

$$\vec{F}: \mathbb{A}^3 \times \mathbb{R}^3 \rightarrow \mathbb{R}^3$$

$$\vec{F} = f_1 \vec{r} + f_2 \vec{p} + f_3 [\vec{r} \times \vec{p}]$$

$$\{x_i, p_j\} = \delta_{ij}$$

$$a) \{ \vec{F}, (\vec{M} \cdot \vec{n}) \} y$$

$$\vec{M} = [\vec{r} \times \vec{p}] = \epsilon_{ijk} r_i p_j$$

f_i - скалярное пр-вие от $\vec{r}^2, \vec{p}^2, (\vec{r}, \vec{p})$

$$(M g h) = \epsilon_{ijk} r_j p_k h_i$$

$$\{f_1 \vec{r} + f_2 \vec{p} + f_3 [\vec{r} \times \vec{p}], (\vec{M} \cdot \vec{n})\}$$

$$\textcircled{1} \quad \{f_1 \vec{r}, (\vec{M} \cdot \vec{n})\} y = f_1 \{\vec{r}, (\vec{M} \cdot \vec{n})\} + \vec{r} \{f_1, (\vec{M} \cdot \vec{n})\}$$

$$-f_1 \{r_e, \epsilon_{ijk} r_j p_k h_i\} y + \vec{r} \left(\frac{\partial f_1}{\partial \vec{r}^2} \{\vec{r}_e^2, (\vec{M} \cdot \vec{n})\} + \frac{\partial f_1}{\partial \vec{p}^2} \{\vec{p}_e^2, (\vec{M} \cdot \vec{n})\} + \frac{\partial f_1}{\partial (\vec{r} \cdot \vec{p})} \{(\vec{r} \cdot \vec{p}), (\vec{M} \cdot \vec{n})\} \right)$$

~~$$\textcircled{2} \quad \{f_1 \vec{r}, \vec{p}_e, \epsilon_{ijk} r_j p_k h_i\} = f_1 \epsilon_{ijk} r_j \{r_e, p_k\} h_i + \vec{r} \left(\frac{\partial f_1}{\partial \vec{r}^2} \{\vec{r}_e^2, \epsilon_{ijk} r_j p_k h_i\} + \right.$$~~

$$+ \frac{\partial f_1}{\partial \vec{p}^2} \{\vec{p}_e^2, \epsilon_{ijk} r_j p_k h_i\} + \frac{\partial f_1}{\partial (\vec{r} \cdot \vec{p})} \{(\vec{r} \cdot \vec{p}), \epsilon_{ijk} r_j p_k h_i\} =$$

$$= f_1 \epsilon_{ijk} r_j h_i + \vec{r} \left(\frac{\partial f_1}{\partial \vec{r}^2} \cdot 2 \vec{r}_e \{r_e, \epsilon_{ijk} r_j p_k h_i\} + \frac{\partial f_1}{\partial \vec{p}^2} \cdot 2 \vec{p}_e \{p_e, \epsilon_{ijk} r_j p_k h_i\} + \right. \\ \left. + \frac{\partial f_1}{\partial (\vec{r} \cdot \vec{p})} \left(\vec{r}_e \{p_e, \epsilon_{ijk} r_j p_k h_i\} + \vec{p}_e \{r_e, \epsilon_{ijk} r_j p_k h_i\} \right) \right) =$$

$$= -f_1 \epsilon_{ijk} r_j h_i + \vec{r} \left(\frac{\partial f_1}{\partial \vec{r}^2} \cdot 2 \vec{r}_e \cdot \epsilon_{ijk} r_j h_i + \frac{\partial f_1}{\partial \vec{p}^2} \cdot 2 \vec{p}_e \cdot (-\epsilon_{ijk} p_k h_i) + \right. \\ \left. + \frac{\partial f_1}{\partial (\vec{r} \cdot \vec{p})} \left(\vec{r}_e \cdot (-\epsilon_{ijk} p_k h_i) + \vec{p}_e \cdot \epsilon_{ijk} r_j h_i \right) \right) =$$

$$= -f_1 \cdot [\vec{r} \times \vec{n}]_e + \vec{r} \left(\frac{\partial f_1}{\partial \vec{r}^2} \cdot 2 \vec{r}_e (-[\vec{r} \times \vec{n}]_e) + \frac{\partial f_1}{\partial \vec{p}^2} \cdot 2 \vec{p}_e (-[\vec{p} \times \vec{n}]_e) + \right. \\ \left. + \frac{\partial f_1}{\partial (\vec{r} \cdot \vec{p})} \cdot (r_e (-[\vec{p} \times \vec{n}]_e) + p_e \cdot [\vec{r} \times \vec{n}]_e) \right) =$$

$$\textcircled{3} \Rightarrow \{f_1 \vec{p}, (\vec{M} \cdot \vec{n})\} y = -f_1 [\vec{r} \times \vec{n}] + \vec{r} \left(\frac{\partial f_1}{\partial \vec{r}^2} (-2) \cdot (\vec{r}, [\vec{r} \times \vec{n}]) + \right. \\ \left. + \frac{\partial f_1}{\partial \vec{p}^2} \cdot (-2) \cdot (\vec{p}, [\vec{p} \times \vec{n}]) + \frac{\partial f_1}{\partial (\vec{r} \cdot \vec{p})} \left(-(\vec{r}, [\vec{p} \times \vec{n}]) - (\vec{p}, [\vec{r} \times \vec{n}]) \right) \right) = -f_1 [\vec{r} \times \vec{n}]$$

$$\textcircled{4} \quad \{f_2 \vec{p}, (\vec{M} \cdot \vec{n})\} y = f_2 \{\vec{p}, (\vec{M} \cdot \vec{n})\} y + \vec{p} \{f_2, (\vec{M} \cdot \vec{n})\} y.$$

