Вариант 6

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лист 1

№1

$Q = \{0<x<\pi,\ 0<y<\pi,\ 0<t<+\infty\}$

$u_{tt} - \Delta u = -\cos(t)\cos\left(\frac{3y}{2}\right)\cos x$

$\begin{cases} u|_{t=0} = 0 \\ u_t|_{t=0} = -2\cos\left(\frac{3y}{2}\right)\cos x \end{cases}$ $\begin{cases} \frac{\partial u}{\partial x}\Big|_{x=0} = 0 & \frac{\partial u}{\partial x}\Big|_{x=\pi} = 0 \\ \frac{\partial u}{\partial y}\Big|_{y=0} = 0 & u|_{y=\pi} = 0 \end{cases}$

Ищем решение в виде

$$u(x,y,t) = \sum_{\substack{k\geq 0\\ l\geq 1}} T_{kl}(t)\cos(kx)\cdot\cos\left(\frac{2l-1}{2}y\right)$$

$$\sum_{\substack{k\geq 0\\ l\geq 1}} T_{kl}'' \cos(kx)\cos\left(\frac{2l-1}{2}y\right) + \left(k^2 + \left(\frac{2l-1}{2}\right)^2\right) T_{kl} \cos(kx)\cos\left(\frac{2l-1}{2}y\right) = -\cos t\cos\frac{3y}{2}\cos x$$

$$T_{1,2}^{(2)} + \frac{13}{4}T_{1,2} = -\cos t, \quad T_{1,2} = A_{1,2}\cos\sqrt{\tfrac{13}{4}}t + B_{1,2}\sin\sqrt{\tfrac{13}{4}}t - \frac{4}{9}\cos t$$

$$u(t,x,y) = \left(A_{1,2}\cos\frac{\sqrt{13}}{2}t + B_{1,2}\sin\frac{\sqrt{13}}{2}t - \frac{4}{9}\cos t\right)\cos x\cos\frac{3y}{2}$$

$$u(0,x,y) = \left(A_{1,2}\cdot 1 + B_{1,2}\cdot 0 - \frac{4}{9}\cdot 1\right)\cos x\cos\frac{3y}{2} = 0$$

$$u_t(0,x,y) = \left(A_{1,2}\cdot 0 + B_{1,2}\cdot\frac{\sqrt{13}}{2}\cdot 1 - \frac{4}{9}\cdot 0\right)\cos x\cos\frac{3y}{2} = -2\cos\frac{3y}{2}\cos x$$

$$A_{1,2} = \frac{4}{9} \quad B_{1,2} = -\frac{4}{\sqrt{13}}$$

$$u(t,x,y) = \left(\frac{4}{9}\cos\frac{\sqrt{13}}{2}t - \frac{4}{\sqrt{13}}\sin\frac{\sqrt{13}}{2}t - \frac{4}{9}\cos t\right)\cos\frac{3y}{2}\cos x$$

$(k,l) \neq (1,2)$

$$T_{kl}^{(2)} + \left(k^2 + \left(\frac{2l-1}{2}\right)^2\right)T_{kl} = 0 \qquad T_{kl} = A_{kl}\cos\left(\sqrt{k^2+\left(\frac{2l-1}{2}\right)^2}\,t\right) + B_{kl}\sin\left(\sqrt{k^2+\left(\frac{2l-1}{2}\right)^2}\,t\right)$$

$$u(0,x,y) = \sum A_{kl}\cos kx\cos\frac{2l-1}{2}y = 0 \Rightarrow A = 0$$

$$u_t(0,x,y) = \sum B_{kl}\sqrt{k^2+\left(\frac{2l-1}{2}\right)^2}\cos kx\cos\frac{2l-1}{2}y = -2\cos x\cos\frac{3y}{2} \Rightarrow B_{kl}\neq 0 \text{ только при } k=1,\ l=2$$

Ответ: $u(t,x,y) = \left(\frac{4}{9}\cos\frac{\sqrt{13}}{2}t - \frac{4}{\sqrt{13}}\sin\frac{\sqrt{13}}{2}t - \frac{4}{9}\cos t\right)\cos\frac{3y}{2}\cos x$

$Q = \{0 < x < \pi, 0 < y < \pi, 0 < t < +\infty\}$

$u_{tt} - \Delta u = -2\sin(3t) f(x,y)$

$\begin{cases} u|_{t=0} = 0 \\ u_t|_{t=0} \end{cases}$ $\begin{cases} \frac{\partial u}{\partial x}\Big|_{x=0} = 0 \\ u|_{y=0} = 0 \end{cases}$ $\frac{\partial u}{\partial x}\Big|_{x=\pi} = 0$ $\frac{\partial u}{\partial y}\Big|_{y=\pi} = 0$

$$u(t,x,y) = \sum_{\substack{k\geq 1 \\ l \geq 0}} T_{kl}(t) \cos(kx) \sin(l-\tfrac{1}{2})y$$

$$\sum T_{kl}^{(2)}(t)\cos(kx)\sin(l-\tfrac{1}{2})y + \left(k^2 + (l-\tfrac{1}{2})^2\right) T_{kl}(t)\cos(kx)\sin(l-\tfrac{1}{2})y = -2\sin 3t\, f(x,y)$$

$T_{kl} = A_{kl}\cos\sqrt{k^2+(l-\frac{1}{2})^2}\,t + B_{kl}\sin\sqrt{k^2+(l-\frac{1}{2})^2}\,t +$ (частное решение)

