FRESNEL FIELD TO FAR FIELD TRANSFORMATION USING SPARSE FIELD SAMPLES

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***Abstract* – A method of antenna pattern reconstruction using sampling field data in sparse grid of points in Fresnel zone is presented. According to this method, antenna far field pattern in a desired section is reconstructed from several sections of the field in Fresnel region. Fresnel field to far field transformation is based on two-dimensional Fourier series expansion. Main features of the method are discussed. Computer simulation results are given.**

I. INTRODUCTION

Reconstruction of antenna characteristics from amplitude and phase in near region is widely used in practice. For example it allows to determine characteristics of large antennas measured in relatively small test ranges, e.g. in anechoic chambers. In particular, a method of measurements on a sphere encompassing the antenna can be implemented using a two-axes angular positioner used for far field measurements. However, near field methods require a large amount of points, which is due to a large angular sector (about half-sphere) and small spacing between samples (about half-wavelength).

An alternative method is a method of far field reconstruction from Fresnel field data on a sphere. It is known that in this case data from a limited angular sector are sufficient [1], [2]. Also, in Fresnel region a different algorithm of far field reconstruction can be used. In this approach the spacing between samples is much larger than half-wavelength [3]-[10]. Using this approach the amount of measurements can be significantly reduced, one far field section can be reconstructed from a small amount of sections in Fresnel region.

In the current paper the method of Fresnel field to far field transformation is presented and main features of the method are considered.

II. MATHEMATICAL MODEL

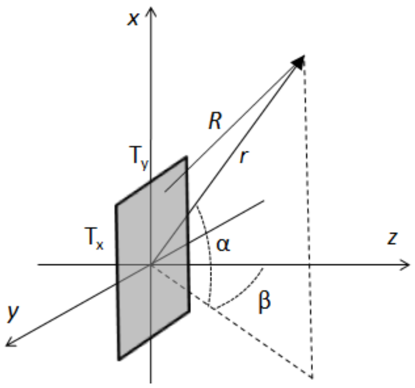

Fig. 1. Antenna aperture and the reference frame.

Consider an antenna with dimensions at least several wavelengths. Let the antenna be in the left half-space near the plane z=0 (Fig.1), where it produces directional radiation to the right half-space z>0. The antenna is behind a rectangle T_xxT_y in the plane z=0. The rectangle can be a little larger than the antenna. Also let the antenna be positioned in such way that the center of the rectangle coincides with the origin.

To describe the field at z>0 mathematically we'll use Kirchhoff integral with the Green's function, chosen to eliminate the term with the field in the integral. The field at z=0 outside the T_xxT_y rectangle is neglected. Then the electric field in an arbitrary point in the right half-space can be written in terms of the normal derivative of the field at z=0 plane as follows (e.g. see [11]):

$$\mathbf{E}(u,v,r) = -\frac{1}{2\pi}\iint_{T_x\times T_y}\frac{\partial \mathbf{E}}{\partial z}(x,y,0)\frac{e^{-jkR(x,y)}}{R(x,y)}dxdy\,, \qquad (1)$$

where **E** is complex electric field amplitude; λ is wavelength, k=2π/λ; u=sinα, v=cosα sinβ, w=cosα cosβ are direction cosines of the direction of observation, β and α are azimuth and elevation of the direction of observation, r is the distance between the origin and observation point; R is the distance between integration point and observation point.

When T_x,T_y →∞, formula (1) is strict. The error of (1) is due to limited size of the chosen aperture and it's determined by the same integral over the region $\mathbb{R}^2 \setminus$ T_xxT_y. If the rectangle T_xxT_y includes all points with significant (large enough) amplitude of the field, then the error is small. The analysis of errors of such representation in terms of geometrical theory of diffraction is given e.g. in [12].

