Уравнения Максвелла

$$rot\ \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} + \boldsymbol{J}$$

$$rot\ \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$div\ \boldsymbol{D} = \rho$$

$$div\ \boldsymbol{B} = 0$$

$$\boldsymbol{D} = \varepsilon\varepsilon_0 \boldsymbol{E}$$

$$\boldsymbol{B} = \mu\mu_0 \boldsymbol{H}$$

$$\boldsymbol{J} = \sigma(\boldsymbol{E} - \boldsymbol{E}_{\text{ст}}) = \sigma \boldsymbol{E} + \boldsymbol{J}_{\text{ст}}$$

$$rot\ \boldsymbol{A} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$rot\ \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} + \boldsymbol{J} = \varepsilon\varepsilon_0 \frac{\partial\ \boldsymbol{E}}{\partial t} + \boldsymbol{J} = \varepsilon\varepsilon_0 \frac{\partial\ \boldsymbol{E}}{\partial t} + \sigma \boldsymbol{E} + \boldsymbol{J}_{\text{ст}}$$

$$\frac{\partial\ \boldsymbol{E}}{\partial t} = \frac{1}{\varepsilon\varepsilon_0} rot\ \boldsymbol{H} - \frac{\sigma}{\varepsilon\varepsilon_0}\boldsymbol{E} + \frac{1}{\varepsilon\varepsilon_0}\boldsymbol{J}_{\text{ст}} = \frac{1}{\varepsilon\varepsilon_0} rot\ \boldsymbol{H} - \frac{1}{\varepsilon\varepsilon_0}\boldsymbol{J}$$

$$rot\ \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} = -\mu\mu_0 \frac{\partial \boldsymbol{H}}{\partial t}$$

$$rot\ \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} = -\mu\mu_0 \frac{\partial \boldsymbol{H}}{\partial t}$$

Тейлор в точке +- $\frac{\delta}{2}$

$$f\left(x + \frac{\delta}{2}\right) = f(x) + \frac{\delta}{2} f'(x) + \frac{1}{2!}\left(\frac{\delta}{4}\right)^2 \dots$$

$$f\left(x - \frac{\delta}{2}\right) = f(x) - \frac{\delta}{2} f'(x) + \frac{1}{2!}\left(\frac{\delta}{4}\right)^2 \dots$$

$$f\left(x + \frac{\delta}{2}\right) - f\left(x - \frac{\delta}{2}\right) = f(x) + \frac{\delta}{2} f'(x) + \frac{1}{2!}\left(\frac{\delta}{4}\right)^2 \dots - f(x) + \frac{\delta}{2} f'(x) - \frac{1}{2!}\left(\frac{\delta}{4}\right)^2 \dots \approx \delta f'(x)$$

$$f'(x) \approx \frac{f\left(x + \frac{\delta}{2}\right) - f\left(x - \frac{\delta}{2}\right)}{\delta}$$

2d TMz

$$\frac{\partial \boldsymbol{H}}{\partial t} = -\frac{1}{\mu\mu_0}\, rot\, \boldsymbol{E} - \frac{1}{\mu\mu_0}\, \sigma_m \boldsymbol{H}$$

$$\frac{\partial\, \boldsymbol{E}}{\partial t} = \frac{1}{\varepsilon\varepsilon_0} rot\, \boldsymbol{H} - \frac{1}{\varepsilon\varepsilon_0} \sigma \boldsymbol{E}$$

$$rot\, \boldsymbol{E} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & E_z \end{vmatrix} = \boldsymbol{i}\frac{\partial E_z}{\partial y} - \boldsymbol{j}\frac{\partial E_z}{\partial x}$$

$$rot\, \boldsymbol{H} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0 \\ H_x & H_y & 0 \end{vmatrix} = \boldsymbol{k}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)$$