частное решение выражается как $\frac{2f_{kl}}{(k^2+(l-\frac{1}{2})^2)-9}\sin(3t)$

$$u(x,y,t) = \left(A_{kl}\cos\sqrt{k^2+(l-\tfrac{1}{2})^2}\,t + B_{kl}\sin\sqrt{k^2+(l-\tfrac{1}{2})^2}\,t - \frac{2f_{kl}}{(k^2+(l-\frac{1}{2})^2)-9}\sin(3t)\right)\cos kx \sin(l-\tfrac{1}{2})y$$

н.у.

$$u(x,y,0) = \left(A_{kl}\cos\left(\sqrt{k^2+(l-\tfrac{1}{2})^2}\cdot 0\right)\right)\cos kx\sin(l-\tfrac{1}{2})y = 0$$

$$u_t(x,y,0) = \left(B_{kl}\sqrt{k^2+(l-\tfrac{1}{2})^2} - \frac{6f_{kl}}{(k^2+(l-\frac{1}{2})^2)-9}\right)\cos kx\sin(l-\tfrac{1}{2})y = 0$$

$A_{kl} = 0$ $\quad B_{kl} = \frac{6f_{kl}}{\sqrt{k^2+(l-\frac{1}{2})^2}\,(k^2+(l-\frac{1}{2})^2-9)}$

$$u(x,y,t) = \sum_{\substack{k\geq 0 \\ l\geq 0}}\left(\frac{6f_{kl}}{\sqrt{k^2+(l-\frac{1}{2})^2}\,(k^2+(l-\frac{1}{2})^2-9)}\sin\sqrt{k^2+(l-\tfrac{1}{2})^2}\,t - \frac{2f_{kl}}{(k^2+(l-\frac{1}{2})^2)-9}\sin 3t\right)\cos kx\sin(l-\tfrac{1}{2})y =$$

$$= \sum_{\substack{k\geq 1 \\ l\geq 0}} \frac{2f_{kl}}{k^2+(l-\frac{1}{2})^2-9}\left(\frac{3}{\sqrt{k^2+(l-\frac{1}{2})^2}}\sin\sqrt{k^2+(l-\tfrac{1}{2})^2}\,t - \sin 3t\right)\cos kx\sin(l-\tfrac{1}{2})y$$

№4 МИП 3

$u(x,t)=\sum_{k\geq 1} T_k(t)\sin((2k-1)x)$

$\sum T_k' \sin((2k-1)x)=\sum T_k(t)\cdot -(2k-1)^2 \sin((2k-1)x)+T_k(t)\sin((2k-1)x)-2\cos 2x \cdot \sin x$

$-\sin 3x+\sin x=-2\cos 2x \sin x$

и ~~начальное условие~~

$T_2'=-9T_2\sin 3x+T_2\sin 3x-\sin 3x$

$T_2'=-8T_2-1$

$T_2=c_2\cdot e^{-8t}-\frac{1}{8}$

$T_1'=-T_1+T_1+1$

$T_1'=1$

$T_1=c_1+t$

$T_k'(t)=-(2k-1)^2T_k(t)+T_k(t)$

$T_k'=4k(1-k)T_k$

$T_k=c_k e^{4(1-k)kt}$

$u(0,x)=c_k\sin((2k-1)x)=0$

$\Rightarrow u(t,x)=\frac{1}{8}(e^{-8t}-1)\sin 3x+t\sin x$

№ 3

$$\langle f, \varphi' \rangle = \sum_k \int_{-\frac{\pi}{2}+2\pi k}^{\frac{\pi}{2}+2\pi k} \cos x\, \varphi'(x)\,dx - \sum_k \int_{\frac{\pi}{2}+2\pi k}^{\frac{3\pi}{2}+2\pi k} \cos x\, \varphi'(x)\,dx - \sum_k \int_{-\frac{\pi}{2}+2\pi k}^{\frac{\pi}{2}+2\pi k} \cos x\, d\varphi(x) -$$

$$- \sum_k \int_{\frac{\pi}{2}+2\pi k}^{\frac{3\pi}{2}+2\pi k} \cos x\, d\varphi(x) = \sum_k \cos x\, \varphi(x) \Big|_{-\frac{\pi}{2}+2\pi k}^{\frac{\pi}{2}+2\pi k} - \sum_k \cos x\, \varphi(x) \Big|_{\frac{\pi}{2}+2\pi k}^{\frac{3\pi}{2}+2\pi k}$$

$$+ \sum_k \int_{-\frac{\pi}{2}+2\pi k}^{\frac{\pi}{2}+2\pi k} \sin x\, \varphi(x)\,dx - \sum_k \int_{\frac{\pi}{2}+2\pi k}^{\frac{3\pi}{2}+2\pi k} \sin x\, \varphi(x)\,dx = -\langle f', \varphi \rangle$$

$$\cos x\, \varphi(x) \Big|_{-\frac{\pi}{2}+2\pi k}^{\frac{\pi}{2}+2\pi k} + \sin x\, \varphi(x) \Big|_{\frac{\pi}{2}+2\pi k}^{\frac{3\pi}{2}+2\pi k}$$

$\varphi(x)$ - пробная

$$\langle f'', \varphi \rangle = -\langle f', \varphi' \rangle = \langle f, \varphi'' \rangle$$