How Much Financial Flexibility Is Optimal? A Closed-Form Capital Structure Model for Investment-Grade Firms

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Abstract

Investment-grade (IG) firms often manage capital structure by preserving financial flexibility and rating resilience rather than targeting a precise leverage ratio. I develop a tractable continuous-time model in which the key financing choice is contractual headroom: the firm selects a trigger threshold V_g at which debt pricing steps up and reverts upon recovery. Optimal V_g trades off tax benefits against expected trigger-based tightening costs and an issuance-time concession tied to headroom, yielding a closed-form interior solution. Implied leverage follows endogenously, and a simple Debt/EBITDA mapping expresses the policy in familiar covenant terms. Comparative statics provide clear predictions for how headroom and implied leverage vary with taxes, repricing severity, and debt-service burden.

Keywords: capital structure, financial flexibility, investment-grade, financing policy, debt capacity, closed-form

1 Introduction

Classic capital structure models, such as Leland (1994), frame financing as a trade-off between the tax benefits of debt and the expected costs of financial distress. For investment-grade (IG) firms, however, a large share of financing practice is organized around a different object: preserving financial flexibility and maintaining rating resilience – avoiding state-contingent contract tightening well before severe distress becomes the binding constraint.

Evidence from corporate practice supports this framing. Financing decisions are often discussed in terms of flexibility buffers and ratings rather than a precise "optimal" leverage target (Graham, 2022; Graham & Harvey, 2001). Relatedly, Kisgen (2006) documents that credit ratings can exert discrete effects on corporate financing behavior, consistent with threshold-driven discipline in the IG regime.

Debt contracts provide concrete mechanisms for such discipline. Empirical work shows that covenant violations have sharp real effects and are followed by meaningful changes in corporate policy (Chava & Roberts, 2008; Roberts & Sufi, 2009). Contract terms also embed pricing channels: performance-pricing provisions adjust spreads mechanically as borrower conditions change (Asquith et al., 2005). More broadly, the selection of covenant thresholds contains economically meaningful information (Demiroglu & James, 2010). Together, these findings suggest that a central policy question for IG firms is how much flexibility buffer to maintain before contractual terms tighten. We use the term "trigger" to refer primarily to performance-pricing and rating-grid provisions that mechanically tighten pricing when a contractual node is crossed. Covenant thresholds that mechanically reprice debt can be viewed as a special case of the same trigger-based tightening mechanism.

Mapping. We interpret the trigger as a *pricing-grid node*—a contractual threshold at which spreads or coupons adjust mechanically (e.g., performance-pricing provisions; Asquith et al. (2005)). In the model, this node is represented in value space by a single threshold V_g: when V falls below V_g, pricing steps up by $(1 + s)$, and it reverts upon recovery. Accordingly, V_g summarizes *contractual headroom* (distance to the tightening node), while s captures repricing severity. When contracts are written in accounting ratios (e.g., Debt/EBITDA), we treat the translation into value space as a measurement layer rather than a separate mechanism.

Motivated by this evidence, we propose a tractable continuous-time model that shifts the focus from choosing "optimal debt" to choosing an optimal flexibility buffer. The firm selects a contractual trigger threshold V_g that summarizes this buffer: a lower V_g implies more flexibility (i.e., more distance to the tightening event). The costs of flexibility operate on two margins. First, moving the threshold changes the implicit "strike" in the contract, requiring an ex-ante concession to debtholders for weaker trigger protection (captured as an up-front pricing concession required for weaker trigger protection). Second, crossing the threshold generates an ex-post, state-contingent repricing penalty (a temporary coupon step-up that reverts upon recovery). This trade-off also admits a real-options interpretation: choosing V_g selects the effective strike of trigger protection, i.e., the contractual node at which pricing steps up (and reverts upon recovery). The contribution is a closed-form IG benchmark in which V_g is the single financing control and leverage is implied endogenously from the optimal trigger policy.

The model yields an interior trigger threshold V_g and implied leverage in closed form. Importantly, leverage is not chosen directly; debt capacity and implied leverage emerge endogenously from the chosen flexibility policy. The framework is designed to isolate the *investment-grade pre-default contracting margin*: leverage is disciplined by trigger-based repricing, and the firm trades off headroom against expected repricing costs.[1]

This focus is empirically natural, as long-run default studies document that IG defaults are rare; see, e.g., (S&P Global Ratings, 2025).

