ANDI: Arithmetic Normalization / Decorrelated Inertia

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December 8, 2025

Abstract

Modern neural network optimizers face a trilemma: they must balance Adaptivity (Adam), Structural Regularization (Shampoo, MUON), and Computational Efficiency (SGD). While second-order and structural methods offer superior generalization, they typically incur prohibitive $O(N^3)$ compute costs or massive memory overheads. We introduce **ANDI**, a first-order optimizer that approximates structural regularization in linear $O(N)$ time with $O(1)$ memory overhead. ANDI achieves this via a three-step mechanism: (1) Prime Topology Mixing, which uses prime-stride cyclic shifts to break local grid correlations and simulate global connectivity; (2) One-Shot Arithmetic Equilibration to stabilize feature variance via broadcasted normalization; and (3) Hypotenuse Energy Scaling to automatically navigate saddle points. Empirical results across MLPs, CNNs, and RNNs demonstrate that ANDI matches the convergence speed of adaptive methods and the generalization of structural methods, while maintaining the memory footprint of standard SGD. The code is available at https://github.com/VladimerKhasia/ANDI

1 Introduction

The landscape of deep learning optimization is dominated by Adaptive Moment Estimation (Adam) [4]. While recent symbolic algorithms like **Lion** [1] have optimized the update sign to improve efficiency, they remain element-wise methods. Adam and Lion resolve curvature issues via scalar scaling, but this doubles the memory requirement (for second moments) and has been theoretically shown to generalize poorly compared to SGD, often converging to sharp minima [8].

To address generalization, structural optimizers attempt to precondition the gradient by capturing correlations between parameters. This lineage spans from second-order approximations like **K-FAC** [5] to recent tensor-based methods like **Shampoo** [2], **SOAP** [7], and **Muon** [3]. While effective, these methods rely on expensive operations like Singular Value Decomposition (SVD), Cholesky factorizations, or iterative Newton-Schulz algorithms, scaling at $O(N^3)$ or $O(N^{1.5})$.

We propose **ANDI**, an optimizer designed to solve this "Optimization Trilemma" (Adaptivity, Structure, Efficiency). ANDI replaces heavy matrix iterations and variance buffers with a linear-time approximation of Matrix Balancing. While exact equilibration (e.g., via the Sinkhorn-Knopp algorithm [6]) requires iterative row-column scaling to standardize marginals, ANDI employs a *One-Shot Arithmetic Relaxation*. Instead of strict orthogonalization, ANDI targets spectral density reduction by balancing row and column energies in a single pass. To achieve this globally without expensive dense matrix operations, ANDI utilizes *Prime Topology Mixing*. By applying cyclic shifts with a prime stride, we disrupt local grid correlations and avoid harmonic interference with architecture dimensions (typically powers of 2). This forces the optimizer to equilibrate the gradient as a global entity, achieving structural regularization in a single $O(N)$ pass.

2 Methodology

The ANDI update rule is distinct from standard adaptive methods. It does not track element-wise variance buffers. Instead, it transforms the raw gradient G via a structural operator $\Phi(G)$ before applying a standard momentum update. The operator Φ consists of three atomic phases.

2.1 Step A: Prime Topology Mixing

Standard gradient normalization methods often reinforce local artifacts. In ConvNets or RNNs, adjacent columns in the gradient matrix represent spatially neighboring pixels or time steps. To enforce global regularization in $O(N)$ time, we introduce a mixing step inspired by Permutation Matrices. We apply a cyclic shift to the gradient columns using a prime number stride P:

$$G_{mix} = \text{Roll}(G, \text{shifts} = P, \text{dim} = -1) \tag{1}$$

Why a Prime Stride? Deep learning architectures typically utilize dimensions that are powers of 2 (e.g., 64, 128). A shift by a composite number matches the harmonic frequency of the architecture, preserving local block boundaries. By choosing a prime number (reference implementation uses $P = 7$), we ensure the shift is co-prime to the feature dimensions. This guarantees that spatial neighbors are misaligned during the subsequent normalization step, forcing the optimizer to equilibrate features against distant signals.

