0.5 1

Transpose

1.

$$\underset{2\times 2}{A} = \begin{bmatrix} 3 & -5 \\ -2 & 7 \end{bmatrix} \qquad \underset{2\times 2}{A^T} = \begin{bmatrix} 3 & -2 \\ -5 & 7 \end{bmatrix}$$

$$\underset{2\times 2}{B} = \begin{bmatrix} 2 & -3 & 4 \\ -5 & 6 & 7 \\ -8 & 9 & 1 \end{bmatrix} \qquad \underset{3\times 3}{B^T} = \begin{bmatrix} 2 & -5 & 8 \\ -3 & 6 & 9 \\ 4 & 7 & 1 \end{bmatrix}$$

2.

$$\underset{3\times 3}{A} = \begin{bmatrix} 3 & 2 & -1 \\ -2 & 7 & 4 \\ 1 & 6 & 8 \end{bmatrix} \qquad \underset{3\times 3}{B} = \begin{bmatrix} 2 & -3 & -4 \\ -5 & -6 & 7 \\ -8 & 9 & 1 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 3.2 & 2(-3) & -1(-4) \\ (-2)(-5) & 7(-6) & 4(7) \\ 1.(-8) & 6.9 & 8.1 \end{bmatrix} = \begin{bmatrix} 6 & -6 & 4 \\ 10 & -42 & 28 \\ -8 & 54 & 8 \end{bmatrix}$$ - Element-wise product

Matrix product of A and B

$$AB = \begin{bmatrix} 3.2 + 2(-5) + (-1).(-8) & 3.(-3) + 2.(-6) + (-1).9 & 3.(-4) + 2.7 + (-1).1 \\ -2.2 + 7.(-5) + 4.(-8) & -2(-3) + 7.(-6) + 4.9 & (-2)(-4) + 7.7 + 4.1 \\ 1.2 + 6.(-5) + 8.(-8) & 1.(-3) + 6(-6) + 8.9 & 1.(-4) + 6.7 + 8.1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 6 - 10 + 8 & -9 - 12 - 9 & -12 + 14 - 1 \\ -4 - 35 - 32 & 6 - 42 + 36 & 8 + 49 + 4 \\ 2 - 30 - 64 & -3 - 36 + 72 & -4 + 42 + 8 \end{bmatrix}$$

$$AB = \begin{bmatrix} 4 & -30 & 1 \\ -71 & 0 & 61 \\ -92 & 33 & 46 \end{bmatrix}$$

Element-wise product of B and A

$$B = \begin{bmatrix} 2 & -3 & -4 \\ -5 & -6 & 7 \\ -8 & 9 & 1 \end{bmatrix} \qquad A = \begin{bmatrix} 3 & 2 & -1 \\ -2 & 7 & 4 \\ 1 & 6 & 8 \end{bmatrix}$$

$$B \cdot A = \begin{bmatrix} 2.3 & (-3).2 & (-4).(-1) \\ (-5).(-2) & (-6).7 & 7.4 \\ (-8).1 & 9.6 & 1.8 \end{bmatrix} \qquad A.B = \begin{bmatrix} 6 & -6 & 4 \\ 10 & -42 & 28 \\ -8 & 54 & 8 \end{bmatrix}$$

Matrix product of B and A

$$BA = \begin{bmatrix} 2(3)+(-3)(-2)+(-4)(1) & 2.2+(-3).7+(-4).6 & 2.(-1)+(-3).4+(-4).8 \\ -5(3)+(-6)(-2)+7.(1) & (-5)2+(-6).7+7.6 & -5(-1)+(-6).4+7.8 \\ -8(3)+9(-2)+1(1) & (-8).2+9.7+1.6 & -8(-1)+9.4+1.8 \end{bmatrix}$$

$$BA = \begin{bmatrix} 6+6-4 & 4-21-24 & -2-12-32 \\ -15+12+7 & -10-42+42 & 5-24+56 \\ -24-18+1 & -16+63+6 & 8+36+8 \end{bmatrix}$$

$$BA = \begin{bmatrix} 8 & 41 & -46 \\ 4 & 10 & 37 \\ -41 & 53 & 52 \end{bmatrix} \Rightarrow BA \neq AB$$

The element-wise products ~~are~~ A.B and B.A are the same
The matrix products AB and BA are not the same

$AB \neq BA$, ~~AB~~ $A.B = B.A$

The order of multiplication matters
matrix multiplication is ~~commutative~~ not commutative

$$A = \begin{bmatrix} 3 & 2 & -1 \\ -2 & 7 & 4 \\ 1 & 6 & 8 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & -2 & 1 \\ 2 & 7 & 6 \\ -1 & 4 & 8 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 6 & 7 \\ 4 & 12 & 8 \end{bmatrix}$$

~~$|A| = 3 \begin{vmatrix} 7 & 4 \\ 6 & 8 \end{vmatrix} - 2$~~

$$A = 3.\det\begin{bmatrix} 7 & 4 \\ 6 & 8 \end{bmatrix} - 2.\det\begin{bmatrix} -2 & 4 \\ 1 & 8 \end{bmatrix} + (-1).\det\begin{bmatrix} -2 & 7 \\ 1 & 6 \end{bmatrix} =$$

