Just Finished Event			Variables			Attributes: Arrival Times		Statistical Accumulators										Event Calendar
Entity #	Time t	Event Type	Q(t)	I(t)	B(t)	(In Queue)	In Sevice	P	N	∑WQ	WQ*	∑TS	TS*	∫I	∫Q	Q*	∫B	[Ent. #, Time, Type]
3	8.05	Dep	1	1	1	4.41	3.79	3	4	7.01	4.26	10.8	4.97	8.05	10.65	3	8.05	[4, 12.57, Dep] [6, 18.0, Arr] **[-, 18.5, Failure]** [- , 20.0, End]
4	12.57	Dep	0	1	1		4.41	4	5	15.17	8.16	19.58	8.78	12.57	15.17	3	12.57	[5, 17.03, Dep] [6, 18.0, Arr] **[-, 18.5, Failure]** [- , 20.0, End]
5	17.03	Dep	0	1	0			5	5	15.17	8.16	32.2	12.62	17.03	15.17	3	17.03	[6, 18.0, Arr] **[-, 18.5, Failure]** [- , 20.0, End]
6	18	Arr		1														[7, , Arr] [6, 20.77, Dep] [-, 18.5, Fail [-, 20.00 end
-	18.5	Failure		0														

Next Random Values:

Service Times	Arrival Times	Repair Times
Job 6: 2.77	Job 7: 20.1	2.1

Name: ____________________

Problem IV. Manual Simulation (24 pts.)

IE 415 ONLY

The table on the next page (page 10) shows a portion of the manual simulation table that is similar to Model 03-01 as executed in class except now the drill press may randomly fail while processing. Two additional events can occur – drill press failure, and drill press repair. The drill press can go down only if it is operating. It will not go down if it is up but idle so the simulation must take this into account. Once down the repair time is random and the job in the drill press is pre-empted. After the repair is completed the job continues processing from the point it was pre-empted. Complete the manual simulation for the next two events. Random times needed to complete these events are shown below the table. Make sure all events on the event list are shown even if the scheduled events occur after the "End" event.

The additional variable I indicates drill press status. $I = 1$ if the press is up, $I = 0$ if the press is down. $\int I$ is the area under the drill press status curve.

IE 515 ONLY

a. (20 pts.) The IE 415 problem above.
b. (4 pts.) Assume job arrivals to the drill press occur at a rate of 50 jobs per hour, and the drill press processes jobs at a rate of 60 jobs per hour when it is up and busy. If the drill press is up 90% of the time when it holds a job what is the long-run utilization of the drill press (i.e., the fraction of time it is holding a job)?

Name: ____________________

Problem III. Arena (2 pts. each)

a.) Describe what a Create module has been used for in the Arena models examined. List two parameters of the Create module that can be changed to model different situations.

generates a stream of arrivals into the system
• name • entity type • time between arrivals
• entities per arrival • units(time)

b.) Describe what a Process module has been used for in the Arena models examined. List two parameters of the Process module that can be changed to model different situations.

"Processes" or serves entities within the system
• name • type • action • priority
• delay type • resources

c.) Describe what an Assign module has been used for in the Arena models examined.

used by entities to assign values to attributes

Name: ____________________

Problem III – Simulation Concepts (16 pts. Total – 2pts. each).

a. In a discrete event simulation implementation the Event Calander indicates the sequence of events and the time that they occur.

b. Briefly (10 words or less) describe what an observer of the simulation clock would see with respect to how simulated time advances.

Time will advance at the rate of when events occure

→ not uniform

c. Briefly describe two uses of entity attributes in a simulation model.

parameters that arespecific to an entity

Job type / name

processing time

d. The "modeling view" adopted and used in Arena is called __________________.

e. List two of the recommended steps to complete in a simulation study before simulation model development begins.

- Define the objectives of the study
- write a description of the system
- list assumptions
- Identify Data collection needs

Name: ____________________

Problem II – Monte Carlo Simulation (workspace)

Name: ____________________

Problem II – Monte Carlo Simulation (25 pts. total)

A function of two random variables (*X* and *Y*) needs to be evaluated. *X* is a lognormal random variable with mean = 25 and standard deviation = 5. *Y* is a uniform random variable with minimum = 40 and maximum = 90. The function is

$$Z = \frac{(X-25)^2}{5} + 2Y^2$$

Trial values	Lognormal (Mean = 25, StdDev =5)	Uniform(40,90)	Z	Z^2
1	28.36	62.62	7844.78	61540573.25
2	31.73	82.81	13724.05	188349548.4
3	23.15	52.75	5565.81	30978240.96
4	27.62	77.63	12054.2	145303737.6
5	32.97	87.08	15178.56	230388683.7
			$\Sigma = 54367.4$	656560783.9

i. (10 pts.) Use the five realizations of *X* and *Y* to conduct a Monte Carlo simulation of *Z*. Based on these results estimate *E[Z]* and *Var[Z]*.