2 Model Setup

We work in a continuous-time, pre-default investment-grade (IG) setting. Under the risk-neutral measure Q, firm value follows a geometric Brownian motion,

$$\frac{\mathrm{d}V_t}{V_t} = r\,\mathrm{d}t + \sigma\,\mathrm{d}W_t^Q. \tag{1}$$

The analysis is stationary: the firm selects a single contractual trigger threshold V_g (headroom) given the contract form and parameters; we do not model repeated time-varying re-optimization of leverage.

Financial flexibility as the decision. The firm's control is the trig-

[1] A default boundary can be appended in the standard structural way, but it introduces an orthogonal channel and would obscure the closed-form benchmark. The paper should be read as a tractable IG benchmark for trigger-based tightening rather than a model of bankruptcy.

ger threshold V_g (contractual headroom). Under break-even pricing for debt with trigger-based repricing, this choice maps into the corresponding debt capacity and leverage. Intuitively, V_g represents the next pricing-grid (or rating-grid) node: if V falls below it, contractual pricing tightens mechanically. A lower V_g implies more headroom (a larger distance to the tightening event) and therefore more financial flexibility.

Contractual repricing (penalty) with recovery. Following a Leland-style steady-state setup, we model perpetual debt with coupon C to preserve tractability. When V drops below V_g, debt becomes temporarily more expensive: the coupon increases from C to $C(1+s)$, where $s > 0$ captures the severity of the repricing penalty. When V recovers above V_g, the coupon reverts to C.

Taxes. Coupons generate a tax shield at rate τ. Because the coupon is higher in the penalty region, the tax shield also increases proportionally.

Up-front pricing concession for weaker trigger protection. Looser trigger protection is not free: a lower V_g weakens creditor protection and, in practice, must be compensated at issuance through pricing and terms (e.g., spreads, fees, or a weaker grid). We capture this issuance-time concession by an up-front term k/V_g borne by equity. We take $k > 0$, so the concession increases as V_g decreases (looser triggers require a larger up-front concession).

We do not model an explicit bargaining game between lenders and shareholders. Instead, we summarize the issuance-time price of obtaining a looser trigger by the one-parameter term k/V_g. The Supplementary Appendix C motivates this functional form by linking it to the strike sensitivity of an

American cash-or-nothing put that represents trigger protection. The goal is tractability: the k/V_g specification preserves the correct monotonicity (looser triggers require a larger concession) while keeping the model in closed form.

We interpret k/V_g as an issuance/negotiation cost borne up front by equityholders (fees or concessions), not as an additional coupon stream.

This term parsimoniously captures that looser triggers require an up-front concession to lenders.

3 Valuation and Optimal Flexibility

Solving the stationary two-region valuation problem yields the following closed-form expressions (derivations in the Supplementary Appendix A).

Given the repricing trigger, debt value and the present value of the tax shield can be written in closed form (for $V > V_g$):

$$D(V) = \frac{C}{r} + \frac{Cs}{r(1+x)}\left(\frac{V_g}{V}\right)^x, \qquad TB(V) = \frac{C\tau}{r} + \frac{Cs\tau}{r(1+x)}\left(\frac{V_g}{V}\right)^x, \quad (2)$$

where r is the risk-free rate and $x = \frac{2r}{\sigma^2}$.

Equity equals firm value plus tax benefits minus debt value and the ex-ante concession cost:

$$E(V) = V + TB(V) - D(V) - \frac{k}{V_g}. \quad (3)$$

We take $k > 0$, so a lower V_g (more headroom) requires a larger up-front concession.

We take the standard shareholder perspective: the firm selects its financing policy to maximize equity value. For any chosen trigger V_g, debt is competitively priced given the coupon and the repricing rule. Separately, we capture the up-front concession required to obtain weaker trigger protection by the term k/V_g. The equity objective therefore trades off (i) tax benefits of debt against (ii) expected repricing costs and (iii) the ex-ante price of flexibility, delivering an interior optimal trigger threshold:

$$V_g^* = V^{\frac{x}{x+1}} \left(\frac{k\, r(1+x)}{Cs(1-\tau)\, x} \right)^{\frac{1}{x+1}}. \tag{4}$$

A lower V_g corresponds to more financial flexibility (more contractual headroom), while a higher V_g implies tighter trigger protection and higher implied leverage.

4 Implied Leverage from a Debt/EBITDA Covenant

To connect the trigger to a standard Debt/EBITDA covenant, assume EBITDA $= \alpha V$, where $\alpha \equiv \text{EBITDA}/V$ is an EBITDA "yield" (a profitability proxy). With covenant multiple K, the implied debt capacity is

$$D^{\text{cap}} = \alpha K V_g. \tag{5}$$

This proportionality is used only as a translation layer; it does not affect the optimality condition for V_g. Because α is an EBITDA-yield (profitability) proxy, higher profitability mechanically increases the covenant-implied debt capacity $D^{\text{cap}}/V = \alpha K(V_g/V)$ for a given trigger policy.

