Math 231
Midterm Exam
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Turn off and put away your cell phone. No notes, books, calculators, or or other assistance is allowed. Read each question carefully, answer each question completely, and show all of your work. Write your solutions clearly and legibly – no credit will be given for illegible solutions.

#	Points	Score
1	4	3
2	4	4
3	4	2
4	4	0
5	4	2
6	4	1
7	4	0
8	4	2
9	4	4
Σ	36	18

1. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ be the universal set, and consider the subsets $A = \{2, 4, 6, 8\}$ and $B = \{1, 2, 3, 4\}$ of U.

   (a) Find $(A \cup B) - (A \cap B)$.

   $A \cup B = \{1, 2, 3, 4, 6, 8\}$ $\qquad$ $A \cap B = \{2, 4\}$

   $(A \cup B) - (A \cap B) = \{1, 3, 6, 8\}$

   (b) Find $B - A$.

   $B - A = \{1, 3\}$

   (c) Find $\overline{A \cup B}$

   $\overline{A \cup B} = \{5, 7\}$

   (d) Find $|\mathcal{P}(B)|$. $\cancel{= 4}$

2. Assume that p is a true proposition, q is a true proposition, and r is a false proposition. Find the truth value of:

$$(\neg p \to (q \wedge \neg r)) \vee r$$

$(\neg T \to (T \wedge \neg F)) \vee F$

$(F \to \underbrace{(T \wedge T)}_{T}) \vee F$

T

$T \vee F$

$\boxed{T}$

3. Let the domain of discourse be $\mathbb{R} \times \mathbb{R}$. Determine the truth values of the following quantified statements:

(a) $\forall x \, \forall y \, (x + y = 2)$ F, $3+5 \neq 2$

(b) $\exists x \, \forall y \, (x + y = 2)$ ~~T~~, $3 - 1 = 2$

(c) $\forall x \, \exists y \, (x + y = 2)$ ~~F~~ $3+0 \neq 2$

(d) $\exists x \, \exists y \, (x + y = 2)$ T $2+0=2$

4. Simplify the following proposition:

p = it is cloudy

q = it is raining

$$p \wedge ((p \wedge q) \rightarrow (p \rightarrow q))$$

$p \wedge ((p \leftrightarrow q) \rightarrow (p \rightarrow q))$

$p \leftrightarrow ((p \leftrightarrow q) \rightarrow (p \rightarrow q))$

It is cloudy ((... cloudy ∧ raining)) → (It is cloudy

Cloudy ∧ ((Cloudy ∧ raining) → (Cloudy → raining))

~~$(p \vee q)$~~

5. Write the contrapositive of the following statement (in plain English):
If n is a prime and $n \neq 2$, then n is odd.

$p \rightarrow q$

$\neg(q \rightarrow p)$

$\neg q \rightarrow \neg p$

n is even if n is not a prime and n = 2

p if q $\neq$ if p then q

6. Prove that $A \cap \overline{(A - B)} = A \cap B$.

$\overline{(A-B)} = \{x \mid x \notin A \wedge x \in B\}$

$A \cap B = \{x \mid x \in A \wedge x \in B\}$

$A \cap \overline{(A-B)} = \{x \mid x \in A \wedge (x \notin A \wedge x \in B)\}$

$\{x \mid (x \in A \wedge (x \notin A \wedge x \in B)\} = \{x \mid x \in A \wedge x \in B\}$

Explain?

Are you proving the statement?

7. Prove by induction on n that $2n < 2^n$ for $n = 3, 4, 5, 6, \ldots$.

$\sum_{i=3}^{n} 2i < \sum_{i=3}^{n} 2^i$

Base case: $n = 3$

$2n < 2^n$

$2 \cdot 3 < 2^3$

$6 < 8$

Inductive step:

Prove: $\sum_{i=3}^{n+1} 2i < \sum_{i=3}^{n+1} 2^i$

$\sum_{i=3}^{n} 2i + 2(n+1) \quad \sum_{i=3}^{n} 2^i + 2^{(n+1)}$

$2n + 2(n+1) < 2^{(n+1)} + 2^n$

$2n + 2n + 2 < 2 \cdot 2^n + 2^n$

$4n + 2 <$

$2(2n+1) <$

8. Prove or disprove the following statement:
For all integers m and n, if m is odd and n is even, then mn is even.

$m = 2k+1$
$n = 2k$

These values of k are potentially different.

$m \cdot n = (2k+1)(2k)$

$2k^2 + 2k$

$2(k^2+k) = 2k$?

~~By a proof by equivalence~~ $m \cdot n$ is even $\boxed{3}$

9. Prove or disprove the following statement:
For all integers $n \geq 2$, $2^n - 1$ is prime.

$2 \cdot 2 = 4$
$2 \cdot 4 = 8$
$2 \cdot 8 = 16$

$2^4 = 16$

$2^4 - 1 = 15$

Prime is anything that only divides by itself or one

$\frac{15}{3} = 5$

Counterexample.

By a ~~proof by contradiction~~ $2^4 - 1 = 15$ and 15 divides by 3, 5, 15, 1 which is more than itself or one so the statement $n \geq 2$, $2^n - 1$ is a Prime is false $\boxed{3}$