Task
Let f_n be the Fibonacci sequence $f_1 = 1,\ f_2 = 2$
Prove that each positive integer admits a unique representation in a form
$a_1 f_1 + a_2 f_2 + \ldots + a_n f_n + \ldots$ such that

- each a_i is either 0 or 1
- there are finitely many numbers a_i equal to 1
- no two consequent numbers a_i are equal to 1

Solution
We'll proof this statement by induction
If we can represent $1, \ldots, n$ as sum of Fibonacci numbers, we also can represent $n + 1$ Basis:

$$1 = f_1$$
$$2 = f_2$$
$$3 = f_3$$
$$4 = f_1 + f_3$$

Step:
If $n + 1$ is a Fibonacci number, it is itself representation.
Suppose that $n + 1$ isn't a Fibonacci number, then

$$\exists i \in \mathbb{Z} : F_i < n + 1 < F_{i+1}$$
$$n + 1 - F_i = m$$
$$m < F_i$$

Then we can represent $n + 1$ as representation of m plus F_i

Then proof the uniqueness
$n \in \mathbb{N}$ and n has 2 representations: A_1, A_2
Let $A_1' = A_1/A_2,\ A_2' = A_2/A_1$, so $A_1' \cap A_2' = \varnothing$ and $A_1',\ A_2'$ represent the same number.
Suppose the the largest element of A_1' larger then the largest element of A_2', but it means that A_1' represent larger number, cause sum of elements of A_2' less then the largest element of A_1' (else A_2' should have this number in it's representation). It follows that there cannot be 2 different representations.