Thus, leverage is not chosen directly: it follows endogenously from the optimal trigger policy through D^{cap}/V.

5 Comparative Statics and a Simple Illustration

5.1 What moves financial flexibility (directional predictions)

The model yields a closed-form optimal trigger threshold V_g and an implied leverage ratio D^{cap}/V (via the Debt/EBITDA mapping).

Rather than listing derivatives, we summarize the key directional predictions; formal expressions are reported in the Supplementary Appendix E.

- **Repricing severity ($s \uparrow$) $\Rightarrow$ more headroom.** If a trigger event becomes more expensive, the firm prefers to remain further away from it. The optimal trigger threshold falls ($V_g^* \downarrow$), and implied leverage decreases.

- **Tax rate ($\tau \uparrow$) $\Rightarrow$ tighter trigger (less headroom).** A higher tax rate increases the value of the interest tax shield, making debt more attractive. The model therefore shifts toward a tighter flexibility policy ($V_g^* \uparrow$), which raises implied leverage.

- **Debt-service burden ($C \uparrow$) $\Rightarrow$ more headroom.** A higher coupon makes debt service more expensive in all states and increases the cost of entering the repricing region. The firm responds by choosing more headroom ($V_g^* \downarrow$), reducing implied leverage.

The effects of r and σ operate through the trigger-hitting sensitivity in the valuation formula and need not have a uniform sign for all parameter values; we therefore report them numerically relative to a baseline calibration.

5.2 Baseline magnitudes (BBB, U.S.)

To show magnitudes, consider a representative BBB calibration: $r = 4\%$, $\tau = 21\%$, $\sigma = 22\%$, $C = 5\%$, $s = 0.12$, and $k = 0.05$. We set s to a representative magnitude of contractual repricing (step-up) observed in practice; details are discussed in the Supplementary Appendix D.

The model implies $\frac{V_g^*}{V} \approx 0.86$, i.e., the firm can absorb roughly a 14% decline in asset value before the trigger is crossed and repricing applies. Using the Debt/EBITDA mapping, implied leverage is $D^{\text{cap}}/V \approx 0.35$.

Two simple checks illustrate how the mechanism works. First, increasing repricing severity from $s = 0.12$ to $s = 0.20$ lowers the optimal threshold level to $\frac{V_g^*}{V} \approx 0.71$ and reduces implied leverage to $D^{\text{cap}}/V \approx 0.285$. Second, increasing the corporate tax rate from $\tau = 0.21$ to $\tau = 0.30$ raises implied leverage from about 0.35 to about 0.36, reflecting the stronger tax incentive to use debt.

Finally, the Debt/EBITDA mapping delivers transparent scaling effects: holding V_g fixed, implied debt scales linearly with αK, so a 25% increase in the covenant multiple K (or profitability α) raises D^{cap}/V by 25%.

Extended scenario tables and derivative expressions are provided in the Supplementary Appendix E.

6 Scope and limitations

We focus on the pre-default investment-grade (IG) regime, where a large part of financing policy is about avoiding contractual tightening. Accordingly, we do not model explicit default, bankruptcy, or strategic renegotiation. Contract frictions are summarized parsimoniously: s captures the severity of the repricing step-up, and k/V_g (with $k > 0$) captures the issuance-time pricing concession required to obtain more headroom (a lower trigger threshold). Derivations and extended checks are provided in the Supplementary Appendix A–E.

7 Conclusion

This paper proposes a simple reframing of investment-grade (IG) capital structure: firms do not choose an "optimal debt" target directly; they choose how much *financial flexibility* to preserve. In the model, this choice is summarized by a contractual trigger threshold V_g (the breach threshold for contractual tightening), and leverage follows endogenously as a consequence of the chosen trigger policy.

The key mechanism is a contract-based repricing channel that is natural for IG firms: when conditions deteriorate enough to cross V_g, debt service becomes temporarily more expensive, and it reverts upon recovery. This yields a real-options interpretation: selecting V_g makes the firm effectively short a put option on its own assets, with V_g acting as the strike. The payoff is a fully tractable framework with closed-form valuation, transparent comparative statics, and parameters that map directly to economically interpretable objects (tax incentives, debt service burden, and repricing severity).

