Phd-2
Status
Review

Spore-Based Path Planning for Two-Wheeled Robot Control Systems

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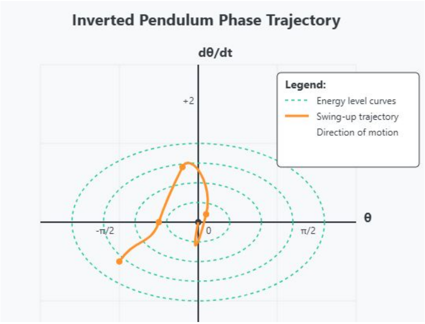
Skoltech

Background: Control and Path Planning: Unifying Perspectives

Key Insight: Control policy derivation and path planning are fundamentally linked problems with complementary approaches.



Unifying Concepts:

- Both seek trajectories
- Involve search in high-dimensional spaces
- Optimal control can be viewed as finding minimum-cost paths in state-control space [1, 10]

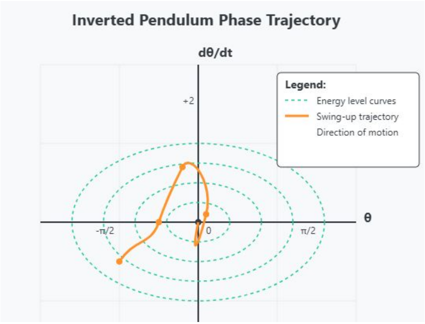
Background: Control and Path Planning: Unifying Perspectives

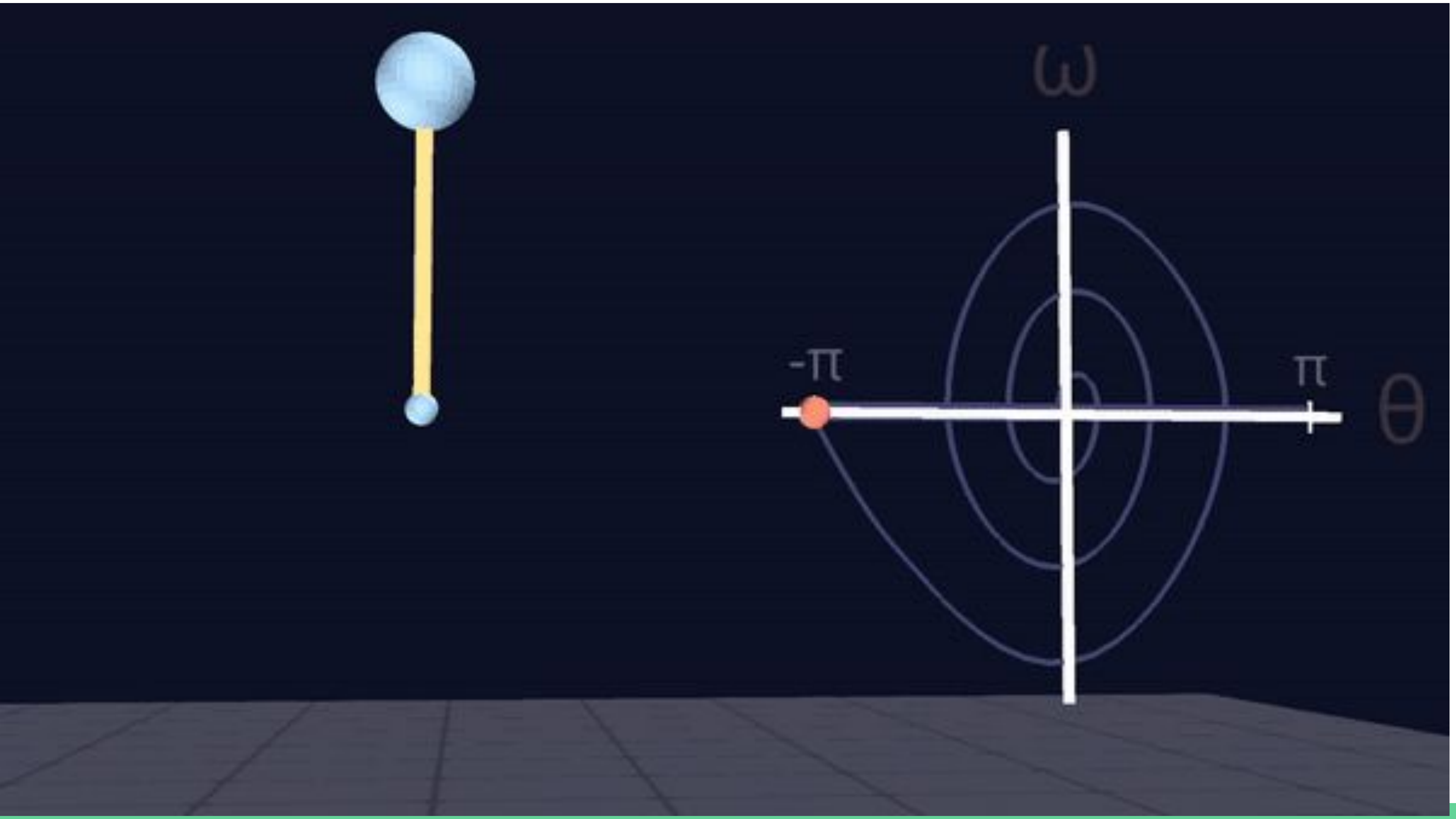
Advantages of Unification:

