Monte Carlo Simulation

Monte Carlo Simulation

- Simulations where random values are used but the explicit passage of time is not modeled
  - Static simulation
- Introduction
  - Simulation of the maximum value when rolling two fair die

Monte Carlo Simulation

- IE 425 – Function optimization
  - Simulated annealing
  - Genetic algorithms
- IE 415/515
  - Engineering economic analysis
  - Probability models
  - Integration
  - Project network simulation

Monte Carlo Simulation

- Lab 1 – use Monte Carlo simulation to estimate the true confidence level of confidence intervals.

History

- Developed by Manhattan Project scientists near the end of WWII.
- Monte Carlo, Monaco has been associated with casino gambling, which is based on randomization procedures and games of chance.
- Since the new simulation techniques relied on randomization procedures it was given the name “Monte Carlo” simulation.

Monte Carlo Simulation

- Inputs to an "analysis" are unpredictable
- Outputs of the analysis are

Monte Carlo Simulation

- How do we represent unpredictability?

Monte Carlo Simulation

Diagram

Assumptions

- Assume that we have methods for generating observations from different probability distributions.
  - How this is accomplished will be covered later.

Summarizing/Characterizing Risk in Engineering Economic Calculations

- Much of the cash flow data used in engineering economic analysis are "best estimates".
- In reality, we do not know what the actual cash flows will be.

Terminology

- Risk –
- Uncertainty –
-

Quick Review of NPV

- NPV – Net Present Value
  - Applicable to a series of cash flows over time.
  - Computes the value of all cash flows today (the present).
- We will only consider years as time periods with given annual interest rates.

Quick Review of NPV

Quick Review of NPV

Quick Review of NPV

In-Class Exercise

- Two alternative heating systems are being considered, gas and electric, for a temporary building to be used for 5 years. The gas system will cost $6K to install (at year 0). It is estimated that this system will have a salvage value of $500 after 5 years, and will have annual fuel and maintenance costs of $1K. The electric system will cost $8K to install and has an estimated 5 year salvage value of $1.5K. The estimated annual fuel and maintenance costs are $750 per year. The assumed MARR (interest rate) = 6%.

