

$$k_1 = k_4 = 3k$$

$$k_2 = k_3 = 2k$$

$$a) \begin{cases} m\ddot{x} = -k_1x - k_2(x-y) \\ m\ddot{y} = k_2(x-y) - k_3(y-z) \\ m\ddot{z} = k_3(y-z) - k_4z \end{cases}$$

$$b) \ddot{X} = -AX$$

$$A = \frac{1}{m} \begin{pmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3+k_4 \end{pmatrix} = \frac{k}{m} \begin{pmatrix} 5 & -2 & 0 \\ -2 & 4 & -2 \\ 0 & -2 & 5 \end{pmatrix}$$

$$\begin{vmatrix} 5-\lambda & -2 & 0 \\ -2 & 4-\lambda & -2 \\ 0 & -2 & 5-\lambda \end{vmatrix} = -\lambda^3 + 14\lambda^2 - 57\lambda + 60 = -(\lambda-5)(\lambda^2 - 9\lambda + 12)$$

$$\lambda_1 = 5$$

$$\lambda_{2,3} = \frac{9 \pm \sqrt{33}}{2}$$

$$\omega_i^2 = \frac{\lambda_i k}{m} \Rightarrow \omega_1 = \sqrt{\frac{5k}{m}}, \omega_2 = \sqrt{\frac{(9+\sqrt{33})k}{2m}}, \omega_3 = \sqrt{\frac{(9-\sqrt{33})k}{2m}}$$

$$\lambda_1) \begin{pmatrix} 0 & -2 & 0 \\ -2 & 1 & -2 \\ 0 & -2 & 0 \end{pmatrix} = \begin{pmatrix} 2 & -1 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{cases} 2x_1 - x_2 + 2x_3 = 0 \\ x_2 = 0 \end{cases} \Rightarrow \psi_1 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

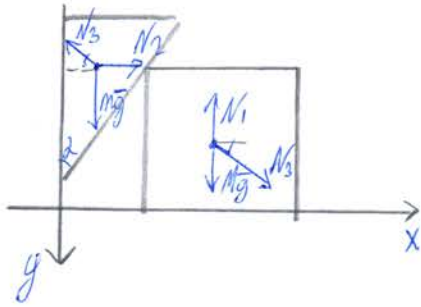
$$\lambda_2) \begin{pmatrix} 5-\lambda_2 & -2 & 0 \\ -2 & 4-\lambda_2 & -2 \\ 0 & -2 & 5-\lambda_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & \frac{\sqrt{33}-1}{4} \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{cases} x_1 - x_3 = 0 \\ x_2 + \frac{\sqrt{33}-1}{4}x_3 = 0 \end{cases} \Rightarrow \psi_2 = \begin{pmatrix} +1 \\ \frac{1-\sqrt{33}}{4} \\ 1 \end{pmatrix}$$

$$\lambda_3) \begin{pmatrix} 5-\lambda_3 & -2 & 0 \\ -2 & 4-\lambda_3 & -2 \\ 0 & -2 & 5-\lambda_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -\frac{\sqrt{33}+1}{4} \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{cases} x_1 - x_3 = 0 \\ x_2 - \frac{\sqrt{33}+1}{4}x_3 = 0 \end{cases} \Rightarrow \psi_3 = \begin{pmatrix} +1 \\ \frac{1+\sqrt{33}}{4} \\ 1 \end{pmatrix}$$

Orbital 15:

$$\omega_1 = \sqrt{\frac{5k}{m}}, \omega_2 = \sqrt{\frac{(9+\sqrt{33})k}{2m}}, \omega_3 = \sqrt{\frac{(9-\sqrt{33})k}{2m}}$$

$$\psi_1 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \psi_2 = \begin{pmatrix} +1 \\ \frac{1-\sqrt{33}}{4} \\ 1 \end{pmatrix}, \psi_3 = \begin{pmatrix} +1 \\ \frac{1+\sqrt{33}}{4} \\ 1 \end{pmatrix}$$