$$-f_2 \epsilon_{ijk} p_k h_i + \vec{p} \left[\frac{\partial f_2}{\partial \vec{p}^2} \cdot 2 \vec{p}_e \epsilon_{ijk} p_k h_i + \frac{\partial f_2}{\partial \vec{r}^2} \cdot 2 \vec{p}_e \cdot \epsilon_{ijk} r_j h_i + \frac{\partial f_2}{\partial (\vec{r} \cdot \vec{p})} (r_e \cdot (-\epsilon_{ijk} p_k h_i) + \right. \\ \left. + p_e \cdot \epsilon_{ijk} r_j h_i) \right] =$$

$$= -f_2 \cdot [\vec{p} \times \vec{n}]_e + \vec{p} \cdot \left[\frac{\partial f_2}{\partial \vec{p}^2} \cdot 2 \vec{p}_e (-[\vec{p} \times \vec{n}]_e) + \frac{\partial f_2}{\partial \vec{r}^2} \cdot 2 \vec{p}_e (-[\vec{r} \times \vec{n}]_e) + \right. \\ \left. + \frac{\partial f_2}{\partial (\vec{r} \cdot \vec{p})} \cdot (r_e \cdot (-[\vec{p} \times \vec{n}]_e) + p_e \cdot (-[\vec{r} \times \vec{n}]_e)) \right] =$$

$$\Rightarrow \{f_2 \vec{p}, (\vec{M} \cdot \vec{n})\} y = -f_2 [\vec{p} \times \vec{n}] + \vec{p} \left[\frac{\partial f_2}{\partial \vec{r}^2} (-2) (\vec{r}, [\vec{r} \times \vec{n}]) + \right. \\ \left. + \frac{\partial f_2}{\partial \vec{p}^2} \cdot (-2) \cdot (\vec{p}, [\vec{p} \times \vec{n}]) + \frac{\partial f_2}{\partial (\vec{r} \cdot \vec{p})} \cdot \left(-(\vec{r}, [\vec{p} \times \vec{n}]) - (\vec{p}, [\vec{r} \times \vec{n}]) \right) \right] = -f_2 [\vec{p} \times \vec{n}]$$

$$\textcircled{3} \quad \underbrace{\{f_3 \vec{M}, (\vec{M} \times \vec{n})\}}_{* f_3 \{ \vec{M}, \vec{M} \cdot \vec{n} \} + M \{ f_3, (\vec{M} \cdot \vec{n}) \}} = f_3 \{ M, (\vec{M} \cdot \vec{n}) \} + M \{ f_3, (\vec{M} \cdot \vec{n}) \} *$$

$$f_3 \{ M_e, M_k \} \frac{\partial(\vec{M} \cdot \vec{n})}{\partial M_k} + \vec{M} \cdot \left(\frac{\partial f_3}{\partial \vec{e}^2} \{ \vec{e}^2, (\vec{M} \times \vec{n}) \} + \frac{\partial f_3}{\partial \vec{p}^2} \{ \vec{p}^2, (\vec{M} \cdot \vec{n}) \} + \frac{\partial f_3}{\partial (\vec{e} \cdot \vec{p})} \cdot \{ (\vec{e} \cdot \vec{p}), (\vec{M} \cdot \vec{n}) \} \right).$$

$$f_3 \{ M_e, M_k \} \frac{\partial(\vec{M} \cdot \vec{n})}{\partial M_k} = f_3 \cdot \epsilon_{ek_i} \cdot M_i \cdot n_k = - f_3 \epsilon_{eik} M_i \cdot n_k = - f_3 [M \times n]_k$$

$$\Rightarrow f_3 \{ N, (\vec{M} \cdot \vec{n}) \} = - f_3 \cdot [M \times n].$$

Orter $\{F, (\vec{M} \cdot \vec{n})\} = - f_1 [\vec{e} \times \vec{n}] - f_3 [\vec{M} \times \vec{n}] - f_2 [\vec{p} \times \vec{n}]$

$$\textcircled{1} \quad \{ \vec{F}, \vec{M}^2 \} = \{ f_1 \vec{z} + f_2 \vec{p} + f_3 [\vec{z} \times \vec{p}], \vec{M}^2 \}$$

$$= f_1 \{ \vec{z}, \vec{M}^2 \} + \vec{z} \{ f_1, \vec{M}^2 \} =$$

$$= f_1 \cdot \underbrace{\{ \vec{z}, \vec{M}^2 \}}_{\{ z, M_i \} \cdot 2M_i} + \vec{z} \cdot \left(\frac{\partial f_1}{\partial \vec{z}^2} \{ \vec{z}^2, \vec{M}^2 \} + \frac{\partial f_1}{\partial \vec{p}^2} \{ \vec{p}^2, \vec{M}^2 \} + \frac{\partial f_1}{\partial (\vec{z} \cdot \vec{p})} \{ (\vec{z} \cdot \vec{p}), \vec{M}^2 \} \right)$$

$$- f_1 \cdot \underbrace{2M_i \{ z_e, \epsilon_{ijk} z_j p_k \}}_{\{ z_e, M_i \} \cdot 2M_i} + \vec{z} \cdot 2M_i \left(\frac{\partial f_1}{\partial \vec{z}^2} \{ \vec{z}_e^2, \epsilon_{ijk} z_j p_k \} + \frac{\partial f_1}{\partial \vec{p}^2} \{ \vec{p}_e^2, \epsilon_{ijk} z_j p_k \} + \frac{\partial f_1}{\partial (\vec{z} \cdot \vec{p})} \{ (\vec{z}_e \cdot \vec{p}_e), \epsilon_{ijk} z_j p_k \} \right) +$$

$$+ \frac{\partial f_1}{\partial (\vec{z} \cdot \vec{p})} \{ \vec{z}_e p_e, \epsilon_{ijk} z_j p_k \} =$$