Example

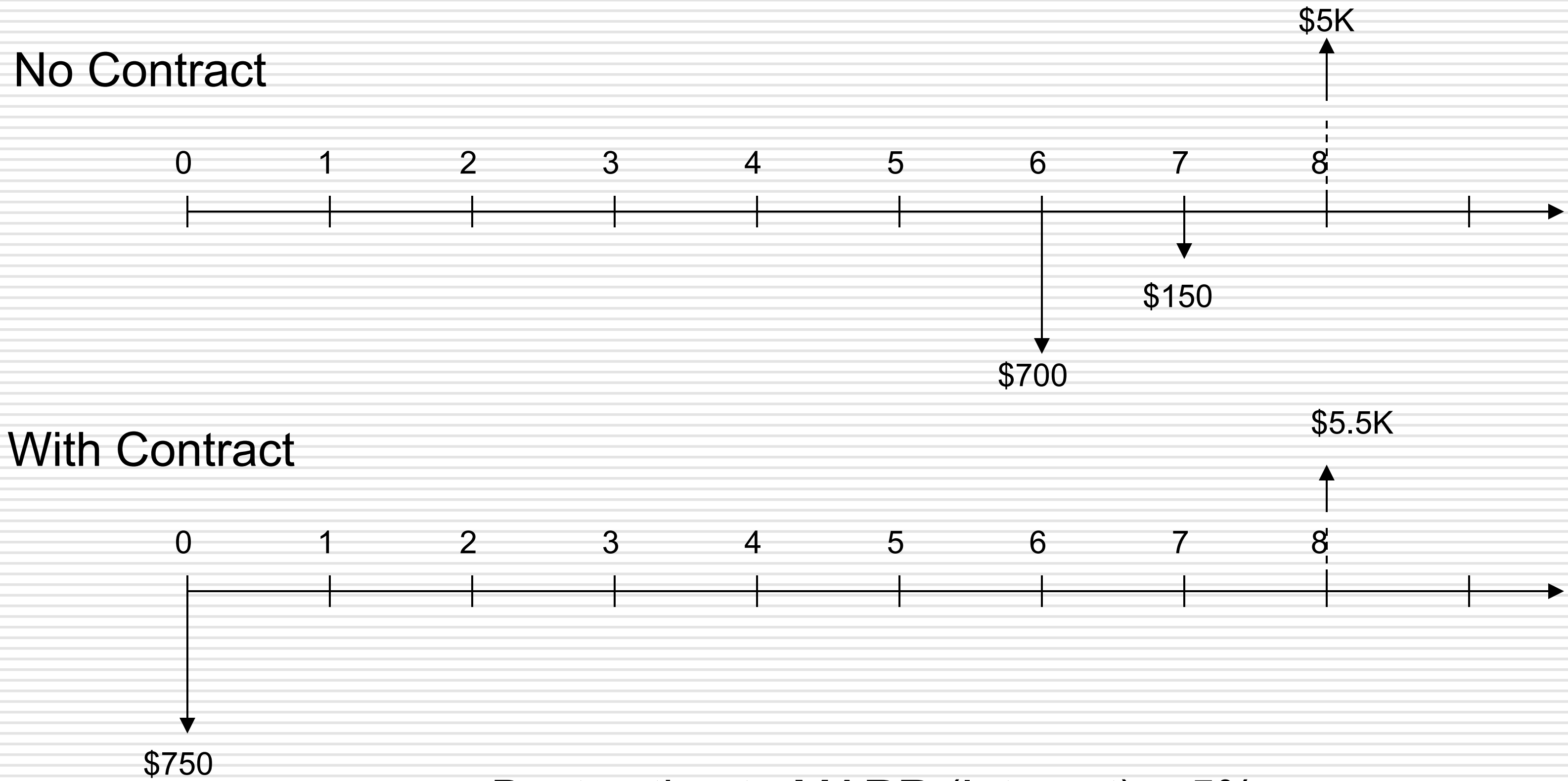
- Should you purchase a service contract with a new vehicle?
  - Your plan is to keep the vehicle for 8 years.
  - The service contract begins after the warranty expires (5 yrs.) and it lasts for 5 years. It covers the same items as the manufacturers warranty.
  - The cost is $750 at the time of purchase and it is transferable when the vehicle is sold.

Example

- What cash flows do you need to consider?
- Why only these cash flows?

Example

- Analysis using best estimates of cash flow.



Best estimate MARR (Interest) = 5%

Example

Example

- What do we know and don't know with certainty?
  - Known
    -
  - Unknown

Addressing Risk Using Monte Carlo Simulation

- Basic Approach
  -
  -
- No approach can eliminate risk/ uncertainty.

Addressing Risk Using Monte Carlo Simulation

- Replace best estimates with probability distributions.
- Generate an observation from each distribution and perform the engineering economic calculation – repeat.
- The answer is now in the form of a histogram.

Implementation

- What is required to make implementation practical?

MS Excel Capability

- Data Tab
  - Data Analysis → Random Number Generation
  - Requires Analysis ToolPak installation

Simulation Demonstration

- Excel
  - Year 6 maintenance cost
    - Uniform( )
  - Year 7 maintenance cost
    - Uniform( )
  - Salvage Value
    - Normal( )

Simulation Demonstration

- Excel
- Crystal Ball
  - Automates/expands the capabilities within Excel to conduct Monte Carlo simulations

In-Class Discussion

- Generate ideas for making this simulation “better” (more realistic).
  -

Independent Random Variables

- Two random variables X and Y are independent if

$$P_{X,Y}(X \le x, Y \le y) = P_X(X \le x) * P_Y(Y \le y)$$

$$F_{X,Y}(x,y) = F_X(x) * F_Y(y)$$

  - Continuous random variables

$$f_{X,Y}(x,y) = f_X(x) * f_Y(y)$$

  - Discrete random variables

$$p_{X,Y}(X = x, Y = y) = p_X(X = x) * p_Y(Y = y)$$

Independent Random Variables

- Expected values for independent random variables X and Y.