A baseline BBB calibration provides a useful sanity check: the model implies a plausible flexibility buffer (about a 14% asset-value decline, i.e., $\frac{V_g^*}{V} \approx 0.86$, before tightening) and a plausible implied debt-to-value ratio (around 0.35). These magnitudes help illustrate how repricing severity, taxes, and debt-service costs jointly shape financing outcomes.

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Guide to the Supplementary Appendix

This Supplementary Appendix provides derivations, calibration details, and additional numerical results that support the short main text.

- **Appendix A** contains derivations and proofs for the closed-form expressions in the model.
- **Appendix B** reports a numerical (LSM-based) estimation of the reduced-form wedge parameter k.
- **Appendix C** provides an analytical motivation for the k/V_g term using a simple option-based representation.
- **Appendix D** describes the calibration of the covenant-violation penalty parameter s from rating-spread data.
- **Appendix E** reports comparative statics (partial derivatives) and extended scenario tables.

Appendix A: Detailed Derivation of optimal Vg

Step 1: Simplify the PDEs

We model the firm's asset value V using a Geometric Brownian Motion (GBM). The value of the firm's debt $D(V)$ satisfies the following partial differential equations (PDEs):

- **For $V > V_g$ (no covenant violation):**

$$\frac{1}{2}\sigma^2 V^2 D''(V) + rVD'(V) - rD(V) + C = 0 \tag{6}$$

- **For $V \leq V_g$ (covenant violation):**

$$\frac{1}{2}\sigma^2 V^2 D''(V) + rVD'(V) - rD(V) + C(1+s) = 0 \tag{7}$$

Here:

- $D(V)$ is the value of the firm's debt.
- σ is the volatility of the firm's assets.
- r is the risk-free rate.
- C is the coupon payment.
- s is the proportional increase in the coupon payment upon covenant violation.

Since the problem is time-independent (steady state), the PDEs reduce to ordinary differential equations (ODEs) in terms of V.

Step 2: Transform the ODEs

Divide each term by $\frac{1}{2}\sigma^2 V^2$ to simplify the equations:

$$D''(V) + \frac{2r}{\sigma^2 V} D'(V) - \frac{2r}{\sigma^2 V^2} D(V) + \frac{2C}{\sigma^2 V^2} = 0 \tag{8}$$

For $V \leq V_g$, replace C with $C(1+s)$ in the last term.

Step 3: Solve the Homogeneous Equation

Consider the homogeneous part of the equation:

$$D''(V) + \frac{2r}{\sigma^2 V} D'(V) - \frac{2r}{\sigma^2 V^2} D(V) = 0 \tag{9}$$

Assume a solution of the form $D(V) = V^\beta$. Compute the derivatives:

$$D'(V) = \beta V^{\beta-1} \tag{10}$$

$$D''(V) = \beta(\beta-1) V^{\beta-2} \tag{11}$$

Substitute into the homogeneous equation:

$$\beta(\beta-1)V^{\beta-2} + \frac{2r}{\sigma^2 V}\beta V^{\beta-1} - \frac{2r}{\sigma^2 V^2} V^\beta = 0 \tag{12}$$

Simplify:

$$\left[\beta(\beta-1) + \frac{2r\beta}{\sigma^2} - \frac{2r}{\sigma^2}\right] V^{\beta-2} = 0 \tag{13}$$

Set the coefficient to zero:

$$\beta^2 - \beta + \frac{2r\beta}{\sigma^2} - \frac{2r}{\sigma^2} = 0 \tag{14}$$

Multiply both sides by σ^2:

$$\sigma^2\beta^2 + (2r - \sigma^2)\beta - 2r = 0 \tag{15}$$

Let $x = \dfrac{2r}{\sigma^2}$, then the equation becomes:

$$\beta^2 + (x-1)\beta - x = 0 \tag{16}$$

Solve for β using the quadratic formula:

$$\beta = \frac{-(x-1) \pm \sqrt{(x-1)^2 + 4x}}{2} \tag{17}$$

Compute the discriminant:

$$\Delta = (x-1)^2 + 4x = x^2 - 2x + 1 + 4x = x^2 + 2x + 1 = (x+1)^2 \tag{18}$$

Therefore, $\sqrt{\Delta} = x + 1$. The roots are:

$$\beta_1 = \frac{-(x-1) + (x+1)}{2} = 1 \tag{19}$$

$$\beta_2 = \frac{-(x-1) - (x+1)}{2} = -x \tag{20}$$

Step 4: Write the Homogeneous Solution

The general solution to the homogeneous equation is:

$$D_{\text{hom}}(V) = AV^1 + BV^{-x} \tag{21}$$

where A and B are constants.