Let us make a simplification for the case, when the observation point is at least as far as in the Fresnel region:

$$r > 0.62\sqrt{D^3/\lambda}\ . \tag{2}$$

In this case (1) can be rewritten in the following way:

$$\mathbf{E}(u,v,r) = -\frac{e^{-jkr}}{2\pi r}\iint_{T_x\times T_y}\frac{\partial \mathbf{E}}{\partial z}(x,y,0)e^{-j\Phi_r(x,y,u,v)}dxdy\ , \tag{3}$$

where

$$\Phi_r(x,y,u,v) = k\sqrt{(ru-x)^2+(rv-y)^2+r^2(1-u^2-v^2)} - kr \approx \Phi^{(1)}(x,y,u,v)+\Phi_r^{(2)}(x,y,u,v)\ , \tag{4}$$

$$\Phi^{(1)}(x,y,u,v) = -k(xu+yv)\ , \tag{5}$$

$$\Phi_r^{(2)}(x,y,u,v) = \frac{k}{2r}\left(x^2(1-u^2)+y^2\left(1-v^2\right)-2xyuv\right). \tag{6}$$

III. Field Transformation

Consider the following problem: given field samples at a sphere r_1 in the Fresnel region, we need to determine field values at a sphere r_2 (Fig.2). The problem of reconstructing far field corresponds to $r_2 \to \infty$.

Consider the following representation of exponential of the phase function from (3) for the sphere r_2:

$$e^{-j\Phi_{r2}(x,y,u,v)} = g(x,y,u,v)e^{-j\Phi_{r1}(x,y,u,v)}\ , \tag{7}$$

where

$$g(x,y,u,v) = \exp\left(j\frac{k}{2}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\left(x^2(1-u^2)+y^2\left(1-v^2\right)-2xyuv\right)\right). \tag{8}$$

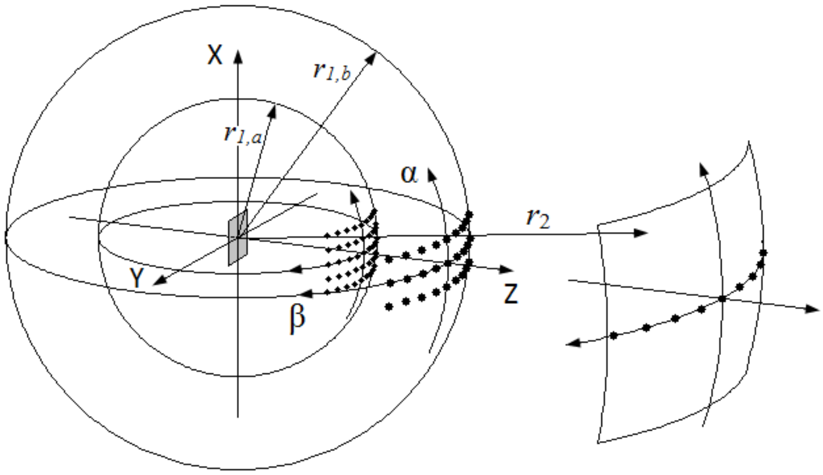

Fig. 2. Field samples in Fresnel zone (spheres $r_{1,a}$ or $r_{1,b}$) and reconstructed points in far zone (sphere r_2).

Let us expand (8) into Fourier series as a function of (x,y) at a rectangle $[-T_x/2, T_x/2]\mathrm{x}[-T_y/2, T_y/2]$ for fixed u and v:

$$g(x,y,u,v) = \sum_{m,n} k_{mn}(u,v)e^{jm\frac{2\pi}{T_x}x}e^{jn\frac{2\pi}{T_y}y}\ , \tag{9}$$

where $$k_{mn}(u,v) = \frac{1}{T_xT_y}\iint_{T_x\times T_y} e^{j\frac{k}{2}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\left(x^2(1-u^2)+y^2\left(1-v^2\right)-2xyuv\right)}e^{-jm\frac{2\pi}{T_x}x}e^{-jn\frac{2\pi}{T_y}y}dxdy\ . \tag{10}$$