- More efficient search for **nonlinear control** policies
- Better handling of **underactuated** systems
- Improved scaling to **higher dimensions**
- Clear theoretical connections to **reachability analysis** [5, 6]

Bridging the Gap:

- Path planning methods typically ignore **dynamics**
- Control methods often struggle with **global search**
- Leverage dynamical systems theory to guide efficient exploration of **control space**




ω
-π
π
θ

Research Focus and Goals

Main Focus:

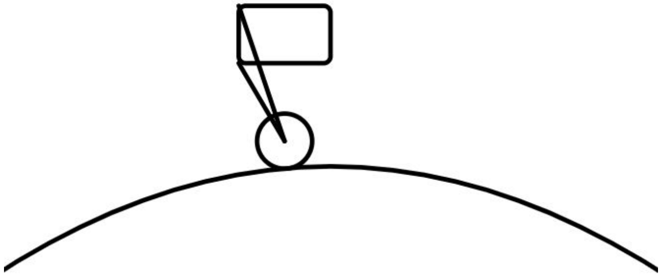
- Dynamic balance control
- Adaptive suspension
- Efficient exploration of high-dimensional phase spaces
- Real-time control policy

Main goals:

- Develop mathematical foundations for algorytm
- Develop workable algorytm

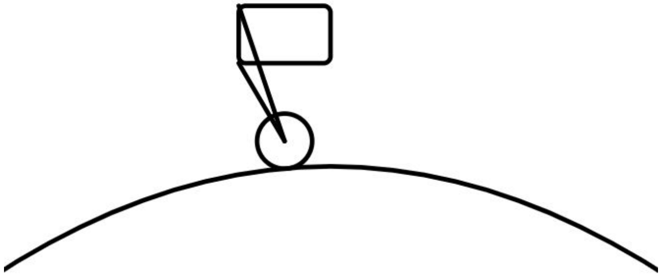
Research Gap

- Control methods for two-wheeled robots face significant challenges:
    - Complex nonlinear dynamics and underactuation [3]
    - Coupling between balancing and navigation objectives [8]
    - Traditional approaches struggle with stability-maneuverability tradeoffs
    - Difficult to find globally optimal solutions across varied terrains [9]



Research Gap

- Current approaches to two-wheeled robot control:
  - Linear control methods (LQR, PID) provide limited operational range
  - Model predictive control requires accurate models and high computation [10]
  - Learning-based methods need extensive training data/time
  - Sampling-based planners rarely address dynamic balance constraints [1, 2]



Research Gap

Gap: Existing methods fail to effectively connect the phase space structure of two-wheeled robot dynamics to control policy derivation for real-world deployment [4, 6].

My approach: Using Hessian tensor analysis to guide exploration of control-state coupling in two-wheeled robots, enabling more efficient discovery of robust control policies [7].

Hessian Matrix Definition

The Hessian of a function $f : \mathbb{R}^n \to \mathbb{R}$ is a square matrix H of second-order partial derivatives:

$$H(f)(\mathbf{x}) = \begin{pmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

where $H_{ij} = \frac{\partial^2 f}{\partial x_i \partial x_j}$

Research Question

Can spore-based path planning algorithm significantly improve the search for optimal control policies in two-wheeled robots with adaptive suspension, addressing the challenges terrain adaptation?

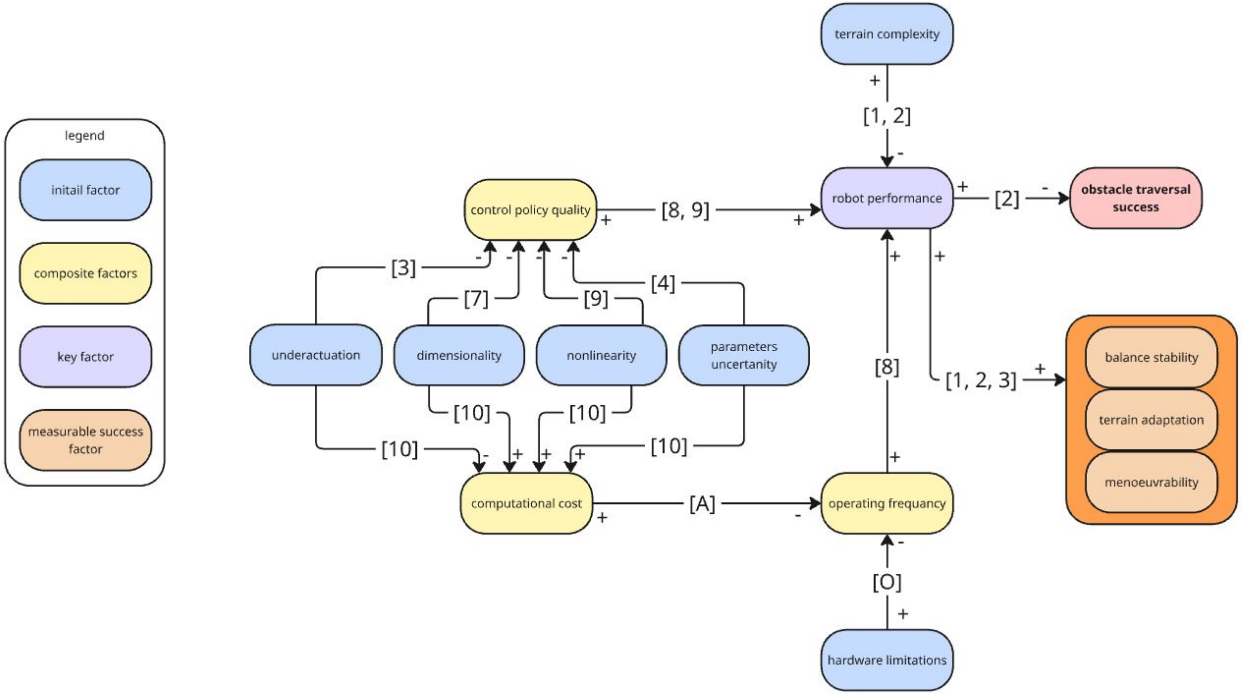
Developing a novel control framework for prototype implementation with experimental validation across multiple mechanical configurations