$$= f_1 \cdot 2M_i \{ z_e, \epsilon_{ijk} z_j p_k \} + \vec{z} \cdot 2M_i \left(\frac{\partial f_1}{\partial \vec{z}^2} 2z_e \{ \vec{z}_e, \epsilon_{ijk} z_j p_k \} + \frac{\partial f_1}{\partial \vec{p}^2} 2p_e \{ p_e, \epsilon_{ijk} z_j p_k \} + \frac{\partial f_1}{\partial (\vec{z} \cdot \vec{p})} (z_e \{ p_e, \epsilon_{ijk} z_j p_k \} + p_e \{ z_e, \epsilon_{ijk} z_j p_k \}) \right)$$

$$+ \frac{\partial f_1}{\partial (\vec{z} \cdot \vec{p})} (z_e \{ p_e, \epsilon_{ijk} z_j p_k \} + p_e \{ z_e, \epsilon_{ijk} z_j p_k \})$$

$$\textcircled{2} \quad f_1 \cdot \underbrace{2M_i \epsilon_{ijl} z_j}_{2\epsilon_{lij} M_i z_j} + \vec{z} \cdot 2 \left(\frac{\partial f_1}{\partial \vec{z}^2} \cdot 2z_e \epsilon_{ijl} M_i z_j - \frac{\partial f_1}{\partial \vec{p}^2} 2p_e \epsilon_{ijl} M_i z_j + \frac{\partial f_1}{\partial (\vec{z} \cdot \vec{p})} (-z_e \epsilon_{ijl} M_i p_k + p_e \epsilon_{ijl} M_i z_j) \right)$$

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$$\textcircled{3} \quad \{ f_1 \vec{z}, \vec{M}^2 \} = 2f_1 \cdot [\vec{M} \times \vec{z}] + \vec{z} \cdot \left(4 \frac{\partial f_1}{\partial \vec{z}^2} ([\vec{z}, [\vec{M} \times \vec{z}]] + 4 \frac{\partial f_1}{\partial \vec{p}^2} ([\vec{p}, [\vec{M} \times \vec{p}]]) + 2 \frac{\partial f_1}{\partial (\vec{z} \cdot \vec{p})} ([\vec{z}, [\vec{M} \times \vec{p}]] + [\vec{p}, [\vec{M} \times \vec{z}]]) \right) = 2f_1 [\vec{M} \times \vec{z}]$$

$$\textcircled{4} \quad \{ f_2 \vec{p}, \vec{M}^2 \} = 2f_2 \cdot [\vec{M} \times \vec{p}] + \vec{p} \left(4 \frac{\partial f_2}{\partial \vec{z}^2} ([\vec{z}, [\vec{M} \times \vec{p}]] + 4 \frac{\partial f_2}{\partial \vec{p}^2} ([\vec{p}, [\vec{M} \times \vec{p}]]) + 2 \frac{\partial f_2}{\partial (\vec{z} \cdot \vec{p})} ([\vec{z}, [\vec{M} \times \vec{p}]] + [\vec{p}, [\vec{M} \times \vec{z}]]) \right)$$

$$\textcircled{5} \quad \{ f_3 \vec{M}, \vec{M}^2 \} = f_3 \{ \vec{M}, \vec{M}^2 \} + \vec{M} \{ f_3, \vec{M}^2 \} =$$

$$= f_3 \cdot (\vec{M}, \vec{M}^2) + \vec{M} \left(\frac{\partial f_3}{\partial \vec{z}^2} \cdot \{ \vec{z}^2, \vec{M}^2 \} + \frac{\partial f_3}{\partial (\vec{z} \cdot \vec{p})} \{ (\vec{z} \cdot \vec{p}), \vec{M}^2 \} + \frac{\partial f_3}{\partial \vec{p}^2} \{ \vec{p}^2, \vec{M}^2 \} \right)$$

$$+ f_3 \underbrace{2 \cdot \epsilon_{ijk} M_k M_j}_{\{ M_i, M_j \} \cdot 2M_i} = -2f_3 \cdot \epsilon_{ijk} M_k M_j + \underbrace{2 \frac{\partial f_3}{\partial (\vec{z} \cdot \vec{p})} \cdot (([\vec{z}, [\vec{M} \times \vec{p}]] + [\vec{p}, [\vec{M} \times \vec{z}]]))}_{\textcircled{6}}$$

$$\textcircled{7} \quad -2f_3 [\vec{M} \times \vec{M}]_i + \vec{M} \left(4 \frac{\partial f_3}{\partial \vec{z}^2} ([\vec{z}, [\vec{M} \times \vec{z}]] + [\vec{p}, [\vec{M} \times \vec{p}]]) + 2 \frac{\partial f_3}{\partial (\vec{z} \cdot \vec{p})} ([\vec{z}, [\vec{M} \times \vec{p}]] + [\vec{p}, [\vec{M} \times \vec{z}]]) \right)$$

$$\Rightarrow \{ f_3 \vec{M}, \vec{M}^2 \} = 0$$

$$\textcircled{8} \quad \begin{aligned} \beta^a \underbrace{\epsilon_{ijk}}_{i=1,2,3} &= \underbrace{\frac{\partial f_1}{\partial \vec{z}^2} ([\vec{z}, [\vec{M} \times \vec{z}]] + [\vec{p}, [\vec{M} \times \vec{p}]] + [\vec{z}, [\vec{M} \times \vec{p}]] + [\vec{p}, [\vec{M} \times \vec{z}]])}_{\textcircled{9}} \\ &\quad + \underbrace{\frac{\partial f_2}{\partial \vec{p}^2} ([\vec{z}, [\vec{M} \times \vec{p}]] + [\vec{p}, [\vec{M} \times \vec{z}]] + [\vec{z}, [\vec{M} \times \vec{p}]] + [\vec{p}, [\vec{M} \times \vec{z}]])}_{\textcircled{10}} \end{aligned}$$

$$\text{Target: } 2f_1 [\vec{M} \times \vec{z}] + 2f_2 [\vec{M} \times \vec{p}]$$

$$L = \frac{m\dot{u}^2}{2} - \frac{mw^2 u^2}{2}$$

N2.

