① $\begin{cases} S[y] = 2y^2(\pi) + \int_0^{\pi} dx \, \underbrace{((y')^2 - y^2 + 3y\cos 2x)}_{L} \\ y \in C^2[0,\pi], \quad y(0) = 0 \end{cases}$ $\boxed{\text{Кр №2}}$

$$S[y] = \int_a^b dx \, L(y, y', y'', \dots; x)$$

$$\boxed{\delta S = \int_a^b dx \left(\frac{\partial L}{\partial y} \delta y + \frac{\partial L}{\partial y'} \delta y' + \frac{\partial L}{\partial y''} \delta y'' + \dots \right)}$$

В нашем случае

$$2(y(\pi) + \delta y(\pi))^2 - 2y(\pi)^2$$

$$\delta S[y] = 4y(\pi)\,\delta y(\pi) + \int_0^{\pi} dx \, (2y'\delta y' - 2y\delta y + 3\cos 2x \, \delta y) =$$

$$= \underbrace{4y(\pi)\delta y(\pi) + 2y'\delta y \Big|_0^{\pi}}_{} + \int_0^{\pi} dx \, (3\cos 2x - 2y - 2y'')\,\delta y$$

$$y(0) = 0 \Rightarrow \delta y(0) = 0$$

$$\underbrace{(4y(\pi) + 2y'(\pi))}_{=0} \delta y(\pi) + \int_0^{\pi} dx \underbrace{(\dots)}_{0} \delta y$$

$$\begin{cases} y'' + y = \frac{3}{2}\cos 2x \\ y(0) = 0, \quad 2y(\pi) + y'(\pi) = 0 \end{cases}$$

$$y_0'' + y_0 = 0 \Rightarrow \lambda^2 + 1 = 0 \Rightarrow \lambda_{1,2} = \pm i$$

$$\begin{cases} y_0(x) = A\cos x + B\sin x \\ y_r(x) = -\frac{1}{2}\cos 2x \end{cases}$$

$$A = \frac{1}{2} \Leftarrow y(0) = 0$$

$$-2A - 1 - B = 0 \Rightarrow B = -2.$$

$$\boxed{y(x) = \frac{1}{2}\cos x - 2\sin x - \frac{1}{2}\cos 2x}$$

② $$S[y] = \int_0^{\pi/2} dx\left((y'')^2 - 81y^2 + 18xy'\right)$$

$$y(0)=0, \quad y\left(\frac{\pi}{2}\right) = \frac{1}{9}, \quad y'(0) = 0$$

$$\begin{cases} y^{(4)} - 81y = 9 \\ y''\left(\frac{\pi}{2}\right) = 0 \end{cases} \quad \Big| \quad y = Ae^{3x} + Be^{-3x} + C\cos 3x + D\sin 3x - \frac{1}{9}$$

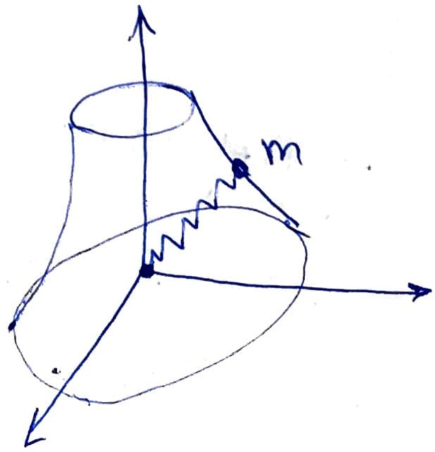
$$S[y] = \int dx\sqrt{1+(y')^2}$$

$$\delta S[y] = \int \frac{y'\,\delta y'}{\sqrt{1+(y')^2}}\,dx$$

$$S[y] = \int dx\left(\sin(y^2 y') + y^2 y''\right)$$

$$\delta S[y] = \int dx\left[\cos(y^2y')(2yy'\delta y + y^2\delta y') + 2yy''\delta y + y^2\delta y''\right]$$

③ $$z = \frac{1}{2(x^2+y^2)} \qquad U = \frac{kl^2}{2}$$



$$l^2 = z^2 + x^2 + y^2$$

$$\begin{cases} x = \rho\cos\varphi \\ y = \rho\sin\varphi \end{cases} \quad \Big| \quad z = \frac{1}{2\rho^2}$$

$$\Rightarrow \dot{z} = -\frac{\dot\rho}{\rho^3}$$

$$L = T - U = \frac{m}{2}\left[\dot\rho^2\left(1+\frac{1}{\rho^6}\right) + \rho^2\dot\varphi^2\right] - \frac{k}{2}\left(\rho^2 + \frac{1}{4\rho^4}\right)$$

$$\frac{\partial L}{\partial \varphi} = 0 \Rightarrow \frac{d}{dt}\frac{\partial L}{\partial \dot\varphi} = 0 \Rightarrow \boxed{\frac{\partial L}{\partial \dot\varphi} = m\rho^2\varphi = J = \text{const}}$$

$$E = T + U = \text{const}$$

$$\boxed{E = \frac{m}{2}\left(\dot\rho^2(1+\rho^{-6}) + \rho^2\dot\varphi^2\right) + \frac{k}{2}\left(\rho + \frac{1}{4\rho}\right)}$$

$$z = z_0 \Rightarrow \rho = \rho_0$$

$$L_\rho: \frac{d}{dt}\left(m\dot\rho(1+\rho^{-6})\right) - m\rho\left(\dot\varphi^2 - 3\dot\rho^2\rho^{-7}\right) + k\rho\left(1 - \frac{1}{2\rho^6}\right) = 0$$

$$\rho = \rho_0 > 0 \qquad \boxed{\dot\varphi_0^2 = \frac{k}{m}\left(1 - \frac{1}{2\rho^6}\right)} \geq 0$$

$$\rho_0 \geq \frac{1}{\sqrt[6]{2}} \longrightarrow 0 < z_0 \leq \frac{1}{\sqrt[3]{4}}$$

$$\rho_0 = \frac{1}{\sqrt[6]{2}} \longmapsto \min U_{\text{эфф}} \Rightarrow \dot\varphi = 0$$

$$0 \leq \dot\varphi^2 < \frac{k}{m}$$