Sampling Methods

- Sampling refers to how observations are “selected” from a probability distribution when the simulation is run.

Sampling Methods

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Sampling Methods

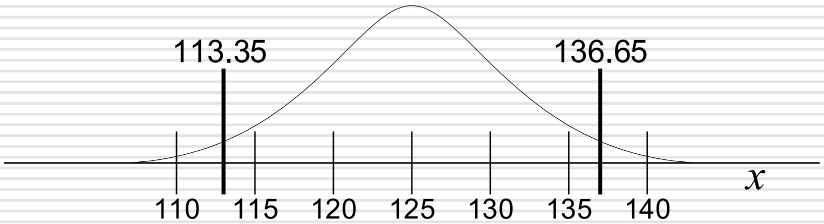
- Pure random sampling.
  - The quantity of interest is a function of N random variables $X_1, \ldots, X_N$. That is we are interested in the function

$$g(\mathbf{X}) \text{ where } \mathbf{X} = (X_1, X_2, \ldots, X_N)$$

  - The random variables $X_1, \ldots, X_N$ follow some joint distribution F.

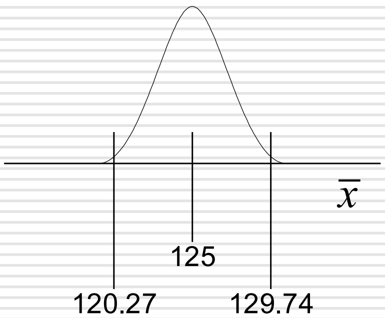
Sampling Methods

- Random sampling generates an observation "randomly" from F .
  - What observations are more likely?



$$r.v.\ \ x \sim N(\mu = 125, \sigma = 5)$$

- Depending on the number of trials you may or may not observe values in the "tails".



$$r.v.\ \ \bar{x} \sim N(\mu = 125, \sigma_{\bar{x}} = 5/\sqrt{10} = 1.58)$$

Latin Hypercube Sampling

Latin Hypercube Sampling

- The range of each random variable $X_1, \ldots, X_N$ is divided up into n equal probability non-overlapping intervals.
- E.g., normal, uniform, exponential.

Latin Hypercube Sampling

- Generate an observation from each interval using the conditional distribution.
- Example – Uniform.
- Do this for all $X_1, \ldots, X_N$.

Latin Hypercube Sampling

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Latin Hypercube Sampling

- One value from each of the n observations are randomly matched to form a realization of

$$\mathbf{X} = (X_1, X_2, \ldots, X_N)$$

- Example with 2 random variables (n = 5).

		X2				
		1	2	3	4	5
X1	1			X		
	2	X				
	3					X
	4		X			
	5				X	

Latin Hypercube Sampling

- Crystal Ball demo.

Sampling Methods

- Random sampling will always work and may give you a better idea of the variability you may observe.
- Latin hypercube sampling should give better estimates of mean values (less variance).

Monte Carlo Simulation Applications

- The evaluation of probability modeling problems

Probability Modeling

1. Containers of boxes are delivered to the receiving area of retail business and the boxes must be placed in a temporary storage facility until they can be moved to store shelves. There is one delivery every two days. Each container in a delivery contains the same number of boxes, which are taken out of the container and stored on the floor. A box requires 4 sq. ft. of storage space and can be stacked no more than two-high. The number of boxes in a container (the same for all containers in a delivery) follows a discrete uniform distribution with minimum = 8, and maximum = 16. The number of containers in a delivery has a Poisson distribution with a mean = 5.
   - What is the expected value and variance of the storage space required for a delivery? For a Poisson random variable X, E[X] = Var[X]. Clearly state any assumptions you make.

Probability Modeling

2. *p* denotes the probability that an inspected part in a lot of parts is defective and is independent of the other parts. A lot of parts contains 100 parts and an inspector inspects every part in the lot. It takes *T* time units to inspect a single part and $T \sim$ Uniform[a,b]. If a defective part is discovered an additional *R* time units is required to prepare the defective to be returned and $R \sim$ Uniform[c,d]. What is the expected value and variance of the time required to complete the inspection of a lot?

Developing Monte Carlo Simulations

- A certain amount of “art” or creativity within the constraints of the software being used is required.
- Crystal Ball/Excel examples
  - Integration
  - Generating points distributed uniformly in a circle
  - Stochastic Project Network

Integration

- Developed by Manhattan Project scientists near the end of WWII.
- A-Bomb development.
- Will consider a simple example.
  - Applied to more complex integration where other numerical methods do not work as well.

Integration

Integration

Integration

- To estimate I use Monte Carlo simulation
  -

Crystal Ball Example

$$I = \int_0^{\pi} \sin x dx = 2$$

Generating Points Uniformly in a Circle

- HW #2 Consider the *x*-*y* plane and a circle of radius = 1, centered at *x=2, y=2*. An algorithm for generating random points within this circle is as follows:

$$
\begin{aligned}
&1.\ \text{Generate a random angle } \theta \text{ that is uniformly distributed between } -\pi, \text{ and } +\pi.\\
&2.\ \text{Generate a random distance } r \text{ from the center of the circle where } r \sim U(0,1).\\
&3.\ \text{Compute the coordinates of the point}\\
&\qquad x = 2 + r * \cos(\theta),\\
&\qquad y = 2 + r * \sin(\theta).
\end{aligned}
$$