Substitute (9) into (7) taking into account denotations (5) and (6):

$$e^{-j\Phi_{r2}(x,y,u,v)} = \sum_{m,n} k_{mn}(u,v) e^{jm\frac{2\pi}{T_x}x} e^{jn\frac{2\pi}{T_y}y} e^{-j\Phi_{r1}(x,y,u,v)} \approx \sum_{m,n} k_{mn}(u,v) e^{-j\Phi^{(1)}(x,y,u+m\Delta u,v+n\Delta v)} e^{-j\Phi_{r1}^{(2)}(x,y,u,v)} , \tag{11}$$

where

$$\Delta u = \lambda / T_x , \ \Delta v = \lambda / T_y . \tag{12}$$

For r_1 in the Fresnel region, in the vicinity of u, v the following inequality takes place:

$$|\Phi_{r_1}^{(2)}(x,y,u,v) - \Phi_{r_1}^{(2)}\left(x,y,u+m\Delta u,v+n\Delta v\right)| << 1 . \tag{13}$$

With (13), the relation (11) can be rewritten as:

$$e^{-j\Phi_{r2}(x,y,u,v)} \approx \sum_{m,n} k_{mn}(u,v) e^{-j\Phi_{r1}(x,y,u+m\Delta u,v+n\Delta v)} . \tag{14}$$

Now substitute (14) into (3). After changing order of summation and integration, we have:

$$\mathbf{E}(u,v,r_2) \approx e^{-jk(r_2-r_1)} \frac{r_1}{r_2} \sum_{m,n} k_{mn}(u,v) \mathbf{E}(u+m\Delta u, v+n\Delta v, r_1) . \tag{15}$$

In sum (15) only several terms contribute significantly, so other terms can be dropped. This issue will be considered in more detail below.

Note, that a formula analogous to (15) was originally obtained in [13] for a two-dimensional problem and later in [14] for a three-dimensional problem for the case when antenna is illuminated by a nonplanar wave. However in these publications the formulas were applied only to nonplanar waves of a compact range.

Now let us consider a point u_2, v_2, which is located between nodes $u_1+m\Delta u$, $v_1+n\Delta v$. Let the node u_1, v_1 be the closest node of the mentioned grid's nodes to u_2, v_2. Expression (7) can be rewritten as:

$$e^{-j\Phi_{r2}(x,y,u_2,v_2)} = g(x,y,u_1,v_1,u_2,v_2) e^{-j\Phi_{r1}(x,y,u_1,v_1)} . \tag{16}$$

After transformations similar to (9)-(15), we obtain:

$$\mathbf{E}(u_2,v_2,r_2) \approx e^{-jk(r_2-r_1)} \frac{r_1}{r_2} \sum_{m,n} k_{mn}(u_1,v_1,u_2,v_2) \mathbf{E}(u_1+m\Delta u, v_1+n\Delta v, r_1) , \tag{17}$$

where

$$k_{mn}(u_1,v_1,u_2,v_2) = \frac{1}{T_x T_y} \iint_{T_x \times T_y} e^{j\Phi_{r1}^{(2)}(x,y,u_1,v_1) - j\Phi_{r2}^{(2)}(x,y,u_2,v_2)} e^{jk(x(u_2-u_1)+y(v_2-v_1))} e^{-jm\frac{2\pi}{T_x}x} e^{-jn\frac{2\pi}{T_y}y} dxdy . \tag{18}$$

Note, that in (17) the largest contribution into sum is made by the field samples near (u_2,v_2), because coefficients (18) have largest magnitudes in this area.

Thereby, using field values in the angular grid $u_1+m\Delta u$, $v_1+n\Delta v$ on a sphere in Fresnel region or far region, one can reconstruct field values in every point of the half-space in Fresnel region or far region.

The formulas can also be applied to samples equally spaced in azimuth-elevation coordinates. Let us show it for the case when either α≈0 or β≈0. In this case the phase doesn't have an *xy* term, so the two-dimensional integral (18) becomes a product of two one-dimensional integrals and the formula takes simpler form.

