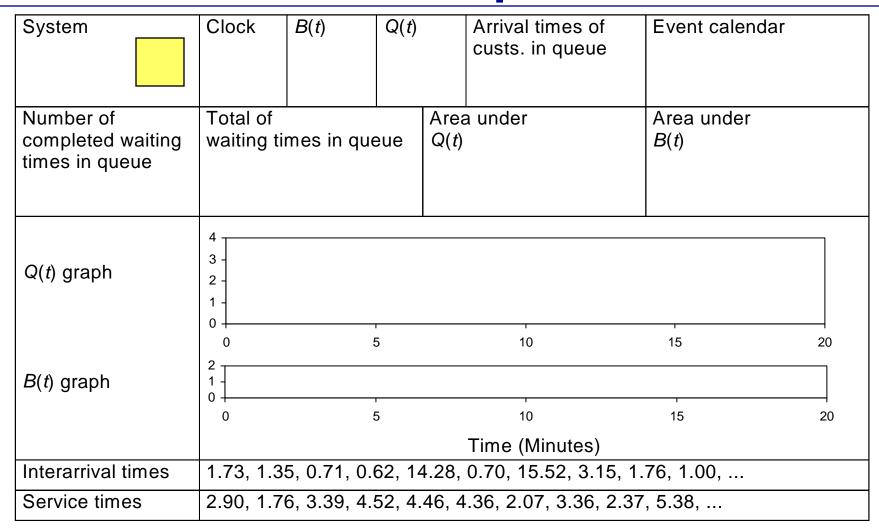
#### Simulation by Hand

- Manually track state variables, statistical accumulators
- Use "given" interarrival, service times
- Keep track of event calendar
- "Lurch" clock from one event to the next
- Will omit times in system, "max" computations here (see text for complete details)

### Simulation by Hand: Setup



## Simulation by Hand: t = 0.00, Initialize

System	Clock	B(t)	Q(t)		Arrival times of custs. in queue	Eve [1,	nt calend 0.00,	ar Arr]	
	0.00	0	0		<empty></empty>	I = '	20.00,	End]	
Number of completed waiting times in queue	Total of waiting ti	mes in que	eue	Area Q(t)	a under	Area B(t)	a under		
0	0.00			0.00		0.00	)		
Q(t) graph	4 3 - 2 - 1 - 0								
B(t) graph	0 2 1 0 0		5		10	15		20	
	0 5 10 15 20 Time (Minutes)								
Interarrival times	1.73, 1.35, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00,								
Service times	2.90, 1.7	2.90, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38,							

## Simulation by Hand: t = 0.00, Arrival of Part 1

System	Clock	B(t)	Q(t)		Arrival times of custs. in queue	[2,	nt calenda 1.73,	Arr]
	0.00	1	0		<empty></empty>	[1,   [–,	2.90, 20.00,	Dep] End]
Number of	Total of				a under		a under	
completed waiting times in queue	waiting ti	mes in que	eue	Q(t)		B(t)		
1	0.00			0.00		0.00	)	
	4					•		
Q(t) graph	3 - 2 -							
	1 - 0							
	0	!	5		10	15		20
B(t) graph	2 1 0							
	0	!	5		10	15		20
	Time (Minutes)							
Interarrival times	1,73, 1.35, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00,							
Service times	280, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38,							

## Simulation by Hand: t = 1.73, Arrival of Part 2

System	Clock	B(t)	Q(t)		Arrival times of custs. in queue	Eve [1,	nt calenda 2.90,	ar Dep]
2 1	1.73	1	1		(1.73)	-	•	Arr] End]
Number of	Total of				a under		a under	
completed waiting times in queue	waiting ti	mes in que	eue	Q(t)		B(t)		
1	0.00			0.00		1.73	3	
	4							
Q(t) graph	3 - 2 -							
	1 - 0							
	0	ţ	5		10	15		20
B(t) graph	2 1 0							
	0	Ę	5		10	15		20
	Time (Minutes)							
Interarrival times	1,73, 1,25, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00,							
Service times	280, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38,							

### Simulation by Hand: t = 2.90, Departure of Part 1

System	Clock $B(t)$ $Q(t)$		Q(t)		Arrival times of custs. in queue	Eve [3,	2, 4.66, Dep] -, 20.00, End] rea under	
2	2.90	1	0		<empty></empty>	[2, [–,	4.66,	Dep]
Number of completed waiting times in queue	Total of waiting ti	mes in que	eue	Area Q(t)	a under	Area B(t)	a under	
2	1.17			1.17	•	2.90	)	
Q(t) graph	4 3 - 2 - 1 - 0							
B(t) graph	0 2 1 0 0		5		10	15		20
	Time (Minutes)							
Interarrival times	1,73, 1,25, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00,							
Service times	2.80, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38,							