Research Hypothesis

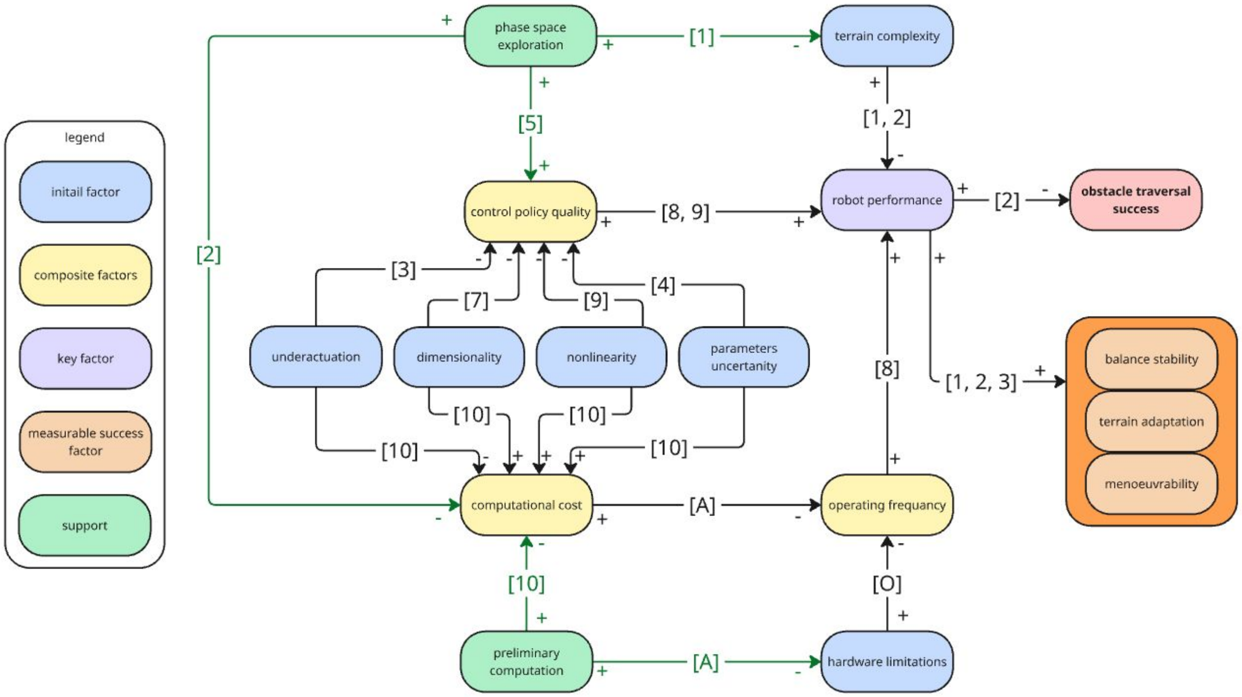
Hypothesis: A spore-based path planning approach operating directly in the phase space of a two-wheeled robot can efficiently discover robust control policies by:

1. Exploiting the **natural dynamic flow** of the two-wheeled system [3, 8]
2. Building **sparse approximations** of reachable states in the robot's phase space [5, 6]
3. Constructing feasible trajectories with **stability guarantees** for varied terrains [4]
4. **Scaling more effectively** than traditional control methods as conditions vary [10]

