

## 5. Доказательство леммы 5.2.

```
> restart; read("newlib.m"); with(mylib): with(LinearAlgebra):
```

Результат замены с  $s_1=0$  в исходной системе:

```
> zamproc(0,q1,0,0,p2,q2,1,0, r1,0,r2,s2):
```

$$\begin{matrix} q1\ r1\ r2, q1\ r1\ s2, 0, 0 \\ \frac{r1\ (p2\ r1^2 + q2\ r1\ r2 - r2^2\ (q1 - 1))}{s2}, -r1\ (-q2\ r1 + r2\ (q1 - 2)), r1\ s2, 0 \end{matrix}$$

1)  $a_1=0 \Leftrightarrow r_2=0$

```
> zamproc(0,q1,0,0,p2,q2,1,0, r1,0,0,s2):
```

$$\begin{matrix} 0, q1\ r1\ s2, 0, 0 \\ \frac{r1^3\ p2}{s2}, q2\ r1^2, r1\ s2, 0 \end{matrix}$$

1<sub>1</sub>)  $b_2=0 \Leftrightarrow q_2=0$

```
> zamproc(0,q1,0,0,p2,0,1,0, r1,0,0,s2):
```

$$\begin{matrix} 0, q1\ r1\ s2, 0, 0 \\ \frac{r1^3\ p2}{s2}, 0, r1\ s2, 0 \end{matrix}$$

```
> r11 := 1/(q1*s2):
  zamproc(0,q1,0,0,p2,0,1,0, r11,0,0,s2):
```

$$\begin{matrix} 0, 1, 0, 0 \\ \frac{p2}{q1^3\ s2^4}, 0, \frac{1}{q1}, 0 \end{matrix}$$

```
> s21 := abs(p2)^(1/4)*abs(q1)^(-3/4):
  r11 := 1/(q1*s21):
  zamproc(0,q1,0,0,p2,0,1,0, r11,0,0,s21):
```

$$\begin{matrix} 0, 1, 0, 0 \\ \frac{|q1|^3\ p2}{q1^3\ |p2|}, 0, \frac{1}{q1}, 0 \end{matrix}$$

1<sub>2</sub>)  $b_2 \neq 0 \Leftrightarrow q_2 \neq 0$

```
> zamproc(0,q1,0,0,p2,q2,1,0, r1,0,0,s2):
```

$$\begin{matrix} 0, q1\ r1\ s2, 0, 0 \\ \frac{r1^3\ p2}{s2}, q2\ r1^2, r1\ s2, 0 \end{matrix}$$

```
> r11 := abs(q2)^(-1/2):
  zamproc(0,q1,0,0,p2,q2,1,0, r11,0,0,s2):
```

$$\begin{matrix} 0, \frac{q1\ s2}{\sqrt{|q2|}}, 0, 0 \\ \frac{p2}{|q2|^3\ |s2|^2}, \frac{q2}{|q2|}, \frac{s2}{\sqrt{|q2|}}, 0 \end{matrix}$$

```
> s21 := q2*abs(q2)^(-1/2)/q1:
  zamproc(0,q1,0,0,p2,q2,1,0, r11,0,0,s21):
```

$$\begin{matrix} 0, \frac{q2}{|q2|}, 0, 0 \\ \frac{p2\ q1}{|q2|\ q2}, \frac{q2}{|q2|}, \frac{q2}{|q2|\ q1}, 0 \end{matrix}$$

```
> u = 1/q1;
  v = p2*q1/(q2^2);
```

$$\begin{matrix} u = \frac{1}{q1} \\ v = \frac{p2\ q1}{q2^2} \end{matrix}$$

2)  $a_1 \neq 0 \Leftrightarrow r_2 \neq 0$

2<sub>1</sub>)  $q_1=2$

```
> zamproc(0,2,0,0,p2,q2,1,0, r1,0,r2,s2):
```

$$\begin{matrix} 2\ r1\ r2, 2\ r1\ s2, 0, 0 \\ \frac{r1\ (p2\ r1^2 + q2\ r1\ r2 - r2^2)}{s2}, q2\ r1^2, r1\ s2, 0 \end{matrix}$$