IE 415 ONLY

ii. (15 pts.) You were told that *X* and *Y* in the above formula are independent. Estimate the rank correlation between the *X* and *Y* realizations and conduct a statistical test to evaluate the null hypothesis that the rank correlation is zero (statistical tables on page 11). Show all of your work.

IE 515 ONLY

ii. (15 pts.) Find *E[Z]* (not an estimate but the exact value).

Hint: $E[X_1 + X_2] = E[X_1] + E[X_2]$. Apply variance formulas.

$$E[Z] = \frac{1}{n}\sum x_i = \frac{1}{5}(54367.4) = 10873.48$$

$$Var[Z] = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1} = 1634948.84$$

$$r_s = \frac{1}{n-1}\frac{\sum (R-\bar{R})(S-\bar{S})}{S_R S_S}$$

	x	y
1	3	4
2	2	2
3	5	5
4	4	3
5	1	1

Name: ____________________

Problem I - Probability/Statistics Concepts (35 pts. total)

IE 515 ONLY

iv. (10 pts.) A normally distributed random variable has an unknown mean μ and a known variance $\sigma^2 = 9$. Find the sample size required to construct a 95 percent confidence interval on the mean, that has total length of 1.0.

Name: ____________________

Problem I - Probability/Statistics Concepts (35 pts. total)

iii. (10 pts.) $x_1, x_2, \cdots, x_n$ are observations of a normally distributed random variable X with mean μ_X. $y_1, y_2, \cdots, y_n$ are observations of a normally distributed random variable Y with mean μ_Y. In the table below various statistics computed from the observations are shown. Indicate the probability distribution of the statistic in the appropriate column. $\bar{x}$ and $\bar{y}$, and s_X and s_Y are sample averages and sample standard deviations of the x_i and y_i. All parameters of the distribution must be specified.

Statistic	Additional Assumptions	Sampling Distribution of the Statistic	
$\frac{\bar{x}-\mu_X}{s/\sqrt{n}}$	None	normal	t
$\bar{x}$	Variance of X is known and equals σ^2	normal	z
$\frac{(n-1)s^2}{\sigma^2}$	Variance of X is known and equals σ^2	chi-squared	
$\frac{s_X^2/\sigma_X^2}{s_Y^2/\sigma_Y^2}$	Variance of X is known and equals σ_X^2. Variance of Y is known and equals σ_Y^2.	f	
$\frac{\bar{x}-\mu_X}{\sigma/\sqrt{n}}$	Variance of X is known and equals σ^2	Z	

IE 415 ONLY $(X^2)P$

iv. (10 pts.) The random variable X has the following probability mass function.

$$P(X=i)=\begin{cases} .10 & i=2 \\ .10 & i=4 \\ .25 & i=5 \\ .15 & i=10 \\ .40 & i=11 \end{cases}$$

	X	X^2
.4	.2	.04
1.6	.4	.16
6.25	1.25	1.5625
15	1.5	2.25
118.4	4.4	19.36
71.65 ~~71.65~~	7.75	23.37

$23.37 - 7.75^2$ no!

Compute the following (**Show any formulas you use**):

a. E[X] $= \sum x_i p(x) = .1(2) + .1(4) + .25(5) + .15(10) + .40(11) = 7.75$

b. Var[X] $= E(X^2) - [E(X)]^2 = [.1(2)^2 + .1(4^2) + .25(5^2) + .15(10^2) + .4(11^2)] - (7.75)^2 = 11.59$ 71.65

c. Write out the cumulative distribution function $F(i)$.

$$F(x)=\begin{cases} 0 & x<2 \\ .1 & 2\le x<4 \\ .2 & 4\le x<5 \\ .45 & 5\le x<10 \\ .60 & 10\le x<11 \\ 1 & x\ge 11 \end{cases}$$

use $Var[x] = \sum (x_i - \mu)^2 p(x)$

IE 415/515: Winter 2010. Midterm Exam, February 8, 2011

Name: ____________________

Problem I - Probability/Statistics Concepts (35 pts. total)

i. (10 pts.) Values computed from a sample of data (realizations of random variable X) are shown below. Compute the values of the missing quantities (all missing quantities are sample statistics, e.g., Average is the sample average). Show your work for each quantity computed.