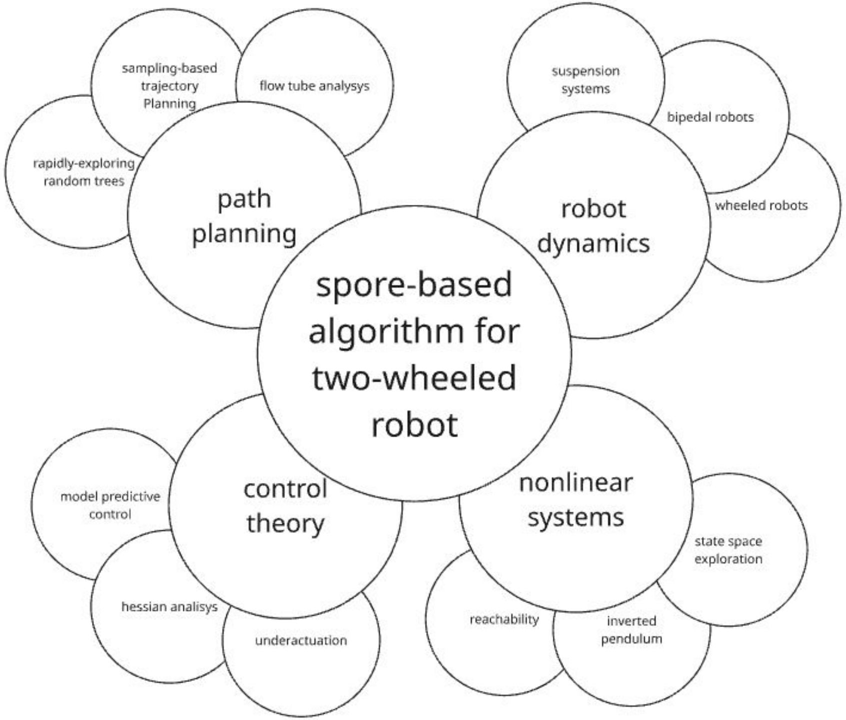
Initial Reference Diagram



Initial Reference Diagram



ARC diagram


sampling-based trajectory Planning
flow tube analysys
rapidly-exploring random trees
path planning
suspension systems
bipedal robots
wheeled robots
robot dynamics
spore-based algorithm for two-wheeled robot
model predictive control
control theory
hessian analisys
underactuation
nonlinear systems
state space exploration
reachability
inverted pendulum

Spore-Based Control Algorithm

Core Concept: Incrementally build a graph of reachable states in the two-wheeled robot's phase space through targeted random exploration.

Key Components:

1. **Neighborhood-based exploration** in phase space
2. **Random action sequences** tested for reachability and stability
3. Incremental **connection graph** with trajectory validation
4. **Path reconstruction** with control policy extraction

Spore-Based Control Algorithm

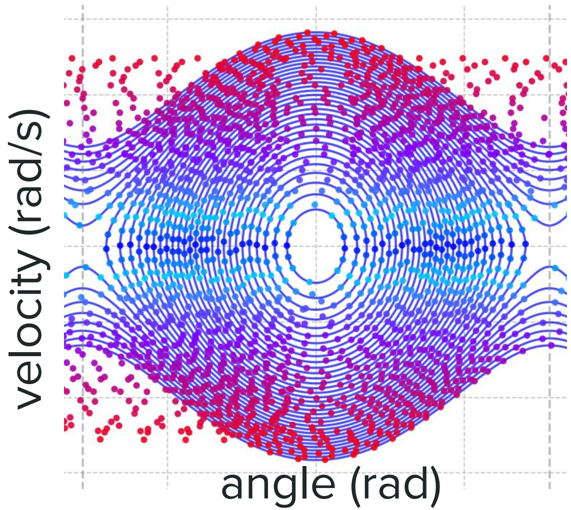
Novelty: Combines concepts from multiple planning and control algorithms:

- **RRT (Rapidly-exploring Random Trees)** [1]:
  Incrementally builds a tree by sampling random states and connecting them to the nearest node in the tree. My approach adapts this to the phase space of dynamical systems.
- **STP (Sampling-based Trajectory Planning)** [2]:
  Tests random control sequences for feasibility. I extend this to specifically track reachable sets in phase space.
- **Flow Tube analysis** [5, 6]:
  Studies the flow of dynamical systems through state space. My method uses this concept to identify natural dynamic corridors.

Spore-Based Control Algorithm

Steps:

1. Generate **random spores** in expanded state space
2. Identify **critical regions** via Hessian analysis

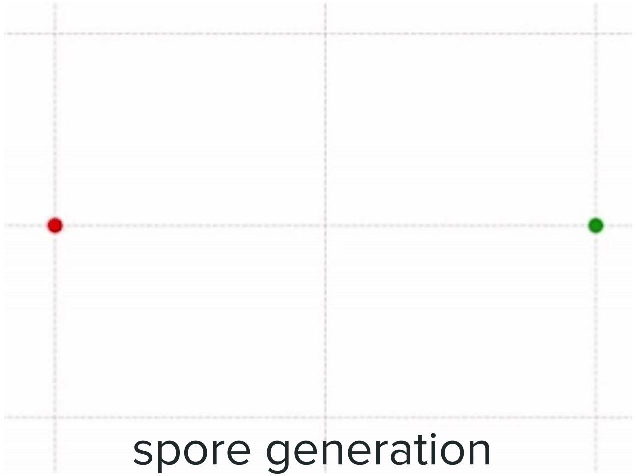


Inverted pendulum phase space
nonlinearity heatmap
blue - low level
red - high level
lines - energy levels

Spore-Based Control Algorithm

Steps:

1. Generate **random spores** in expanded state space
2. Identify **critical regions** via Hessian analysis
3. Apply targeted **random torque** sequences
4. Track **reachable states**
5. Build multi-modal **graph** with distinct edges
6. Extract **control trajectory**