2<sub>1</sub>)  $b_2=0 \Leftrightarrow q_2=0$

```
> zamproc(0,2,0,0,p2,0,1,0, r1,0,r2,s2):
```

$$\frac{2 r l r 2, 2 r l s 2, 0, 0}{r l (p 2 r l^2 - r 2^2)}, 0, r l s 2, 0$$

$$2_1^{1a}) a_2 = 0 \Rightarrow p_2 > 0$$

```
> r21 := sqrt(p2)*r1:
  zamproc(0,2,0,0,p2,0,1,0, r1,0,r21,s2):
```

$$\frac{2 r l^2 \sqrt{p 2}, 2 r l s 2, 0, 0}{0, 0, r l s 2, 0}$$

```
> r11 := (4*p2)^(-1/4):
  r21 := sqrt(p2)*r11:
  s21 := 1/r11:
  zamproc(0,2,0,0,p2,0,1,0, r11,0,r21,s21):
```

$$1, 2, 0, 0$$

$$0, 0, 1, 0$$

$$2_1^{1b}) p_2 < 0$$

```
> zamproc(0,2,0,0,p2,0,1,0, r1,0,0,s2):
```

$$0, 2 r l s 2, 0, 0$$

$$\frac{p 2 r l^3}{s 2}, 0, r l s 2, 0$$

$$2_1^2) b_2 \neq 0 \Leftrightarrow q_2 \neq 0$$

$$2_1^{2a}) a_2 = 0$$

```
> zamproc(0,2,0,0,p2,q2,1,0, r1,0,r2,s2):
```

$$\frac{2 r l r 2, 2 r l s 2, 0, 0}{r l (p 2 r l^2 + q 2 r l r 2 - r 2^2)}, q 2 r l^2, r l s 2, 0$$

```
> solve(p2*r1^2+q2*r1*r2-r2^2, r2);
```

$$\left( \frac{q 2}{2} + \frac{\sqrt{q 2^2 + 4 p 2}}{2} \right) r l, \left( \frac{q 2}{2} - \frac{\sqrt{q 2^2 + 4 p 2}}{2} \right) r l$$

```
> r21 := ((1/2)+(1/2)*sqrt(q2^2+4*p2)*abs(q2)^(-1))*q2*r1:
  zamproc(0,2,0,0,p2,q2,1,0, r1,0,r21,s2):
```

$$\frac{r l^2 (|q 2| + \sqrt{q 2^2 + 4 p 2}) q 2}{|q 2|}, 2 r l s 2, 0, 0$$

$$\frac{r l^3 (q 2^2 + 4 p 2) (|q 2|^2 - q 2^2)}{4 |q 2|^2 s 2}, q 2 r l^2, r l s 2, 0$$

```
> r11 := abs(q2)^(-1/2):
  s21 := q2*r11:
  r21 := ((1/2)+(1/2)*sqrt(q2^2+4*p2)*abs(q2)^(-1))*q2*r11:
  zamproc(0,2,0,0,p2,q2,1,0, r11,0,r21,s21):
```

$$\frac{q 2 (|q 2| + \sqrt{q 2^2 + 4 p 2})}{|q 2|^2}, \frac{2 q 2}{|q 2|}, 0, 0$$

$$\frac{(q 2^2 + 4 p 2) (|q 2|^2 - q 2^2)}{4 |q 2|^3 q 2}, \frac{q 2}{|q 2|}, \frac{q 2}{|q 2|}, 0$$

```
> u = (abs(q2)+sqrt(q2^2+4*p2))/abs(q2);
  v = 2;
```

$$u = \frac{|q 2| + \sqrt{q 2^2 + 4 p 2}}{|q 2|}$$

$$v = 2$$

$$2_2) q_1 \neq 2$$

$$2_2^{1}) b_2 = 0$$

```
> r21 := q2*r1*(q1-2)^(-1):
  zamproc(0,q1,0,0,p2,q2,1,0, r1,0,r21,s2):
```