First, consider α≈0 (when field is measured in azimuth sections). Then field values are given in a rectangular grid in α-β:

$$\alpha = \alpha_1 + m\Delta u , \beta = \beta_1 + n\Delta v , \ \alpha_1 \approx 0 . \tag{19}$$

These angles correspond to the following u and v:

$$u \approx \alpha_1 + m\Delta u , v = \sin(\beta_1 + n\Delta v) , \tag{20}$$

i.e. v spacing becomes smaller when azimuth increases. Based on (12), to have a smaller spacing for v we need to increase the area for integration: $\tilde{T}_y = T_y / \cos(\beta)$. Substituting it into (18) yields:

$$k_{mn} = \frac{1}{T_x T_y} \int_{-T_x/2}^{T_x/2} e^{j\frac{k}{2}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)x^2 + jkx(\alpha_2-\alpha_1)} e^{-jm\frac{2\pi}{T_x}x} dx \int_{-T_y/2}^{T_y/2} e^{j\frac{k}{2}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)y^2 + jky(\beta_2-\beta_1)} e^{-jn\frac{2\pi}{T_y}y} dy . \tag{21}$$

Using same considerations one can obtain the formula for β≈0 (elevation section):

$$k_{mn} = \frac{1}{T_x T_y} \int_{-T_x/2}^{T_x/2} e^{j\frac{k}{2}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)x^2 + jkx(\alpha_2-\alpha_1)} e^{-jm\frac{2\pi}{T_x}x} dx \int_{-T_y/2}^{T_y/2} e^{j\frac{k}{2}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)y^2/\cos^2\alpha_1 + jky(\beta_2-\beta_1)/\cos\alpha_1} e^{-jn\frac{2\pi}{T_y}y} dy . \tag{22}$$

Also note, that all formulas were written in vector form so far. However, one doesn't need to have all three components of the field. Firstly, it is not necessary to have radial component since in Fresnel region it is small and in addition its influence on far field is reduced due to the fact that in (17) radial components are almost orthogonal to transversal components of the far field.

Also, (17) can be used in scalar form separately for co and cross components of the field. It is also due to the fact that in (17) only points with close angular directions are used, so basis vectors of co and cross polarizations are almost orthogonal, regardless of which pair of co and cross polarizations is used.

IV. The Amount Of Sections

The k_{mn} coefficients rapidly decrease with m and n, as can be seen from Fig.3. Also, field in Fresnel region decreases as the observation point moves away from field maximum. Therefore, in sums (15), (17) a finite number of terms can be used. Let us estimate minimum amount of terms for correct far field reconstruction.

At first, consider a well-focused antenna with radiation maximum in the direction of z-axis, when central azimuth section is reconstructed. For field reconstruction M azimuth sections of the field in the Fresnel region are used. Based on the estimation for minimum angular sector [2], the minimum amount of sections can be estimated as:

$$M_{\min} = 2\left[\frac{T_x^2}{2\lambda r_1} + \frac{3}{2}\sqrt{\frac{T_x^2}{2\lambda r_1}}\right] + 1 . \tag{23}$$

Note, that this estimation is a little larger than the estimation in [8], because it takes into account not only geometrical optics boundaries, but semishadow zones too.

Thereby, the amount of sections in Fresnel region decreases with the increase of radius of the sphere where the samples are taken (Fig.2). It allows to significantly reduce the amount of points in Fresnel region to reconstruct the far field comparing to reconstruction from near field.

However, the amount of n-terms shouldn't be limited with this estimation. It is due to contribution of the points with largest field amplitudes in Fresnel region to sums (15), (17) even for points distant from field maximum in far zone. Such terms have small k_{mn} but large $E(u+m\Delta u,v+n\Delta v,r_1)$, so they should also be included into the summation.

A similar effect takes place when a reconstructed section doesn't go through field maximum: in this case in addition to (23) it might be required to measure consider the sections which go through Fresnel field maximum. Also (23) cannot be applied to badly focused antennas (e.g. antennas with contoured beams), in this case the minimum amount of sections might be larger.

