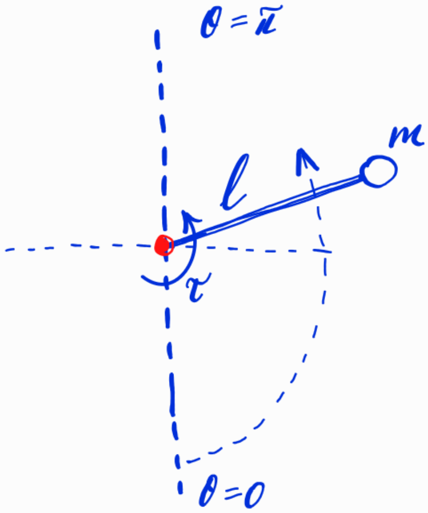
Introduction to adaptive control



Plant: $s=\begin{pmatrix}\theta\\ \dot\theta\end{pmatrix}$, $a=\tau$

$c=$ const (fric. coef.)

$$\dot s_1 = s_2$$

$$\dot s_2 = -\frac{g}{l}\sin s_1 + \frac{a}{ml^2} + c\, s_2^2 \operatorname{sgn}(s_2)$$

c is unknown!

$c\, s_2^2 \operatorname{sgn}(s_2)$ is a viscous friction torque

$$E_{tot} = \underbrace{\frac{1}{2} m l^2 s_2^2}_{E_{kin}} + \underbrace{mgl(1-\cos s_1)}_{E_{pot}}$$

$$E_{des} = 2mgl$$

Let's say $\bar\tau$ is the applied torque bound.

Energy upswing control:

1. $L := \frac{1}{2}\left(E_{des}-E_{tot}\right)^2$

   $L=0 \Rightarrow E_{tot}=E_{des}$ (we want this)

   $L \geq 0$ always

   So, we want $\dot L < 0$

2. Compute $\dot L$:

   Check that!

   $$\dot L = \left(E_{des}-E_{tot}\right)\left(-a s_2 + c m l^2 |s_2|^3\right)$$

Could do:

$$a \longleftarrow \bar{\tau}\,\mathrm{sgn}\big((E_{des}-E_{tot})S_2\big) + cml^2|S_2|\cdot S_2$$

Check it!

Want $\dot{L} < 0$

Problem! c is unknown!

The suggestion: let's substitute the unknown c in the designed control for some estimate $\hat{c}$

We will try to find $\hat{c}$ in such a way that would make the whole closed loop stabilized

Core idea of adaptive control:

$$L_c := L + \frac{1}{2\alpha}(\hat{c}-c)^2$$

Complemented cand. LF

Learning rate

Leads to certainty-equivalence control

$$\dot{L}_c = \underbrace{(E_{des}-E_{tot})}_{=:\Delta E}\big(-aS_2 + \underset{\hat{c}-\Delta c\,!}{\overset{}{c}}ml^2|S_2|^3\big) + \underbrace{\tfrac{1}{\alpha}(\hat{c}-c)}_{=:\Delta c}\dot{\hat{c}}$$

$$= \langle \text{something neg.} \rangle + \Delta E \Delta c\, m \cdot l^2 |S_2|^3 + \frac{1}{\alpha}\Delta c \cdot \dot{\hat{c}}$$

if we plugged in the certainty-equivalent control:

$$a \longleftarrow \bar{\tau}\, \mathrm{sgn}\big((E_{des} - E_{tot}) S_2\big) + \hat{c} m l^2 |S_2| \cdot S_2$$

