Найти ограничение скобку Пуассона

На лист $\begin{cases} h^2+u^2-v^2=0 \\ v \geqslant 0 \end{cases}$

(*) 1) Найти удобную параметризацию пов-ти и вычислить ск. П. в терм. этой парам.

$$\cancel{\begin{cases} h = r\,\mathrm{ch}\, s \cos\varphi \\ u = r\,\mathrm{ch}\, s \sin\varphi \\ v = r\,\mathrm{sh}\, s \end{cases}} \qquad \begin{cases} h = r\cos\varphi \\ u = r\sin\varphi \\ v = r \end{cases}$$

$\cancel{r = R > 0 = const}$

$\cancel{r^2 = h^2+u^2-v^2}$

$r, \varphi: \quad r>0, \quad \varphi \in [0, 2\pi)$

$r = R = const > 0$

$\{h, u\} = \{h, x+y\} = 2v$

$\{h, v\} = 2u$

$\{u, v\} = -2h$

$\operatorname{tg}\varphi = \frac{u}{h}$

$\{r, \varphi\} = \{v, \varphi\}$

$\{v, \operatorname{tg}\varphi\} = \{v, \frac{u}{h}\} = \left(\frac{1}{h}\{v,u\} - \frac{u}{h^2}\{v,h\}\right) =$

$= \frac{1}{h}\cdot 2h - \frac{u}{h^2}\cdot(-2u) = \underline{2 + \frac{2u^2}{h^2}}$

$\{r, \operatorname{tg}\varphi\} = \{r, \varphi\}\,\frac{1}{\cos^2\varphi}$

$$\{r, \varphi\} = \cos^2\varphi \left(2 + \frac{2u^2}{h^2}\right) = \cos^2\varphi \left(\frac{2(h^2+u^2)}{h^2}\right) =$$

$$= \frac{\cos^2\varphi \cdot 2r^2}{r^2\cos^2\varphi} = 2.$$

$y \in C^\infty[0,1], \quad y(0) = A$

$$S[y] = \int_0^1 dx \Big(\underbrace{\tfrac{1}{4} (y')^2 \log(y^2) + xy' + e^{\cos y}}_{L} \Big)$$

a) $$\delta S[y] = \int_0^1 dx \left(\frac{\partial L}{\partial y} \delta y + \frac{\partial L}{\partial y'} \delta y' \right)$$

$$\frac{\partial L}{\partial y} = \frac{1}{4} (y')^2 \cdot \frac{1}{y^2} \cdot 2y + e^{\cos y} (-\sin y) =$$

$$= \frac{1}{2} \cdot \frac{1}{y} (y')^2 - \sin y \, e^{\cos y}$$

$$\frac{\partial L}{\partial y'} = \frac{1}{4} \cdot 2y' \ln y^2 + x = \frac{1}{2} y' \ln y^2 + x$$

$$\delta S[y] = \int_0^1 dx \left[\left(\frac{(y')^2}{2y} - \sin y \, e^{\cos y} \right) \delta y + \left(\frac{1}{2} y' \ln y^2 + x \right) \delta y' \right] =$$

$$= \int_0^1 \left(\frac{(y')^2}{2y} - \sin y \, e^{\cos y} \right) \delta y \, dx + \left(\frac{1}{2} y' \ln y^2 + x \right) \delta y \Big|_0^1 -$$

$$- \int_0^1 \left[\frac{1}{2} \left(y'' \ln y^2 + y' \cdot \frac{2}{y} \right) + 1 \right] \delta y \, dx =$$

$$= \int_0^1 \left(\frac{(y')^2}{2y} - \sin y \, e^{\cos y} - \frac{1}{2} y'' \ln y^2 - \frac{y'}{y} - 1 \right) \delta y \, dx +$$

$$+ \left(\frac{1}{2} y' \ln y^2 + x \right) \delta y \Big|_0^1$$

б) $y(0) = A \Rightarrow \delta y(0) = 0$

$$\delta y(1) \ \forall \Rightarrow \boxed{ \frac{1}{2} y' \ln y^2 + x \Big|_{x=1} = 0 }$$

$F[y]$ на $C^2[0,1]$: $y(1)=0$

$$F[y] = \int_0^1 dx\left((y')^2 - 2xy\right)$$

$$\Delta F[y] = \Delta \int_0^1 dx\left((y')^2 - 2xy\right) =$$

$$= \int_0^1 dx\left((y'+(\delta y)')^2 - 2x(y+\delta y)\right) - \int_0^1 dx\left((y')^2 - 2xy\right) =$$

$$= \int_0^1 dx\left\{\cancel{(y')^2} + 2y'\delta y' + (\delta y')^2 \cancel{-2xy} - 2x\delta y \cancel{-(y')^2} \cancel{+2xy}\right\} =$$

$$= \int_0^1 dx\left(2y'(\delta y)' + (\delta y')^2 - 2x\delta y\right)$$

$$\delta F[y] = \int_0^1 dx\left(2y'(\delta y)' - 2x\delta y\right) =$$

$$= \int_0^1 2y'(\delta y)'\,dx - \int_0^1 2x\delta y\,dx =$$

$$= \int_0^1 (2y'\delta y)'\,dx - \int_0^1 2y''\delta y\,dx - \int_0^1 2x\delta y\,dx =$$

