Econ 362. Macroeconomic Theory
Final Review

Vladimir Marquez Stone

Binghamton University

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Savings Rate in the Solow Model

From Our Goods Market Equilibrium:

$$Y = C + I$$

- Y: total output
- C: consumption
- I: investment

Households save a fixed fraction s of output, which funds investment:

$$S = sY = I$$

Interpretation: The savings rate determines the share of output devoted to capital accumulation each period.

Solow Model Setup

- Production function (per worker): $y = f(k)$, with $f(k) = k^\alpha$
- Capital accumulation: $\dot{k} = sf(k) - (n + \delta)k$
- Steady state: $\dot{k} = 0 \Rightarrow sf(k) = (n + \delta)k$

$$\Rightarrow sk^\alpha = (n + \delta)k \Rightarrow k^* = \left(\frac{s}{n + \delta}\right)^{\frac{1}{1-\alpha}}$$

Steady-State Output and Consumption

$$k^* = \left(\frac{s}{n+\delta}\right)^{\frac{1}{1-\alpha}}$$

$$y^* = (k^*)^\alpha = \left(\frac{s}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

$$c^* = y^* - (n+\delta)k^*$$

Interpretation:

- Higher $s \Rightarrow$ more capital per worker $\rightarrow$ higher output!
- But also more investment needed $\rightarrow$ possible consumption trade-off

Golden Rule Capital Accumulation

We maximize steady-state consumption:

$$c = f(k) - (n+\delta)k$$

By taking the first derivative of the expression above:

$$f'(k_G) = n + \delta$$

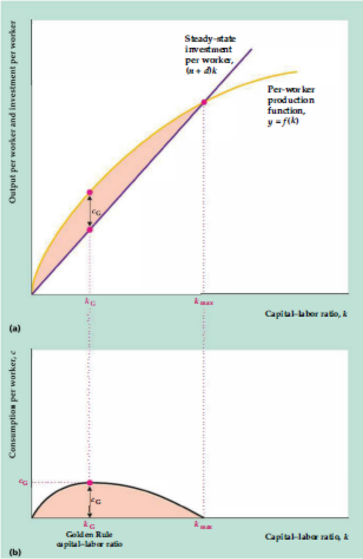
- At Golden Rule: marginal product of capital equals population + depreciation growth
- Overaccumulation: $f'(k) < n + \delta \Rightarrow c \downarrow$

Summary of Long-Run Living Standards

- **Savings rate ↑** $k^* \uparrow, y^* \uparrow, c^* \uparrow$ — up to the Golden Rule
- **Population growth ↑** $k^* \downarrow, y^* \downarrow, c^* \downarrow$
- **Productivity growth ↑** Sustained growth in y_t, c_t

$$\boxed{\text{Golden Rule: } f'(k) = n + \delta}$$

Graph: The Production Function



A Numerical Example of Finding the Steady State

- Production function: $y = k^{0.5}$
- Savings rate: $s = 0.25$
- Depreciation: $\delta = 0.05$
- Population growth: $n = 0.02$

Steady-state capital per worker:

$$k^* = \left(\frac{s}{n+\delta}\right)^2 = \left(\frac{0.25}{0.07}\right)^2 \approx 12.76$$

Steady-state output per worker:

$$y^* = (k^*)^{0.5} = \sqrt{12.76} \approx 3.57$$

Steady-state consumption per worker:

$$c^* = (1-s)y^* = 0.75 \times 3.57 \approx 2.68$$

Comparing Steady State to the Golden Rule

Golden Rule condition:

$$f'(k_G) = n + \delta = 0.07 \Rightarrow \frac{1}{2\sqrt{k_G}} = 0.07 \Rightarrow k_G = \left(\frac{1}{2 \cdot 0.07}\right)^2 \approx 51.0$$

Interpretation:

- Current $k^* = 12.76 < k_G = 51.0$
- Capital is **below** the Golden Rule economy is **under-accumulating**
- Increasing s would raise steady-state consumption

Cyclical Behavior of Macroeconomic Variables

Three key dimensions:

- **Direction:** Procyclical, Countercyclical, Acyclical
- **Timing:** Leading, Lagging, Coincident
- **Categories:** Production, Expenditure, Labor Market, Money and Inflation, Financial

Cyclical Behavior: Direction and Examples

Type	**Examples**
Procyclical	GDP, employment, consumption, investment
Countercyclical	Unemployment, government transfers
Acyclical	Real interest rate, certain components of imports

Table: *

Direction of movement relative to the business cycle.

Cyclical Behavior: Timing and Examples

Timing Type	Examples
Leading	Stock market, housing starts, consumer sentiment
Lagging	Inflation, interest rates, unemployment duration
Coincident	GDP, employment, industrial production

Table: *

Timing of changes relative to the business cycle.

Real Business Cycle (RBC) Theory

- **Key idea:** Business cycles result from real (not monetary) shocks
- **Key example:** productivity shocks (**A** in a Cobb-Douglas production function)
- Unlike Solow, this model is grounded in optimization by firms and households across time.

RBC Baseline (For Illustration)

Representative household maximizes:

$$U = \sum_{t=0}^{\infty} \beta^t u(C_t, 1 - N_t)$$

Subject to:

$$C_t + K_{t+1} = w_t N_t + r_t K_t + (1 - \delta) K_t$$

Firm: Cobb-Douglas Production Function:

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$$

Solow Residual and Utilization Rates

Measured TFP is the Solow Residual:

$$A_t = \frac{Y_t}{K_t^{\alpha} N_t^{1-\alpha}}$$

But if "utilization rate" varies the PF becomes:

$$Y_t = A_t (u_t^K K_t)^{\alpha} (u_t^N N_t)^{1-\alpha}$$

Interpretation:

- Solow residual includes both technology and intensity of use

Classical Causation in Business Cycle Theory

- Output is driven by supply-side factors: productivity (A), labor supply, and capital
- Fluctuations reflect optimal responses to real shocks (not market failures).
- Prices and wages are flexible, so aggregate demand adjusts passively

- This contrasts with Keynesianism, where demand shocks are primary drivers of cycles.
- Price and wage rigidities prevent automatic adjustment
- Active policy is often needed to stabilize output and employment