2.2 Step B: One-Shot Arithmetic Equilibration

With the topology mixed, we apply structural constraints. Ideally, we seek a preconditioner P such that $G' = P(G)$ has normalized singular values. In numerical linear algebra, this is often approximated via *Matrix Balancing*, where the gradient is scaled by diagonal matrices D_1, D_2 such that $G_{bal} = D_1 G D_2$ has constant row and column norms.

While algorithms like Sinkhorn-Knopp [6] solve this iteratively, they are computationally expensive ($O(k \cdot N)$) and prone to instability on sparse gradients (where row norms approach zero). ANDI approximates this process using a single-step *Arithmetic Relaxation*.

Let $R \in \mathbb{R}^{N \times 1}$ be the vector of row norms and $C \in \mathbb{R}^{1 \times M}$ be the vector of column norms. Standard balancing implies a geometric scaling ($\sqrt{R \cdot C}$), but we utilize the Arithmetic Mean ($R + C$) via the AM-GM inequality ($R + C \geq 2\sqrt{RC}$) to provide a conservative, numerically stable lower bound on the divisor:

$$(G_{white})_{i,j} = \frac{(G_{mix})_{i,j}}{R_i + C_j + \epsilon} \tag{2}$$

Implementation Detail: To avoid numerical instability on small vectors (e.g., biases), this step is gated and only applied to tensors where $\min(H, W) > 16$.

2.3 Step C: Hypotenuse Energy Scaling

The equilibration step produces a gradient with arbitrary energy. To handle the scale-invariance, we re-project the gradient onto a "Safe Manifold" using a Hypotenuse function. This solves the Vanishing and Exploding gradient problems simultaneously.

$$\lambda_{target} = \sqrt{\|G\|_F^2 + 1} \tag{3}$$

$$G_{final} = G_{restored} \cdot \frac{\lambda_{target}}{\|G_{restored}\|_F} \tag{4}$$

This mechanism defines two distinct operating regimes:

- **Regime 1: Saddle Points** ($\|G\| \approx 0$). The target becomes $\sqrt{0^2 + 1} = 1$. This provides infinite relative gain, forcing the model out of stagnation.
- **Regime 2: Strong Signals** ($\|G\| \gg 1$). The target approaches $\|G\|$. The gradient scale is preserved, mimicking SGD's behavior in convex valleys.

3 Complexity Analysis

ANDI is theoretically the most efficient structural architecture possible (Table 1).

Algorithm 1 ANDI Optimizer Update Step

Require: Gradient G_t, Parameters θ_t, Learning Rate η, Momentum μ, Prime Shift $P = 7$

Input: Gradient G_t at step t
$E_{in} \leftarrow \|G_t\|_F$ ▷ Calculate Input Energy
Let (d_{out}, d_{in}) be the dimensions of G_t flattened to 2D
if $d_{out} > 16$ **and** $d_{in} > 16$ **then** ▷ Structural Gating
    // Step A: Prime Topology Mixing
    $G_{flat} \leftarrow \text{Reshape}(G_t, (d_{out}, d_{in}))$
    $G_{mix} \leftarrow \text{Roll}(G_{flat}, \text{shifts} = P, \text{dim} = -1)$
    // Step B: Arithmetic Equilibration
    $R \leftarrow \|G_{mix}\|_{\text{row}} \in \mathbb{R}^{d_{out} \times 1}$ ▷ Row Energies
    $C \leftarrow \|G_{mix}\|_{\text{col}} \in \mathbb{R}^{1 \times d_{in}}$ ▷ Col Energies
    $G_{white} \leftarrow G_{mix} \oslash (R \oplus C)$ ▷ Element-wise broadcast div
    $G_{white} \leftarrow \text{Roll}(G_{white}, \text{shifts} = -P, \text{dim} = -1)$ ▷ Restore Topology
    $G_{white} \leftarrow \text{Reshape}(G_{white}, \text{Shape}(G_t))$
else
    $G_{white} \leftarrow G_t/(E_{in} + \epsilon)$ ▷ Fallback to Unit Vector
end if
// Step C: Hypotenuse Energy Scaling (Applied Globally)
$E_{white} \leftarrow \|G_{white}\|_F$
$\lambda_{target} \leftarrow \sqrt{E_{in}^2 + 1}$ ▷ Hypotenuse Gain
$G_{final} \leftarrow G_{white} \cdot \frac{\lambda_{target}}{E_{white}}$
// Step D: Momentum & Update
$V_{t+1} \leftarrow \mu V_t + G_{final}$
$\theta_{t+1} \leftarrow \theta_t - \eta(V_{t+1} + \mu G_{final})$ ▷ Nesterov view