N2

a) # уравнений движения

$$2-1=1$$

$$\begin{cases} M\ddot{x} = N_3 \cos \alpha \\ 0 = N_2 - N_3 \cos \alpha \\ 0 = N_1 - N_3 \sin \alpha - Mg \\ m\ddot{y} = mg - N_3 \sin \alpha \\ \ddot{x} = \ddot{y} \tan \alpha \end{cases}$$

b) $N_2 = N_3 \cos \alpha$

$$\ddot{x} = \frac{N_3 \cos \alpha}{M}$$

$$N_1 = N_3 \sin \alpha + Mg$$

$$\ddot{y} = g - \frac{N_3}{m} \sin \alpha$$

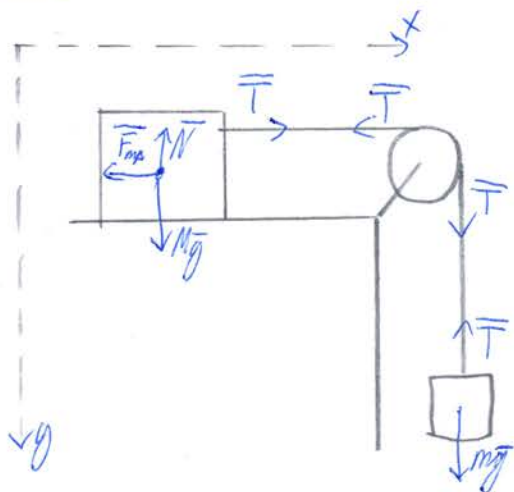
$$\frac{N_3 \cos \alpha}{M} = g \tan \alpha - \frac{N_3}{m} \sin \alpha \tan \alpha$$

$$N_3 \left(\frac{\cos \alpha}{M} + \frac{\sin \alpha \tan \alpha}{m} \right) = g \tan \alpha \Rightarrow N_3 = \frac{g \tan \alpha M m}{m \cos \alpha + M \sin \alpha \tan \alpha}$$

$$\ddot{x} = \frac{mg \sin \alpha}{m \cos \alpha + M \sin \alpha \tan \alpha}$$

$$\ddot{y} = \frac{mg \cos \alpha + Mg \sin \alpha \tan \alpha - Mg \sin \alpha \tan \alpha}{m \cos \alpha + M \sin \alpha \tan \alpha} = \frac{mg \cos \alpha}{m \cos \alpha + M \sin \alpha \tan \alpha} = \frac{mg \cos^2 \alpha}{m \cos^2 \alpha + M \sin^2 \alpha}$$

1/3



если ~~μN~~ $|\mu N| > |T|$, то движение не происходит и

$$T = mg \quad N = Mg$$

$$\mu Mg > mg, \quad \mu > \frac{m}{M}$$

а) # степеней свободы $2 - 1 = 1$

$$д) \begin{cases} m\ddot{y} = mg - T \\ M\ddot{x} = T - \mu N \\ N = mg \\ \ddot{x} = \ddot{y} \end{cases}$$

$$б) \begin{cases} \ddot{y} = g - \frac{T}{m} \\ \ddot{x} = \frac{T}{M} - \frac{\mu Mg}{M} \end{cases}$$

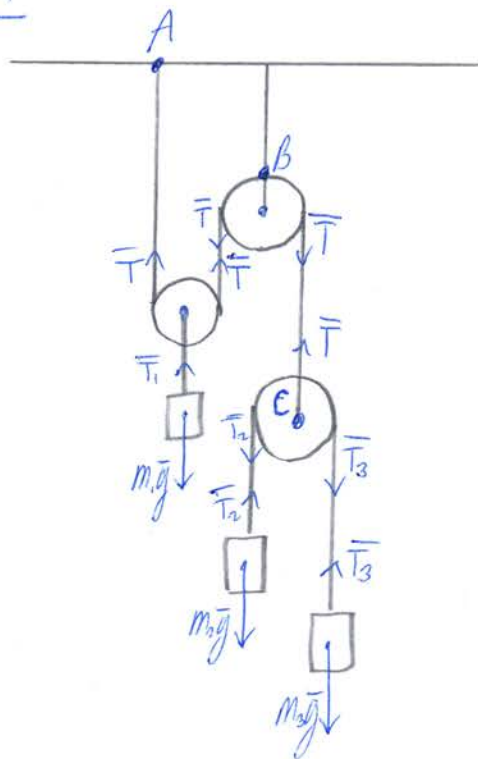
$$g - \frac{T}{m} = \frac{T}{M} - \mu g$$

$$T\left(\frac{1}{m} + \frac{1}{M}\right) = g(1 + \mu)$$

$$T = \frac{mMg(1 + \mu)}{m + M}$$

$$|\ddot{x} = \ddot{y} = g - \frac{Mg(1 + \mu)}{m + M} = \frac{mg + Mg - Mg - Mg\mu}{m + M} = \frac{m - M\mu}{m + M}g$$

N4



$$\begin{aligned} m_1 \ddot{x}_1 &= m_1 g - T_1 \\ m_2 \ddot{x}_2 &= m_2 g - T_2 \\ m_3 \ddot{x}_3 &= m_3 g - T_3 \\ T_2 &= T_3 = \frac{T}{2} \\ T_1 &= 2T \end{aligned}$$