$$\frac{q l r l^2 q 2}{q l - 2}, q l r l s 2, 0, 0$$

$$\frac{(p 2 (q l - 2)^2 - q 2^2) r l^3}{(q l - 2)^2 s 2}, 0, r l s 2, 0$$

$$2_2^{1a}) a_2 = 0$$

```
> p21 := q2^2/(q1-2)^2:
  r11 := abs(q1-2)^(1/2)*abs(q1)^(-1/2)*abs(q2)^(-1/2):
```

r21 := q2\*r11\*(q1-2)^(-1):

s21 := q1\*q2\*r11/(q1-2):

zamproc(0,q1,0,0,p21,q2,1,0, r11,0,r21,s21):

$$\frac{q1\ q2\ \left| \frac{q1-2}{q1\ q2} \right|}{q1-2}, \frac{q2\ q1^2\ \left| \frac{q1-2}{q1\ q2} \right|}{q1-2}, 0, 0$$

$$0, 0, \frac{q1\ q2\ \left| \frac{q1-2}{q1\ q2} \right|}{q1-2}, 0$$

$2_2^{1b})\ a_2 \neq 0$

> r11 := abs(q1-2)^(1/2)\*abs(q1)^(-1/2)\*abs(q2)^(-1/2):

r21 := q2\*r11\*(q1-2)^(-1):

s21 := q2\*r11/(q1-2):

zamproc(0,q1,0,0,p2,q2,1,0, r11,0,r21,s21):

$$\frac{q1\ q2\ \left| \frac{q1-2}{q1\ q2} \right|}{q1-2}, \frac{q1\ q2\ \left| \frac{q1-2}{q1\ q2} \right|}{q1-2}, 0, 0$$

$$\frac{(p2\ (q1-2)^2 - q2^2)\ \left| \frac{q1-2}{q1\ q2} \right|}{(q1-2)\ q2}, 0, \frac{q2\ \left| \frac{q1-2}{q1\ q2} \right|}{q1-2}, 0$$

> u = 1/q1;

v = (p2\*(q1-2)^2-q2^2)/(q1\*q2^2);

$$u = \frac{1}{q1}$$

$$v = \frac{p2\ (q1-2)^2 - q2^2}{q2^2\ q1}$$

$2_2^2)\ b_2 \neq 0$

$2_2^{2a})\ a_2 = 0$

> zamproc(0,q1,0,0,p2,q2,1,0, r1,0,r2,s2):

$$\frac{q1\ r1\ r2, q1\ r1\ s2, 0, 0}{s2}, \frac{(p2\ r1^2 + q2\ r1\ r2 - r2^2\ (q1-1))\ r1}{s2}, -r1\ (-q2\ r1 + r2\ (q1-2)), r1\ s2, 0$$

> solve(p2\*r1^2+q2\*r1\*r2-r2^2\*(q1-1), r1);

$$\frac{(-q2 + \sqrt{4\ p2\ q1 + q2^2 - 4\ p2})\ r2}{2\ p2}, -\frac{(q2 + \sqrt{4\ p2\ q1 + q2^2 - 4\ p2})\ r2}{2\ p2}$$

> r11 := -q2\*(1+sqrt(4\*p2\*q1+q2^2-4\*p2))\*abs(q2)^(-1)\*r2/(2\*p2):

zamproc(0,q1,0,0,p2,q2,1,0, r11,0,r2,s2):

$$-\frac{q1\ q2\ (|q2| + \sqrt{(4\ q1 - 4)\ p2 + q2^2})\ r2^2}{2\ |q2|\ p2}, -\frac{q1\ q2\ (|q2| + \sqrt{(4\ q1 - 4)\ p2 + q2^2})\ r2\ s2}{2\ |q2|\ p2}, 0, 0$$

$$\frac{q2\ (|q2| + \sqrt{(4\ q1 - 4)\ p2 + q2^2})\ (4\ p2\ q1 + q2^2 - 4\ p2)\ r2^3\ (|q2|^2 - q2^2)}{8\ p2^2\ |q2|^3\ s2},$$

$$\frac{q2\ (|q2| + \sqrt{(4\ q1 - 4)\ p2 + q2^2})\ ((q2^2 + 2\ (q1-2)\ p2)\ |q2| + q2^2\ \sqrt{(4\ q1 - 4)\ p2 + q2^2})\ r2^2}{4\ p2^2\ |q2|^2}, -\frac{q2\ (|q2| + \sqrt{(4\ q1 - 4)\ p2 + q2^2})\ r2\ s2}{2\ |q2|\ p2}, 0$$