$$\ddot{u} = \frac{du}{dt}$$

$$a) p = \frac{\partial L}{\partial \dot{u}} = m\dot{u} \quad \dot{u} = \frac{p}{m}$$

$$H = p \cdot \dot{u} - L = \frac{p^2}{m} - L = \frac{p^2}{m} - \frac{m \cdot \frac{p^2}{m^2}}{2} + \frac{mw^2 u^2}{2} = \frac{p^2}{m} - \frac{p^2}{2m} + \frac{mw^2 u^2}{2} =$$

$$= \boxed{\frac{p^2}{2m} + \frac{mw^2 u^2}{2}}$$

$$\delta) a = \sqrt{\frac{mw}{2}} \left(u + i \cdot \frac{p}{mw} \right)$$

$$\bar{a} = \sqrt{\frac{mw}{2}} \left(u - i \cdot \frac{p}{mw} \right)$$

$$a \cdot \bar{a} = \frac{mw}{2} \left(u^2 + \frac{p^2}{m^2 w^2} \right) = \frac{mw u^2}{2} + \frac{p^2}{2mw}$$

$$H = \boxed{a \cdot \bar{a} \cdot w} = \frac{p^2}{2m} + \frac{mw^2 u^2}{2}$$

$$b) \{a, \bar{a}\} = \frac{\partial a}{\partial x} \{u, \bar{a}\} + \frac{\partial a}{\partial p} \{p, \bar{a}\} = \sqrt{\frac{mw}{2}} \left\{ u, \bar{a} \right\} \stackrel{0}{\frac{\partial \bar{a}}{\partial u}} + \frac{i}{\sqrt{2mw}} \left\{ p, \bar{a} \right\} \stackrel{-1}{\frac{\partial \bar{a}}{\partial u}} + \sqrt{\frac{mw}{2}} \left\{ u, p \right\} \stackrel{1}{\frac{\partial \bar{a}}{\partial p}} + \frac{i}{\sqrt{2mw}} \left\{ p, p \right\} \stackrel{0}{\frac{\partial \bar{a}}{\partial p}} = -\frac{i}{\sqrt{2mw}} + \sqrt{\frac{mw}{2}} \cdot \left(-\frac{i}{\sqrt{2mw}} \right) = -\frac{i}{2} - \frac{i}{2} = \boxed{-i}$$

$$\{a, H\} = \{a, a \bar{a} w\} = \{a, a \bar{a}\} \cdot w = (\bar{a} \{a, a\} + a \{a, \bar{a}\}) \cdot w \quad \textcircled{S}$$

$$2) \begin{cases} \dot{p} = -\frac{\partial H}{\partial x} = -mw^2 u \\ \dot{u} = \frac{\partial H}{\partial p} = \frac{p}{m} \end{cases}$$

$$H = a \cdot \bar{a} \cdot w$$

$$\textcircled{S} \boxed{-a i w}$$

$$\dot{a} = \sqrt{\frac{mw}{2}} \left(\dot{u} + i \frac{\dot{p}}{mw} \right) = \sqrt{\frac{mw}{2}} \cdot \left(\frac{\partial H}{\partial a} \cdot \frac{\partial a}{\partial p} - i \cdot \frac{\partial H}{\partial a} \cdot \frac{\partial a}{\partial u} \cdot \frac{1}{mw} \right) +$$

$$+ \sqrt{\frac{mw}{2}} \cdot \left(\frac{\partial H}{\partial \bar{a}} \cdot \frac{\partial \bar{a}}{\partial p} - i \cdot \frac{\partial H}{\partial \bar{a}} \cdot \frac{\partial \bar{a}}{\partial u} \cdot \frac{1}{mw} \right) =$$

$$= \sqrt{\frac{mw}{2}} \left(\bar{a} w \cdot \frac{1}{\sqrt{2mw}} - \frac{i}{mw} \cdot \bar{a} \cdot w \cdot \sqrt{\frac{mw}{2}} \right) +$$

$$+ \sqrt{\frac{mw}{2}} \cdot \left(a w \cdot \left(-\frac{i}{\sqrt{2mw}} \right) - \frac{i}{mw} a w \cdot \sqrt{\frac{mw}{2}} \right) =$$

$$= \frac{\bar{a} \cdot w}{2} - \frac{i \bar{a} \cdot w}{2} - \frac{i \cdot a w}{2} - \frac{i a w}{2} = -a i w$$

$$\Rightarrow \dot{a} = -a i w$$

$$\boxed{a = e^{i \omega t} e^{-i \omega t}}$$

N3.

$$L = \frac{m}{2} (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{qB}{2c} (x\dot{y} - y\dot{x})$$

$$a) \dot{p}_1 = \frac{\partial L}{\partial \dot{x}} = m\ddot{x} - \frac{qB}{2c} \cdot y \quad \ddot{x} = \frac{p_1 + \frac{qB}{2c} \cdot y}{m}$$

$$p_2 = \frac{\partial L}{\partial \dot{y}} = m\ddot{y} + \frac{qB}{2c} \cdot x \quad \ddot{y} = \frac{p_2 - \frac{qB}{2c} \cdot x}{m}$$

$$p_3 = \frac{\partial L}{\partial \dot{z}} = m\ddot{z} \quad \ddot{z} = \frac{p_3}{m}$$

$$H = p_1 \dot{x} + p_2 \dot{y} + p_3 \dot{z} - \frac{m}{2} (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - \frac{qB}{2c} (x\dot{y} - y\dot{x}) =$$