$\sum X_i$	$\sum X_i^2$	N	Average	Variance	CV(X)	Estimated values for the random variable $\bar{X}$ assuming X is Normally Distributed		
						Mean	Variance	CV
179.64	3825.5	9	$\frac{179.64}{9}$	30.0	.274	19.96	$\frac{\text{Variance}}{n}$	.0914

$\frac{\sqrt{3.33}}{19.96}$

19.96 $\frac{30.0}{9}$ 3.33 $\frac{\sqrt{\text{Variance}}}{\text{mean}}$

$$\text{Variance} = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1} = \frac{3825.5 - \frac{179.64^2}{9}}{8} = 30.0$$

$$CV(x) = \frac{\text{std Dev}(X)}{\text{mean}(x)} = \frac{\sqrt{30}}{19.96} = .274$$

$$\bar{X} \text{ mean} = \frac{1}{n}\sum x_i = \frac{1}{9}(179.64) = 19.96$$

ii. (5 pts.) The shelf life of a packaged food is of interest. Ten samples are randomly selected and tested, and the following results are obtained:

Days	
108	138
124	163
124	159
106	134
115	139

$\sum x_i = 1310$

$\bar{X} = \frac{\sum \text{all}}{10} = \frac{1310}{10} = 131$

$\sum x_i^2 = 175048$

$s^2 = \frac{\sum x_i^2 - \left(\frac{(\sum x_i)^2}{n}\right)}{n-1} = 382$

Assuming that shelf life is normally distributed, compute a 95% confidence interval for the mean shelf life (statistical tables on page 11). Show your work.

$H_0: \mu = 131$

$H_1: \mu \neq 131$

$t_{.025,10-1} = 2.262$

$\left(\bar{X} - t\frac{s}{\sqrt{n}}, \bar{X} + t\frac{s}{\sqrt{n}}\right)$

13.98

$131 - 2.262\frac{\sqrt{382}}{\sqrt{10}}, 131 + 2.262\frac{\sqrt{382}}{\sqrt{10}}$

$(117.02, 144.98)$ contains 131

$\Rightarrow$ would not reject H_0

IE 415/515 - Simulation

Midterm Exam, February 8, 2011

Name: ____________________

	Possible	Score
I. Probability/Statistics	35	______
II. Monte Carlo Simulation	25	______
III. Simulation Concepts & Arena	16	______
IV. Manual Simulation	24	______
Total possible	100	

Work that is hard to read and/or poorly organized on the space provided will be marked down.

IE 415: Winter 2012. Midterm Exam, February 8, 2012

Name: Tasha Larson

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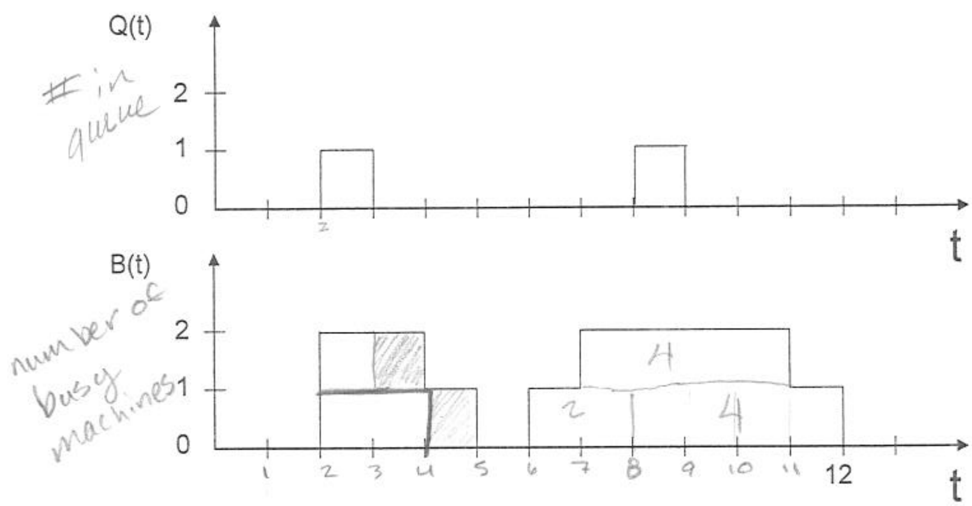
Problem IV. Discrete Event Simulation (25 pts.)

