Problem List 2

(1) (10 points) **Stigler diet**. The diet planning problem is considered. There are 77 foods reached of 9 nutrients. Recommended daily consumption of nutrient i is defined by number k_i. For each food j and nutrient i the amount of nutrient per 1\$ spent on this food is defined by $s_{j,i}$. The goal is to find amounts of money spent for each type of food in 365 days and consume all nutrients in numbers which ratio to required consumption lays in $[1-\delta, 1+\delta]$. Objective – minimize amount of total spent money. Create Linear programming model and find the solutions for given data set and $\delta = 0.1$.

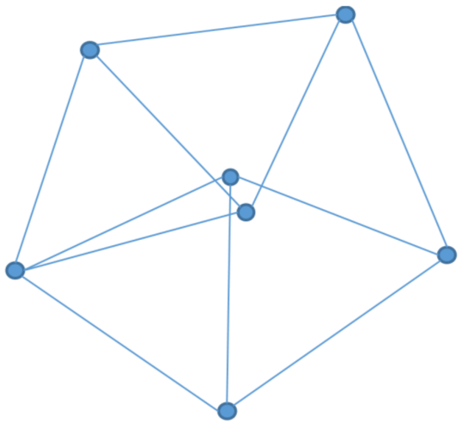
(2) (10 points) **Sudoku puzzle**. Sudoku puzzle objective is to fill a 9×9 grid with digits so that each column, each row, and each of the nine 3×3 boxes contain all of the digits from 1 to 9. Create Integer Programming model and find the solution of the following instance.

			8		1			
							4	3
5								
				7		8		
						1		
6							7	5
		3	4					
			2			6		

(3) (15 points) **Function**. Consider the problem $\max f$, where $f : [0,1] \to R$ subject to $f(x) + f(y) + f(z) \leq xyz + 1$. Create Mixed-Integer Programming model of this problem, consider discretion $\{0/N, 1/N, \ldots, N/N\}$ of function domain $[0,1]$ and find approximate solutions for $N = 5, 8, 30$.

(4) (20 points) **No 4-cycles**. How many edges can be in the graph with n vertices without cycles of the length 4? Create Integer Programming model and find solution for $n = 8, 10, 12$.

(5) (10 points) **Plane chromatic number**. We can prove that chromatic number of plane is not less than 4 using the following graph. Create Integer Programming model to check if the vertices of graph on the picture can be colored in 3 colors such that each edge connect vertices of different color.



(6) (15 points) **Shortest path.** There is a direct graph $G(V, E)$. For each edge $(i, j) \in V$ its length is defined by $l_{ij} = l_{ji}$. If there is no edge from i to j, then $l_{ij} = -1$ and $l_{ii} = 0$ for each $i \in V$. The objective is to find shortest path from a to b. Create Integer Programming model and find solutions for three different pairs of vertices defined in the attached materials.

(7) (20 points) **Assignment with minimal cost.** There is a set of workers W and a set of tasks N, such that $|W| = |N|$ and assignment cost function c_{ij} defined for each $i \in W, j \in N$. The objective is to assign workers on tasks with minimal cost. The correct Integer Programming model is described below.

- Binary variables: $\forall i \in W, j \in N x_{ij}$ equals to 1 iff worker i assigned on task i.
- Objective: $\min \sum_{i \in W} \sum_{j \in N} c_{ij} x_{ij}$.
- Constraints:
  - each worker should be assigned on one task: $\forall i \in W : \sum_{j \in N} x_{ij} = 1$;
  - each task should be assigned to one worker: $\forall j \in N : \sum_{i \in W} x_{ij} = 1$.

Prove that for this model fractional solution of linear relaxation is optimal solution of Integer Programming model. There is no need to implement model, only theoretical proof is required.