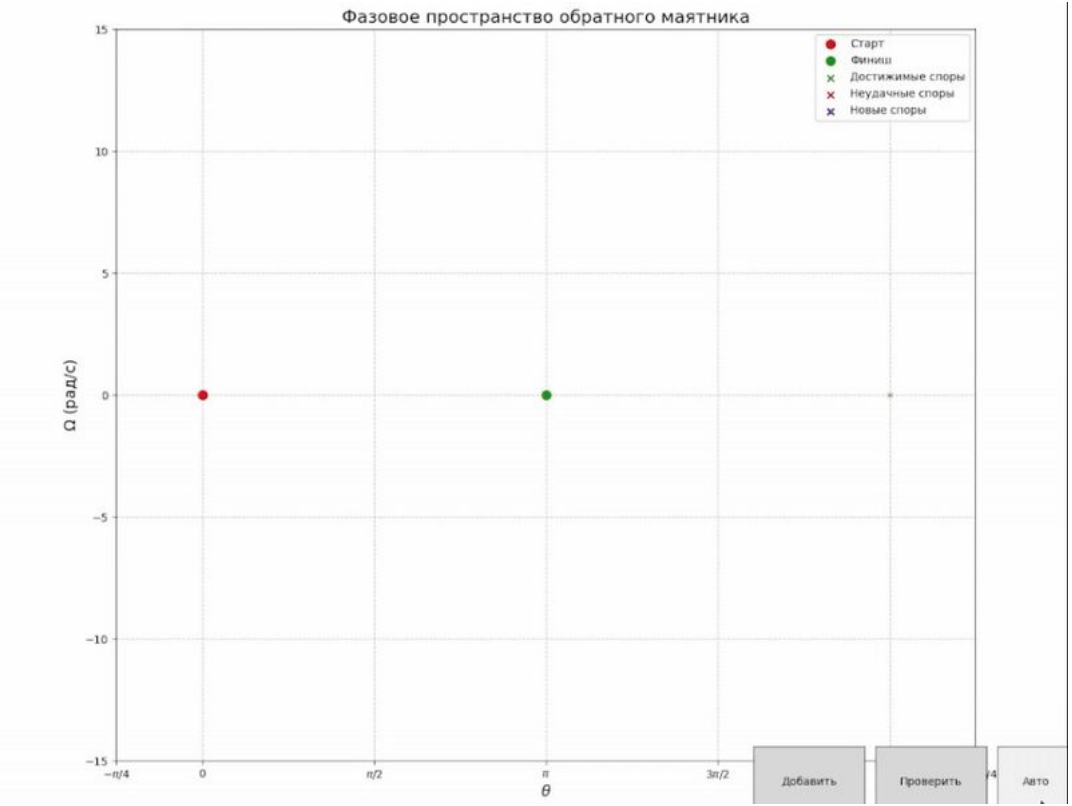
spore generation
red dot - start
green dot - finish

blue cross - new spore
red cross - useless spore
green cross - useful spore

Spore-Based Control Algorithm

Adaptive Neighborhood Selection Concepts:

- Multi-dimensional Hessian tensor computation
- Variable neighborhood sizing
- Region adaptation



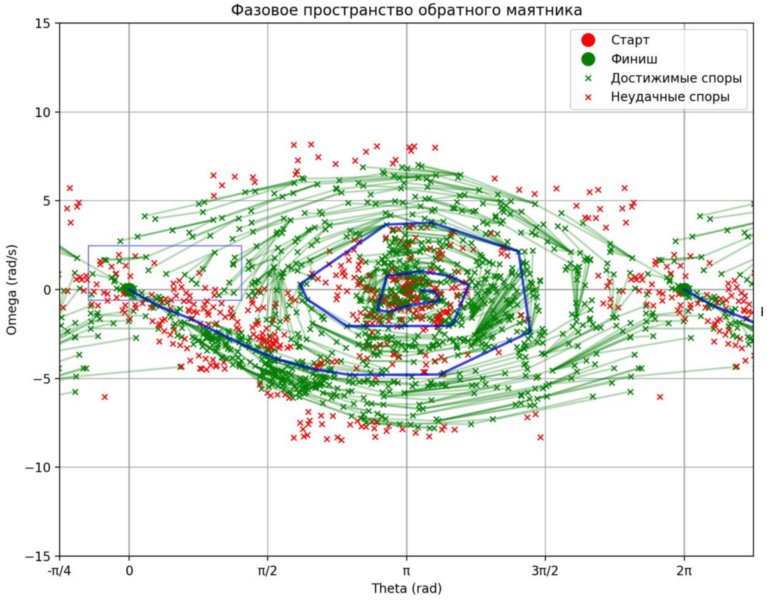
Implementation for Inverted Pendulum

Simple Inverted Pendulum Model:

- Dynamics: $\ddot{\theta} = \frac{g}{l}\sin\theta - b\dot{\theta} + \frac{u}{ml^2}$ [5]
- 2D state space: $[\theta, \dot{\theta}]$ (angle, angular velocity)
- Single control input: u (applied torque)
- Fixed pendulum length and mass distribution