Пусть длина AB - l_{AB} , BC - l_{BC}

Если блок C неподвижен, то $\ddot{x}_2 + \ddot{x}_3 = 0$

$$\begin{cases} x_2 = \tilde{x}_2 + \Delta l_{BC} \Rightarrow \tilde{x}_2 = x_2 - \Delta l_{BC} \\ x_3 = \tilde{x}_3 + \Delta l_{BC} \Rightarrow \tilde{x}_3 = -x_2 + 2\Delta l_{BC} \Rightarrow \Delta l_{BC} = \frac{x_2 + x_3}{2} \\ \ddot{\tilde{x}}_2 + \ddot{\tilde{x}}_3 = 0 \end{cases}$$

Для нити через веревку 2 блока: $l_{AB} + 2x_1 + l_{BC} + \frac{x_2 + x_3}{2} = \text{const}$
 $\Rightarrow 4\ddot{x}_1 + \ddot{x}_2 + \ddot{x}_3 = 0$

$$\begin{aligned} \ddot{x}_1 &= g - \frac{2T}{m_1} \\ \ddot{x}_2 &= g - \frac{T}{2m_2} \\ \ddot{x}_3 &= g - \frac{T}{2m_3} \end{aligned} \Rightarrow 4g - \frac{8T}{m_1} + g - \frac{T}{2m_2} + g - \frac{T}{2m_3} = 0$$

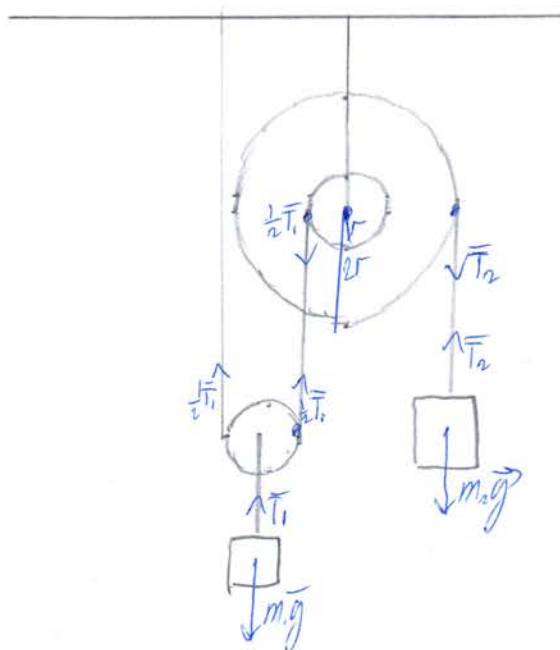
$$\Rightarrow T \left(\frac{8}{m_1} + \frac{1}{2m_2} + \frac{1}{2m_3} \right) = 6g \Rightarrow T = \frac{12 m_1 m_2 m_3 g}{16 m_2 m_3 + m_1 m_3 + m_1 m_2}$$

$$\Rightarrow \ddot{x}_1 = \frac{16 m_2 m_3 g + m_1 m_3 g + m_1 m_2 g - 24 m_2 m_3 g}{16 m_2 m_3 + m_1 m_3 + m_1 m_2} = \frac{m_1 m_3 + m_1 m_2 - 8 m_2 m_3}{16 m_2 m_3 + m_1 m_3 + m_1 m_2} g$$

$$\ddot{x}_2 = \frac{16 m_2 m_3 + m_1 m_2 - 5 m_1 m_3}{16 m_2 m_3 + m_1 m_3 + m_1 m_2} g$$

$$\ddot{x}_3 = \frac{16 m_2 m_3 + m_1 m_3 - 5 m_1 m_2}{16 m_2 m_3 + m_1 m_3 + m_1 m_2} g$$

степеней свободы $3 - 1 = 2$



$$m_1 \ddot{x}_1 = m_1 g - T_1$$

$$m_2 \ddot{x}_2 = m_2 g - T_2$$

$$T_2 = \frac{1}{2} \cdot \frac{1}{2} T_1 = \frac{1}{4} T_1$$

степеней свободы $2 - 1 = 1$

Пусть катушка повернется на φ по часовой, изменив длины нити через малую катушку Δl_1 , через большую Δl_2

$$\Delta l_1 = -r\varphi = -\frac{x_2}{2}$$

$$\Delta l_2 = x_2 = 2r\varphi$$

$$\Rightarrow x_1 = \frac{\Delta l_1}{2} = -\frac{x_2}{4} \Rightarrow 4x_1 = -x_2$$