Step 5: Find Particular Solutions

For $V > V_g$:

Assume a constant particular solution:

$$D_{\text{part}} = \frac{C}{r} \tag{22}$$

Verify by substituting into the ODE:

$$-r\left(\frac{C}{r}\right) + C = 0 \tag{23}$$

For $V \leq V_g$:

Assume:

$$D_{\text{part}} = \frac{C(1+s)}{r} \tag{24}$$

Verify:

$$-r\left(\frac{C(1+s)}{r}\right) + C(1+s) = 0 \tag{25}$$

Step 6: Write the General Solutions

For $V > V_g$:

$$D_1(V) = A_1 V^1 + B_1 V^{-x} + \frac{C}{r} \tag{26}$$

To ensure $D_1(V)$ remains finite as $V \to \infty$, set $A_1 = 0$:

$$D_1(V) = B_1 V^{-x} + \frac{C}{r} \tag{27}$$

For $V \leq V_g$:

$$D_2(V) = A_2 V^1 + B_2 V^{-x} + \frac{C(1+s)}{r} \tag{28}$$

To ensure $D_2(V)$ remains finite as $V \to 0$, set $B_2 = 0$:

$$D_2(V) = A_2 V + \frac{C(1+s)}{r} \tag{29}$$

Step 7: Apply Boundary Conditions

At $V = V_g$, apply:

1. **Value Matching:**

$$D_1(V_g) = D_2(V_g) \tag{30}$$

2. **Smooth Pasting:**

$$D_1'(V_g) = D_2'(V_g) \tag{31}$$

Compute derivatives:

$$D_1'(V) = -xB_1V^{-x-1} \tag{32}$$

$$D_2'(V) = A_2 \tag{33}$$

Step 8: Solve for Constants

From smooth pasting:

$$A_2 = -xB_1V_g^{-x-1} \tag{34}$$

Substitute A_2 into value matching:

$$B_1V_g^{-x} + \frac{C}{r} = \left(-xB_1V_g^{-x-1}\right)V_g + \frac{C(1+s)}{r} \tag{35}$$

Simplify $V_g^{-x-1}V_g = V_g^{-x}$:

$$B_1V_g^{-x} + \frac{C}{r} = -xB_1V_g^{-x} + \frac{C(1+s)}{r} \tag{36}$$

Combine terms:

$$(1+x)B_1V_g^{-x} = \frac{Cs}{r} \tag{37}$$

Solve for B_1:

$$B_1 = \frac{Cs}{r(1+x)}V_g^x \tag{38}$$

Compute A_2:

$$A_2 = -xB_1 V_g^{-x-1} = -\frac{Csx}{r(1+x)} V_g^{-1} \tag{39}$$

Step 9: Final Debt Value Functions

For $V > V_g$:

$$D(V) = \frac{C}{r} + \frac{Cs}{r(1+x)} \left(\frac{V_g}{V}\right)^x \tag{40}$$

For $V \leq V_g$:

$$D(V) = \frac{C(1+s)}{r} - \frac{Csx}{r(1+x)} \left(\frac{V}{V_g}\right) \tag{41}$$

Step 10: Compute Tax Shield Value

The tax shield $TB(V)$ is proportional to the interest payments.

For $V > V_g$:

$$TB(V) = \frac{C\tau}{r} + \frac{Cs\tau}{r(1+x)} \left(\frac{V_g}{V}\right)^x \tag{42}$$

For $V \leq V_g$:

$$TB(V) = \frac{C(1+s)\tau}{r} - \frac{Csx\tau}{r(1+x)} \left(\frac{V}{V_g}\right) \tag{43}$$

Step 11: Equity Value

The equity value is:

$$E(V) = V + TB(V) - D(V) - \frac{k}{V_g} \tag{44}$$

For $V > V_g$:

Substitute $D(V)$ and $TB(V)$:

$$E(V) = V + \left(\frac{C\tau}{r} + \frac{Cs\tau}{r(1+x)}\left(\frac{V_g}{V}\right)^x\right) - \left(\frac{C}{r} + \frac{Cs}{r(1+x)}\left(\frac{V_g}{V}\right)^x\right) - \frac{k}{V_g} \tag{45}$$

$$= V - \frac{C}{r} + \frac{C\tau}{r} - \frac{Cs(1-\tau)}{r(1+x)}\left(\frac{V_g}{V}\right)^x - \frac{k}{V_g} \tag{46}$$

