1 23 Jan

Задача 1.1. Should TXOP length be upper bounded?

Доказательство. Да, так как иначе ожидание может быть сколь угодно большим ☐

Задача 1.2. Let AP have opportunity set arbitrary CWmin and CWmax to each STA in BSS. There are two users in BSS. The first one wants to download file, while the second one has videocall. Which CWmin and CWmax to use for each STA?

Доказательство. У Видеозвонка приоритет выше, соответственно чтобы не было столкновений $\mathrm{CW}_{\mathrm{max\ vid}} < \mathrm{CW}_{\mathrm{min\ file}}$ ☐

Задача 1.3. Let $b_{i,k} = \lim_{t \longrightarrow \infty} P\{s(t) = i, b(t) = k\}, i \in (0, m), k \in (0, W_i - 1)$. Prove that

(a) $b_{i,0} = p^i b_{0,0}$

(b) $b_{m,0} = \frac{p^m}{1-p} b_{0,0}$

(c) $b_{i,k} = \frac{W_i - k}{W_i} \cdot \begin{cases} (1-p) \sum_{j=0}^{m} b_{j,0} & i = 0 \\ p \cdot b_{i-1,0} & 0 < i < m \\ p\left(b_{m-1,0} + b_{m,0}\right) & i = m \end{cases}$

(d) $b_{0,0} = \frac{2(1-2p)(1-p)}{(1-2p)(W_0+1) + pW_0(1-(2p)^m)}$

(e) $\tau = \frac{2(1-2p)}{(1-2p)(W_0+1) + pW_0(1-(2p)^m)}$

Доказательство. ☐

Задача 1.4. Assume constant and independent collision probability of a packet transmitted by each station. Express τ in terms of p.

Доказательство. ☐

Задача 1.5. $X \sim \mathrm{Uniform}(a, b)$ find $\tilde{X}$

Доказательство.

$$\frac{1}{b-a}$$

☐

Задача 1.6. $\mathrm{Exp}(\lambda)$ find $\tilde{X}$

Доказательство.

$$\int_0^\infty a e^{-(s+a)t} dt = -\frac{a}{s+a}(0-1) = \frac{a}{s+a}$$

☐

2 6 Feb

Задача 2.1. Prove that for each random variable X we have

(a) $\widetilde{X}(0) = 1$

(b) $\widehat{X}(1) = 1$

Доказательство. (a)

$$\widetilde{X}(0) = \int\limits_0^\infty e^{-0t} f_x(t)dt = \int\limits_0^\infty f_x(t)dt = 0$$

(b)

$$\widetilde{X}(1) = \sum_{i=0}^\infty P\{x = i\} = 1$$

□

Задача 2.2. Define packet length distribution by the rules below:

$$S = \begin{cases} 0 & \text{with probability } q \\ \frac{1}{1-q} & \text{with probability } 1-q \end{cases}$$

Find ES by using theorems 3 and 5 .

Доказательство.

$$\widetilde{S}(s) = q \cdot \widetilde{A}(s) + (1-q)\widetilde{B}(s) = q \cdot \int\limits_0^\infty e^{st} 0dt + (1-q)\int\limits_0^\infty e^{st}\frac{1}{1-q}dt$$

□

Задача 2.3. Express EB in terms of S, ρ.

Доказательство.

$$\widetilde{B}'(s) = \widetilde{S}'(s + \lambda - \lambda\widetilde{B}(s))(1 - \widetilde{B}'(s))$$
$$\widetilde{B}'(0) = \frac{ES}{1-\rho}$$

□

3 13 Feb

Задача 3.1. Let packet length be distributed by the rule:

$$S = \begin{cases} \frac{1}{q} & \text{with probability } q \\ \frac{1}{1-q} & \text{with probability } 1-q \end{cases}$$

Find ES_e.

Доказательство. □

Задача 3.2. Express $EB = -\widetilde{B}'(0)$ in terms of S, ρ.

Доказательство.

$$\widetilde{B}'(s) = \widetilde{S}'(s + \lambda - \lambda \widetilde{B}(s))(1 - \widetilde{B}'(s))$$
$$\widetilde{B}'(0) = \frac{ES}{1-\rho}$$

□

Задача 3.3. Express $EB^2 = \widetilde{B}''(0)$ in terms of S, ρ.