$$\frac{\partial \boldsymbol{H}}{\partial t} = -\frac{1}{\mu\mu_0}\left(\boldsymbol{i}\frac{\partial E_z}{\partial y} - \boldsymbol{j}\frac{\partial E_z}{\partial x}\right) - \frac{1}{\mu\mu_0}\, \sigma_m \boldsymbol{H}$$

$$\frac{\partial\, \boldsymbol{E}}{\partial t} = \frac{1}{\varepsilon\varepsilon_0} \boldsymbol{k}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) - \frac{1}{\varepsilon\varepsilon_0} \sigma \boldsymbol{E}$$

$$\frac{\partial H_x}{\partial t}\boldsymbol{i} + \frac{\partial H_y}{\partial t}\boldsymbol{j} + \frac{\partial H_z}{\partial t}\boldsymbol{k} = \left(-\frac{1}{\mu\mu_0}\frac{\partial E_z}{\partial y}\boldsymbol{i} + \frac{1}{\mu\mu_0}\frac{\partial E_z}{\partial x}\boldsymbol{j}\right) - \frac{1}{\mu\mu_0}\, \sigma_m H_x \boldsymbol{i} - \frac{1}{\mu\mu_0}\, \sigma_m H_y \boldsymbol{j} - \frac{1}{\mu\mu_0}\, \sigma_m H_z\, \boldsymbol{k}$$

$$\frac{\partial E_x}{\partial t}\boldsymbol{i} + \frac{\partial E_y}{\partial t}\boldsymbol{j} + \frac{\partial E_z}{\partial t}\boldsymbol{k} = \frac{1}{\varepsilon\varepsilon_0}\boldsymbol{k}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) - \frac{1}{\varepsilon\varepsilon_0}\sigma E_x \boldsymbol{i} - \frac{1}{\varepsilon\varepsilon_0}\sigma E_y \boldsymbol{j} - \frac{1}{\varepsilon\varepsilon_0}\sigma E_z \boldsymbol{k}$$

$\frac{\partial H_x}{\partial t} = -\frac{1}{\mu\mu_0}\frac{\partial E_z}{\partial y} - \frac{1}{\mu\mu_0}\, \sigma_m H_x$	$\frac{\partial E_x}{\partial t} = -\frac{1}{\varepsilon\varepsilon_0}\sigma E_x$
$\frac{\partial H_y}{\partial t} = +\frac{1}{\mu\mu_0}\frac{\partial E_z}{\partial x} - \frac{1}{\mu\mu_0}\, \sigma_m H_y$	$\frac{\partial E_y}{\partial t} = -\frac{1}{\varepsilon\varepsilon_0}\sigma E_y$
$\frac{\partial H_z}{\partial t} = -\frac{1}{\mu\mu_0}\, \sigma_m H_z$	$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon\varepsilon_0}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) - \frac{1}{\varepsilon\varepsilon_0}\sigma E_z$

$$\frac{\partial H_x}{\partial t} = -\frac{1}{\mu\mu_0}\frac{\partial E_z}{\partial y} - \frac{1}{\mu\mu_0}\, \sigma_m H_x$$

$$\frac{\partial H_y}{\partial t} = -\frac{1}{\mu\mu_0}\frac{\partial E_z}{\partial x} - \frac{1}{\mu\mu_0}\, \sigma_m H_y$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon\varepsilon_0}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) - \frac{1}{\varepsilon\varepsilon_0}\sigma E_z$$