Step 12: Optimize Equity Value

To find the optimal V_g that maximizes $E(V)$, take the derivative of $E(V)$ with respect to V_g and set it to zero:

$$\frac{dE}{dV_g} = -\frac{Cs(1-\tau)x}{r(1+x)}V^{-x}V_g^{x-1} + \frac{k}{V_g^2} = 0 \tag{47}$$

Solve for V_g:

$$\frac{Cs(1-\tau)x}{r(1+x)}V^{-x}V_g^{x+1} = k \tag{48}$$

Therefore:

$$V_g = V^{\frac{x}{x+1}}\left(\frac{kr(1+x)}{Cs(1-\tau)x}\right)^{\frac{1}{x+1}} \tag{49}$$

Step 13: Second Derivative Test

Compute the second derivative to confirm that V_g maximizes $E(V)$:

$$\frac{d^2E}{dV_g^2} = -\frac{Cs(1-\tau)x(x-1)}{r(1+x)}V^{-x}V_g^{x-2} - \frac{2k}{V_g^3} \tag{50}$$

Since $x > 1$ (assuming $\sigma^2 < 2r$), both terms are negative, confirming that V_g corresponds to a maximum of $E(V)$.

Appendix B: Numerical Estimation of the k Parameter Using the Least Squares Monte Carlo Method

In the context of American cash-or-nothing put options, deriving an analytical expression for the $\frac{k}{V_g}$ cost term is challenging due to the option's early exercise feature. While Appendix C provides an analytical justification using European options, substituting an American option with a European one in our model has limited validity because it neglects the possibility of early exercise, which is central to the valuation of American options.

To address this limitation, we employ the Least Squares Monte Carlo (LSM) method introduced by Longstaff and Schwartz (2001) to numerically estimate the k parameter. The LSM method is a powerful technique for valuing American options by simulating optimal exercise strategies and estimating option prices through regression.

Overview of the LSM Method

The LSM method operates as follows:

1. **Simulate Asset Price Paths:** Generate a large number of possible future paths for the asset price S_t using Monte Carlo simulation based on the geometric Brownian motion model.

2. **Set Up Payoffs:** At each time step, determine if the option is in the money (i.e., $S_t \leq V_g$). If so, record the immediate exercise payoff Q.

3. **Backward Induction:** Starting from the penultimate time step and moving backward:

(a) For paths where the option is in the money, regress the discounted future payoffs on functions of S_t to estimate the continuation value.

(b) Decide whether to exercise the option by comparing the immediate exercise payoff with the estimated continuation value.

(c) Update the option value based on the optimal decision.

4. **Estimate Option Prices:** Calculate the expected option price as the average of the discounted payoffs across all simulated paths.

Estimation of the k Parameter

To estimate the k parameter:

1. **Vary Strike Prices:** Compute option prices V_{put} for a range of strike prices X around V_g.

2. **Calculate Derivatives:** Estimate $\frac{\partial V_{\text{put}}}{\partial X}$ numerically using finite differences.

3. **Linear Regression:** Fit a linear model to approximate the relationship between $\frac{\partial V_{\text{put}}}{\partial X}$ and $1/X$.

4. **Determine k:** Let $\widehat{\beta}$ denote the fitted slope from the regression

$$\frac{\partial V_{\text{put}}}{\partial X} \approx \alpha + \widehat{\beta}\,\frac{1}{X}.$$

Since the regressor is $1/X$, the fitted slope $\widehat{\beta}$ is negative in our baseline calibration. Throughout the paper we report the cost-intensity parameter as $k \equiv |\widehat{\beta}| = -\widehat{\beta} > 0$, so that the term $-k/V_g$ is interpreted as a cost wedge.

Using the LSM method, we calibrated k by pricing American cash-or-nothing put options with strikes from 0.50 to 0.90. Baseline inputs included $r = 0.04$, $\sigma = 0.2$, $T = 5$, and a cash payout $Q = 0.05$, which represents the penalty for covenant violation, expressed as a percentage of the initial firm value. A linear regression of $\frac{dV_{\text{put}}}{dX}$ against $1/X$ produced an estimated slope $\widehat{\beta} = -0.0887$, corresponding to $k = |\widehat{\beta}| = 0.0887$, with an R^2 of 0.977, validating the approximation.

In the calibration exercise, we fixed the initial asset price $S_0 = 1$, the risk-free interest rate $r = 0.04$, and the asset volatility $\sigma = 0.2$. These values represent typical conditions for modeling medium-term corporate debt, allowing us to focus on the impact of varying the cash payout Q and the time to maturity T on the estimated k values.