$$= \frac{p_1^2 + \frac{p_1 qB}{2c} \cdot y}{m} + \frac{p_2^2 - \frac{p_2 qB}{2c} \cdot x}{m} + \frac{p_3^2}{m} - \frac{1}{2m} \left(p_1^2 + p_1 \cdot \frac{qB}{c} \cdot y + \frac{q^2 p_1^2}{4c^2} \cdot y^2 + \right.$$

$$\left. + p_2^2 - p_2 \cdot \frac{qB}{c} \cdot x + \frac{q^2 p_2^2}{4c^2} \cdot x^2 + p_3^2 \right) - \frac{qB}{2c} \left(\frac{p_2 x - \frac{qB}{2c} \cdot u^2}{m} - \frac{p_1 y + \frac{qB}{2c} \cdot y^2}{m} \right) =$$

$$= \frac{1}{m} \left(\frac{p_1^2}{2} + \frac{q^2 p_1^2}{8c^2} \cdot p_1 + \frac{p_2^2}{2} - \frac{q^2 p_2^2}{8c^2} \cdot p_2 + \frac{p_3^2}{2} - \frac{p_1^2}{2} - \frac{q^2 p_1^2}{8c^2} \cdot p_1 - \frac{q^2 p_2^2 y^2}{8c^2} \right) -$$

$$- \frac{p_2^2}{2} + \frac{q^2 p_2^2}{8c^2} \cdot p_2 - \frac{q^2 p_2^2 x^2}{8c^2} - \frac{p_3^2}{2} - \frac{q^2 p_2^2}{8c^2} \cdot p_2 + \frac{q^2 p_2^2 x^2}{8c^2} + \frac{q^2 p_3^2}{8c^2} \cdot p_2 + \frac{q^2 p_3^2 y^2}{8c^2}$$

$$= \frac{1}{m} \left(\frac{p_1^2}{2} + \frac{p_2^2}{2} + \frac{p_3^2}{2} + \frac{q^2 p_2^2 y^2}{8c^2} + \frac{q^2 p_2^2 x^2}{8c^2} - \frac{q^2 p_2 p_2}{2c} - \frac{q^2 p_3 p_3}{2c} \right).$$

$$\begin{cases} \dot{p}_1 = - \frac{\partial H}{\partial x} = \left(- \frac{q^2 p_2^2 x}{4c^2} + \frac{q^2 p_2 p_2}{2c} \right) \cdot \frac{1}{m} \\ \dot{p}_2 = - \frac{\partial H}{\partial y} = \left(- \frac{q^2 p_2^2 y}{4c^2} - \frac{q^2 p_3 p_1}{2c} \right) \cdot \frac{1}{m} \end{cases}$$

$$\begin{cases} \dot{p}_3 = - \frac{\partial H}{\partial z} = 0 \quad p_3 = \text{const}, \text{ w.K. } p_3(0) = p, \text{ so } p_3(t) = p \\ \dot{x} = \frac{\partial H}{\partial p_1} = \frac{1}{m} \left(p_1 + \frac{q^2 p_2 y}{2c} \right) \quad x(0) = y(0) = z(0) = 0 \\ \dot{y} = \frac{\partial H}{\partial p_2} = \frac{1}{m} \left(p_2 - \frac{q^2 p_3 x}{2c} \right) \quad p_1(0) = p_3(0) = p \\ \dot{z} = \frac{\partial H}{\partial p_3} = \frac{1}{m} \cdot p_3 = \frac{p}{m} \quad p_2(0) = 0 \end{cases}$$

$$\text{T.K. } z(0) = 0, \text{ so } z = \frac{p}{m} t$$

$$\text{Durch } A = \begin{pmatrix} x \\ y \\ p_1 \\ p_2 \end{pmatrix}$$

$$\text{Drehzahl } N = \frac{qB}{2cm}$$

$$A = \underbrace{\begin{pmatrix} 0 & N & \frac{1}{m} & 0 \\ -N & 0 & 0 & \frac{1}{m} \\ -mN^2 & 0 & 0 & N \\ 0 & -mN^2 & -N & 0 \end{pmatrix}}_{\text{B}} \cdot A$$

$$-N \cdot \begin{pmatrix} -N & 0 & \frac{1}{m} \\ -mN^2 & 0 & N \\ 0 & -N & 0 \end{pmatrix} = N^3 - N^3 = 0$$

$$\frac{1}{m} \cdot \begin{pmatrix} -N & 0 & \frac{1}{m} \\ -mN^2 & 0 & N \\ 0 & -mN^2 & 0 \end{pmatrix} = mN^4 - mN^4 = 0$$

$$\chi_B(\lambda) = \lambda^4 + (2N^2 + N^2 + N^2)\lambda^2 = \lambda^4 + 4N^2\lambda^2 = \lambda^2(\lambda^2 + 4N^2)$$

$$\Rightarrow \lambda_1 = 0, \lambda_2 = 2iN, \lambda_3 = -2iN$$

$$\lambda_1 = 0;$$

$$Bu = \lambda_1 u$$

$$u := \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

$$\begin{pmatrix} BN + \frac{c}{m} \\ -aN + \frac{d}{m} \\ -amN^2 + Nd \\ -bmN^2 - Nc \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

zugehöriger:

$$\begin{pmatrix} 1 \\ mN \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -\frac{1}{mN} \\ 1 \\ 0 \end{pmatrix}.$$

$$\lambda_2 = 2Ni: Bu = \lambda_2 u$$

$$\begin{pmatrix} BN + \frac{c}{m} \\ -aN + \frac{d}{m} \\ -amN^2 + Nd \\ -bmN^2 - Nc \end{pmatrix} = \begin{pmatrix} 2Ni \cdot a \\ 2Ni \cdot b \\ 2Ni \cdot c \\ 2Ni \cdot d \end{pmatrix}$$

zugehöriger:

$$\begin{pmatrix} -i \\ 1 \\ mN \\ mNi \end{pmatrix}$$

$$\lambda_3 = -2Ni: Bu = \lambda_3 u$$

$$\begin{pmatrix} BN + \frac{c}{m} \\ -aN + \frac{d}{m} \\ -amN^2 + Nd \\ -bmN^2 - Nc \end{pmatrix} = \begin{pmatrix} -2Ni \cdot a \\ -2Ni \cdot b \\ -2Ni \cdot c \\ -2Ni \cdot d \end{pmatrix}$$

$$\text{zugehöriger: } \begin{pmatrix} i \\ +1 \\ +mN \\ -mNi \end{pmatrix}$$

$$A = \begin{pmatrix} x \\ y \\ p_1 \\ p_2 \end{pmatrix} = C_0 \cdot \begin{pmatrix} \frac{1}{mN} \\ 0 \\ 1 \\ 1 \end{pmatrix} + C_1 \cdot \begin{pmatrix} 0 \\ -\frac{1}{mN} \\ 1 \\ 0 \end{pmatrix} + C_2 \cdot \begin{pmatrix} -i \\ 1 \\ mN \\ mNi \end{pmatrix}^{2Ni t} + C_3 \cdot \begin{pmatrix} i \\ 1 \\ mN \\ -mNi \end{pmatrix}^{-2Ni t} \cdot e^{2Ni t}$$

$$M(0) = C_0 \cdot \frac{1}{mN} - C_2 \cdot i + C_3 \cdot i = 0$$

$$y(0) = -C_1 \cdot \frac{1}{mN} + C_2 + C_3 = 0$$

$$p_1(0) = C_1 + C_2 \cdot mN + C_3 \cdot mN = p$$

$$p_2(0) = C_0 + C_2 \cdot mNi - C_3 \cdot mNi = 0$$

$$C_0 \cdot \frac{1}{mN} = i(C_2 - C_3)$$

$$C_0 = mNi(C_3 - C_2)$$

$$mNi(C_2 - C_3) = mNi(C_3 - C_2)$$

$$2C_2 = 2C_3 \quad |C_2 = C_3$$

$$+ \begin{cases} -C_1 \cdot \frac{1}{mN} + C_3 \cdot mN = 0 \\ C_1 + C_3 \cdot mN = p \end{cases}$$

$$2C_3 \cdot mN = p$$

$$C_3 = \frac{p}{2mN}$$

$$-\frac{C_1}{mN} + \frac{p}{2mN} = 0$$

$$-2C_1 + p = 0 \quad |C_1 = \frac{p}{2}$$

$$C_0 = mNi \left(\frac{p}{2mN} - 0 \right) = \frac{p \cdot i}{2}$$

$$i(C_2 + C_3) = C_0 \cdot \frac{1}{mN}$$

$$C_2 + C_3 = C_1 \cdot \frac{1}{mN}$$

$$\Rightarrow \frac{C_0}{mNi} = \frac{C_1}{mNi}$$

$$C_0 = iC_1$$

$$mNi(C_2 + C_3) = C_1$$

$$C_2 + C_3 = \frac{-C_1}{mN} = \frac{C_1}{mN} \Rightarrow C_1 = 0$$

$$C_2 = -C_3$$

$$+ C_1 + 2C_2 \cdot mN = p$$

$$C_0 = 0$$

$$-\frac{C_1}{mN} + 2C_2 = 0$$

$$-C_1 + 2C_2 \cdot mN = 0$$

$$4C_2 \cdot mN = p$$

$$C_2 = \frac{p}{4mN} = C_3$$

$$C_1 = \frac{p}{2}$$

mit

$$\begin{pmatrix} u \\ y \\ p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} -\frac{pi}{4mN} \cdot e^{2Nit} + \frac{pi}{4mN} \cdot e^{-2Nit} \\ \frac{p}{4mN} \cdot e^{2Nit} + \frac{p}{4mN} \cdot e^{-2Nit} - \frac{p}{2mN} \\ \frac{p}{4} \cdot e^{2Nit} + \frac{p}{4} \cdot e^{-2Nit} + \frac{p}{2} \\ \frac{pi}{4} \cdot e^{2Nit} - \frac{pi}{4} \cdot e^{-2Nit} \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{p}{8mN} \cdot \sin(2Nt) \\ \frac{p}{2mN} \cdot \cos(2Nt) - \frac{p}{2mN} \\ \frac{p}{2} \cdot \cos(2Nt) + \frac{p}{2} \\ -\frac{p}{2} \cdot \sin(2Nt) \end{pmatrix} = \begin{pmatrix} \frac{pc}{2\beta} \sin\left(\frac{q\beta}{mc} t\right) \\ \frac{pc}{2\beta} \left(\cos\left(\frac{q\beta}{mc} t\right) - 1\right) \\ \frac{p}{2} \left(\cos\left(\frac{q\beta}{mc} t\right) + 1\right) \\ -\frac{p}{2} \cdot \sin\left(\frac{q\beta}{mc} t\right) \end{pmatrix}$$

$$b) \vec{v} = (v_1, v_2, v_3) = (\dot{x}, \dot{y}, \dot{z})$$

$$\{\dot{x}, \dot{y}\} = \frac{1}{m^2} \left\{ p_1 + \frac{q\beta y}{2c}, p_2 - \frac{q\beta x}{2c} \right\} = \\ = \frac{1}{m^2} \cdot \frac{q\beta}{2c} \{y, p_2\} - \frac{1}{m^2} \cdot \frac{q\beta}{2c} \{p_1, x\} = \frac{2}{m^2} \cdot \frac{q\beta}{2c} = \frac{q\beta}{c \cdot m^2}$$

$$\{\dot{y}, \dot{z}\} = \frac{1}{m^2} \left\{ p_2 - \frac{q\beta x}{2c}, p_3 \right\} = \frac{1}{m^2} \{p_2, p_3\} - \frac{q\beta x}{2cm^2} \{x, p_3\} = 0$$

$$\{\dot{x}, \dot{z}\} = \frac{1}{m^2} \left\{ p_1 + \frac{q\beta y}{2c}, p_3 \right\} = 0$$