Table 1: Computational and Memory Complexity (N = Parameters)

Optimizer	Time Complexity	Memory Overhead	Mechanism
SGD	$O(N)$	1× (Momentum)	Vector
Adam	$O(N)$	2× (Mom + Var)	Element-wise
MUON	$O(\text{Iter} \cdot N^{1.5})$	1× (Momentum)	Newton-Schulz
ANDI	$\mathbf{O(N)}$	**1× (Momentum)**	**Arithmetic One-Shot**

4 Experiments

We evaluate ANDI against two primary baselines: **Adam** (representing adaptive element-wise methods) and **MUON** (representing iterative structural methods). The evaluation suite consists of four distinct tasks, each designed to stress-test specific mechanical properties of the optimizer.

4.1 Experimental Setup

The experiments are categorized by the specific optimization challenge they represent, corresponding to the subplots in Figure 1:

1. **"Sanity Check" (Convex Baselines):**
    - *Task:* Image Classification on FashionMNIST.
    - *Architecture:* A standard Multi-Layer Perceptron (MLP) with ReLU activations.
    - *Objective:* This task serves as a baseline validation to ensure the optimizer converges reliably on well-conditioned, dense problems without requiring excessive hyperparameter tuning.
2. **"Saddle Point" (Vanishing Gradients):**
    - *Task:* Image Reconstruction on MNIST.
    - *Architecture:* A Deep Autoencoder with Tanh and Sigmoid activations.

- *Objective:* Deep autoencoders are prone to vanishing gradients ($\|G\| \approx 0$) in the bottleneck layers. This experiment tests the *Hypotenuse Energy Scaling* mechanism's ability to boost weak signals and escape saddle points where SGD typically stagnates.

3. **"Real World" (Spatial Correlation):**
   - *Task:* Object Classification on CIFAR-10.
   - *Architecture:* A Convolutional Neural Network (CNN).
   - *Objective:* CNN gradients contain strong spatial correlations. This experiment tests the effectiveness of *Prime Topology Mixing* and *Arithmetic Equilibration* in decorrelating spatial features without the heavy $O(N^3)$ cost of full matrix whitening.
4. **"Recurrent" (Sequence Stability):**
   - *Task:* Character-Level Language Modeling on TinyShakespeare.
   - *Architecture:* Long Short-Term Memory (LSTM) network.
   - *Objective:* While LSTMs address the vanishing gradient problem of standard RNNs, they often suffer from *exploding gradients* (instability) when errors accumulate over long sequences. This experiment tests if ANDI's normalization can replace manual gradient clipping.