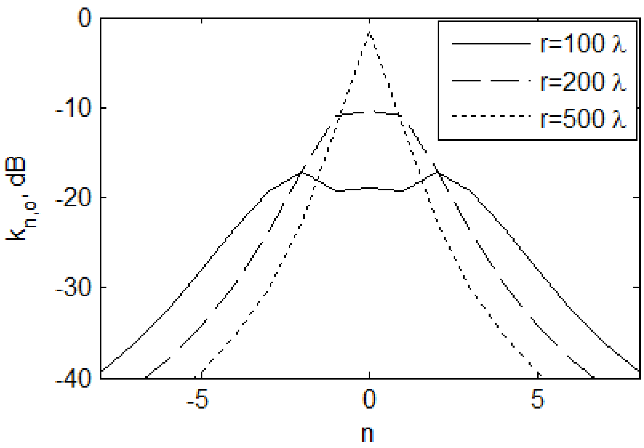

Fig. 3. Magnitudes of $k_{n,0}$ coefficients for antenna with size 30 λ and different r.

V. Generalization To Far Field Reconstruction From Near Field

When the field samples are taken in regular grid in azimuth-elevation coordinates and coefficients are calculated using (21) the method can be applied for smaller distances then indicated by (2). In [9] the analysis of error of the transformation was carried out, the analysis being based on consideration of error of expression similar to (14). The difference of right hand side and left hand side was expanded into Taylor series of x and y, and principle terms of the error were found. The criterion of applicability of the method is that the principle terms of the error are small. On the basis of it the criterion was stated as:

$$\frac{D^4}{50\lambda r^3} << 1,\ \frac{D}{4r} << 1\,. \tag{24}$$

The method can also be generalized for far field reconstruction from near field data measurements. When initial formulas (17), (18) are used for r in near region, the far field is reconstructed with an error which is due to the fact that in formula (18) for coefficients the higher orders of phase function are not taken into account. To take it into account the coefficients can be found as a solution of minimization of error in (14) in L_2 norm [9]:

$$\zeta(\vec{k}) = \iint_S |\, e^{-j\Phi_{r2}(x,y,u,v)} - \sum_{m,n} k_{mn} e^{-j\Phi_{r1}(x,y,u+m\Delta u,v+n\Delta v)} |^2 \, dS \to \min_{\vec{k}}\,. \tag{25}$$

Note, that for more precise solution an amplitude dependency can also be included to (25).

The simplification of (25) leads to the following linear system:

$$\begin{cases} Sx_j + \sum_{n\neq j} I_{jn}^{\mathrm{Re}} x_n + \sum_{n\neq j} I_{jn}^{\mathrm{Im}} y_n = I_j^{\mathrm{Re}}, \\ Sy_j + \sum_{n\neq j} I_{jn}^{\mathrm{Re}} y_n + \sum_{n\neq j} I_{nj}^{\mathrm{Im}} x_n = -I_j^{\mathrm{Im}}, \end{cases} \tag{26}$$

where $x_m + jy_m = k_m$, $I_{mn} = I_{mn}^{\mathrm{Re}} + jI_{mn}^{\mathrm{Im}} = \iint_S e^{-j\Phi_{r1}(x,y,u+i_m\Delta u,v+j_m\Delta v)} e^{j\Phi_{r1}(x,y,u+i_n\Delta u,v+j_n\Delta v)} dS$, $I_m = I_m^{\mathrm{Re}} + jI_m^{\mathrm{Im}}$ $= \iint_S e^{-j\Phi_{r1}(x,y,u+i_m\Delta u,v+j_m\Delta v)} e^{j\Phi_{r2}(x,y,u,v)} dS$, S is aperture area. Also, in this expression the coefficients have a single index (k_m, as opposed to double index k_{mn}).