## Simulation by Hand: t = 3.08, Arrival of Part 3

System	Clock	B(t)	Q(t)		Arrival times of		nt calenda	ar	
3 2	3.08	1	1		custs. in queue (3.08)	[4, [2, [-,	3.79, 4.66, 20.00,	Arr] Dep] End]	
Number of	Total of				a under		a under		
completed waiting times in queue	waiting ti	mes in que	eue	Q(t)		B(t)			
2	1.17			1.17		3.08	3		
Q(t) graph	4 3 - 2 - 1 - 0								
	0	;	5		10	15		20	
B(t) graph	2 1	•••							
	0	;	5		10	15		20	
	Time (Minutes)								
Interarrival times	1,73, 1,25, 0,71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00,								
Service times	2.80, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38,								

## Simulation by Hand: t = 3.79, Arrival of Part 4

System	Clock	B(t)	Q(t)		Arrival times of	_	nt calenda	
432	3.79	1	2		custs. in queue (3.79, 3.08)	[5, [2, [–,	4.41, 4.66, 20.00,	Arr] Dep] End]
Number of completed waiting times in queue	Total of waiting ti	mes in que	eue	Area Q(t)	a under	Area B(t)	a under	
2	1.17			1.88	<b>;</b>	3.79	)	
Q(t) graph	4 3 - 2 - 1 - 0							
B(t) graph	0 2 1 0 0	••••	5		10	15		20
	Time (Minutes)							
Interarrival times	1,73, 1,25, 0,71, 0,82, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00,							
Service times	280, 176, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38,							

## Simulation by Hand: t = 4.41, Arrival of Part 5

System	Clock	B(t)	Q(t)		Arrival times of	Eve	nt calenda	ar		
					custs. in queue	[2,	4.66,	Dep]		
0 4 0 2	4.41	1	3		(4.41, 3.79, 3.08)		18.69, 20.00,	Arr]		
Number of	Total of			۸ro	 a under	[-,	a under	End]		
completed waiting		vaiting times in queue				B(t)				
times in queue	1.17			3.12	2	4.4	I			
	4 —									
Q(t) graph	3 - 2 -	<b>,</b>								
	1 - 0		ī		,	1				
	0	;	5		10	15		20		
B(t) graph	2 1 0	••••								
	0	!	5		10	15		20		
	Time (Minutes)									
Interarrival times	1,75, 1,25, 0,71, 0,82, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00,									
Service times	280, 17	2.80, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38,								

### Simulation by Hand: t = 4.66, Departure of Part 2

System	Clock	B(t)	B(t) $Q(t)$		Arrival times of		nt calenda 8.05,	
5 4 3	4.66	1	2		custs. in queue (4.41, 3.79)	[3, [6, [–,	18.69, 20.00,	Dep] Arr] End]
Number of completed waiting	Total of waiting ti	mes in que	eue	Area Q(t)	a under	Area B(t)	a under	
times in queue 3	2.75			3.87	,	4.66	5	
Q(t) graph	4 3 - 2 - 1 - 0							
B(t) graph	0 2 1		5		10	15		20
	0		5		10	15	i	20
	Time (Minutes)							
Interarrival times	1,78, 1,35, 0,71, 0,62, 14,28, 0.70, 15.52, 3.15, 1.76, 1.00,							
Service times	2,80, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38,							

### Simulation by Hand: t = 8.05, Departure of Part 3

System 5	Clock 8.05	B(t)	Q(t)		Arrival times of custs. in queue (4.41)	[4,	ent calenda 12.57, 18.69, 20.00,	Dep] Arr] End]
Number of completed waiting times in queue 4	Total of waiting ti	mes in que	eue	Area Q(t)	a under	<del>-</del> -	a under	
Q(t) graph	4 3 - 2 - 1 - 0							
B(t) graph	0 2 1 0 0	••••	5		10	15		20
	Time (Minutes)							
Interarrival times	1,78, 1,35, 0,71, 0,82, 14,28, 0.70, 15.52, 3.15, 1.76, 1.00,							
Service times	2.80, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38,							

#### Simulation by Hand: t = 12.57, Departure of Part 4

System	Clock	B(t) $Q(t)$			Arrival times of custs. in queue		Event calenda [5, 17.03,	ar Dep]	
5	12.57	1	0		•	()	[6, 18.69, [–, 20.00,	Arr] End]	
Number of	Total of	mas in aug	2110		a under		Area under		
completed waiting times in queue	waiting ti	mes in que	eue	Q(t)			B(t)		
5	15.17			15.1	7		12.57		
	4 —								
Q(t) graph	3 -	**							
a(t) graph	2 -								
	0		<del>.</del> 5		10		15	20	
D(A)	2 —		<b>.</b>		10				
B(t) graph	1 0	00 0 00	ı		,		ı		
	0		5		10		15	20	
	Time (Minutes)								
Interarrival times	1,75, 1,35, 0,71, 0,62, 14,28, 0.70, 15.52, 3.15, 1.76, 1.00,								
Service times	2,80, 1,76, 3,39, 4,52, 4,46, 4.36, 2.07, 3.36, 2.37, 5.38,								