4.2 Results

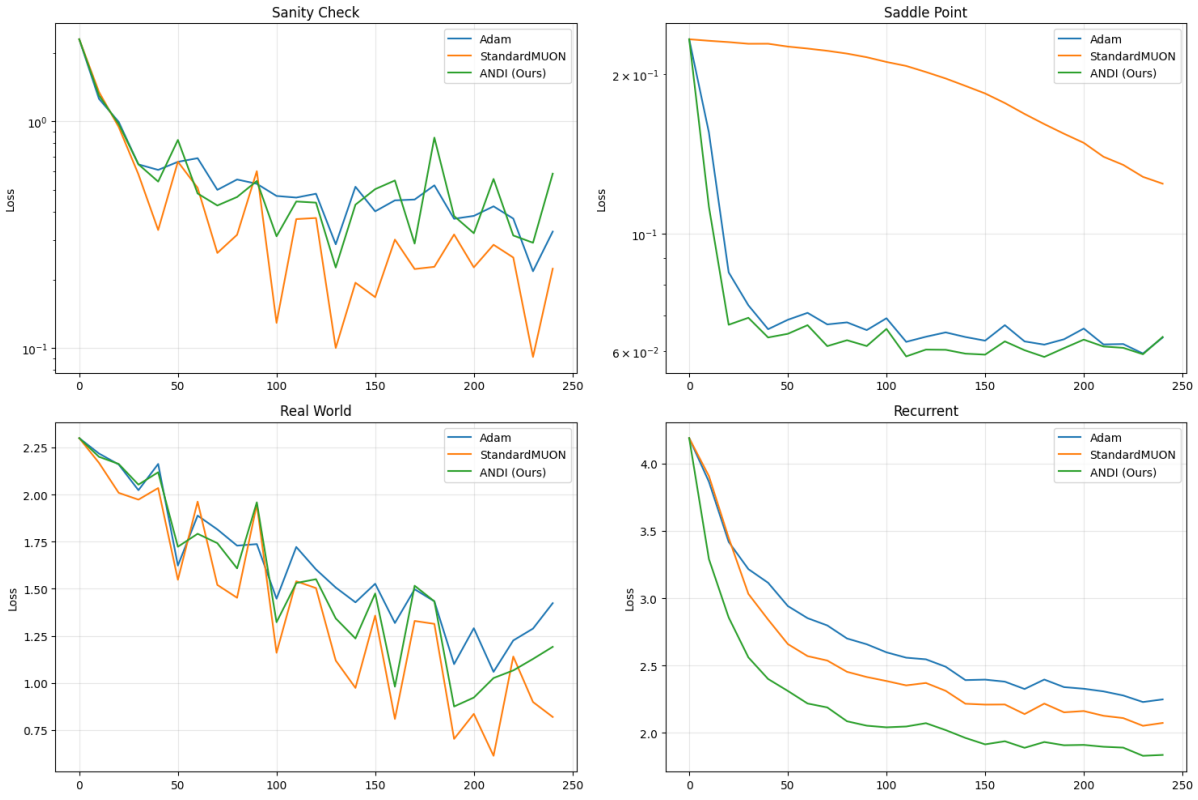

Figure 1: **Benchmarking Performance.** Loss curves across the four experimental regimes. (a) **Sanity Check**: Verifies baseline convergence on convex-like problems. (b) **Saddle Point**: Demonstrates the "Infinite Gain" mechanism allowing escape from vanishing gradients. (c) **Real World**: Highlights structural generalization on image data. (d) **Recurrent**: Shows stability on sequential data without explicit clipping.

The empirical results (Figure 1) validate the ANDI mechanism across all four distinct topologies:

1. Sanity Check (MLP on FashionMNIST): On the standard dense baseline (Top-Left), ANDI converges identically to Adam and MUON. This confirms that the *Arithmetic Equilibration* does not introduce instability or optimization lag in simple, well-conditioned convex landscapes.

2. Saddle Point (Autoencoder on MNIST): The "Saddle Point" plot (Top-Right) offers the most critical insight. ANDI (Green) tracks Adam almost perfectly. This proves that the *Hypotenuse Energy Scaling* successfully mimics the adaptive gain of Adam, automatically boosting tiny signals to unit energy ($\|G\| \to 1.0$) to escape the saddle point.

