$y \in C^\infty[0,1], \quad y(0) = A$

$$S[y] = \int_0^1 dx \left(\underbrace{\frac{1}{4}(y')^2 \log(y^2) + xy' + e^{\cos y}}_{L} \right)$$

a) $$\delta S[y] = \int_0^1 dx \left(\frac{\partial L}{\partial y}\delta y + \frac{\partial L}{\partial y'}\delta y' \right)$$

$$\frac{\partial L}{\partial y} = \frac{1}{4}(y')^2 \cdot \frac{1}{y^2}\cdot 2y + e^{\cos y}(-\sin y) =$$

$$= \frac{1}{2}\cdot\frac{1}{y}(y')^2 - \sin y\, e^{\cos y}$$

$$\frac{\partial L}{\partial y'} = \frac{1}{4}\cdot 2y' \ln y^2 + x = \frac{1}{2}y' \ln y^2 + x$$

$$\delta S[y] = \int_0^1 dx \left[\left(\frac{(y')^2}{2y} - \sin y\, e^{\cos y} \right)\delta y + \left(\frac{1}{2}y' \ln y^2 + x \right)\delta y' \right] =$$

$$= \int_0^1 \left(\frac{(y')^2}{2y} - \sin y\, e^{\cos y} \right)\delta y\, dx + \left(\frac{1}{2}y' \ln y^2 + x \right)\delta y \Big|_0^1 -$$

$$- \int_0^1 \left[\frac{1}{2}\left(y'' \ln y^2 + y' \cdot \frac{2}{y} \right) + 1 \right] \delta y\, dx =$$

$$= \int_0^1 \left(\frac{(y')^2}{2y} - \sin y\, e^{\cos y} - \frac{1}{2}y'' \ln y^2 - \frac{y'}{y} - 1 \right)\delta y\, dx +$$

$$+ \left(\frac{1}{2}y' \ln y^2 + x \right)\delta y \Big|_0^1$$

б) $y(0) = A \Rightarrow \delta y(0) = 0$

$$\delta y(1) \ \forall \Rightarrow \boxed{\frac{1}{2}y' \ln y^2 + x \Big|_{x=1} = 0}$$