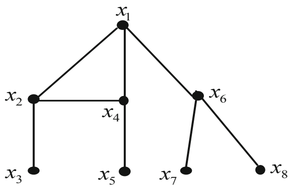

Fig. 1. The graph G

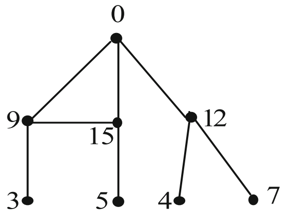

Fig. 2. Radio 4-coloring of G

Give color 0 to the vertex x_1. Minimum element of 1^{st}- row of the matrix C_0 is 3 and a position of this minimum element is at 3^{rd}-column of the matrix C_0. We assign color 3 to the vertex x_3. Replace the 3^{rd}-row of the matrix C_0 by $\max\{R_3^0+3, R_1^0\}$, let this new matrix be C_1. The minimum element in 3^{rd}-row of the matrix C_1 is 4 and a position of this minimum element is at 7^{th} column. We give color 4 to the vertex x_7. Replace the 7^{th}-row of the matrix C_1 by $\max\{R_7^1+4, R_3^1\}$ and we obtained a new matrix C_2. The minimum element in 7^{th}-row of the matrix C_2 is 5 and a position of this minimum element is at 5^{th}-column. We assign color 5 to the vertex x_5. By similar way we can give a coloring of the vertices of G as shown in Fig.2.

$$D: \begin{pmatrix} 0 & 1 & 2 & 1 & 2 & 1 & 2 & 2 \\ 1 & 0 & 1 & 1 & 2 & 2 & 3 & 3 \\ 2 & 1 & 0 & 2 & 3 & 3 & 4 & 4 \\ 1 & 1 & 2 & 0 & 1 & 2 & 3 & 3 \\ 2 & 2 & 3 & 1 & 0 & 3 & 4 & 4 \\ 1 & 2 & 3 & 2 & 3 & 0 & 1 & 1 \\ 2 & 3 & 4 & 3 & 4 & 1 & 0 & 2 \\ 2 & 3 & 4 & 3 & 4 & 1 & 2 & 0 \end{pmatrix} \quad C_0: \begin{pmatrix} \infty & 4 & \boxed{3} & 4 & 3 & 4 & 3 & 3 \\ 4 & \infty & 4 & 4 & 3 & 3 & 2 & 2 \\ 3 & 4 & \infty & 3 & 2 & 2 & 1 & 1 \\ 4 & 4 & 3 & \infty & 4 & 3 & 2 & 2 \\ 3 & 3 & 2 & 4 & \infty & 2 & 1 & 1 \\ 4 & 3 & 2 & 3 & 2 & \infty & 4 & 4 \\ 3 & 2 & 1 & 2 & 1 & 4 & \infty & 3 \\ 3 & 2 & 1 & 2 & 1 & 4 & 3 & \infty \end{pmatrix}.$$

$$C_1: \begin{pmatrix} \infty & 4 & 3 & 4 & 3 & 4 & 3 & 3 \\ 4 & \infty & 4 & 4 & 3 & 3 & 2 & 2 \\ \infty & 7 & \infty & 6 & 5 & 5 & \boxed{4} & 4 \\ 4 & 4 & 3 & \infty & 4 & 3 & 2 & 2 \\ 3 & 3 & 2 & 4 & \infty & 2 & 1 & 1 \\ 4 & 3 & 2 & 3 & 2 & \infty & 4 & 4 \\ 3 & 2 & 1 & 2 & 1 & 4 & \infty & 3 \\ 3 & 2 & 1 & 2 & 1 & 4 & 3 & \infty \end{pmatrix} \quad C_2: \begin{pmatrix} \infty & 4 & 3 & 4 & 3 & 4 & 3 & 3 \\ 4 & \infty & 4 & 4 & 3 & 3 & 2 & 2 \\ \infty & 7 & \infty & 6 & 6 & 5 & 4 & 4 \\ 4 & 4 & 3 & \infty & 4 & 3 & 2 & 2 \\ 3 & 3 & 2 & 4 & \infty & 2 & 1 & 1 \\ 4 & 3 & 2 & 3 & 2 & \infty & 4 & 4 \\ \infty & 7 & \infty & 6 & \boxed{5} & 8 & \infty & 7 \\ 3 & 2 & 1 & 2 & 1 & 4 & 3 & \infty \end{pmatrix}$$

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