~~$= 3(7.8) - 3(4.6) - 2(-2.8) + 2(4.1)$~~

~~$= 3(7.8) - 3(4.6)$~~

$= 3(7.8 - 4.6) - 2[(-2).8 - 4.1] - 1((-2).6 - 7.1) =$

$= 3(32) - 2(-20) - 1(-19) = 96 + 40 + 19 =$
$= 100 + 15 + 40 = 155$

$$B = 3.\det\begin{bmatrix} 7 & 6 \\ 4 & 8 \end{bmatrix} + 2\begin{bmatrix} 2 & 6 \\ -1 & 8 \end{bmatrix} + 1\begin{bmatrix} 2 & 7 \\ -1 & 4 \end{bmatrix} = 3(7.8 - 6.4) + 2(2.8 + 6) + 1(8 + 7) =$$

$= 3.32 + 44 + 15 = 96 + 44 + 15 = 100 + 40 + 15$
$= 155$

$$C = 1.\det\begin{bmatrix} 6 & 7 \\ 12 & 8 \end{bmatrix} - 3.\det\begin{bmatrix} 2 & 7 \\ 4 & 8 \end{bmatrix} + 5.\det\begin{bmatrix} 2 & 6 \\ 4 & 12 \end{bmatrix} = 1(6.8 - 7.12) - 3(2.8 - 2.4) + 5.(2.12 - 6.4) =$$

$= -36 - 3(-12) + 5(0) = -36 + 36 = 0$

~~det~~ $[A] = [B] \neq [C]$

~~det~~

$$C = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 6 & 7 \\ 4 & 12 & 8 \end{bmatrix}$$

The first column of C is $\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ and the second $\begin{bmatrix} 3 \\ 6 \\ 12 \end{bmatrix}$ is $3 \times \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$

The second columns is 3 times the first column.

$A = \begin{bmatrix} 3 & 2 \\ -2 & 7 \end{bmatrix}$ $\quad B = \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix}$

$$A^{-1} = \frac{1}{\det[A]} \begin{bmatrix} 7 & -2 \\ 2 & 3 \end{bmatrix} = \frac{1}{3.7+4} \begin{bmatrix} 7 & -2 \\ 2 & 3 \end{bmatrix} = \frac{1}{25} \begin{bmatrix} 7 & -2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} \frac{7}{25} & -\frac{2}{25} \\ \frac{2}{25} & \frac{3}{25} \end{bmatrix}$$

$$B^{-1} = \frac{1}{\det[B]} \begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix} = \frac{1}{5.4-0} \begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix} = \frac{1}{20} \cdot \begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} \frac{5}{20} & \frac{0}{20} \\ \frac{0}{20} & \frac{4}{20} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{5} \end{bmatrix}$$

$$A^{-1} B^{-1} = \begin{bmatrix} \frac{7}{25} \cdot \frac{1}{4} + \left(-\frac{2}{25}\right) \cdot 0 & \frac{7}{25} \cdot 0 + \left(-\frac{2}{25}\right) \cdot \frac{1}{5} \\ \frac{2}{25} \cdot \frac{1}{4} + \frac{3}{25} \cdot 0 & \frac{2}{25} \cdot 0 + \frac{3}{25} \cdot \frac{1}{5} \end{bmatrix} =$$

$$A^{-1} B^{-1} = \begin{bmatrix} \frac{7}{100} & -\frac{2}{125} \\ \frac{2}{100} & \frac{3}{125} \end{bmatrix} = \begin{bmatrix} \frac{7}{100} & -\frac{2}{125} \\ \frac{1}{50} & \frac{3}{125} \end{bmatrix}$$

$$BA = \begin{bmatrix} 4.3 + 0.(-2) & 4.(2) + 0.(7) \\ 0.3 + 5.(-2) & 0.(2) + 5.7 \end{bmatrix} = \begin{bmatrix} 12 + 0 & 8 + 0 \\ 0 + (-10) & 0 + 35 \end{bmatrix} = \begin{bmatrix} 12 & 8 \\ 0-10 & 35 \end{bmatrix} =$$

$$= \begin{bmatrix} 12 & 8 \\ -10 & 35 \end{bmatrix}$$

$$(BA)^{-1} = \frac{1}{\det(BA)} \begin{bmatrix} 35 & -8 \\ 10 & 12 \end{bmatrix}$$

$$(BA)^{-1} = \frac{1}{35.12+8.10} \begin{bmatrix} 35 & -8 \\ 10 & 12 \end{bmatrix} =$$

$$= \frac{1}{420+80} \begin{bmatrix} 35 & -8 \\ 10 & 12 \end{bmatrix} = \frac{1}{500} \begin{bmatrix} 35 & -8 \\ 10 & 12 \end{bmatrix}$$

$$(BA)^{-1} = \frac{1}{500} \begin{bmatrix} 35 & -8 \\ 10 & 12 \end{bmatrix}$$

$$(BA)^{-1} = \begin{bmatrix} \frac{35}{500} & \frac{-8}{500} \\ \frac{10}{500} & \frac{12}{500} \end{bmatrix} \quad BA^{-1} = \begin{bmatrix} \frac{7}{100} & -\frac{2}{125} \\ \frac{1}{50} & \frac{3}{125} \end{bmatrix} \Rightarrow A^{-1} B^{-1} = (BA)^{-1}$$

From the comparison between B and B^{-1} we can see that $B^{-1} = \frac{1}{B}$