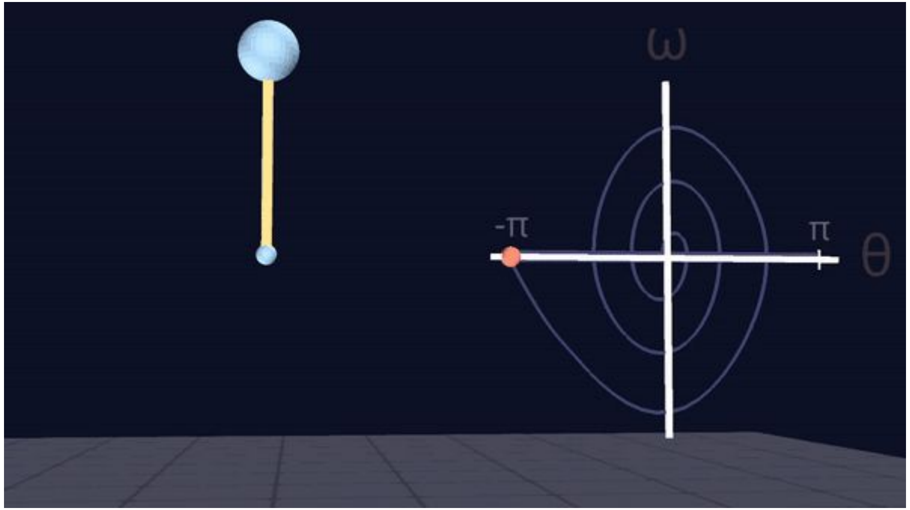
Algorithm Application:

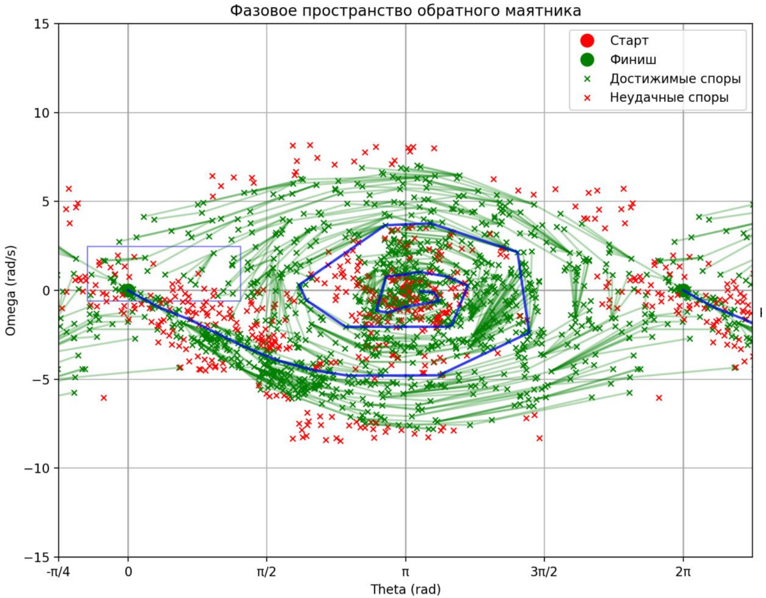
- Used as simplified model for initial validation
- Established proof-of-concept for Hessian-guided exploration
- Demonstrated successful swing-up policy discovery



Agent was able to find swing up trajectory

Demonstration





Optimal controller

spore-based trajectory

Found trajectory stabilizes pendulum in upper position with close to optimal policy

Relationship to Existing Algorithms

Algorithm	Similarities	Key Differences	Why Spore-Based is Potentially Superior	Why Spore-Based is Potentially Inferior
RRT for Robotic Planning [1, 2]	- Random sampling - Tree structure - Incremental exploration	- Works in robot's phase space - Uses wheel torque sequences	**- Captures balancing dynamics naturally** - More efficient in underactuated wheeled systems	- **Higher initial computational overhead** - Requires accurate system dynamics model
PRM for Mobile Robots	- Graph representation - Connection validation - Reusable structure	- Dynamic balance validation - Adaptive sampling based on Hessian - Control-oriented connections	**- Explicitly considers torque limitations** - Adapts to balance-critical regions - Provides control policies for real deployment	**- May require parameter tuning for different terrain types** - Graph construction can be memory-intensive
MPC for Two-Wheeled Robots [10]	- Optimality focus - Predictive horizon - Constraints handling	- Sparse, targeted sampling - Avoids full dynamics integration - Progressive refinement	**- Potentially lower computation requirements** - Better scalability to terrain variations - Handles continuous state transitions	**- Probabilistic completeness vs. deterministic optimization** - Less theoretical guarantees than traditional MPC
RL for Balancing Robots	- Policy optimization - Iterative improvement - Solution robustness	- No extensive training data needed - Focuses on phase space connectivity - Adapts to robot dynamics structure	**- Higher interpretability** - More systematic exploration - Reusable solution components	**- May require manual tuning of exploration parameters** - Lacks end-to-end learning capabilities

Expected deliverables and contribution

1. Novel control algorithm
2. Way of understanding structure of phase-space relative to control limitations
3. Time to deploy robot reduced

Future Work: Suspension-Specific Extensions

Short-term (2 months):

- Complete formal convergence proofs for adaptive suspension systems
- Benchmark against advanced control methods on physical prototype
- Optimize neighborhood selection strategy for different suspension configurations

Medium-term (3-6 months):

- Develop automated suspension parameter tuning based on terrain characteristics
- Implement predictive suspension control based on terrain sensing

Long-term (7-12 months):

- Extend to more complex leg designs with additional degrees of freedom
- Develop integrated planning for locomotion and manipulation tasks
- Create a unified framework for legged and wheeled robots with variable compliance

Theoretical Extensions:

- Formalize the relationship between suspension dynamics and controllability
- Develop metrics for quantifying suspension-balance coupling strength
- Establish optimal sampling distributions for suspension-dominated states

