If the complexity of the required algorithm is not given, you shall find an algorithm with as good complexity as you can. If your algorithm would be far from optimal, the grade will be decreased.

1 (2). Give a counterexample to the conjecture that if a directed graph G contains a path from u to v, and if $s[u] < s[v]$ in a depth-first search of G, then v is a descendant of u in the depth-first forest produced.

2 (2). You need to get out of the maze. You don't know its map and how many rooms it has. All corridors can be freely moved in both directions, all rooms and corridors look the same (rooms can differ only in the number of corridors). Let m be the number of corridors between rooms. Suggest an algorithm that finds a way out of a maze (or proves that there is none) in $O(m)$ transitions between rooms. You have an unlimited number of coins at your disposal that you can leave in the rooms, and you know that apart from your coins, there are no others in the labyrinth, and you are alone in it.

3(3). A directed graph $G(V, E)$ is *semiconnected* if, for all pairs of vertices $u, v \in V$, we have that u is reachable from v or v is reachable from u. Give an efficient algorithm to determine whether or not G is semiconnected. Prove that your algorithm is correct, and analyze its running time.

4 (3). Let $G(V, E)$ be a directed graph in which each vertex $u \in V$ is labeled with a unique integer $L(u)$ from the set $\{1, 2, \ldots, |V|\}$. For each vertex $u \in V$, let $R(u) = \{v \in V : u \rightsquigarrow v\}$ be the set of vertices that are reachable from u. Define $\min(u)$ to be the vertex in $R(u)$ whose label is minimum, i.e., $\min(u)$ is the vertex v such that $L(v) = \min\{L(w) : w \in R(w)\}$. Give an $O(|V| + |E|)$-time algorithm that computes $\min(u)$ for all vertices $u \in V$.

5 (4). Prove that every tournament on n vertices contains a (simple) path of length $n - 1$. Construct an algorithm that finds such a path in a tournament, and estimate its complexity.

Recall that a directed graph is a *tournament* if it can be obtained from a complete undirected graph by setting directions of edges.

6 (4). Given a digraph on n vertices ($V = \{1, \ldots, n\}$), which is obtained from a path graph (an edge leading from i to $i + 1$) by adding m arrows connecting arbitrary pairs of vertices. Find the number of strongly connected components in $O(m \log m)$.

7 (4) [Lemma 22]. Let S be a set of n disjoint (pairwise non intersecting) plane segments. For two segments, $s, s' \in S$ we say that s lies below s' denoted as $s \prec s'$ if and only if there are points $p \in s, p' \in s'$ such that $p_x = p_{x'}$ and $p_y < p_{y'}$. For a set S of disjoint segments consider the directed graph with $V = S$ and arrow $s \to s'$ whenever $s \prec s'$. Prove that this graph is a DAG.