$2y'(\delta y)' + 2y''\delta y$

$$-2\int_0^1 \delta y\,dx\,(y''+x)$$

$$= -2 \int_0^1 dx \, (y'' + x)\, \delta y + 2 y' \delta y \Big|_0^1 = 0$$

$$y(1) = 0 \Rightarrow \delta y \big|_{x=1} = 0$$

$$y'' + x = 0$$

$$y'' = -x$$

$$y' = -\frac{x^2}{2} + C_1$$

$$y = -\int \frac{x^2}{2} dx + C_1 x = -\frac{1}{2} \cdot \frac{x^3}{3} + C_1 x + C_0$$

$$y(x) = -\frac{x^3}{6} + C_1 x + C_0 \quad (*)$$

Граничное условие в т. $x = 0$

$$2y' \big|_{x=0} = 0 \qquad (\delta y \text{ – произв. в т. } x = 0)$$

$$y(1) = -\frac{1}{6} + C_1 + C_0 = 0 \qquad \Rightarrow C_0 = \frac{1}{6}.$$

$$y'(0) = -\frac{x^2}{2} + C_1 \Big|_{x=0} = +C_1 = 0 \Rightarrow C_1 = 0$$

$$\Rightarrow \boxed{y(x) = -\frac{x^3}{6} + \frac{1}{6}}$$

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мат. т. в $\mathbb{R}^2$ движ под действ F: $\vec{F} = (F_x, F_y)$

$$\forall x, y \in \mathbb{R} : \begin{cases} F_x = -2xy - \dfrac{(1+x)^2}{1+x^2} \\ F_y = -x^2 + \dfrac{2y}{1+y^2} \end{cases}$$

а) Показать: $\vec{F}$ - потенциальна $\Rightarrow$ найти $U(x,y) = ?$

б) Работа $\vec{F}$ при движ. по дуге окр

$x^2+y^2=1$ от $P(1,0)$ до $Q(0,1)$ $= ?$

а) $\dfrac{\partial F_y}{\partial x} = \dfrac{\partial F_x}{\partial y}$ $(*)$

$\dfrac{\partial F_y}{\partial x} = -2x$

$\Rightarrow (*)$ выполнено

$\dfrac{\partial F_x}{\partial y} = -2x$

пр-во $\mathbb{R}^2$ односвязно $\Big\} \Rightarrow$

$\Rightarrow$ по лемме Пуанкаре $(*)$ явл-ся достаточным условием $\Rightarrow$

$\Rightarrow$ сила $\vec{F}$ потенциальна.

$$\exists U(x,y) : \quad \frac{\partial U}{\partial x} = -F_x = 2xy + \frac{(1+x)^2}{1+x^2}$$

$$\frac{\partial U}{\partial y} = -F_y = x^2 - \frac{2y}{1+y^2}$$

$\int \left(1 + \dfrac{2x}{1+x^2}\right) dx =$

$= x + \int \dfrac{dx^2}{1+x^2}$

$$U(x,y) = +\int dx \left(2xy + \frac{(1+x)^2}{1+x^2}\right) = \cancel{2}y\frac{x^2}{\cancel{2}} + \int \frac{(1+x)^2}{1+x^2}\, dx =$$

$$= x^2y + \ln(x^2+1) + x + C(y)$$

$$\cancel{x^2} + C'(y) = \cancel{x^2} - \frac{2y}{1+y^2}$$

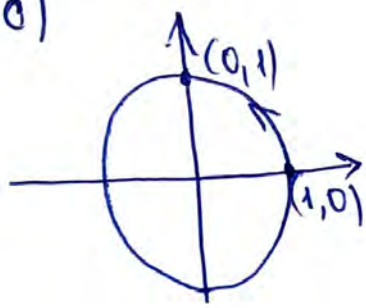
$$C'(y) = -\frac{2y}{1+y^2}$$

$$-\int \frac{dy^2}{1+y^2}$$

$$C(y) = -2\int \frac{y}{1+y^2}\,dy = -\ln(y^2+1) + C$$

$$\boxed{U(x,y) = x^2 y + \ln(x^2+1) + x - \ln(y^2+1) + C}$$

б)



