$y \in C^\infty[0,1], \quad y(0) = A$

$$S[y] = \int_0^1 dx \Big(\underbrace{\tfrac{1}{4} (y')^2 \log(y^2) + x y' + e^{\cos y}}_{L} \Big)$$

a) $$\delta S[y] = \int_0^1 dx \left(\frac{\partial L}{\partial y} \delta y + \frac{\partial L}{\partial y'} \delta y' \right)$$

$$\frac{\partial L}{\partial y} = \frac{1}{4} (y')^2 \cdot \frac{1}{y^2} \cdot 2y + e^{\cos y}(-\sin y) =$$

$$= \frac{1}{2} \cdot \frac{1}{y} (y')^2 - \sin y \, e^{\cos y}$$

$$\frac{\partial L}{\partial y'} = \frac{1}{4} \cdot 2 y' \ln y^2 + x = \frac{1}{2} y' \ln y^2 + x$$

$$\delta S[y] = \int_0^1 dx \left[\left(\frac{(y')^2}{2y} - \sin y \, e^{\cos y} \right) \delta y + \left(\frac{1}{2} y' \ln y^2 + x \right) \delta y' \right] =$$

$$= \int_0^1 \left(\frac{(y')^2}{2y} - \sin y \, e^{\cos y} \right) \delta y \, dx + \left(\frac{1}{2} y' \ln y^2 + x \right) \delta y \Big|_0^1 -$$

$$- \int_0^1 \left[\frac{1}{2} \left(y'' \ln y^2 + y' \cdot \frac{2}{y} \right) + 1 \right] \delta y \, dx =$$

$$= \int_0^1 \boxed{\left(\frac{(y')^2}{2y} - \sin y \, e^{\cos y} - \frac{1}{2} y'' \ln y^2 - \frac{y'}{y} - 1 \right)} \delta y \, dx +$$

$$+ \left(\frac{1}{2} y' \ln y^2 + x \right) \delta y \Big|_0^1$$

б) $y(0) = A \Rightarrow \delta y(0) = 0$

$$\delta y(1) \ \forall \Rightarrow \boxed{\frac{1}{2} y' \ln y^2 + x \Big|_{x=1} = 0}$$