Research type

Research Clarification	Descriptive Study I	Prescriptive Study	Descriptive Study II
1. Review-based →	Comprehensive		
2. Review-based →	Comprehensive →	Initial	
3. Review-based →	Review-based →	Comprehensive →	Initial
4. Review-based →	Review-based →	Review-based → Initial/ Comprehensive ←	Comprehensive
5. Review-based →	Comprehensive →	Comprehensive →	Initial
6. Review-based →	Review-based →	Comprehensive →	Comprehensive
7. Review-based →	Comprehensive →	Comprehensive →	Comprehensive

Requirements

- Courses:
  - Path planning
  - Research methodology
- TA'ship:
  - ISP course
- Publications:
  - The Impact of Noise on the Quality of the Second-order Curve Recognition by Random Sample Consensus algorithm
    IEEE Access 2025
    Ilya Osokin, Ilya Ryakin, Grigory Yaremenko, Sina Moghimi, Sergei Davidenko, Vladimir Guneavoy, Mikhail Patrikeev, Pavel Osinenko

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[4] Manchester, I. R., & Slotine, J. J. E. (2017). "Control contraction metrics: Convex and intrinsic criteria for nonlinear feedback design." *IEEE Transactions on Automatic Control*, 62(6), 3046-3053.

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[6] Bansal, S., Chen, M., Herbert, S., & Tomlin, C. J. (2017). "Hamilton-Jacobi reachability: A brief overview and recent advances." *IEEE Conference on Decision and Control*, 2242-2253.

[7] Bullo, F., & Lewis, A. D. (2019). *Geometric Control of Mechanical Systems*. Springer.