Доказательство.

$$\widetilde{B}''(s) = \widetilde{S}''(s + \lambda - \lambda \widetilde{B}(s))(1 - \widetilde{B}'(s))^2 - \widetilde{S}'(s + \lambda - \lambda \widetilde{B}(s))\lambda \widetilde{B}'(s)$$
$$\widetilde{B}''(0) = \widetilde{S}''(0 + \lambda - \lambda \widetilde{B}(0))(1 - \widetilde{B}'(0))^2 - \widetilde{S}'(0 + \lambda - \lambda \widetilde{B}(0))\lambda \widetilde{B}'(0)$$
$$= \widetilde{S}''(0 + \lambda - \lambda \frac{ES}{\rho - 1})(1 - \frac{ES}{\rho-1})^2 - \widetilde{S}'(0 + \lambda - \lambda \frac{ES}{\rho-1})\lambda \frac{ES}{\rho-1} = ES^2 \cdot (1 + \lambda \frac{ES}{1-\rho}) + ES\lambda \widetilde{B}''(0)$$
$$= \frac{ES^2(1 + \frac{\lambda ES}{1-\rho})^2}{1 - \lambda ES} = \frac{ES^2(1 - \frac{\rho}{1-\rho})^2}{(1-\rho)} = \frac{ES^2}{(1-\rho)^3}$$

□

Задача 3.4. Express $-EB^3 = \widetilde{B}'''(0)$ in terms of S, ρ.

Доказательство.

$$\widetilde{B}'''(s) = \widetilde{S}'''(s + \lambda - \lambda \widetilde{B}(s))(1 - \lambda \widetilde{B}(s))^3 - \widetilde{S}''(s + \lambda - \lambda \widetilde{B}(s))2(1 - \lambda \widetilde{B}(s))\lambda \widetilde{B}''(s)$$
$$-\widetilde{S}'(s + \lambda - \lambda \widetilde{B}(s))\lambda \widetilde{B}'''(s) - \widetilde{S}''(s + \lambda - \lambda \widetilde{B}(s))(1 - \lambda \widetilde{B}'(s))\lambda \widetilde{B}''(s)$$
$$\widetilde{B}'''(0) = -\frac{ES^3(1-\rho) + 3\lambda(ES^2)^2}{(1-\rho)^5}$$

□

4 20 Feb

Задача 4.1. Express $\mathrm{Var}(B)$ in terms of S and ρ

Доказательство.

$$\mathrm{Var}(B) = E(B^2) - (EB)^2$$
$$= -(-\widetilde{B}'(0))^2 + \widetilde{B}''(0)$$
$$= -\left(-\frac{ES}{1-\rho}\right)^2 + \frac{ES^2}{(1-\rho)^3}$$
$$= -\frac{ES^2}{(1-\rho)^2} + \frac{E(S^2)}{(1-\rho)^3}$$
$$= \frac{-(1-\rho)(ES)^2 + E(S^2)}{(1-\rho)^3}$$

□

Задача 4.2. Express $ET_Q^{FCFS} = ET_Q^{LCFS}$ in terms of S and ρ

Доказательство.

$$ET_Q^{FCFS} = \frac{\left(-s\lambda\widetilde{B}'(s) - \lambda + \lambda\widetilde{B}(s)\right)''}{\left((s+\lambda-\lambda\widetilde{B}(s))^2\right)''} = \frac{-\lambda(s\widetilde{B}'''(s) + \widetilde{B}'')}{2(1-\lambda\widetilde{B}'(s))^2 - 2\lambda(\lambda(-\widetilde{B}(s)) + \lambda + s)\widetilde{B}''(s)}$$

$$= \frac{-\lambda(0\widetilde{B}'''(0) + \widetilde{B}''(0))}{2(1-\lambda\widetilde{B}'(0))^2 - 2\lambda(\lambda(-\widetilde{B}(0)) + \lambda + s)\widetilde{B}''(0)} = \frac{-\lambda\frac{ES^2}{(1-\rho)^3}}{2(1-\lambda\frac{ES}{1-\rho})^2 - 2\lambda(\lambda(-\frac{ES}{1-\rho}) + \lambda)\frac{ES^2}{(1-\rho)^3}}$$

$$= \frac{-\lambda\frac{ES^2}{(1-\rho)^3}}{2(1-\lambda\frac{ES}{1-\rho})^2 - 2\lambda^2\frac{ES}{-1+\rho}\frac{ES^2}{(1-\rho)^3}}$$

□