> simplify((q2\*(abs(q2)+sqrt((4\*q1-4)\*p2+q2^2)))\*((q2^2+(2\*(q1-2))\*p2)\*abs(q2)+q2^2\*sqrt((4\*q1-4)\*p2+q2^2))\*r2^2/(4\*abs(q2)^2\*p2^2))/(-q2\*(1+sqrt(4\*p2\*q1+q2^2-4\*p2))\*abs(q2)^(-1))/(2\*p2));

$$-\frac{((q2^2 + (2\ q1 - 4)\ p2)\ |q2| + q2^2\ \sqrt{(4\ q1 - 4)\ p2 + q2^2})\ r2^2}{2\ |q2|\ p2}$$

> p2 = solve((q2^2+(2\*q1-4)\*p2)\*q2+q2^2\*sqrt((4\*q1-4)\*p2+q2^2), p2);

$$p2 = \left( 0, \frac{q2^2}{q1^2 - 4\ q1 + 4} \right)$$

> #simplify(subs(p2 = q2^2/(q1^2-4\*q1+4), (q2^2+(2\*q1-4)\*p2)\*abs(q2)+q2^2\*sqrt((4\*q1-4)\*p2+q2^2)));

> r21 := (abs(q2)+sqrt((4\*q1-4)\*p2+q2^2))^(1/2)\*((q2^2+(2\*(q1-2))\*p2)\*abs(q2)+q2^2\*sqrt((4\*q1-4)\*p2+q2^2))^(1/2)\*2\*p2\*abs(q2)^(1/2):

s21 := -2\*p2/((abs(q2)+sqrt((4\*q1-4)\*p2+q2^2))\*r21):

r11 := -q2\*(1+sqrt(4\*p2\*q1+q2^2-4\*p2))\*abs(q2)^(-1)\*r21/(2\*p2):

zamproc(0,q1,0,0,p2,q2,1,0, r11,0,r21,s21):

$$-\frac{2\ p2\ q2\ q1}{(q2^2 + 2\ (q1-2)\ p2)\ |q2| + q2^2\ \sqrt{(4\ q1 - 4)\ p2 + q2^2}}, \frac{q2\ q1}{|q2|}, 0, 0$$

$$\left( 8 \left( \left( (q1-2)p2 + \frac{q2^2}{2} \right) |q2|^2 + ((q1-2)p2 + q2^2) |q2| \sqrt{(4q1-4)p2 + q2^2} + 2q2^2 \left( (q1-1)p2 + \frac{q2^2}{4} \right) \right) q2 p2 \left( (q1-1)p2 + \frac{q2^2}{4} \right) (q2 + |q2|) (q2 - |q2|) \right) / \left( \left( |q2| + \sqrt{(4q1-4)p2 + q2^2} \right) |q2| \left( (q2^2 + (2q1-4)p2) |q2| + q2^2 \sqrt{(4q1-4)p2 + q2^2} \right)^3 \right), \frac{q2}{|q2|}, \frac{q2^2}{|q2|}, 0$$

```
> u1 := -2*p2*abs(q2)*q1/((q2^2+(2*(q1-2))*p2)*abs(q2)+q2^2*sqrt((4*q1-4)*p2+q2^2));
v1 := q1;
```

$$u1 := - \frac{2 p2 |q2| q1}{(q2^2 + 2 (q1 - 2) p2) |q2| + q2^2 \sqrt{(4 q1 - 4) p2 + q2^2}}$$

