I the natural system of units from 2d case

$$i\frac{\partial u}{\partial t} = Hu = -\frac{1}{2}\Delta u + v(x,y,t)u = -\frac{1}{2}\frac{\mathrm{d}^2 u}{\mathrm{dx}^2} - \frac{1}{2}\frac{\mathrm{d}^2 u}{\mathrm{dy}^2} + v(x,y,t)u$$

$$\frac{\partial v(x,y,t)}{\partial t} = 0 => v = v(x,y)$$

$$i\frac{\partial u}{\partial t} = -\frac{1}{2}\frac{\mathrm{d}^2 u}{\mathrm{dx}^2} - \frac{1}{2}\frac{\mathrm{d}^2 u}{\mathrm{dy}^2} + v(x,y)u$$

$$\frac{i}{dt}\left(u_{i,j}^{n+1} - u_{i,j}^{n}\right) = -\frac{\left(u_{i+1,j}^{n} - 2u_{i,j}^{n} + u_{i-1,j}^{n}\right)}{2\mathrm{dx}^2} - \frac{\left(u_{i,j+1}^{n} - 2u_{i,j}^{n} + u_{i,j-1}^{n}\right)}{2\mathrm{dy}^2} + v(x_i,y_j)u_{i,j}^{n}$$

$$u_{i,j}^{n+1} - u_{i,j}^{n} = \frac{idt}{2\mathrm{dx}^2}\left(u_{i+1,j}^{n} - 2u_{i,j}^{n} + u_{i-1,j}^{n}\right) + \frac{idt}{2\mathrm{dy}^2}\left(u_{i,j+1}^{n} - 2u_{i,j}^{n} + u_{i,j-1}^{n}\right) - idt * v(x_i,y_j)u_{i,j}^{n}$$

$$u_{i,j}^{n+1} = \frac{idt}{2\mathrm{dx}^2}\left(u_{i+1,j}^{n} - 2u_{i,j}^{n} + u_{i-1,j}^{n}\right) + \frac{idt}{2\mathrm{dy}^2}\left(u_{i,j+1}^{n} - 2u_{i,j}^{n} + u_{i,j-1}^{n}\right) - idt * v(x_i,y_j)u_{i,j}^{n} + u_{i,j}^{n}$$

$$u_{i,j}^{n+1} = \frac{idt}{2\mathrm{dx}^2}u_{i+1,j}^{n} - \frac{idt}{\mathrm{dx}^2}u_{i,j}^{n} + \frac{idt}{2\mathrm{dx}^2}u_{i-1,j}^{n} + \frac{idt}{2\mathrm{dy}^2}u_{i,j+1}^{n} - \frac{idt}{\mathrm{dy}^2}u_{i,j}^{n} + \frac{idt}{2\mathrm{dy}^2}u_{i,j-1}^{n} - idt * v(x_i,y_j)u_{i,j}^{n} + u_{i,j}^{n}$$

$$u_{i,j}^{n+1} = \frac{idt}{2\mathrm{dx}^2}u_{i+1,j}^{n} + \frac{idt}{2\mathrm{dx}^2}u_{i-1,j}^{n} + \frac{idt}{2\mathrm{dy}^2}u_{i,j+1}^{n} + \frac{idt}{2\mathrm{dy}^2}u_{i,j-1}^{n} + \left(-\frac{idt}{\mathrm{dx}^2} - \frac{idt}{\mathrm{dy}^2} - idt * v(x_i,y_j) + 1\right)u_{i,j}^{n}$$

For one step on t

$$\frac{dt}{\mathrm{dx}^2} = a; \frac{dt}{\mathrm{dy}^2} = b$$

$$u_{i,j}^{n+1} = \frac{i}{2}au_{i+1,j}^{n} + \frac{i}{2}au_{i-1,j}^{n} + \frac{i}{2}bu_{i,j+1}^{n} + \frac{i}{2}bu_{i,j-1}^{n} + \left(-ia - ib - idt * v(x_i,y_j) + 1\right)u_{i,j}^{n}$$

$$i = 0 \ldots I; j = 0 \ldots J$$

$$u_{i,j}^{n+1}=\frac{i}{2}a\begin{pmatrix}u_{1,1}^n\\u_{2,1}^n\\\vdots\\u_{I+1,1}^n\\u_{1,2}^n\\\vdots\\u_{I+1,J}^n\end{pmatrix}+\frac{i}{2}a\begin{pmatrix}u_{-1,1}^n\\u_{0,1}^n\\\vdots\\u_{I-1,1}^n\\u_{-1,2}^n\\\vdots\\u_{I-1,J}^n\end{pmatrix}+\frac{i}{2}bu_{i,j+1}^n+\frac{i}{2}bu_{i,j-1}^n+\left(-ia-ib-idt*v\left(x_i,y_j\right)+1\right)u_{i,j}^n$$

