## Math 231 Midterm Exam February 4, 2011

Name: Tyler Gildings
Student ID: 931-734-154

Turn off and put away your cell phone. No notes, books, calculators, or or other assistance is allowed. Read each question carefully, answer each question completely, and show all of your work. Write your solutions clearly and legibly – no credit will be given for illegible solutions.

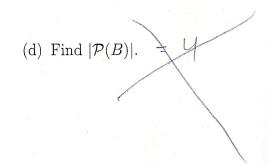
#	Points	Score
1	4	3
2	4	4
3	4	2
4	4	0
5	4	2
6	4	
7	4	0
8	4	2
9	4	4
$\Sigma$	36	18

- 1. Let  $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$  be the universal set, and consider the subsets  $A = \{2, 4, 6, 8\}$  and  $B = \{1, 2, 3, 4\}$  of U.
  - (a) Find  $(A \cup B) (A \cap B)$ .

$$A \cup B = \{1, 2, 3, 4, 6, 8\}$$
  $A \cap B = \{2, 4\}$   
 $(A \vee B) - (A \cap B) = \{1, 3, 6, 8\}$ 

(b) Find B - A.

(c) Find  $\overline{A \cup B}$   $\overline{A \lor B} = \{ S, 7 \}$ 



2. Assume that p is a true proposition, q is a true proposition, and r is a false proposition. Find the truth value of:

$$(\neg p \to (q \land \neg r)) \lor r$$

$$(\neg T \to (T \land \neg F)) \lor F$$

$$(F \to (T \land T)) \lor F$$

$$T$$

$$T$$

$$T$$

$$T$$

$$T$$

3. Let the domain of discourse be  $\mathbb{R} \times \mathbb{R}$ . Determine the truth values of the following quantified statements:

(a) 
$$\forall x \forall y (x + y = 2)$$
  $\forall x \forall y (x + y = 2)$ 

quantified statements.

(a) 
$$\forall x \forall y (x + y = 2)$$
  $\Rightarrow 3 + 5 \neq 2$ 

(b)  $\exists x \forall y (x + y = 2)$   $\Rightarrow 3 + 5 \neq 2$ 

(c)  $\forall x \exists y (x + y = 2)$   $\Rightarrow 3 + 5 \neq 2$ 

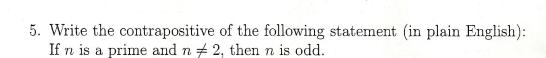
(c) 
$$\forall x \exists y (x + y = 2)$$

(d) 
$$\exists x \, \exists y \, (x+y=2) \, \top$$

4. Simplify the following proposition:

$$p \land ((p \land q) \rightarrow (p \rightarrow q))$$





6. Prove that 
$$A \cap \overline{(A-B)} = A \cap B$$
.

$$(A-B) = \{ \times | \times \neq A \cap X \in B \} \}$$

$$A \cap B = \{ \times | \times \neq A \cap X \in B \} \}$$

$$A \cap (A-B) = \{ \times | \times \neq A \cap X \in B \} \}$$

$$A \cap (A-B) = \{ \times | \times \neq A \cap X \in B \} \}$$

7. Prove by induction on n that  $2n < 2^n$  for  $n = 3, 4, 5, 6, \ldots$ 

Base case 
$$n = \frac{3}{2}2^n$$

Base case  $n = \frac{3}{2}2^n$ 
 $2^n$ 
 $2^n$ 

8. Prove or disprove the following statement: For all integers m and n, if m is odd and n is even, then mn is even.

M= 2kt | Tree values of k are potentially different.

 $M \cdot V = (2k+1)(5k)$ 2(x2+x) = 2x 7

By a frost y environce M.N. 15 even 3

9. Prove or disprove the following statement: For all integers  $n \geq 2$ ,  $2^n - 1$  is prime.

24-1=15

24 = 16 Prime is anything the tonly 24 = 15 Rivilis by itself or one

15 = 5 Canterexample.

By a froot by total order 24-1=15 and 15 divides by 3,5,15,1 which is workful itself orders so the statement NZZ, 2-1 is a Prime is false