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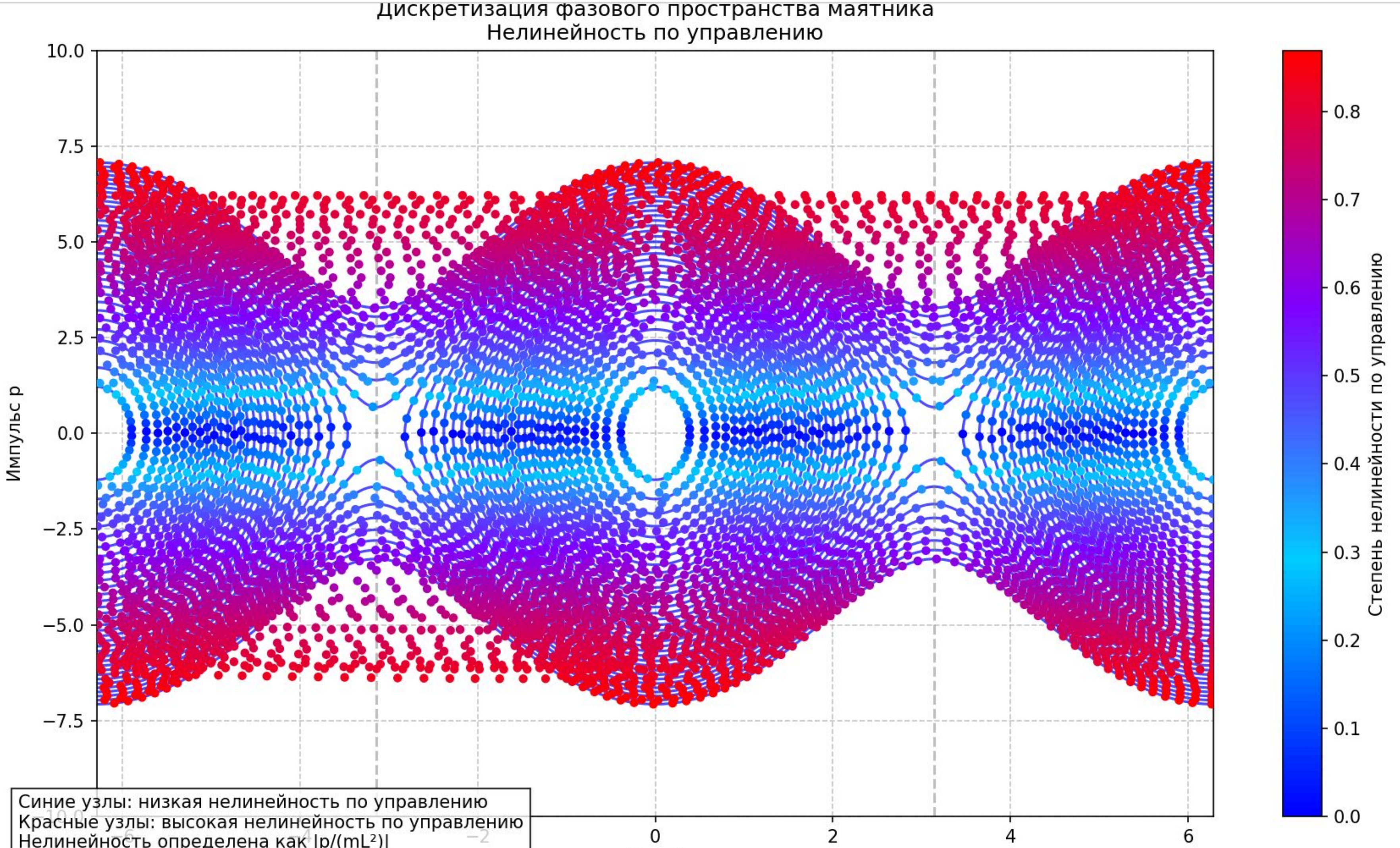
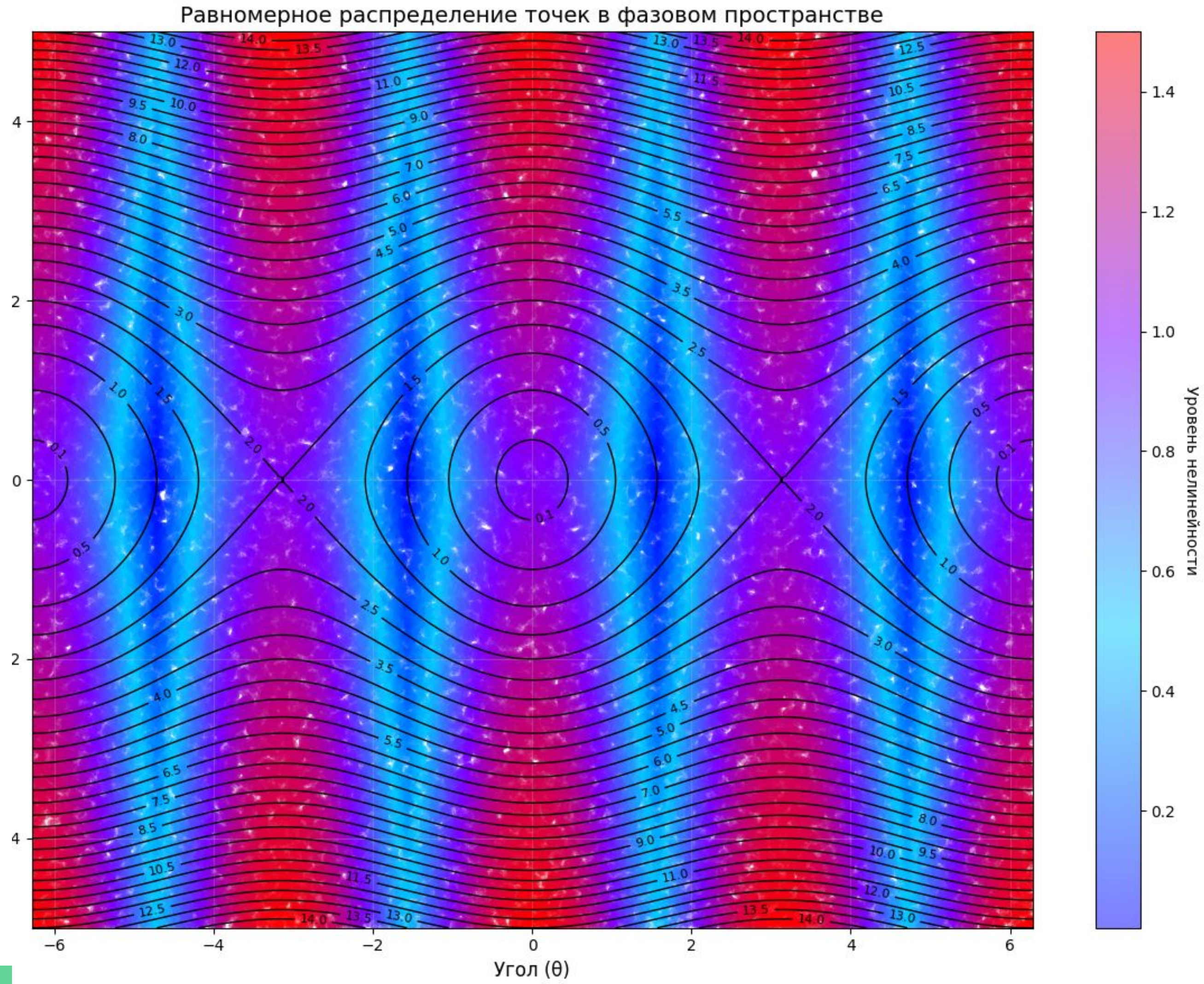
Comparative Analysis with RL Approaches:

Aspect	Spore-Based Method	Deep RL	Actor-Critic	Policy Gradient
Training Data	No offline data needed	Large datasets	Moderate datasets	Moderate datasets
Interpretability	High (explicit graph)	Low (black box)	Medium-Low	Medium-Low
Adaptation	Natural adaptation to varied dynamics	Requires retraining	Requires retraining	Requires retraining
Computational Profile	Front-loaded computation	Extensive offline training	Extensive offline training	Moderate-High training
Theoretical Guarantees	Probabilistic completeness	Limited	Limited	Asymptotic convergence
Sensitivity to Hyperparameters	Moderate	High	High	High



Дискретизация фазового пространства маятника
Нелинейность по управлению
Импульс p
Степень нелинейности по управлению
Синие узлы: низкая нелинейность по управлению
Красные узлы: высокая нелинейность по управлению
Нелинейность определена как |p/(mL²)|



Равномерное распределение точек в фазовом пространстве
Угол (θ)
Уровень нелинейности

Metric	SPORE Search	MPC	PPO
Average control computation time (s)	0.012	5.247	0.435
Maximum computation time (s)	0.002	3.856	0.047
Minimum computation time (s)	0.0008	0.615	0.018
Pendulum stabilization time (s)	3.24	2.87	3.13
Energy efficiency (J·s)	12.34	14.76	11.89
Required computational power	Low	High	Medium