$$u_{i,j}^{n+1}=\hat{A}u_{i,j}^n$$

From case $i=0\ldots2;j=0\ldots2$

$$\begin{pmatrix}u_{0,0}^{n+1}\\u_{1,0}^{n+1}\\u_{2,0}^{n+1}\\u_{0,1}^{n+1}\\u_{1,1}^{n+1}\\u_{2,1}^{n+1}\\u_{0,2}^{n+1}\\u_{1,2}^{n+1}\\u_{2,2}^{n+1}\end{pmatrix}=\frac{i}{2}a\begin{pmatrix}u_{1,0}^n\\u_{2,0}^n\\u_{3,0}^n\\u_{1,1}^n\\u_{2,1}^n\\u_{3,1}^n\\u_{1,2}^n\\u_{2,2}^n\\u_{3,2}^n\end{pmatrix}+\frac{i}{2}a\begin{pmatrix}u_{-1,0}^n\\u_{0,0}^n\\u_{1,0}^n\\u_{-1,1}^n\\u_{0,1}^n\\u_{1,1}^n\\u_{-1,2}^n\\u_{0,2}^n\\u_{1,2}^n\end{pmatrix}+\frac{i}{2}b\begin{pmatrix}u_{0,1}^n\\u_{1,1}^n\\u_{2,1}^n\\u_{0,2}^n\\u_{1,2}^n\\u_{2,2}^n\\u_{0,3}^n\\u_{1,3}^n\\u_{2,3}^n\end{pmatrix}+\frac{i}{2}b\begin{pmatrix}u_{0,-1}^n\\u_{1,-1}^n\\u_{2,-1}^n\\u_{0,0}^n\\u_{1,0}^n\\u_{2,0}^n\\u_{0,1}^n\\u_{1,1}^n\\u_{2,1}^n\end{pmatrix}+\left(-ia-ib-idt*v\left(x_i,y_j\right)+1\right)\begin{pmatrix}u_{0,0}^n\\u_{1,0}^n\\u_{2,0}^n\\u_{0,1}^n\\u_{1,1}^n\\u_{2,1}^n\\u_{0,2}^n\\u_{1,2}^n\\u_{2,2}^n\end{pmatrix}$$

$$\left(-ia-ib-idt*v\left(x_i,y_j\right)+1\right)=c_{i,j}$$

$$\frac{i}{2}a\begin{pmatrix} u_{1,0}^n \\ u_{2,0}^n \\ u_{3,0}^n \\ u_{1,1}^n \\ u_{2,1}^n \\ u_{3,1}^n \\ u_{1,2}^n \\ u_{2,2}^n \\ u_{3,2}^n \end{pmatrix} + \frac{i}{2}a\begin{pmatrix} u_{-1,0}^n \\ u_{0,0}^n \\ u_{1,0}^n \\ u_{-1,1}^n \\ u_{0,1}^n \\ u_{1,1}^n \\ u_{-1,2}^n \\ u_{0,2}^n \\ u_{1,2}^n \end{pmatrix} + \frac{i}{2}b\begin{pmatrix} u_{0,1}^n \\ u_{1,1}^n \\ u_{2,1}^n \\ u_{0,2}^n \\ u_{1,2}^n \\ u_{2,2}^n \\ u_{0,3}^n \\ u_{1,3}^n \\ u_{2,3}^n \end{pmatrix} + \frac{i}{2}b\begin{pmatrix} u_{0,-1}^n \\ u_{1,-1}^n \\ u_{2,-1}^n \\ u_{0,0}^n \\ u_{1,0}^n \\ u_{2,0}^n \\ u_{0,1}^n \\ u_{1,1}^n \\ u_{2,1}^n \end{pmatrix} + \left(-ia - ib - idt * v\left(x_i, y_j\right) + 1\right)\begin{pmatrix} u_{0,0}^n \\ u_{1,0}^n \\ u_{2,0}^n \\ u_{0,1}^n \\ u_{1,1}^n \\ u_{2,1}^n \\ u_{0,2}^n \\ u_{1,2}^n \\ u_{2,2}^n \end{pmatrix}$$