# Simulation by Hand: t = 17.03, Departure of Part 5

System	Clock 17.03	B(t) 0	Q(t) 0		Arrival times of custs. in queue ()	Eve [6, [-,	nt calenda 18.69, 20.00,	ar Arr] End]	
Number of completed waiting times in queue	Total of waiting ti	mes in que	eue	Area Q(t)	a under	Are B(t)	a under		
5	15.17	15.17			7	17.0	)3		
Q(t) graph	4 3 - 2 - 1 - 0 0	3 - 2 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1						20	
B(t) graph	2 1 0 0	••••	5		10	15		20	
	Time (Minutes)								
Interarrival times	1,73, 1,25, 0,71, 0,82, 14,28, 0.70, 15.52, 3.15, 1.76, 1.00,								
Service times	2,00, 1,76, 3,29, 4,52, 4,46, 4.36, 2.07, 3.36, 2.37, 5.38,								

#### Simulation by Hand: t = 18.69, Arrival of Part 6

System	Clock	B(t)	Q(t)		Arrival times of	ival times of Event ca							
					custs. in queue	[7,	19.39,	Arr]					
6	18.69	1	0		()	[-,	20.00,	End]					
						[6,	23.05,	Dep]					
Number of	Total of			Area	a under	Are	a under						
completed waiting times in queue	waiting ti	times in queue				B(t)							
6	15.17			15.1	7	17.0	03						
	4 —					<u> </u>							
Q(t) graph	3 - 2 -				•								
	1 - 0						•						
	0	:	5		10	15		20					
B(t) graph	2	••••			•		•	•					
	0		<del> </del>		10	15	:	20					
		,	J			15	,	20					
	1 70 1 0		A 4	ı af	Time (Minutes)	70 1							
Interarrival times	1,13, 1,2	5, 0, 1, 0,	52, 14	<b>2</b> 8,	0. <b>1</b> 0, 15.52, 3.15, 1	./6, 1	.00,						
Service times	280, 17	$6, \overline{3,39,4}$	<del>2</del> , 4	46, 4	<b>26</b> , 2.07, 3.36, 2.37	7, 5.38	2.80, 1.76, 3.39, 4.52, 4.46, 4.86, 2.07, 3.36, 2.37, 5.38,						

#### Simulation by Hand: t = 19.39, Arrival of Part 7

System 7 6	Clock 19.39	B(t)	Q(t) 1		Arrival times of custs. in queue (19.39)	[–, [6,	nt calenda 20.00, 23.05,	End] Dep]	
Number of completed waiting times in queue 6	Total of waiting til	aiting times in queue			Area under <i>Q(t)</i> 15.17		[8, 34.91, Arr]  Area under  B(t)  17.73		
Q(t) graph	4 3 - 2 - 1 - 0								
B(t) graph	0 2 1 0 0	•••••			10	15	1	20	
Interarrival times	Time (Minutes) 1,73, 1,25, 0,71, 0,82, 14,28, 0,10, 15,52, 3,15, 1,76, 1,00,								
Service times		· • • · •			<b>3</b> 6, 2.07, 3.36, 2.37,				

## Simulation by Hand: t = 20.00, The End

System	Clock	B(t)			Arrival times of custs. in queue	Event calendar [6, 23.05, Dep]		ar Dep]		
76	20.00	1	1		(19.39)	_	34.91,	Arr]		
Number of completed waiting times in queue	Total of waiting times in queue			Area under Q(t)		Area under <i>B(t)</i>				
6	15.17			15.7	8	18.34				
Q(t) graph	4 3 - 2 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1									
B(t) graph	0 2 1 0 0	•••••	5		10	15		20		
	Time (Minutes)									
Interarrival times	1,73, 1,25, 0,71, 0,82, 14.28, 0.10, 15.52, 3.15, 1.76, 1.00,									
Service times	2.80, 1.76, 3.89, 4.52, 4.46, 4.86, 2.07, 3.36, 2.37, 5.38,									

### Simulation by Hand: Finishing Up

• Average waiting time in queue:

Total of times in queue No. of times in queue 
$$=\frac{15.17}{6}=2.53$$
 minutes per part

• Time-average number in queue:

$$\frac{\text{Area under }Q(t)\text{ curve}}{\text{Final clock value}} = \frac{15.78}{20} = 0.79 \text{ part}$$

• Utilization of drill press:

Area under 
$$B(t)$$
 curve Final clock value =  $\frac{18.34}{20}$  = 0.92 (dimensionless)