$$v1 := q1$$

```
> simplify(v1-u1);
```

$$\frac{q1 \left( 2 |q2| p2 q1 + |q2| q2^2 + q2^2 \sqrt{(4 q1 - 4) p2 + q2^2} - 2 |q2| p2 \right)}{2 |q2| p2 q1 + |q2| q2^2 + q2^2 \sqrt{(4 q1 - 4) p2 + q2^2} - 4 |q2| p2}$$

```
> simplify(v1-(2*u1-1)*u1^(-1));
```

$$\frac{((2 q1^2 - 6 q1 + 4) p2 - q2^2) |q2| - q2^2 \sqrt{(4 q1 - 4) p2 + q2^2}}{2 |q2| p2 q1}$$

```
> solve(((2*q1^2-6*q1+4)*p2-q2^2)-q2*sqrt((4*q1-4)*p2+q2^2), p2);
```

$$0, \frac{q2^2}{q1^2 - 4 q1 + 4}$$

```
> simplify(subs(p2 = q2^2/(q1^2-4*q1+4), ((2*q1^2-6*q1+4)*p2-q2^2)-abs(q2)*sqrt((4*q1-4)*p2+q2^2)));
```

$$\frac{-(q1-2) |q2| \sqrt{\frac{q1^2 q2^2}{(q1-2)^2}} + q1 q2^2}{q1-2}$$

```
> simplify(v1-2*u1*(u1+1)^(-1));
```

$$\frac{(|q2| + \sqrt{(4 q1 - 4) p2 + q2^2}) q1 q2^2}{(q2^2 - 4 p2) |q2| + q2^2 \sqrt{(4 q1 - 4) p2 + q2^2}}$$

Сведение к системам из II части списка.

Результат произвольной замены в исходной системе:

$$\begin{aligned} & \text{M} := \text{zamproc}(0, q1, 0, 0, p2, q2, 1, 0, r1, s1, r2, s2): \\ & - \frac{r1 (p2 r1^2 s1 - r2 (q1 s2 - q2 s1) r1 + r2^2 s1)}{r1 s2 - s1 r2}, \frac{(-3 p2 s1^2 + q1 s2^2 - q2 s1 s2) r1^2 + 2 s1 r2 (-q2 s1 + s2 (q1 - 1)) r1 - s1^2 r2^2}{r1 s2 - s1 r2}, \\ & - \frac{3 s1 \left( \left( p2 r1 + \frac{q2 r2}{3} \right) s1^2 - \frac{s2 (-2 q2 r1 + r2 (q1 - 2)) s1}{3} - \frac{2 \left( q1 - \frac{1}{2} \right) s2^2 r1}{3} \right)}{r1 s2 - s1 r2}, \frac{-s1^2 (p2 s1^2 + q2 s1 s2 - s2^2 (q1 - 1))}{r1 s2 - s1 r2}, \\ & \frac{(p2 r1^2 + q2 r1 r2 - r2^2 (q1 - 1)) r1^2}{r1 s2 - s1 r2}, \frac{3 r1 \left( \left( p2 s1 + \frac{q2 s2}{3} \right) r1^2 - \frac{r2 (-2 q2 s1 + s2 (q1 - 2)) r1}{3} - \frac{2 \left( q1 - \frac{1}{2} \right) s1 r2^2}{3} \right)}{r1 s2 - s1 r2}, \\ & \frac{(3 p2 s1^2 + 2 q2 s1 s2 + s2^2) r1^2 - 2 s1 \left( -\frac{q2 s1}{2} + s2 (q1 - 1) \right) r2 r1 - q1 s1^2 r2^2}{r1 s2 - s1 r2}, \frac{s1 (p2 r1 s1^2 - s2 (q1 r2 - q2 r1) s1 + r1 s2^2)}{r1 s2 - s1 r2} \end{aligned}$$