Заменяем конечноразностной схемой во временной области

$$H_x(x,y,t) = H_x\left(m\Delta_x, n\Delta_y, q\Delta_t\right) = H_x^q[m,n]$$

$$H_y(x,y,t) = H_y\left(m\Delta_x, n\Delta_y, q\Delta_t\right) = H_y^q[m,n]$$

$$E_z(x,y,t) = E_z\left(m\Delta_x, n\Delta_y, q\Delta_t\right) = E_z^q[m,n]$$

$$\frac{\partial H_x}{\partial t} = -\frac{1}{\mu\mu_0}\frac{\partial E_z}{\partial y} - \frac{1}{\mu\mu_0}\sigma_m H_x$$

$$\frac{H_x^{q+\frac{1}{2}}[m,n+1] - H_x^{q-\frac{1}{2}}[m,n+1]}{\Delta_t} = -\frac{1}{\mu\mu_0}\left(\frac{E_z^q[m,n+1] - E_z^q[m,n]}{\Delta_y}\right) - \frac{1}{\mu\mu_0}\sigma_m H_x^q[m,n]$$

$$H_x^q[m,n] - \text{надо усреднить}$$

$$\frac{H_x^{q+1/2}\left[m,n+\frac{1}{2}\right] - H_x^{q-1/2}\left[m,n+\frac{1}{2}\right]}{\Delta_t} = -\frac{1}{\mu\mu_0}\left(\frac{E_z^q[m,n+1] - E_z^q[m,n]}{\Delta_y}\right) - \frac{1}{\mu\mu_0}\frac{\sigma_m\left(H_x^{q+\frac{1}{2}}\left[m,n+\frac{1}{2}\right] + H_x^{q-\frac{1}{2}}\left[m,n+\frac{1}{2}\right]\right)}{2}$$

упростим

$$H_x^{q+1/2}\left[m,n+\frac{1}{2}\right] - H_x^{q-1/2}\left[m,n+\frac{1}{2}\right] = -\frac{\Delta_t}{\mu\mu_0}\left(\frac{E_z^q[m,n+1] - E_z^q[m,n]}{\Delta_y}\right) - \frac{\Delta_t}{\mu\mu_0}\frac{\sigma_m}{2}H_x^{q+\frac{1}{2}}\left[m,n+\frac{1}{2}\right] - \frac{\Delta_t}{\mu\mu_0}\frac{\sigma_m}{2}H_x^{q-\frac{1}{2}}\left[m,n+\frac{1}{2}\right]$$

$$H_x^{q+1/2}\left[m,n+\frac{1}{2}\right] + \frac{\Delta_t}{\mu\mu_0}\frac{\sigma_m}{2}H_x^{q+\frac{1}{2}}\left[m,n+\frac{1}{2}\right] - H_x^{q-\frac{1}{2}}\left[m,n+\frac{1}{2}\right] + \frac{\Delta_t}{\mu\mu_0}\frac{\sigma_m}{2}H_x^{q-\frac{1}{2}}\left[m,n+\frac{1}{2}\right] = -\frac{\Delta_t}{\mu\mu_0}\left(\frac{E_z^q[m,n+1] - E_z^q[m,n]}{\Delta_y}\right)$$

$$\left(1 + \frac{\Delta_t}{\mu\mu_0}\frac{\sigma_m}{2}\right)H_x^{q+\frac{1}{2}}\left[m,n+\frac{1}{2}\right] - \left(1 - \frac{\Delta_t}{\mu\mu_0}\frac{\sigma_m}{2}\right)H_x^{q-\frac{1}{2}}\left[m,n+\frac{1}{2}\right] = -\frac{\Delta_t}{\mu\mu_0\Delta_y}E_z^q[m,n+1] + \frac{\Delta_t}{\mu\mu_0\Delta_y}E_z^q[m,n]$$

$$H_x^{q+\frac{1}{2}}\left[m,n+\frac{1}{2}\right] = \frac{\left(1 - \frac{\Delta_t}{\mu\mu_0}\frac{\sigma_m}{2}\right)}{\left(1 + \frac{\Delta_t}{\mu\mu_0}\frac{\sigma_m}{2}\right)}H_x^{q-\frac{1}{2}}\left[m,n+\frac{1}{2}\right] - \frac{\Delta_t}{\mu\mu_0\Delta_y}\frac{1}{\left(1 + \frac{\Delta_t}{\mu\mu_0}\frac{\sigma_m}{2}\right)}\left(E_z^q[m,n+1] + E_z^q[m,n]\right)$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu\mu_0}\frac{\partial E_z}{\partial x} - \frac{1}{\mu\mu_0}\sigma_m H_y$$