$$\Rightarrow 4\ddot{x}_1 = -\ddot{x}_2$$

$$\ddot{x}_1 = g - \frac{T_1}{m_1} = g - \frac{4T_2}{m_1}$$

$$\ddot{x}_2 = g - \frac{T_2}{m_2}$$

$$4g - \frac{16T_2}{m_1} = -g + \frac{T_2}{m_2} \Rightarrow 5g = T_2 \left(\frac{16}{m_1} + \frac{1}{m_2} \right) \Rightarrow T_2 = \frac{5m_1 m_2 g}{16m_2 + m_1}$$

$$\ddot{x}_1 = g - \frac{20m_2 g}{16m_2 + m_1} = \frac{m_1 - 4m_2}{16m_2 + m_1} g$$

$$\ddot{x}_2 = g - \frac{5m_1 g}{16m_2 + m_1} = \frac{4(4m_2 - m_1)}{16m_2 + m_1} g$$

Nb

$$(\bar{e}_r, \bar{e}_\theta, \bar{e}_\varphi) = (\bar{e}_x, \bar{e}_y, \bar{e}_z) \begin{pmatrix} \sin \theta \cos \varphi & \cos \theta \cos \varphi & -\sin \varphi \\ \sin \theta \sin \varphi & \cos \theta \sin \varphi & \cos \varphi \\ \cos \theta & -\sin \theta & 0 \end{pmatrix}$$

$$r = (x^2 + y^2 + z^2)^{\frac{1}{2}} \quad \tan \theta = \frac{(x^2 + y^2)^{\frac{1}{2}}}{z} \quad \tan \varphi = \frac{y}{x}$$

$$\bar{e}_r = \bar{e}_x \sin \theta \cos \varphi + \bar{e}_y \sin \theta \sin \varphi + \bar{e}_z \cos \theta$$

$$\begin{aligned} \dot{\bar{e}}_r &= \bar{e}_x (\cos \theta \cos \varphi \dot{\theta} - \sin \theta \sin \varphi \dot{\varphi}) \\ &\quad + \bar{e}_y (\cos \theta \sin \varphi \dot{\theta} + \sin \theta \cos \varphi \dot{\varphi}) \\ &\quad + \bar{e}_z (-\sin \theta \dot{\theta}) \end{aligned}$$

$$= \dot{\theta} (\bar{e}_x \cos \theta \cos \varphi + \bar{e}_y \cos \theta \sin \varphi - \bar{e}_z \sin \theta) + \dot{\varphi} (-\bar{e}_x \sin \theta \sin \varphi + \bar{e}_y \sin \theta \cos \varphi)$$

$$= \dot{\theta} \bar{e}_\theta + \dot{\varphi} \sin \theta \bar{e}_\varphi \Rightarrow \underline{\dot{\bar{e}}_r = \dot{\theta} \bar{e}_\theta + \dot{\varphi} \sin \theta \bar{e}_\varphi}$$

$$\bar{e}_\theta = \bar{e}_x \cos \theta \cos \varphi + \bar{e}_y \cos \theta \sin \varphi - \bar{e}_z \sin \theta$$

$$\begin{aligned} \dot{\bar{e}}_\theta &= \bar{e}_x (-\sin \theta \cos \varphi \dot{\theta} - \cos \theta \sin \varphi \dot{\varphi}) \\ &\quad + \bar{e}_y (-\sin \theta \sin \varphi \dot{\theta} + \cos \theta \cos \varphi \dot{\varphi}) \\ &\quad - \bar{e}_z \cos \theta \dot{\theta} \end{aligned}$$

$$= -\dot{\theta} (\bar{e}_x \sin \theta \cos \varphi + \bar{e}_y \sin \theta \sin \varphi + \bar{e}_z \cos \theta)$$

$$+ \dot{\varphi} \cos \theta (-\bar{e}_x \sin \varphi + \bar{e}_y \cos \varphi)$$

$$= -\dot{\theta} \bar{e}_r + \dot{\varphi} \cos \theta \bar{e}_\varphi \Rightarrow \underline{\dot{\bar{e}}_\theta = -\dot{\theta} \bar{e}_r + \dot{\varphi} \cos \theta \bar{e}_\varphi}$$