$$E(X * Y) = E(X) * E(Y)$$

Correlations Between Variables

- Covariance Cov_{XY} is a measure of the dependence between two random variables

$$Cov_{X,Y} = E(X*Y) - E(X)*E(Y)$$

  - Cov_{XY} can be positive or negative.
  - Since Cov_{XY} is not dimensionless, it’s magnitude is relative. <u>Correlation</u> is “normalized” covariance.

$$-1 \le \rho_{XY} = \frac{Cov_{XY}}{\sqrt{\sigma_X^2 \sigma_Y^2}} \le 1$$

Estimates of Correlation

- Pearson's correlation coefficient

$$r = \frac{1}{n-1} \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{s_X s_Y}$$

- A measure of the linear relationship.
  - MS Excel function CORREL(…)

Estimates of Correlation

- Pearson’s correlation coefficient
  - Not a good measure to determine statistical significance of the correlation.

Estimates of Correlation

- Rank correlation – Spearman’s rank-order correlation coefficient.

Estimates of Correlation

- Estimating rank correlation – Spearman’s rank-order correlation coefficient estimated from data.
  - Pearson’s correlation coefficient estimate applied to the ranks.

$$r_S = \frac{1}{n-1} \frac{\sum_{i=1}^{n} (R_i - \bar{R})(S_i - \bar{S})}{s_R s_S}$$

where R_i is the rank of x_i and S_i is the rank of y_i.

Estimates of Correlation

- Another formula for an estimate

$$D = \sum_{i=1}^{N} (R_i - S_i)^2$$

$D =$ Sum of the squared difference in ranks.

With no ties in the ranks of the X or Y

then

$$r_S = 1 - \frac{6D}{N^3 - N}$$

Example

Pearson's Correlation Coefficient
-0.007218

Spearman's Rank-Order Correlation Coefficient
-0.020312

i	X	Y	*Rank X*	*Rank Y*	D_i^2
1	4.399536	9.967498	24	1	529
2	2.444634	7.696615	42	24	324
3	5.488515	5.944395	16	46	900
4	7.552947	6.07242	8	42	1156
5	7.3967	6.041749	9	43	1156
6	8.466266	7.668233	5	25	400
7	0.632825	7.900632	50	22	784
8	4.531638	5.224921	23	49	676
9	7.190045	6.616718	10	36	676
10	2.826599	7.559893	41	29	144
11	3.619592	6.237678	36	40	16
12	1.619135	7.59331	46	27	361
13	1.306178	8.124332	48	20	784
14	3.044741	9.94293	40	2	1444
15	3.452986	5.138859	38	50	144
16	0.764138	7.436903	49	31	324
17	3.86415	9.859462	34	3	961
18	4.191905	8.214515	31	19	144
19	5.269706	8.0665	17	21	16
20	4.269014	5.905026	28	48	400
21	4.346019	6.267586	26	39	169
22	4.259519	7.859279	29	23	36
23	7.685283	9.605243	7	8	1
24	4.829431	8.630177	21	14	49
25	4.627685	9.78988	22	6	256

Last 25 rows not shown

In-class Exercise

- Compute Spearman's rank-order correlation coefficient for the following paired X,Y observations.

i	X	Y
1	34	26
2	46	16
3	1	-26
4	27	-1
5	39	13
6	11	6
7	38	25
8	30	2
9	17	-12
10	8	-1

Estimates of Correlation

- The test of a non-zero r_S uses the test statistic

$$t = r_S \sqrt{\frac{N-2}{1-r_S^2}}$$ which is distributed approximately as a t distribution

with N-2 degrees of freedom.

In-class Exercise

- Test whether the r_S value computed in the last in-class exercise is significantly different from zero at alpha=0.05 ($t_{8,0.025}$= 2.31).

Extended Warranty Example

- Crystal Ball simulates correlation using rank correlation.
  - Demo