$$H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2},n\right] - H_y^{q-\frac{1}{2}}[m+\frac{1}{2},n]$$
$$= \frac{\Delta_t}{\mu\mu_0\Delta_x}\left(E_z^q[m+1,n] - E_z^q[m,n]\right) - \frac{\Delta_t\sigma_m}{2\mu\mu_0}\left(H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2},n\right] + H_y^{q-\frac{1}{2}}\left[m+\frac{1}{2},n\right]\right)$$

$$H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2},n\right] + \frac{\Delta_t\sigma_m}{\mu\mu_0}H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2},n\right] - H_y^{q-\frac{1}{2}}\left[m+\frac{1}{2},n\right] + \frac{\Delta_t\sigma_m}{2\mu\mu_0}H_y^{q-\frac{1}{2}}\left[m+\frac{1}{2},n\right]$$
$$= \frac{\Delta_t}{\mu\mu_0\Delta_x}\left(E_z^q[m+1,n] - E_z^q[m,n]\right)$$

$$\left(1+\frac{\Delta_t\sigma_m}{2\mu\mu_0}\right)H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2},n\right] - \left(1-\frac{\Delta_t\sigma_m}{2\mu\mu_0}\right)H_y^{q-\frac{1}{2}}\left[m+\frac{1}{2},n\right] = \frac{\Delta_t}{\mu\mu_0\Delta_x}\left(E_z^q[m+1,n] - E_z^q[m,n]\right)$$

$$H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2},n\right] = \frac{\left(1-\frac{\Delta_t\sigma_m}{2\mu\mu_0}\right)}{\left(1+\frac{\Delta_t\sigma_m}{2\mu\mu_0}\right)}H_y^{q-\frac{1}{2}}\left[m+\frac{1}{2},n\right] + \frac{\Delta_t}{\mu\mu_0\Delta_x}\frac{1}{\left(1+\frac{\Delta_t\sigma_m}{2\mu\mu_0}\right)}\left(E_z^q[m+1,n] - E_z^q[m,n]\right)$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon\varepsilon_0}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) - \frac{1}{\varepsilon\varepsilon_0}\sigma E_z$$

$$\frac{E_z^{q+1}[m,n] - E_z^q[m,n]}{\Delta_t} = \frac{1}{\varepsilon\varepsilon_0}\frac{H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2},n\right] - H_y^{q+\frac{1}{2}}[m-\frac{1}{2},n]}{\Delta_x}$$

$$-\frac{1}{\varepsilon\varepsilon_0}\frac{H_y^{q+\frac{1}{2}}\left[m,n+\frac{1}{2}\right] - H_y^{q+\frac{1}{2}}\left[m,n-\frac{1}{2}\right]}{\Delta_y} - \frac{\sigma}{2\varepsilon\varepsilon_0}\left(E_z^{q+1}[m,n] + E_z^q[m,n]\right)$$

$$E_z^{q+1}[m,n] - E_z^q[m,n] = \frac{\Delta_t}{\Delta_x\varepsilon\varepsilon_0}H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2},n\right] - \frac{\Delta_t}{\Delta_x\varepsilon\varepsilon_0}H_y^{q+\frac{1}{2}}[m-\frac{1}{2},n]$$

$$-\frac{\Delta_t}{\Delta_y\varepsilon\varepsilon_0}H_y^{q+\frac{1}{2}}\left[m,n+\frac{1}{2}\right] + \frac{\Delta_t}{\Delta_y\varepsilon\varepsilon_0}H_y^{q+\frac{1}{2}}\left[m,n-\frac{1}{2}\right] - \frac{\Delta_t\sigma}{2\varepsilon\varepsilon_0}E_z^{q+1}[m,n] - \frac{\Delta_t\sigma}{2\varepsilon\varepsilon_0}E_z^q[m,n]$$