This approach also allows to reconstruct far field using irregular angular grid. The disadvantage of this method is its computational difficulty, because a large amount of two-dimensional integrals must be calculated to construct the linear system.

Consider the extreme case when the samples are taken on a sphere with radius equal to radius of the antenna: r_1=D/2. Since angular spacing is λ/D the linear spacing for samples with small azimuth and elevation is $r_1\lambda/D$=λ/2, which is the same as in conventional near field measurement methods. However, according to (23) the angular span $M\Delta u$ is larger than 2, which means that far field cannot be reconstructed correctly from such a small sphere using this method. Traditional methods (look, for example [14]) should be applied in this case.

VI. Computer Simulation Results

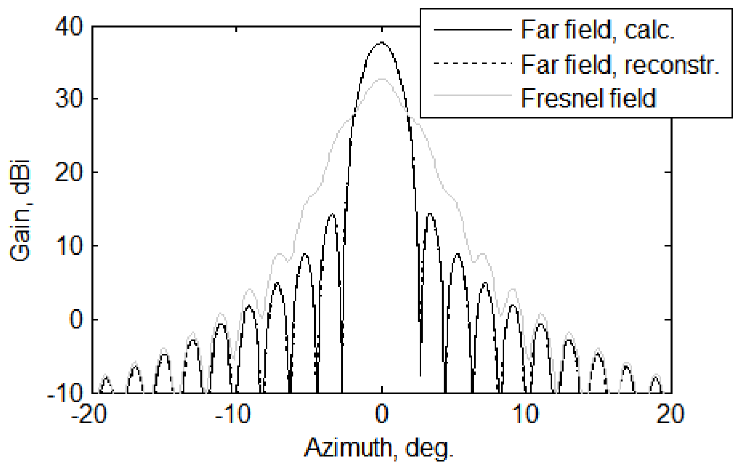

Fig. 4. Computer simulation results using 11 sections in Fresnel region.

In this section the results of computer simulation are presented. For simulation an axisymmetrical reflector antenna was used, D=30λ ($2D^2/\lambda$=1800λ). Using PO approximation several azimuth sections of Fresnel field at 200λ and a central section of far field were calculated. Elevation spacing was 1.8° (T_x=31.8 λ). Based on (17), (18) far field was reconstructed and compared to the calculated far field. Fig.4 shows the results of field reconstruction using 11 sections of Fresnel field (according to (23)). The gain error is 0.01 dB, first side lobe level error is 0.07 dB.

VII. Experimental Verification And Practical Application

For verification of the method a set of experiments was carried out [10]. Field patterns of a reflector antenna were measured in anechoic chamber in Fresnel region and in far region. Then radiation pattern was reconstructed based on Fresnel region measurements and the result was compared to far field measurements. The experiments showed good agreement.

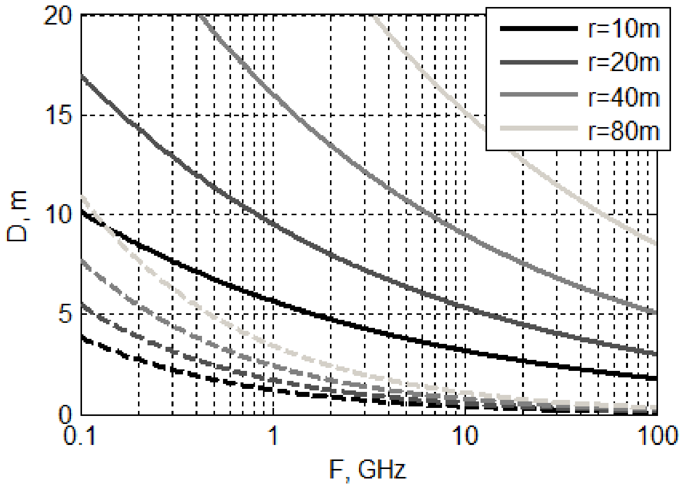

Fig. 5. Maximum size of antennas, which can be tested in anechoic chambers of different size, versus frequency.