N4.

$$\begin{aligned} L &= \frac{m}{2}(\dot{x}^2 + \dot{y}^2) - \frac{mw^2}{2}(x^2 + y^2) \\ p_1 &= \frac{\partial L}{\partial \dot{x}} = m\dot{x} \quad p_2 = \frac{\partial L}{\partial \dot{y}} = m\dot{y} \\ \dot{x} &= \frac{p_1}{m} \quad \dot{y} = \frac{p_2}{m} \end{aligned}$$

$$\text{Nur } p_x = p_1 \\ p_y = p_2$$

$$\begin{aligned} H &= p_1 \dot{x} + p_2 \dot{y} - \frac{m}{2}(\dot{x}^2 + \dot{y}^2) + \frac{mw^2}{2}(x^2 + y^2) = \\ &= \frac{p_1^2}{m} + \frac{p_2^2}{m} - \frac{m}{2} \left(\frac{p_1^2}{m^2} + \frac{p_2^2}{m^2} \right) + \frac{mw^2}{2}(x^2 + y^2) = \\ &= \frac{p_1^2}{m} + \frac{p_2^2}{m} - \frac{p_1^2}{2m} - \frac{p_2^2}{2m} + \frac{mw^2}{2}(x^2 + y^2) = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{mw^2}{2}(x^2 + y^2) \end{aligned}$$

$$\delta) J_1 = \frac{1}{2m} (p_1^2 - p_2^2) + \frac{mw^2}{2}(x^2 - y^2)$$

$$J_2 = \frac{1}{m} p_1 p_2 + mw^2 x y$$

$$J_3 = w(xp_2 - yp_1)$$

$$\begin{aligned} \dot{p}_1 &= -\frac{\partial H}{\partial x} = -mw^2 x \quad \dot{p}_2 = -\frac{\partial H}{\partial y} = -mw^2 y \\ \dot{x} &= \frac{\partial H}{\partial p_1} = \frac{p_1}{m} \quad \dot{y} = \frac{\partial H}{\partial p_2} = \frac{p_2}{m} \end{aligned}$$

unseren gleichen zahlen yzoba: $\frac{df}{dt} = \frac{df}{dt} + \sum_{i=1}^n \left(\frac{\partial f}{\partial q_i} \cdot \frac{dq_i}{dt} + \frac{\partial f}{\partial p_i} \cdot \frac{dp_i}{dt} \right) = 0$

$$0 = \frac{dJ_1}{dt} = \frac{\partial J_1}{\partial x} \cdot \dot{x} + \frac{\partial J_1}{\partial p_1} \cdot \dot{p}_1 + \frac{\partial J_1}{\partial y} \cdot \dot{y} + \frac{\partial J_1}{\partial p_2} \cdot \dot{p}_2 =$$

$$= mw^2 x \cdot \dot{x} + \frac{p_1}{m} \cdot \dot{p}_1 - mw^2 y \cdot \dot{y} - \frac{p_2}{2m} \cdot \dot{p}_2 =$$

$$= mw^2 x \cdot \frac{p_1}{m} + \frac{p_1}{m} \cdot (-mw^2 x) - mw^2 y \cdot \frac{p_2}{m} - \frac{p_2}{2m} \cdot (-mw^2 y) = 0$$

$\Rightarrow J_1$ - unserer gleichen

$$0 = \frac{dJ_2}{dt} = \frac{\partial J_2}{\partial x} \cdot \dot{x} + \frac{\partial J_2}{\partial p_1} \cdot \dot{p}_1 + \frac{\partial J_2}{\partial y} \cdot \dot{y} + \frac{\partial J_2}{\partial p_2} \cdot \dot{p}_2 =$$

$$= mw^2 y \cdot \frac{p_1}{m} + \frac{p_2}{m} \cdot (-mw^2 x) + mw^2 x \cdot \frac{p_2}{m} + \frac{p_1}{m} \cdot (-mw^2 y) = 0$$

$\Rightarrow J_2$ - unserer gleichen

$$0 = \frac{dJ_3}{dt} = \frac{\partial J_3}{\partial x} \cdot \dot{x} + \frac{\partial J_3}{\partial p_1} \cdot \dot{p}_1 + \frac{\partial J_3}{\partial y} \cdot \dot{y} + \frac{\partial J_3}{\partial p_2} \cdot \dot{p}_2 =$$

$$= p_2 w \cdot \frac{p_1}{m} + (-w y) \cdot (-mw^2 x) + (-w p_1) \cdot \frac{p_2}{m} + w x \cdot (-mw^2 y) = 0$$

$\Rightarrow J_3$ - unserer gleichen

б) лин. оболочка, порождённая всеобщим лин. коммутатором групп J_i , изважающая отор скобки Пуассона.

Достаточно показать, что $\{J_i, J_j\} \in \mathcal{L}$

$$\begin{aligned}\{J_1, J_2\} &= \left\{ \frac{1}{2m}(p_1^2 - p_2^2) + \frac{mw^2}{2}(x^2 - y^2), \frac{1}{m}p_1p_2 + mw^2xy \right\} = \\ &= \frac{mw^2y}{2m} \{p_1^2, xy\} - \frac{mw^2x}{2m} \{p_2^2, y\} + \frac{mw^2}{2} \cdot \frac{p_2}{m} \{x^2, p_1y\} - \frac{mw^2}{2} \cdot \frac{p_1}{m} \{y^2, p_2\} = \\ &= \frac{\omega^2y}{2} \cdot 2p_1 \{p_1, xy\} - \frac{\omega^2x}{2} \cdot 2p_2 \{p_2, y\} + \frac{\omega^2p_2}{2} \cdot 2x \{x^2, p_1y\} - \frac{\omega^2p_1}{2} \cdot 2y \{y^2, p_2\} = \\ &= -\omega^2yp_1 + \omega^2xp_2 + \omega^2xp_2 - \omega^2yp_1 = -2\omega^2yp_1 + 2\omega^2xp_2 = \\ &= 2\omega^2(xp_2 - yp_1) = 2\omega J_3 \Rightarrow \{J_1, J_2\} \in \mathcal{L}\end{aligned}$$