Q\T	**5**	**7**	**10**
0.01	0.018	0.014	0.010
0.03	0.053	0.042	0.033
0.05	0.089	0.069	0.054

Table 1: Estimated k values for different combinations of Q and T

Appendix C: Analytical Justification of the $\frac{k}{V_g}$ Cost Term Using European Cash-or-Nothing Options

From option pricing theory, we know that the price of an American put option can be expressed as the sum of the price of a European put option and the value of the early exercise feature. Therefore, the difference between the prices of two American put options can be written as:

$$P_A(K_2) - P_A(K_1) = [P_E(K_2) - P_E(K_1)] + [\text{EE}(K_2) - \text{EE}(K_1)] \, .$$

For simplicity, we assume the second term in RHS to be approximately zero for very small changes in the strike price, which simplifies the mathematics behind the analytical derivations. However, it is always possible to estimate the entire difference numerically using Monte Carlo simulations. Our goal is to justify the cost term in the model, and the simplification to European-type options makes the analysis clearer and less complex.

To justify the $\frac{k}{V_g}$ cost term, we employ the European cash-or-nothing put option framework, as discussed in Hull (2017) , Chapter 25.9. The price of a European cash-or-nothing put option can be expressed as:

$$V_{\text{put}} = Qe^{-rT}N(-d_2),$$

where:

$$d_2 = \frac{\ln\left(\frac{S}{X}\right) + \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}.$$

For small changes in the strike price X, the change in the option value can be approximated by the derivative of the put option price with respect to X. This derivative provides an estimation of the marginal value of looser covenants. By "looser covenants" we mean a lower breach threshold V_g (more covenant headroom).

The derivative of the option price with respect to the strike is:

$$\frac{\partial V_{\text{put}}}{\partial X} = Qe^{-rT}\frac{\phi(-d_2)}{X\sigma\sqrt{T}},$$

where $\phi(-d_2)$ is the probability density function of the standard normal distribution. This expression quantifies how the value of the put option changes as the strike price varies.

For simplicity, we assume that $\phi(-d_2)$ is approximately constant and does not vary significantly with the strike, as it is less sensitive to strike changes compared to the $\frac{1}{X}$ term. Next, all the other terms except $\frac{1}{X}$ can be treated as constants. This allows us to approximate the derivative of the put price with respect to the strike as a constant of the form $\frac{k}{X}$, thereby analytically justifying the choice of the financial flexibility cost function $\frac{k}{V_g}$.

This approach simplifies the model while capturing the essential economic interpretation of the cost associated with looser covenants. Thus, the derivative of the put price with respect to the strike can be approximated as:

$$\frac{\partial V_{\text{put}}}{\partial X} \approx \frac{k}{X},$$

where k is treated as a constant, justifying the analytical form of the financial flexibility cost. Appendix B implements this approximation for American options by regressing $\partial V_{\text{put}}/\partial X$ on $1/X$ with an intercept and mapping the fitted slope into the positive constant k used in the wedge.

Appendix D: Anchoring the Penalty Parameter s Using Rating-Based Yield Differences

The penalty parameter s represents the proportional increase in the coupon payment upon a trigger-based tightening event (a repricing step-up) and is intended to capture the magnitude of contractual tightening costs in an investment-grade setting. To anchor the order of magnitude of s, we use rating-based yield differences as a proxy for repricing associated with a one-notch deterioration. This exercise is illustrative rather than identificational, since rating spreads reflect multiple components beyond contractual repricing.

We use the following formula to calculate s for each downgrade from a higher credit rating to the next lower rating:

$$s = \frac{\text{Yield}_{\text{lower rating}} - \text{Yield}_{\text{higher rating}}}{\text{Yield}_{\text{higher rating}}}$$

The yield data for U.S. corporate bonds across different credit ratings are obtained from the Federal Reserve Economic Data (FRED) database as of October 22, 2024. Because the model targets investment-grade firms and abstracts from default, we anchor s using investment-grade yields (AAA–BBB); we report BB only as a near-investment-grade boundary stress case and do not use it for the baseline calibration.

These values provide an illustrative magnitude anchor for s in the baseline calibration and comparative statics.

The yield data are sourced from the Federal Reserve Economic Data

Table 2: Yields (and implied penalty parameter) by rating as of October 22, 2024. Calibration uses AAA–BBB; BB shown for near-boundary stress only.

Credit Rating	**Yield (%)**	**Parameter** s
AAA	4.61	0.02
AA	4.71	0.05
A	4.94	0.07
BBB	5.27	0.11
BB	5.85	0.18

(FRED) database: https://fred.stlouisfed.org/.