The method described in this article has been used for antenna measurements in an anechoic chamber in company Radiofizika (measurements distance up to 80 m) for several years. Using this method many antennas were tested, with far field distance of hundreds of meters and even kilometers. Fig.5 shows the increase of maximum size of antennas, which can be tested in anechoic chambers with measurement distance 10 m, 20 m, 40 m and 80 m, that is gained when Fresnel field measurements instead of far field measurements are used. The dashed lines correspond to the standard far field criterion $r>2D^2/\lambda$, the solid lines correspond to criterion (24).

VIII. Conclusion

A method of far field reconstruction from samples on a sphere in Fresnel zone was presented. According to this method the amount of points in Fresnel zone required for far field reconstruction is significantly smaller compared to reconstruction from near field. Computer simulation as well as experimental results were presented. The results suggest confirm that the method ensures good quality of far field reconstruction in a large dynamic range.

References

[1.] R. Mittra, W. Imbriale, "Gain and pattern measurements of large aperture antennas in the Fresnel zone," in *Proc. Antennas and Propagation Society International Symposium*, 1969, vol. 7, pp. 40-42.

[2.] Yu.A. Kolosov, A.P. Kurochkin, "Radiation pattern reconstruction using Fresnel field known in a limited angular sector," *Antennas*, vol.16, 1972, pp. 25-37 (in Russian).

[3.] G. D'Elia, G. Leone, and R. Pierri, "A new approach in the near field-far field transformation," *Digest of IEEE-APS Symposium*, Houston, USA, 1983, pp. 495-498.

[4.] G. D'Elia, G. Leone, R. Pierri, G. Schirinzi, "New Method of Far-Field Reconstruction from Fresnel Field," *Electronics Letters*, vol. 20, no. 8, Apr. 1984, pp. 342-343.

[5.] G. Evans, "Far Field Correction for Short Antenna Ranges", in *Proc. AMTA Symposium*, 1985, pp. 34.1–34.9.

[6.] G. Evans, *Antenna Measurement Techniques*, Artech House, 1990.

[7.] K. Wu, S. Parekh, "A method of transforming Fresnel field to far field for circular aperture antennas," in *Proc. of IEEE APS Digest*, May 1990, pp. 216–219.

[8.] I.L. Vilenko, A.A. Meduhin, Yu.A. Suserov, A.K. Tobolev and A.V. Shishlov, "Reconstruction of antenna radiation pattern by using data of measurements in a Fresnel region with test facility intended for far-field measurements," *Antennas*, vol. 1(92), pp. 46–52, 2005 (in Russian).

[9.] Yu.V. Krivosheev and A.V. Shishlov "Development of the method of antenna radiation pattern reconstruction using measurements in the Fresnel zone," in *Proc. Radars and Communication*, Moscow, Russia, November 2010, pp. 320-324 (in Russian).

[10.] Yu.V. Krivosheev, A.V. Shishlov, A.K. Tobolev and I.L. Vilenko. "Fresnel Field to Far Field Transformation Based on Two-Dimensional Fourier Series Expansion," in *Proc. AES 2012*, Paris, pp.200-207.

[11.] A. Ishimaru. *Electromagnetic Wave Propagation, Radiation, and Scattering*, Prentice Hall, New Jersey, 1991.

[12.] V.A. Borovikov, B.E. Kinber. *Geometrical theory of diffraction*, Svyaz, Moscow, 1978 (in Russian).

[13.] L.D. Bakhrakh, I.V. Kaplun and A.P. Kurochkin, "Determination of Parameters of Antennas Illuminated by a Nonplanar Wave," *Radio Engineering and Electronic Physics*, vol. 20, Dec. 1975, pp. 1-8. Translation from Russian.

[14.] L.D. Bakhrakh, S.D. Kremenetskiy, A.P. Kurochkin, V.A. Usin, Ya.S. Shifrin, *Near field antenna measurements*, Nauka, 1985 (in Russian).