CS 325 - Homework 4

1. Which of the following can we infer from the fact that the traveling salesperson problem is NP-complete, if we assume that P is not equal to NP? Explain

   a. There does not exist an algorithm that solves arbitrary instances of the TSP problem.
   b. There does not exist an algorithm that solves an arbitrary instance of the TSP problem in polynomial time.
   c. There exists an algorithm that solves arbitrary instances of the TSP problem in polynomial time, but no one has been able to find it.
   d. The TSP is not in P.
   e. All algorithms that are guaranteed to solve the TSP run in polynomial time for some of the inputs.
   f. All algorithms that are guaranteed to solve the TSP run in exponential time for all of the inputs.

2. Let X and Y be two decision problems. Suppose we know that X reduces to Y. Which of the following can we infer? Explain

   a. If Y is NP-complete then so is X.
   b. If X is NP-complete then so is Y.
   c. If Y is NP-complete and X is in NP then X is NP-complete.
   d. If X is NP-complete and Y is in NP then Y is NP-complete.
   e. X and Y can't both be NP-complete.
   f. If X is in P, then Y is in P.
   g. If Y is in P, then X is in P.

3. A Hamiltonian path in a graph is a simple path that visits every vertex exactly once. Show that HAM-PATH = { (G, u, v): there is a Hamiltonian path from u to v in G} is NP-complete. You may use the fact that HAM-CYCLE is NP-complete

4. LONG-PATH is the problem of, given (G, u, v, k) where G is a graph, u and v vertices and k an integer, determining if there is a simple path in G from u to v of length at least k. Show that LONG-PATH is NP-complete.

5. Graph Coloring. Mapmakers try to use as few colors as possible when coloring countries on a map, as long as no two countries that share a border have the same color. We can model this problem with an undirected graph G = (V,E) in which each vertex represents a country and vertices whose respective countries share a border are adjacent. The a k-coloring is a function c: V -> {1, 2, … , k} such that $c(u) \neq c(v)$ for every edge $(u,v) \in E$. In other words the number 1, 2, .., k represent the k colors and adjacent vertices must have different colors. The graph coloring problem is to determine the minimum number of colors needed to color a given graph.

a. Give an efficient algorithm to determine a 2-coloring of a graph, if one exists.
b. Cast the graph coloring problem as a decision problem K-COLOR. Show that your decision problem is solvable in polynomial time if and only of the graph-coloring problem is solvable in polynomial time.
c. Show that K-COLOR is in NP. That is if you are given the coloring function c, you can verify that the graph is K-colorable.

It has been shown that 3-COLOR is NP-complete by using a reduction from SAT. SAT is NP-complete and there exists a polynomial time algorithm that maps satisfiable formulas to 3-colorable graphs and non-satisfiable formulas to non-3-colorable graphs.

d. Use the fact that 3-COLOR is NP-complete to show that 4-COLOR is NP-complete.