i) Consider the simple example system we used in class (Model 03-01). Suppose (as in a HW problem) that there are now two machines. The graphs below show how the number of busy machines ($B(t)$) and the number in queue ($Q(t)$) changes over time. In this system, it is possible for more than one part to arrive at the same time, but job completions do not occur in the time units when arrivals occur. (Each time mark is one time unit and the t axis starts at zero).

a) (3 pts.) Determine the time when each part arrived.
b) (3 Pts.) Determine the times when parts completed service.
c) (6 pts.) Compute the average number of machines busy, the average number in queue, and the average number in system from time 0 to time 12.
d) (3 pts.) Compute the average time in system for the parts processed by time 12.



#Arrive	Time
3	2
1	6
1	7
1	8

Depart	Time
1	3
1	4
1	5
1	9
1	11
1	12

15/6 2.5

Avg # machines busy = $\cancel{\frac{12 \cdot 2 - 5}{12} = 1.58}$ machines ? −1

Avg # queue = $\frac{2}{12}$ = .166 parts in queue

Avg # in system = $\frac{6 \text{ parts}}{12 \text{ time}}$ = .5 parts ~~in system~~ −2

−17

Avg time in system = $\frac{1+2+2+2+4+4}{6 \text{ parts}} = \frac{15}{6} = 2.5$ time −1

IE 415: Winter 2012. Midterm Exam, February 8, 2012

Name: Tasha Larson

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Problem III – Monte Carlo Simulation and Arena (25 pts.).

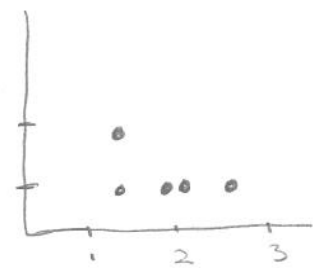
Monte Carlo simulation can be used to evaluate definite integrals, and will be applied to evaluate

$$I = \int_1^3 \frac{e^{2x}}{4 + e^{2x}}$$

The following table of numbers will be used in the simulation.

X-values

Trial	Value
1	1.9
2	2.0
3	1.4
4	2.6
5	1.4



i. (5 pts.) From what distribution are the values in the table obtained?

exsponential

ii. (10 pts.) What is the estimate of I produced by the simulation (show your work)?

9

$$y = (b-a)\,g(x) = (3-1)\frac{e^{2x}}{4+e^{2x}}$$

	X	$g(x)$	y
1	1.9	.918	1.962
2	2.0	.25	.5
3	1.4	.804	1.608
4	2.6	.978	1.956
5	1.4	.804	1.608

Σ 7.634

$$E[y] = I = \frac{1}{n}\Sigma y = \frac{1}{5} \cdot 7.634$$

$$I = 1.53$$

IE 415: Winter 2012. Midterm Exam, February 8, 2012

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Name: Tasha Larson

Problem II – Monte Carlo Simulation (25 pts. total)

A function of two random variables (X and Y) needs to be evaluated. X is a lognormal random variable with mean = 25 and standard deviation = 5. Y is a uniform random variable with minimum = 40 and maximum = 90. The function is

$$Z = \frac{5Y}{\ln(X)} + \ln(Y)$$

Trial values	Lognormal (Mean = 25, StdDev =5) X	Uniform(40,90) y	Z
1	28.4	77.6	120.3
2	27.6	82.8	129.2
3	23.2	87.1	143.0
4	31.7	62.6	94.7
5	33	52.8	79.5
			Σ 566.7

i. (10 pts.) Use the five realizations of X and Y to conduct a Monte Carlo simulation of Z. Based on these results estimate $E[Z]$.