$$= \begin{pmatrix} \frac{i}{2}au_{1,0}^n + \frac{i}{2}au_{-1,0}^n + \frac{i}{2}bu_{0,1}^n + \frac{i}{2}bu_{0,-1}^n + \left(-ia - ib - idt * v\left(x_i, y_j\right) + 1\right)u_{0,0}^n \\ \frac{i}{2}au_{2,0}^n + \frac{i}{2}au_{0,0}^n + \frac{i}{2}bu_{1,1}^n + \frac{i}{2}bu_{1,-1}^n + \left(-ia - ib - idt * v\left(x_i, y_j\right) + 1\right)u_{1,0}^n \\ \frac{i}{2}au_{3,0}^n + \frac{i}{2}au_{1,0}^n + \frac{i}{2}bu_{2,1}^n + \frac{i}{2}bu_{2,-1}^n + \left(-ia - ib - idt * v\left(x_i, y_j\right) + 1\right)u_{2,0}^n \\ \frac{i}{2}au_{1,1}^n + \frac{i}{2}au_{-1,1}^n + \frac{i}{2}bu_{0,2}^n + \frac{i}{2}bu_{0,0}^n + \left(-ia - ib - idt * v\left(x_i, y_j\right) + 1\right)u_{0,1}^n \\ \frac{i}{2}au_{2,1}^n + \frac{i}{2}au_{0,1}^n + \frac{i}{2}bu_{1,2}^n + \frac{i}{2}bu_{1,0}^n + \left(-ia - ib - idt * v\left(x_i, y_j\right) + 1\right)u_{1,1}^n \\ \frac{i}{2}au_{3,1}^n + \frac{i}{2}au_{1,1}^n + bu_{2,2}^n + \frac{i}{2}bu_{2,0}^n + \left(-ia - ib - idt * v\left(x_i, y_j\right) + 1\right)u_{2,1}^n \\ \frac{i}{2}au_{1,2}^n + \frac{i}{2}au_{-1,2}^n + \frac{i}{2}bu_{0,3}^n + \frac{i}{2}bu_{0,1}^n + \left(-ia - ib - idt * v\left(x_i, y_j\right) + 1\right)u_{0,2}^n \\ \frac{i}{2}au_{2,2}^n + \frac{i}{2}au_{0,2}^n + \frac{i}{2}bu_{1,3}^n + \frac{i}{2}bu_{0,1}^n + \left(-ia - ib - idt * v\left(x_i, y_j\right) + 1\right)u_{1,2}^n \\ \frac{i}{2}au_{3,2}^n + \frac{i}{2}au_{1,2}^n + \frac{i}{2}bu_{2,3}^n + \frac{i}{2}bu_{2,1}^n + \left(-ia - ib - idt * v\left(x_i, y_j\right) + 1\right)u_{2,2}^n \end{pmatrix}$$

$$\begin{pmatrix}
\frac{i}{2}au_{1,0}^{n}+\frac{i}{2}au_{-1,0}^{n}+\frac{i}{2}bu_{0,1}^{n}+\frac{i}{2}bu_{0,-1}^{n}+(-ia-ib-idt*v(x_0,y_0)+1)u_{0,0}^{n}\\
\frac{i}{2}au_{2,0}^{n}+\frac{i}{2}au_{0,0}^{n}+\frac{i}{2}bu_{1,1}^{n}+\frac{i}{2}bu_{1,-1}^{n}+(-ia-ib-idt*v(x_1,y_0)+1)u_{1,0}^{n}\\
\frac{i}{2}au_{3,0}^{n}+\frac{i}{2}au_{1,0}^{n}+\frac{i}{2}bu_{2,1}^{n}+\frac{i}{2}bu_{2,-1}^{n}+(-ia-ib-idt*v(x_2,y_0)+1)u_{2,0}^{n}\\
\frac{i}{2}au_{1,1}^{n}+\frac{i}{2}au_{-1,1}^{n}+\frac{i}{2}bu_{0,2}^{n}+\frac{i}{2}bu_{0,0}^{n}+(-ia-ib-idt*v(x_0,y_1)+1)u_{0,1}^{n}\\
\frac{i}{2}au_{2,1}^{n}+\frac{i}{2}au_{0,1}^{n}+\frac{i}{2}bu_{1,2}^{n}+\frac{i}{2}bu_{1,0}^{n}+(-ia-ib-idt*v(x_1,y_1)+1)u_{1,1}^{n}\\
\frac{i}{2}au_{3,1}^{n}+\frac{i}{2}au_{1,1}^{n}+bu_{2,2}^{n}+\frac{i}{2}bu_{2,0}^{n}+(-ia-ib-idt*v(x_2,y_1)+1)u_{2,1}^{n}\\
\frac{i}{2}au_{1,2}^{n}+\frac{i}{2}au_{-1,2}^{n}+\frac{i}{2}bu_{0,3}^{n}+\frac{i}{2}bu_{0,1}^{n}+(-ia-ib-idt*v(x_0,y_2)+1)u_{0,2}^{n}\\
\frac{i}{2}au_{2,2}^{n}+\frac{i}{2}au_{0,2}^{n}+\frac{i}{2}bu_{1,3}^{n}+\frac{i}{2}bu_{0,1}^{n}+(-ia-ib-idt*v(x_1,y_2)+1)u_{1,2}^{n}\\
\frac{i}{2}au_{3,2}^{n}+\frac{i}{2}au_{1,2}^{n}+\frac{i}{2}bu_{2,3}^{n}+\frac{i}{2}bu_{2,1}^{n}+(-ia-ib-idt*v(x_2,y_2)+1)u_{2,2}^{n}
\end{pmatrix}$$