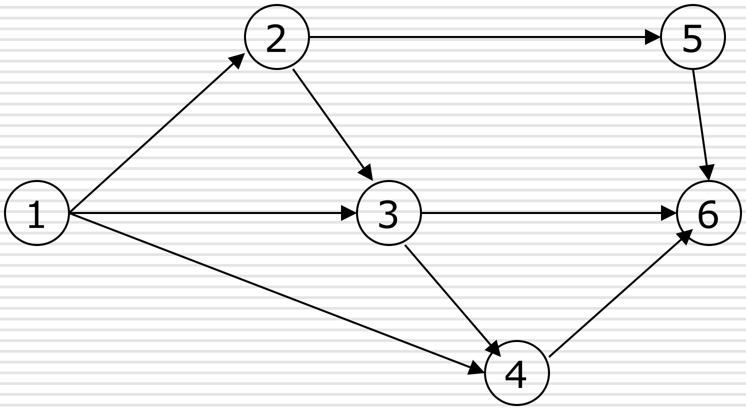
- This does not work.

In-Class Exercise

- Devise a general approach to generate points uniformly distributed in the circle.
  - Hint – Generate points uniformly in a square first.

Stochastic Project Network

- A project network is used to depict the various milestones in a project, the activities needed to achieve the milestones, and the precedence relationships between milestones.



Stochastic Project Network

Stochastic Project Network

- A general n-node simulation model can be developed in Excel.
- Need a general method to represent arbitrary n-node networks.
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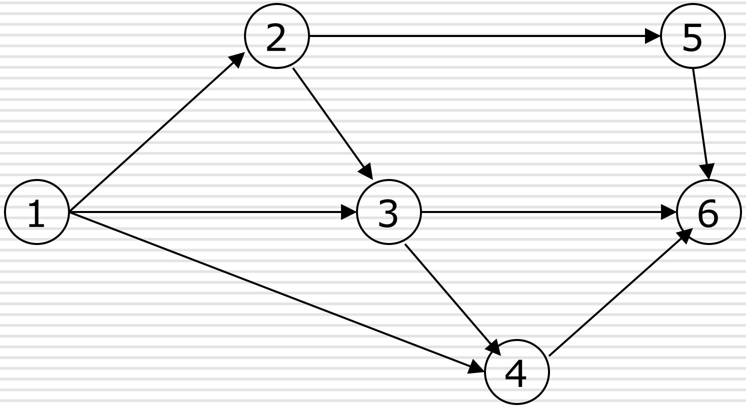
Stochastic Project Network

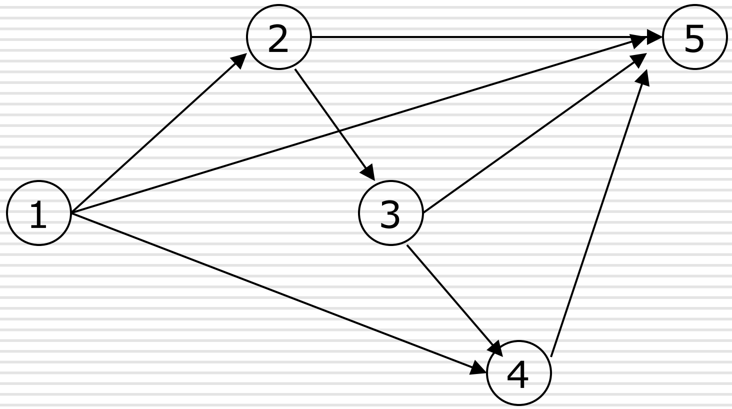


Node-Arc Incidence Matrix

	Arc								
Node	**1-2**	**1-3**	**1-4**	**2-3**	**2-5**	**3-4**	**3-6**	**4-6**	**5-6**
1	1	1	1	0	0	0	0	0	0
2	-1	0	0	1	1	0	0	0	0
3	0	-1	0	-1	0	1	1	0	0
4	0	0	-1	0	0	-1	0	1	0
5	0	0	0	0	-1	0	0	0	1
6	0	0	0	0	0	0	-1	-1	-1

In-class Exercise

- Generate the node-arc incidence matrix for the following network.



In-class Exercise

Stochastic Project Network -Demo

Node-Arc Incidence Matrix

Node	Arc 1-2	1-3	1-4	2-3	2-5	3-4	3-6	4-6	5-6
1	1	1	1	0	0	0	0	0	0
2	-1	0	0	1	1	0	0	0	0
3	0	-1	0	-1	0	1	1	0	0
4	0	0	-1	0	0	-1	0	1	0
5	0	0	0	0	-1	0	0	0	1
6	0	0	0	0	0	0	-1	-1	-1

Node	Arc 1-2	1-3	1-4	2-3	2-5	3-4	3-6	4-6	5-6	Time node/milestone achieved
1	1	1	1	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	1
3	0	2	0	5	0	0	0	0	0	5
4	0	0	3	0	0	11	0	0	0	11
5	0	0	0	0	6	0	0	0	0	6
6	0	0	0	0	0	0	12	19	15	19
Length	1	2	3	4	5	6	7	8	9	
Mean	5	3	2	6	7	11	7	9	10	
Std Dev.	5	3	2	6	7	11	7	9	10	
CV	1	1	1	1	1	1	1	1	1	