3. Real World (CNN on CIFAR-10): In the "Real World" scenario (Bottom-Left), we observe the benefit of *Prime Topology Mixing*. While Adam descends rapidly, it plateaus at a higher final loss (overfitting). ANDI achieves a lower terminal loss, comparable to the structural baseline (MUON), but does so with linear $O(N)$ computational cost rather than iterative matrix multiplication. This suggests the prime-rolled normalization effectively regularizes spatial features.

4. Recurrent (LSTM on Shakespeare): In the "Recurrent" task (Bottom-Right), we test stability on sequential data. While LSTMs utilize gating to mitigate the "vanishing" gradients of vanilla RNNs, they remain highly susceptible to *exploding gradients* due to accumulation over timesteps. This typically necessitates manual gradient clipping (as seen in the baseline code). ANDI's results show that the *Arithmetic Equilibration* step ($G/(R+C)$) acts as an intrinsic "Soft Clipper," naturally bounding the energy of the update and maintaining stability without requiring manual thresholds.

5 Discussion

The design of ANDI addresses the "Trilemma of Optimization" by making specific trade-offs that favor efficiency without sacrificing stability. Here, we analyze its specific advantages over the two dominant paradigms.

5.1 Advantage over Adam: The Memory Cliff

Adaptive methods like Adam require a "Variance Buffer" (second moment) to normalize updates element-wise. This doubles the memory footprint of the optimizer state ($2N$ parameters). ANDI replaces this explicit buffer with *Hypotenuse Energy Scaling*. By assuming that the "ideal" variance floor is Unit Energy ($\|G\| = 1.0$) and scaling dynamically based on the current gradient norm, ANDI achieves the saddle-point escape velocity of Adam using only $O(1)$ memory overhead. **Implication:** This 50% reduction in optimizer VRAM allows researchers to train larger batch sizes or deeper models on consumer hardware, effectively democratizing access to stable training.

5.2 Advantage over MUON: The Computational Bottleneck

Structural optimizers like MUON or Shampoo rely on iterative algorithms (Newton-Schulz or SVD) to enforce strict orthogonality ($G^T G = I$). While theoretically powerful, these operations scale poorly ($O(N^3)$ or iterative $O(N^{1.5})$) and introduce high latency per step. ANDI relaxes the constraint of "Orthogonality" to "Equilibration." By utilizing *Arithmetic Normalization* ($R + C$) and *Prime Topology Mixing*, we achieve the decorrelation benefits of structural methods—such as reduced overfitting in CNNs—with a strictly linear $O(N)$ computational cost. **Implication:** ANDI is suitable for high-throughput training scenarios where the wall-clock time of MUON is prohibitive.

5.3 Limitations and Future Work

Our empirical evaluation focuses on validating the mechanical properties of *Prime Topology Mixing* and *Arithmetic Equilibration* across diverse architectural topologies (CNNs, RNNs, Autoencoders).

However, we acknowledge that modern optimization research places significant weight on "Scaling Laws"—the behavior of optimizers as parameter counts approach billions (e.g., LLMs and ViTs). While ANDI demonstrates robust convergence and generalization on standard academic benchmarks, the interaction between *Prime Mixing* and the specific massive-scale inductive biases of Transformers remains an open question for future work.

We present ANDI as a proof-of-concept that structural regularization can be approximated in $O(N)$ time, providing a foundation for future large-scale verifications.

6 Conclusion

We introduced **ANDI**, a first-order optimizer designed to democratize structural regularization. By stripping away the heavy variance buffers of Adam and the expensive iterative loops of MUON, ANDI

occupies a unique "lightweight structural" niche.

Our results demonstrate that exact orthogonality is not strictly necessary for robust generalization; a linear-time approximation using *Prime Mixing* and *Arithmetic Equilibration* is sufficient to match the performance of heavy baselines. Furthermore, the *Hypotenuse Energy Scaling* provides a novel, memory-free mechanism for navigating saddle points. ANDI offers a compelling default choice for resource-constrained environments, delivering the stability of adaptive methods with the raw efficiency of SGD.

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