Catalan sequence

Task
Let c_n be the Catalan sequence. Find the limit $\lim\limits_{n\to\infty} \frac{c_{n+1}}{c_n}$

Solution

$$c_n = \frac{1}{n+1}\binom{2n}{n}$$

Then

$$\lim_{n\to\infty} \frac{c_{n+1}}{c_n} = \lim_{n\to\infty} \frac{\frac{1}{n+2}\binom{2n+2}{n+1}}{\frac{1}{n+1}\binom{2n}{n}} =$$

$$\lim_{n\to\infty} \frac{\frac{1}{n+2}\frac{(2n+2)!}{(n+1)!((2n+2)-(n+1))!}}{\frac{1}{n+1}\frac{(2n)!}{n!((2n)-(n))!}} = \lim_{n\to\infty} \frac{\frac{1}{n+2}\frac{(2n+2)!}{(n+1)!(n+1)!}}{\frac{1}{n+1}\frac{(2n)!}{n!n!}} =$$

$$\lim_{n\to\infty} \frac{(n+1)\frac{(2n+2)!}{(n+1)!(n+1)!}}{(n+2)\frac{(2n)!}{n!n!}} = \lim_{n\to\infty} \frac{(n+1)(2n+2)!n!n!}{(n+2)(2n)!(n+1)!(n+1)!} =$$

$$\lim_{n\to\infty} \frac{(n+1)(2n+1)(2n+2)}{(n+2)(n+1)(n+1)} = \lim_{n\to\infty} \frac{2(n+1)(2n+1)}{(n+2)(n+1)} =$$

$$\lim_{n\to\infty} \frac{4n+2}{n+2} = \lim_{n\to\infty} \frac{4+\frac{2}{n}}{1+\frac{2}{n}} = 4$$