Appendix E. Comparative statics and extended scenarios

E.1 Sensitivity of the optimal covenant threshold V_g^*

To understand how the optimal covenant threshold V_g^* responds to changes in key parameters, we report partial derivatives with respect to each parameter. For simplicity, we normalize firm value to $V = 1$.

The closed-form expression is

$$V_g^* = \left(\frac{k\, r(1+x)}{C\, s(1-\tau)\, x} \right)^{\frac{1}{x+1}}, \qquad x = \frac{2r}{\sigma^2}. \tag{51}$$

Table 3 summarizes the partial derivatives and their typical signs.

Table 3: Partial derivatives of V_g^* with respect to key parameters (typical signs).

Parameter θ	$\dfrac{\partial V_g^*}{\partial \theta}$	Typical sign
k	$\dfrac{1}{(x+1)k}\, V_g^*$	$+$
σ^2	$V_g^* \left(\dfrac{x}{(x+1)^2\sigma^2} \left(\ln V_g^* + \dfrac{1}{x} \right) \right)$	often $-$
r	$V_g^* \left(\dfrac{1}{(x+1)r} - \dfrac{2}{(x+1)^2(2r+\sigma^2)} \ln V_g^* \right)$	often $+$ if $V_g^* < 1$
s	$-\dfrac{1}{(x+1)s}\, V_g^*$	$-$
C	$-\dfrac{1}{(x+1)C}\, V_g^*$	$-$
τ	$\dfrac{1}{(x+1)(1-\tau)}\, V_g^*$	$+$

Economic interpretation

We interpret V_g^* as the covenant breach threshold (violation occurs when $V \leq V_g^*$). Hence, a lower V_g^* corresponds to looser covenants (more headroom).

- **Covenant cost (k):** Higher k increases V_g^*. Intuitively, a higher "price of flexibility" raises the breach threshold and reduces covenant headroom.

- **Asset volatility (σ^2):** Higher volatility typically lowers V_g^* in the baseline region (the sign condition is captured by Table 3). Economically, volatility raises the likelihood of approaching the breach threshold, so firms choose more covenant headroom (lower V_g^*).

- **Interest rate (r):** A higher r often increases V_g^* when $V_g^* < 1$, i.e., in the relevant investment-grade region.

- **Penalty for covenant violation (s):** Higher s lowers V_g^*: firms choose more covenant headroom (a lower breach threshold) to reduce the chance of paying the penalty.

- **Coupon payment (C):** Higher C lowers V_g^*, as higher debt service increases downside risk and leads the firm to choose more covenant headroom (lower V_g^*).

- **Corporate tax rate (τ):** Higher τ raises V_g^* because the tax benefit of debt becomes more valuable.

E.2 Baseline numerical example (BBB-rated firm)

We illustrate the magnitudes using a representative BBB-rated U.S. firm with the following baseline parameters:

Table 4: Baseline inputs used in the numerical illustration.

Parameter	Value
Normalized firm value V	1.00
Risk-free rate r	0.04
Corporate tax rate τ	0.21
Asset volatility σ	0.22
EBITDA-to-value ratio α	0.10
Debt/EBITDA covenant ratio K	4
Coupon payment C	0.05
Penalty for covenant violation s	0.12
Covenant negotiation cost k	0.05

The model implies a moderate baseline leverage and non-trivial covenant headroom:

Table 5: Baseline outputs.

Output	Value
Optimal threshold V_g/V	0.8634
Optimal leverage D/V	0.3453

E.3 Extended scenario analysis

Table 6 reports the optimal leverage D^* under one-at-a-time parameter changes around the baseline case.

Table 6: Extended scenarios: changes in optimal debt-to-value ratio D^*.

Scenario	New D^*	Change (%)
Baseline	0.3453	0.00
r increases to 5%	0.3693	+6.93
r decreases to 3%	0.3116	-9.76
Tax rate increases to 30%	0.3615	+4.66
σ increases to 25%	0.3529	+2.19
σ decreases to 20%	0.3434	-0.55
α decreases to 0.08	0.2763	-20.00
α increases to 0.12	0.4144	+20.00
K increases to 5	0.4317	+25.00
K decreases to 3	0.2590	-25.00
C increases to 0.06	0.3224	-6.64
C decreases to 0.04	0.3756	+8.78
s increases to 0.20	0.2849	-17.52
s decreases to 0.10	0.3699	+7.11
k decreases to 0.02	0.2445	-29.21
k increases to 0.07	0.3920	+13.52