№18.

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$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$\sigma'(z) = \frac{-(-e^{-z})}{(1+e^{-z})^2} = \frac{e^{-z}}{(1+e^{-z})^2}$$

$$\parallel$$

$$\sigma(1-\sigma) = \frac{1}{1+e^{-z}} \cdot \frac{e^{-z}}{1+e^{-z}} = \frac{e^{-z}}{(1+e^{-z})^2}$$

№19.

1) $g_k(s_1,\dots,s_k) = \dfrac{e^{s_k}}{\sum\limits_{i=1}^{k} e^{s_i}}$; $R^{(i)} = -\sum\limits_{k=1}^{k} I(y^{(i)}=k)$

$-\ln g_k(s_1,\dots,s_k)$

$$\frac{\partial g_k}{\partial s_l} = \begin{cases} e^{s_k}\left(\dfrac{-e^{s_l}}{(\sum^k e^{s_i})^2}\right) = \dfrac{-e^{s_l}}{\sum_i e^{s_i}} = -g_l g_k, & k\neq l \\ \dfrac{e^{s_k}(\sum_i e^{s_i}) - e^{s_k}\cdot e^{s_l}}{(\sum_i e^{s_i})^2} = \dfrac{e^{s_k}}{\sum_i e^{s_i}} - \dfrac{e^{s_k}}{\sum_i e^{s_i}} \cdot \dfrac{e^{s_l}}{\sum_i e^{s_i}} = g_k(1-g_l), & k=l \end{cases}$$

$$I(k=l) = \begin{cases} 1, & k=l \\ 0, & k\neq l \end{cases} \Rightarrow \frac{\partial g_k}{\partial s_l} = g_k\cdot(I(k=l) - g_l)$$

2) $$\frac{\partial R^{(i)}}{\partial g_k} = -I(y^{(i)}=k)(\ln g_k(s_1,\dots,s_k)) = \frac{-I(y^{(i)}=k)}{g_k(s_1,\dots,s_k)}$$

3) $$\frac{\partial R^{(i)}}{\partial s_1} = \sum_k \frac{\partial R^{(i)}}{\partial g_k} \cdot \frac{\partial g_k}{\partial s_1} = -\sum_k \frac{I(y^{(i)} = k)}{g_k(s_1 \ldots s_k)} \cdot g_k(I(k=l) - g_l) = -(I(y^{(i)}=l) - g_l) = g_l - I(y^{(i)} = l)$$