$$\begin{aligned}\{J_1, J_3\} &= \left\{ \frac{1}{2m}(p_1^2 - p_2^2) + \frac{mw^2}{2}(x^2 - y^2), w(xp_2 - yp_1) \right\} = \\ &= \frac{\omega p_2}{2m} \{p_1^2, xy\} + \frac{p_1w}{2m} \{p_2^2, y\} - \frac{mw^3y}{2} \{x^2, p_1y\} - \frac{mw^3x}{2} \{y^2, p_2y\} = \\ &= \frac{\omega p_1p_2}{m} \{p_1, xy\} + \frac{\omega p_1p_2}{m} \{p_2, y\} - mw^3xy \{x, p_1y\} - mw^3xy \{y, p_2y\} = \\ &= -\frac{\omega p_1p_2}{m} - \frac{\omega p_1p_2}{m} - mw^3xy - mw^3xy = -\omega \left(\frac{\omega p_1p_2}{m} + 2mw^2xy \right) = \\ &= -2\omega J_2 \Rightarrow \{J_1, J_3\} \in \mathcal{L}\end{aligned}$$

$$\begin{aligned}\{J_2, J_3\} &= \left\{ \frac{1}{m}p_1p_2 + mw^2xy, w(xp_2 - yp_1) \right\} = \\ &= \frac{\omega p_2^2}{m} \{p_1, xy\} - \frac{\omega p_1^2}{m} \{p_2, y\} - mw^3y^2 \{x, p_1y\} + mw^3x^2y \{y, p_2y\} = \\ &= -\frac{\omega p_2^2}{m} + \frac{\omega p_1^2}{m} - mw^3y^2 + mw^3x^2y = \\ &= \omega \cdot \left(\frac{p_1^2}{m} - \frac{p_2^2}{m} - mw^2(y^2 - x^2) \right) = 2\omega J_1 \Rightarrow \{J_2, J_3\} \in \mathcal{L}\end{aligned}$$

$\{J_1, J_1\} = \{J_2, J_2\} = \{J_3, J_3\} = 0$, т.к. скобка Пуассона
кососимметрична и билинейна

N5.

$$H = \frac{\vec{M}^2}{2I} - \gamma \vec{M} \cdot \vec{B}$$

$$\vec{M} = (M_1, M_2, M_3) \quad \vec{B} = (0, 0, B)$$

$$H = \frac{M_1^2 + M_2^2 + M_3^2}{2I} - \gamma (M_1 B_1 + M_2 B_2 + M_3 B_3)$$

$$1 M_i, M_i y = 0$$

$$\{M_i, M_j\} = \{E_{ijk}, \tau_j p_k, E_{jki}, \tau_k p_i\} = E_{ijk} \cdot E_{jki} \tau_k \overset{-1}{p}_k, \tau_k \overset{-1}{\tau}_j p_i + E_{ikj} \{ \tau_k, p_i \}; E_{jik} \tau_i =$$

$$= -\tau_j p_i + \tau_i p_j = E_{ijk} \tau_i p_j \cdot E_{ijk} = E_{ijk} M_k$$

$$\frac{dM_i}{dt} = \{M_i, H\} = [M_i, \frac{M_1^2 + M_2^2 + M_3^2}{2I}] - \gamma (M_1 B_1 + M_2 B_2 + M_3 B_3) y =$$

$$= \sum_{j \neq i} \frac{\{M_i, M_j\} \cdot 2M_j}{2I} - \gamma \{M_i, M_j\} \cdot B_j = \sum_{j \neq i} \frac{2E_{ijk} M_k M_j}{2I} - \gamma E_{ijk} M_k B_j =$$

$$= \sum_{j \neq i} -E_{ijk} M_k M_j - \gamma E_{ijk} M_k B_j = \sum_{j \neq i} -[\vec{M} \times \vec{B}]_i - \gamma E_{ijk} M_k B_j =$$

$$= \gamma E_{ijk} M_k B_j = \gamma [\vec{M} \times \vec{B}]_i$$

$$\dot{M}_1 = \gamma (M_2 B_3 - M_3 B_2)$$

$$\dot{M}_2 = \gamma (M_3 B_1 - M_1 B_3)$$

$$\dot{M}_3 = \gamma (M_1 B_2 - M_2 B_1)$$

$$\vec{B} = (0, 0, B) : \begin{cases} \dot{M}_1 = \gamma M_2 B \\ \dot{M}_2 = -\gamma M_1 B \\ \dot{M}_3 = 0 \end{cases}$$

$$\text{Ny gaa } M_{12} = \begin{pmatrix} M_1 \\ M_2 \end{pmatrix} : \dot{M}_{12} = \begin{pmatrix} 0 & \gamma \\ -\gamma & 0 \end{pmatrix} M_{12} B$$

$$M_{12} = \exp\left(\begin{pmatrix} 0 & \gamma \\ -\gamma & 0 \end{pmatrix} B t\right) \cdot M_{12}(0) = \begin{pmatrix} \cos \gamma B t & \sin \gamma B t \\ -\sin \gamma B t & \cos \gamma B t \end{pmatrix} \cdot M_{12}(0)$$

$$M_1 = M_1(0) \cos(\gamma B t) + M_2(0) \sin(\gamma B t)$$

$$M_2 = -M_1(0) \sin(\gamma B t) + M_2(0) \cos(\gamma B t)$$

$$M_3 = \text{const}$$