$$\bar{e}_\varphi = -\bar{e}_x \sin \varphi + \bar{e}_y \cos \varphi$$

$$\begin{aligned} \dot{\bar{e}}_\varphi &= -\bar{e}_x \cos \varphi \dot{\varphi} - \bar{e}_y \sin \varphi \dot{\varphi} = -\dot{\varphi} (\sin^2 \theta \cos \varphi \bar{e}_x + \sin^2 \theta \sin \varphi \bar{e}_y \\ &\quad + \sin \theta \cos \theta \bar{e}_z + \cos^2 \theta \cos \varphi \bar{e}_x + \cos^2 \theta \sin \varphi \bar{e}_y - \sin \theta \cos \theta \bar{e}_z) = \end{aligned}$$

$$= -\dot{\varphi} (\cos \varphi + \sin \varphi) = -\dot{\varphi} (\sin \theta \bar{e}_r + \cos \theta \bar{e}_\theta) \Rightarrow \underline{\dot{\bar{e}}_\varphi = -\dot{\varphi} (\sin \theta \bar{e}_r + \cos \theta \bar{e}_\theta)}$$

№6 параметрические

$$\vec{r} = r \vec{e}_r$$

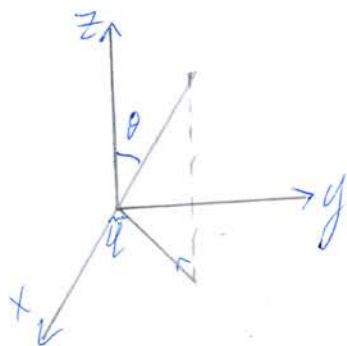
$$\dot{\vec{r}} = \dot{r} \vec{e}_r + r \dot{\vec{e}}_r = \dot{r} \vec{e}_r + r \dot{\theta} \vec{e}_\theta + r \dot{\varphi} \sin \theta \vec{e}_\varphi$$

$$\ddot{\vec{r}} = \ddot{r} \vec{e}_r + \dot{r} (\dot{\theta} \vec{e}_\theta + \dot{\varphi} \sin \theta \vec{e}_\varphi) + \dot{r} \dot{\theta} \vec{e}_\theta + r \ddot{\theta} \vec{e}_\theta + r \dot{\theta} (-\dot{\varphi} \vec{e}_r + \dot{\varphi} \cos \theta \vec{e}_\varphi) + \dot{r} \dot{\varphi} \sin \theta \vec{e}_\varphi +$$

$$+ r \dot{\varphi} \sin \theta \vec{e}_\varphi + r \dot{\varphi} \cos \theta \dot{\theta} \vec{e}_\varphi + r \dot{\varphi} \sin \theta (-\dot{\varphi} \sin \theta \vec{e}_r + \cos \theta \vec{e}_\theta) =$$

$$= \vec{e}_r (\ddot{r} - r \dot{\theta}^2 - r \dot{\varphi}^2 \sin^2 \theta) + \vec{e}_\theta (2 \dot{r} \dot{\theta} + r \ddot{\theta} - r \dot{\varphi}^2 \sin \theta \cos \theta) +$$

$$+ \vec{e}_\varphi (\dot{r} \dot{\varphi} \sin \theta + r \dot{\theta} \dot{\varphi} \cos \theta + \dot{r} \dot{\varphi} \sin \theta + r \ddot{\varphi} \sin \theta + r \dot{\varphi} \dot{\theta} \cos \theta)$$



$$\begin{cases} x = r \sin \theta \cos \varphi & \theta \in [0, \pi] \\ y = r \sin \theta \sin \varphi & \varphi \in [0, 2\pi] \\ z = r \cos \theta \end{cases}$$

$$\left| \frac{\partial(x, y, z)}{\partial(r, \theta, \varphi)} \right| = \begin{vmatrix} \sin \theta \cos \varphi & r \cos \theta \cos \varphi & -r \sin \theta \sin \varphi \\ \sin \theta \sin \varphi & r \cos \theta \sin \varphi & r \sin \theta \cos \varphi \\ \cos \theta & -r \sin \theta & 0 \end{vmatrix} =$$

$$= r^2 \sin \theta \begin{vmatrix} \sin \theta \cos \varphi & \cos \theta \cos \varphi & -\sin \varphi \\ \sin \theta \sin \varphi & \cos \theta \sin \varphi & \cos \varphi \\ \cos \theta & -\sin \theta & 0 \end{vmatrix} = r^2 \sin \theta = 0$$

$$\Leftrightarrow \begin{matrix} r=0 \\ \sin \theta=0 \\ \theta \in [0, \pi] \end{matrix} \Leftrightarrow \begin{matrix} r=0 \\ \theta=0 \\ \theta=\pi \end{matrix}$$

\Rightarrow Переход неинъективен на $[\mathbb{R}^3 \setminus \{0_z\}] \subset \mathbb{R}^3$