$$
= \begin{pmatrix}
\left(\begin{gathered}-ia-ib-\\ idt*v(x_0,y_0)+1\end{gathered}\right) & \frac{i}{2}a & 0 & \frac{i}{2}b & 0 & 0 & 0 & 0 & 0 \\
\frac{i}{2}a & \left(\begin{gathered}-ia-ib-\\ idt*v(x_1,y_0)+1\end{gathered}\right) & \frac{i}{2}a & 0 & \frac{i}{2}b & 0 & 0 & 0 & 0 \\
0 & \frac{i}{2}a & \left(\begin{gathered}-ia-ib-\\ idt*v(x_2,y_0)+1\end{gathered}\right) & 0 & 0 & \frac{i}{2}b & 0 & 0 & 0 \\
\frac{i}{2}b & 0 & 0 & \left(\begin{gathered}-ia-ib-\\ idt*v(x_0,y_1)+1\end{gathered}\right) & \frac{i}{2}a & 0 & \frac{i}{2}b & 0 & 0 \\
0 & \frac{i}{2}b & 0 & \frac{i}{2}a & \left(\begin{gathered}-ia-ib-\\ idt*v(x_1,y_1)+1\end{gathered}\right) & \frac{i}{2}a & 0 & \frac{i}{2}b & 0 \\
0 & 0 & \frac{i}{2}b & 0 & \frac{i}{2}a & \left(\begin{gathered}-ia-ib-\\ idt*v(x_2,y_1)+1\end{gathered}\right) & 0 & 0 & \frac{i}{2}b \\
0 & 0 & 0 & \frac{i}{2}b & 0 & 0 & \left(\begin{gathered}-ia-ib-\\ idt*v(x_0,y_2)+1\end{gathered}\right) & \frac{i}{2}a & 0 \\
0 & 0 & 0 & 0 & \frac{i}{2}b & 0 & \frac{i}{2}a & \left(\begin{gathered}-ia-ib-\\ idt*v(x_1,y_2)+1\end{gathered}\right) & \frac{i}{2}a \\
0 & 0 & 0 & 0 & 0 & \frac{i}{2}b & 0 & \frac{i}{2}a & \left(\begin{gathered}-ia-ib-\\ idt*v(x_2,y_2)+1\end{gathered}\right)
\end{pmatrix}
\begin{pmatrix} u_{0,0}^n \\ u_{1,0}^n \\ u_{2,0}^n \\ u_{0,1}^n \\ u_{1,1}^n \\ u_{2,1}^n \\ u_{0,2}^n \\ u_{1,2}^n \\ u_{2,2}^n \end{pmatrix}
$$

$$
\hat{A} = \begin{pmatrix}
c_{i,j} & \frac{i}{2}a & 0 & & \frac{i}{2}b & 0 & 0 \\
\frac{i}{2}a & c_{i,j} & \frac{i}{2}a & \cdots & 0 & \frac{i}{2}b & 0 \\
0 & \frac{i}{2}a & c_{i,j} & & 0 & 0 & \frac{i}{2}b \\
 & \vdots & & \ddots & & \vdots & \\
\frac{i}{2}b & 0 & 0 & & c_{i,j} & \frac{i}{2}a & 0 \\
0 & \frac{i}{2}b & 0 & \cdots & \frac{i}{2}a & c_{i,j} & \frac{i}{2}a \\
0 & 0 & \frac{i}{2}b & & 0 & \frac{i}{2}a & c_{i,j}
\end{pmatrix}
$$