NSF<sub>1</sub><sup>4,1</sup>

```
> solve([M[1,3],M[1,4],M[2,1],M[2,2]], {q1,p2,q2,r1,s1,r2,s2});
```

$$\left\{ p2 = \frac{s2^2}{s1^2}, q1 = q1, q2 = \frac{s2 (q1 - 2)}{s1}, r1 = 0, r2 = r2, s1 = s1, s2 = s2 \right\}, \left\{ p2 = \frac{r2^2}{r1^2}, q1 = q1, q2 = \frac{r2 (q1 - 2)}{r1}, r1 = r1, r2 = r2, s1 = 0, s2 = s2 \right\}, \left\{ p2 = \frac{s2^2}{s1^2}, q1 = 2, q2 = 0, r1 = r1, r2 = -\frac{r1 s2}{s1}, s1 = s1, s2 = s2 \right\}$$

```
> q11 := 2:
```

```
q21 := 0:
```

```
p21 := s2^2/s1^2:
```

```
r21 := -r1*s2/s1:
```

```
zamproc(0,q11,0,0,p21,q21,1,0, r1,s1,r21,s2):
```

$$- \frac{2 s2 r1^2}{s1}, -2 r1 s2, 0, 0$$

$$0, 0, 2 r1 s2, 2 s1 s2$$

NSF<sub>3</sub><sup>4,1</sup>

```
> solve([M[1,2],M[1,4],M[2,1],M[2,2]], {p2,q1,r1,s1,r2,s2});
```

$$\left\{p2 = \frac{q2^2}{4}, q1 = 0, r1 = -\frac{2r2}{q2}, r2 = r2, s1 = 0, s2 = s2\right\}, \left\{p2 = 4q2^2, q1 = \frac{3}{2}, r1 = -\frac{r2}{2q2}, r2 = r2, s1 = \frac{s2}{4q2}, s2 = s2\right\}$$

```
> q11 := 3/2:
p21 := 4*q2^2:
r21 := -2*q2*r1:
s21 := 4*q2*s1:
zamproc(0,q11,0,0,p21,q2,1,0, r1,s1,r21,s21):
-3 q2 r1^2, 0, 3 s1^2 q2, 0
0, 0, 6 r1 q2 s1, 6 s1^2 q2
```

```
> q11 := 3/2:
p21 := 4*q2^2:
s11 := 1/sqrt(6*abs(q2)):
r11 := s11:
r21 := -2*q2*r11:
s21 := 4*q2*s11:
zamproc(0,q11,0,0,p21,q2,1,0, r11,s11,r21,s21):
- \frac{q2}{2 |q2|}, 0, \frac{q2}{2 |q2|}, 0
0, 0, \frac{q2}{|q2|}, \frac{q2}{|q2|}
```

$NSF_{13}^{4,1}$

```
> solve([M[1,2],M[1,3],M[2,1],M[2,3]], {q1,p2,r1,s1,r2,s2}):
\left\{p2 = \frac{4 q2^2}{9}, q1 = \frac{1}{2}, r1 = -\frac{3 r2}{4 q2}, r2 = r2, s1 = s1, s2 = \frac{2 q2 s1}{3}\right\}
```

```
> p21 := 4*q2^2*(1/9):
q11 := 1/2:
r11 := -3*r2/(4*q2):
s21 := 2*q2*s1*(1/3):
zamproc(0,q11,0,0,p21,q2,1,0, r11,s1,r2,s21):
- \frac{3 r2^2}{8 q2}, 0, 0, \frac{8 q2^2 s1^3}{9 r2}
0, -\frac{9 r2^2}{16 q2}, 0, q2 s1^2
```

```
> p21 := 4*q2^2*(1/9):
q11 := 1/2:
s11 := 1/sqrt(abs(q2)):
r21 := -4*q2/(3*sqrt(abs(q2))):
r11 := -3*r21/(4*q2):
s21 := 2*q2*s11*(1/3):
zamproc(0,q11,0,0,p21,q2,1,0, r11,s11,r21,s21):
r11 := \frac{1}{\sqrt{|q2|}}
- \frac{2 q2}{3 |q2|}, 0, 0, -\frac{2 q2}{3 |q2|}
0, -\frac{q2}{|q2|}, 0, \frac{q2}{|q2|}
```