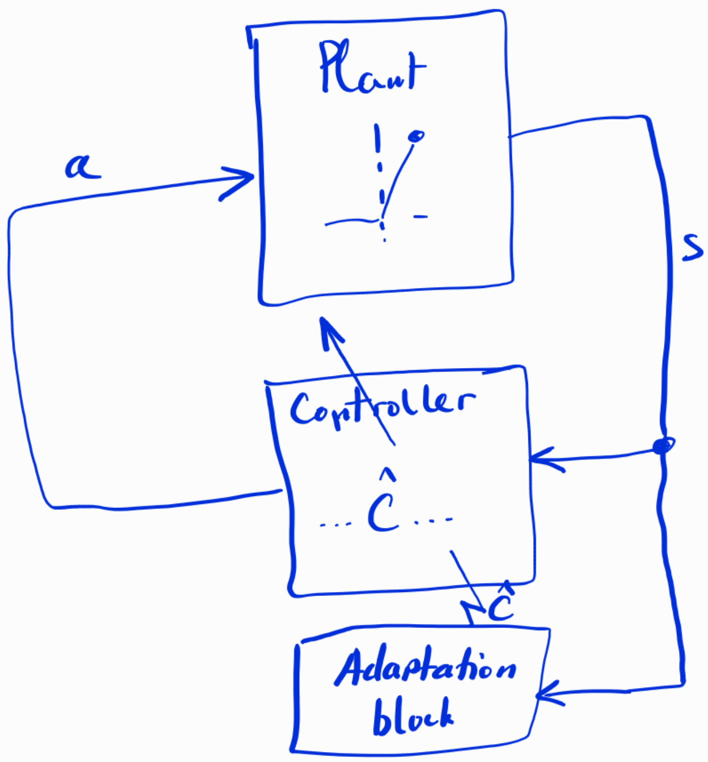
Here is what remains to process:

$$\Delta c \left(\Delta E\, m l^2 |S_2|^3 + \frac{1}{\alpha} \dot{\hat{c}} \right)$$

cancellation

So, adaptation block: $\dot{\hat{c}} := -\alpha\, \Delta E\, m l^2 |S_2|^3$

All in all, the closed loop looks like:



Task: work out a Google Colab about adaptive pendulum control (see example from the lectures), show your tech reqs for LLMs (promt)

Extra! Do the same for the cart pole!

Show your results in the materials topic

General plot:

Plant: $\dot{s} = \underbrace{f(s) + g(s)a}_{\text{Control-affine}}$

Disturbed plant: $\dot{s} = f(s) + g(s)a + \varphi(s)c$

Let's assume we designed a controller $\pi(s|c)$ for the disturbed plant (under known c) and showed that some L was indeed a LF for the closed loop

$$\dot{L} = \underset{\nabla L f}{\mathcal{L}_f L} + \mathcal{L}_g L \, \pi(s|c) + \mathcal{L}_\varphi L \, c \leq -K_d(\|s\|), \forall c$$

Now, the disturbed plant under unknown c! Take, as the theory suggests, a complemented LF:

$$L_c := L + \frac{1}{2\alpha}\|\Delta c\|^2$$

Let's process it:

Estimate for c, not c itself!

$$\dot{L}_c = \mathcal{L}_f L + \mathcal{L}_g L \, \pi(s|\hat{c}) + \mathcal{L}_\varphi L \, c + \frac{1}{\alpha}\dot{\hat{c}} \Delta c$$

$$= \mathcal{L}_f L + \mathcal{L}_g L \, \pi(s|\hat{c}) + \mathcal{L}_\varphi L \, \hat{c} - \mathcal{L}_\varphi L \, \Delta c + \frac{1}{\alpha}\dot{\hat{c}} \Delta c$$

$$\leq -K_d(\|s\|) - \mathcal{L}_\varphi L \, \Delta c + \frac{1}{\alpha}\dot{\hat{c}} \Delta c$$

$$= -K_d(\|s\|) + \Big(\underbrace{\mathcal{L}_\varphi L - \frac{1}{\alpha}\dot{\hat{c}}}_{\text{do the cancellation here}}\Big)\Delta c$$

$$\dot{\hat{c}} = \alpha \mathcal{L}_\varphi L$$

Remarks: we can show $s \to 0$

but we can't show $\hat{c} \to c$