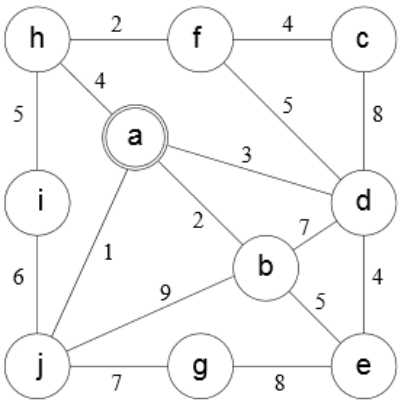
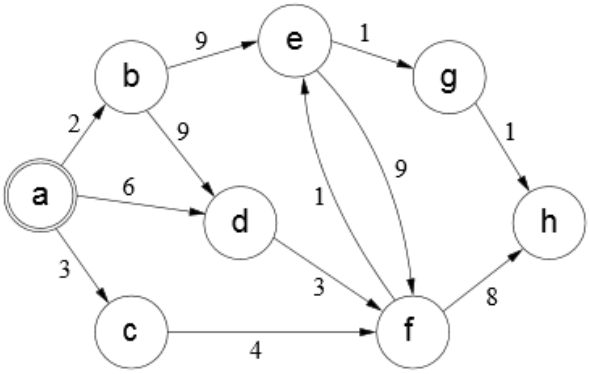
CS 325 - Homework Assignment 3

In questions 1 and 2 you are given a graph and need to show how each algorithm works on the graphs. Write down every change in any of the values that each algorithm maintains. The graphs start from the root vertex a.

1. Demonstrate Prim's algorithm on the graph below. Show your steps.



2. Demonstrate Dijkstra's algorithm on the graph below. Show your steps.



3. Consider an undirected graph G=(V,E) with nonnegative edge weights $w(u,v) \geq 0$. Suppose that you have computed a minimum spanning tree G, and that you have also computed shortest paths to all vertices from vertex $s \in V$.

Now suppose each edge weight is increased by 1: the new weights $w'(u,v) = w(u,v) + 1$.

(a) Does the minimum spanning tree change? Give an example it changes or prove it cannot change.

(b) Do the shortest paths change? Give an example where they change or prove they cannot change.

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4. For the following problem, use the directed unweighted graph given by the following adjacency list. Be sure to consider edges in the given order.

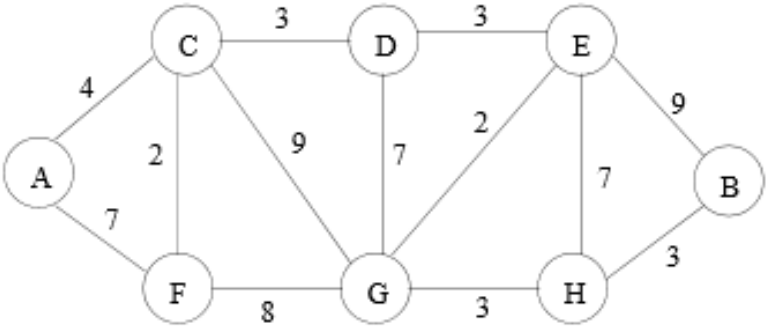
A: C E B
B: E D
C: E
D: C F E
E: F
F:

(a) What is the order in which the vertices are visited by DFS (depth first search)? For each vertex give the discovery and finishing time.

(b) Suppose that this graph represents courses in a CS department with course A being a prerequisite for courses C, E and B; course B a prerequisite for E and D, etc. Using your work above either give a valid order to take the courses or prove there is no valid order.

5. A Hamiltonian path in a graph G=(V,E) is a simple path that includes every vertex in V. Design an algorithm to determine if a directed acyclic graph (DAG) G has a Hamiltonian path. Your algorithm should run in O(V+E). Provide a written description of your algorithm including why it works, pseudocode and an explanation of the running time.

6. A region contains a number of towns connected by roads. Each road is labeled by the average number of minutes required for a fire engine to travel to it. Each intersection is labeled with a circle. Suppose that you work for a city that has decided to place a fire station at location G. (While this problem is small, you want to devise a method to solve much larger problems).



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(a) What algorithm would you recommend be used to find the fastest route from the fire station to each of the intersections? Demonstrate how it would work on the example above if the fire station is placed at G. Show the resulting routes.

(b) Suppose one ”optimal” location (maybe instead of G) must be selected for the fire station such that it minimizes the distance to the farthest intersection. Devise an algorithm to solve this problem given an arbitrary road map. Analyze the time complexity of your algorithm when there are f possible locations for the fire station (which must be at one of the intersections) and r possible roads.

(c) In the above graph what is the “optimal” location to place the fire station? Why?

EXTRA CREDIT: Now suppose you can build two fire stations. Where would you place them to minimize the farthest distance from an intersection to one of the fire stations? Devise an algorithm to solve this problem given an arbitrary road map. Analyze the time complexity of your algorithm when there are f possible locations for the fire station (which must be at one of the intersections) and r possible roads.