$E[Z] = \frac{1}{n} \sum z_i = \frac{1}{5}(566.7) = 113.34$

ii. (5 pts.) What is the sample rank correlation of the X and Y values in the table?

iii. (10 pts.) Assuming the same X and Y values in the table above were generated for a Monte Carlo simulation but now with sample rank correlation of 1, estimate $E[Z]$.

ranks

	x	y	$R_i - S_i$	$(R_i-S_i)^2$
1	3	3	0	0
2	4	2	2	4
3	5	1	4	16
4	2	4	2	4
5	1	5	4	16
				$\Sigma = 40$

$D = \sum (R_i - S_i)^2 = 40$

$r = 1 - \frac{6D}{N^2 - N} = 1 - \frac{6 \cdot 40}{5^3 - 5}$

$r = 1 - 2 = -1$

$E[Z] = \frac{1}{n} \sum z_i = \frac{1}{5}(566.7)$

$E[Z] = 113.34$

IE 415: Winter 2012. Midterm Exam, February 14, 2012

Name: Tasha Larson

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Problem I - Probability/Statistics Concepts (25 pts. total)

A discrete random variable X is characterized by the function $p(i)$ shown below.

$$p(i) = \begin{cases} .3 & \text{for } i = -2 \\ .1 & \text{for } i = 0 \\ .3 & \text{for } i = 2 \\ .2 & \text{for } i = 3 \\ x\ .1 & \text{for } i = 6 \end{cases}$$

$x \cdot p(x)$	$x^2 p_i$
$-.6$	$+1.2$
0	0
$.6$	1.2
$.6$	1.8
$.6$	3.6
$\Sigma\ 1.2$	$\Sigma = 5.4$

7-8

i. (3 pts.) What is the name for the type of function that $p(i)$ is an example?

probability mass function

ii. (2 pts.) What is the value of x?

$1 - .3 - .1 - .3 - .2 = \boxed{x = .1}$

iii. (5 pts.) Find E[X] and Var[X].

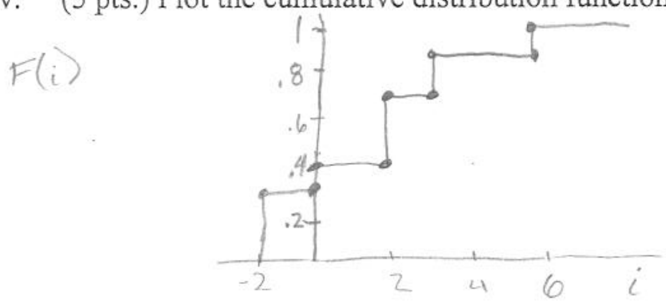
$E[x] = \Sigma\, x_i \cdot p(x_i) = 1.2 \quad Var[X] = E[x^2] - (E[x])^2 = 5.4 - 1.2^2 = 3.96$

-1

iv. (3 pts.) Find the coefficient of variation of X.

$CV = \dfrac{\text{std dev}(x)}{\text{mean}(x)} = \dfrac{\sqrt{3.96}}{1.2} = 1.65$ OK

v. (5 pts.) Plot the cumulative distribution function $F(i)$.



$$F(i) = \begin{cases} .3 & i < -2 \\ .4 & -2 \le i < 0 \\ .7 & 0 \le i \le 2 \\ .9 & 2 \le i \le 3 \\ 1 & i > 6 \end{cases}$$

vi. (7 pts.) A discrete random variable Y is characterized by the function $p_Y(i)$ shown below.

$$p_Y(i) = \begin{cases} .3 & \text{for } i = -2 \\ .1 & \text{for } i = 0 \\ x & \text{for } i = 2 \\ .2 & \text{for } i = 3 \\ z & \text{for } i = 6 \end{cases}$$

$1 = .3 + .1 + x + .2 + z$

$x + z = .4$

$.9 - 3z + z = .4$

$-2z = -.5$

$\boxed{z = .25}$

$x = .4 - .25$

$\boxed{x = .15}$

If E[Y] = 1.8, what are the values of x and z?

$E[y] = .3(-2) + .1(0) + x \cdot 2 + .2(3) + z \cdot 6$

$1.8 = -.6 + 2x + .6 + 6z$

$1.8 = 2x + 6z \,/\, 2 \Rightarrow .9 = x + 3z$

$\boxed{x = .9 - 3z}$

IE 415 - Simulation

Midterm Exam, February 14, 2012

Name: Tasha Larson

		Possible	Score
I. Probability		25	24
II. Monte Carlo Simulation		25	15
III. Monte Carlo Simulation & Arena		25	16
IV. Discrete Event Simulation		25	17
	Total possible	100	72

Work that is hard to read and/or poorly organized on the space provided will be marked down.