$$\left(1+\frac{\Delta_t\sigma}{2\varepsilon\varepsilon_0}\right)E_z^{q+1}[m,n] - \left(1-\frac{\Delta_t\sigma}{2\varepsilon\varepsilon_0}\right)E_z^q[m,n] = \frac{\Delta_t}{\Delta_x\varepsilon\varepsilon_0}H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2},n\right] - \frac{\Delta_t}{\Delta_x\varepsilon\varepsilon_0}H_y^{q+\frac{1}{2}}[m-\frac{1}{2},n]$$
$$-\frac{\Delta_t}{\Delta_y\varepsilon\varepsilon_0}H_y^{q+\frac{1}{2}}\left[m,n+\frac{1}{2}\right] + \frac{\Delta_t}{\Delta_y\varepsilon\varepsilon_0}H_y^{q+\frac{1}{2}}\left[m,n-\frac{1}{2}\right]$$

$$E_z^{q+1}[m,n] = \frac{\left(1-\frac{\Delta_t\sigma}{2\varepsilon\varepsilon_0}\right)}{\left(1+\frac{\Delta_t\sigma}{2\varepsilon\varepsilon_0}\right)}E_z^q[m,n] + \frac{\Delta_t}{\Delta_x\varepsilon\varepsilon_0}\frac{1}{\left(1+\frac{\Delta_t\sigma}{2\varepsilon\varepsilon_0}\right)}\left(H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2},n\right] - H_y^{q+\frac{1}{2}}\left[m-\frac{1}{2},n\right]\right)$$

$$-\frac{\Delta_t}{\Delta_y \varepsilon \varepsilon_0} \frac{1}{\left(1+\frac{\Delta_t \sigma}{2\varepsilon\varepsilon_0}\right)} \left(H_y^{q+\frac{1}{2}}\left[m, n+\frac{1}{2}\right] - H_y^{q+\frac{1}{2}}\left[m, n-\frac{1}{2}\right] \right)$$

$$div\ \boldsymbol{D} = \rho = div\ \varepsilon\varepsilon_0 \boldsymbol{E} = \varepsilon\varepsilon_0 div\ \boldsymbol{E}$$

$$div\ \boldsymbol{E} = \frac{\rho}{\varepsilon\varepsilon_0}$$

$$div\ \ \mu\mu_0 \boldsymbol{H} = 0$$

$$\boldsymbol{D} = \ \varepsilon\varepsilon_0 \boldsymbol{E}$$

$$\boldsymbol{B} = \ \mu\mu_0 \boldsymbol{H}$$

$$rot\ \boldsymbol{H} = \varepsilon\varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} + \boldsymbol{J}$$

$$rot(rot\ \boldsymbol{E}) = \boldsymbol{rot}\left(-\ \mu\mu_0 \frac{\partial \boldsymbol{H}}{\partial t}\right)$$

$$grad(div\boldsymbol{E}) = \boldsymbol{rot}\left(-\ \mu\mu_0 \frac{\partial \boldsymbol{H}}{\partial t}\right)$$

$$grad\left(\frac{\rho}{\varepsilon\varepsilon_0}\right) = -\ \mu\mu_0 rot\left(\frac{\partial \boldsymbol{H}}{\partial t}\right) = -\ \mu\mu_0 \frac{\partial}{\partial t}\big(rot(\boldsymbol{H})\big) = -\ \mu\mu_0 \frac{\partial}{\partial t}\left(\varepsilon\varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} + \boldsymbol{J}\right)$$
$$= -\ \mu\mu_0 \varepsilon\varepsilon_0 \left(\frac{\partial^2 \boldsymbol{E}}{\partial t^2} + \frac{\partial \boldsymbol{J}}{\partial t}\right)$$