

$$L(\vec{x}, \dot{\vec{x}}) = \underbrace{-mc^2}_{\text{константа}} \sqrt{1 - \frac{\dot{\vec{x}}^2}{c^2}}$$

a)

$$p_x = \frac{\partial L}{\partial \dot{x}} = -mc^2 \frac{1/2}{\sqrt{1 - \frac{\dot{x}^2}{c^2}}} \left(- \frac{2\dot{x}}{c^2} \right) =$$

$$= + \frac{mc^2 \dot{x}}{\cancel{c^2} \sqrt{1 - \frac{\dot{x}^2}{c^2}}} \quad \text{~~mc^2 \dot{x} = + mc^2 \dot{x}~~}$$

$$= \frac{m\dot{x}}{\sqrt{1 - \frac{\dot{x}^2}{c^2}}} \Rightarrow \sqrt{1 - \frac{\dot{x}^2}{c^2}} = \frac{m\dot{x}}{p_x}$$

$$H = p_x \dot{x} + L = \frac{m\dot{x}}{\sqrt{1 - \frac{\dot{x}^2}{c^2}}} \dot{x} + mc^2 \sqrt{1 - \frac{\dot{x}^2}{c^2}} \quad \left| \dot{x} = \frac{\sqrt{p_x^2 c^2}}{\sqrt{m^2 c^2 + p_x^2}} \right.$$

$$p_x \sqrt{1 - \frac{\dot{x}^2}{c^2}} = m\dot{x}$$

$$\dot{x} = \sqrt{\frac{p_x^2 c^2}{m^2 c^2 + p_x^2}}$$

$$H = \frac{m}{\sqrt{1 - \frac{p_x^2}{m^2 c^2 + p_x^2}}} \cdot \frac{p_x^2 c^2}{m^2 c^2 + p_x^2} + mc^2 \sqrt{1 - \frac{p_x^2}{m^2 c^2 + p_x^2}} =$$

$$= \cancel{p_x \dot{x}} \quad \cancel{m\dot{x}} \quad \cancel{p_x} + mc^2 \frac{m\dot{x}}{p_x}$$

$$H = \frac{p_x^2 c}{\sqrt{m^2 c^2 + p_x^2}} + \frac{m^2 c^2}{p_x} \cdot \frac{p_x c}{\sqrt{m^2 c^2 + p_x^2}} = \frac{p_x^2 c + m^2 c^3}{\sqrt{m^2 c^2 + p_x^2}}$$

5)

18:15

~~$$\dot{x} = \frac{\partial H}{\partial p_x}$$~~

$$\dot{p}_x = - \frac{\partial H}{\partial x} = 0$$

$$\dot{x} = \frac{\partial H}{\partial p_x} = \frac{(2p_x c) \sqrt{m^2 c^2 + p_x^2} - \frac{1}{2} \frac{2p_x}{\sqrt{m^2 c^2 + p_x^2}} (p_x^2 c + m^2 c^3)}{m^2 c^2 + p_x^2}$$

$$= \frac{2p_x c (m^2 c^2 + p_x^2) - p_x (p_x^2 c + m^2 c^3)}{(m^2 c^2 + p_x^2)^{3/2}}$$

~~$$\dot{x} = 0, \dot{p}_x = 0$$~~

~~$$\dot{x} = 0$$~~