# Remind for the theory of differential equations and scheduling basics

Differential equations theory and exercises

$\tilde{X}(s) := \ Ee^{-sX}$.

Z-transform: $\widehat{X}(z) := \sum\limits_{i=0}^{\infty} Pr\{X = i\} z^i$.

Exercises:

1. $X \sim C_n^k p^i (1-p)^{n-i}$. $\widehat{X}$? $((z-1)p+1)^n$.

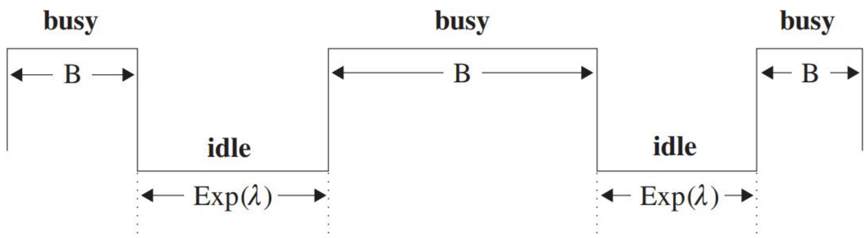
2. $X \sim p(1-p)^i$. $\widehat{X}$? $\frac{p}{1-z(1-p)}$.

3. (26.01:5) $X \sim Uniform(a,b)$. $\tilde{X}$? $\int\limits_0^{\infty} e^{-st} \frac{1}{b-a} dt = \frac{e^{-sa}-e^{-sb}}{s(b-a)}$.

4. (26.01:6) $Exp(\lambda)$. $\tilde{X}$? $\int\limits_0^{\infty} e^{-st} \lambda e^{-\lambda t} dt = \frac{\lambda}{\lambda+s}$.

Calculus of important transforms

Recursive definition of $B$.




Definition of $A_x$, $B(x)$.

Calculus of $\widehat{A_t} = e^{-\lambda t(1-z)}$ (hint: $Pr(A_t = k) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}$).

**Theorem 1** (wo/proof). Let $Z = X + Y$, where $X$, $Y$ be non-negative continuous independent random variables. Then $\widetilde{Z}(s) = \widetilde{X}(s) \cdot \widetilde{Y}(s)$.

**Theorem 2** (wo/proof). Let $Z = \sum\limits_{i=1}^{X}$, where $Y_i \sim Y$ – non-negative continuous independent random variables, $X$ is independent from $Y_i$. Then $\widetilde{Z}(s) = \widehat{X}(\widetilde{Y}(s))$.

Calculus of $\widetilde{B(x)}(s) = e^{-x(s+\lambda-\lambda\widetilde{B}(s))}$.

**Theorem 3** (wo/proof). Let $X$, $A$ и $B$ be non-negative continuous random variables, moreover

$$X = \begin{cases} A & \text{with probability } p \\ B & \text{with probability } 1-p \end{cases}$$

Then we have $\widetilde{X}(s) = p\widetilde{A}(s) + (1-p)\widetilde{B}(s)$.

**Theorem 4** (wo/proof). Let $X_Y$, $Y$ be non-negative random variables, where $X_Y$ is continuously depended on $Y$ and has p.d.f. $f_Y(y)$. Then we have

$$\widetilde{X_Y}(s) = \int\limits_0^{\infty} \widetilde{X_y}(s) f_Y(y) dy.$$

Calculus of $\widetilde{B}(s) = \widetilde{S}(s + \lambda - \lambda\widetilde{B}(s))$.

**Theorem 5**. Let $X$ – be non-negative random variable. Then we have $EX^n = (-1)^n \widetilde{X}^{(n)}\Big|_{s=0}$.

Problems

1. Prove that for each random variable $X$ we have

   (a) $\widetilde{X}(0) = 1$

   (b) $\widehat{X}(1) = 1$

2. Define packet length distribution by the rules below:

$$S = \begin{cases} 0 & \text{with probability } q \\ \frac{1}{1-q} & \text{with probability } 1-q \end{cases}$$

   Find $ES$ by using theorems 3 and 5.

3. Express $EB$ in terms of $S$, $\rho$.