Т.к. сила потенциальна

$A = U_н - U_к =$

$= U(1,0) - U(0,1) = \ln 2 + 1 - \ln 2 = 1.$

20.04.21

$$S[x,y] = \int dt \left(\dot{x}^2 y^{-4} + \dot{y}^2 t^2 - x^2 y^2 t \right)$$

$$\tilde{x} = e^{\varepsilon} x, \quad \tilde{y} = e^{a\varepsilon} y, \quad \tilde{t} = e^{b\varepsilon} t$$

$$S[\tilde{x},\tilde{y}] = \int d\tilde{t}\, e^{-b\varepsilon} \Big[e^{2(b-1)} (\tilde{x}')^2 e^{4a\varepsilon} \tilde{y}^{-4} +$$

$$+ e^{2(b-a)\varepsilon} (\tilde{y}')^2 e^{-2b\varepsilon} \tilde{t}^2 - e^{-2a\varepsilon - 2\varepsilon} \tilde{x}^2 \tilde{y}^2 \cdot e^{-b\varepsilon} \tilde{t} \Big] =$$

$$= \int d\tilde{t} \left(e^{(2b-2-b+4a)\varepsilon} (\tilde{x}')^2 \tilde{t}^2 - e^{(2b-2a-b-2b)\varepsilon} (\tilde{y}')^2 \tilde{t}^2 - \right.$$

$$\left. - e^{-(2a+2b+2)\varepsilon} \tilde{x}^2 \tilde{y}^2 \tilde{t} \right)$$

$$\begin{cases} b + 4a - 2a = 0 \\ 2a + b = 0 \\ 2a + 2b + 2 = 0 \end{cases} \qquad a = 1, \quad b = -2$$

При $\varepsilon = 0$ преобразование тождественны

$$\xi_0 = \frac{\partial \tilde{t}}{\partial \varepsilon}\Big|_{\varepsilon=0} = -2t$$

$$\xi_x = \frac{\partial \tilde{x}}{\partial \varepsilon}\Big|_{\varepsilon=0} = x$$

$$\xi_y = \frac{\partial \tilde{y}}{\partial \varepsilon}\Big|_{\varepsilon=0} = y$$

$$I = \frac{\partial L}{\partial \dot{x}} \xi_x + \frac{\partial L}{\partial \dot{y}} \xi_y + \left(L - \dot{x} \frac{\partial L}{\partial \dot{x}} - \dot{y} \frac{\partial L}{\partial \dot{y}} \right) \xi_0$$

$$I = 2x\dot{x}y^{-4} + 2y\dot{y}t^2 + 2t\dot{x}^2 y^{-4} + 2t^3 \dot{y}^2 + 2x^2 y^2 t^3$$

5-минутка.

$$L = -mc^2\sqrt{1-\frac{\dot{\vec{x}}^2}{c^2}} \qquad L_н \simeq \frac{m\dot{\vec{x}}^2}{2} + O\left(\frac{1}{c}\right)$$

а) $$p_i = \frac{\partial L}{\partial \dot{x}_i} = \frac{m\dot{x}_i}{\sqrt{1-\frac{\dot{\vec{x}}^2}{c^2}}} \Rightarrow \vec{p} = \frac{m\dot{\vec{x}}}{\sqrt{1-\frac{\dot{\vec{x}}^2}{c^2}}}$$

$$\vec{p}^2 = \frac{m^2\dot{\vec{x}}^2}{1-\frac{\dot{\vec{x}}^2}{c^2}} \Rightarrow \dot{\vec{x}}^2 = \frac{\vec{p}^2c^2}{\vec{p}^2+m^2c^2}$$

$$1-\frac{\dot{\vec{x}}^2}{c^2} = \frac{m^2c^2}{\vec{p}^2+m^2c^2} \Rightarrow \vec{p} = \frac{\dot{\vec{x}}}{c}\sqrt{\vec{p}^2+m^2c^2} \Rightarrow$$

$$\Rightarrow \dot{\vec{x}} = \frac{\vec{p}c}{\sqrt{\vec{p}^2+m^2c^2}}$$

$$E = \dot{\vec{x}}\frac{\partial L}{\partial \dot{\vec{x}}} - L = \frac{mc^2}{\sqrt{1-\frac{\dot{\vec{x}}^2}{c^2}}}$$

$$H = E\Big|_{\dot{\vec{x}}=\dot{\vec{x}}(\vec{p})} \Rightarrow \boxed{H = c\sqrt{\vec{p}^2+m^2c^2}}$$

б) $$\begin{cases} \dot{x}_i = \dfrac{\partial H}{\partial p_i} = \dfrac{cp_i}{\sqrt{\vec{p}^2+m^2c^2}} \\ \dot{p}_i = -\dfrac{\partial H}{\partial x_i} = 0 \Rightarrow p_i(t) = p_i = \text{const} \end{cases}$$

в) $$H = c\sqrt{\vec{p}^2+m^2c^2} = \text{const} = \varepsilon$$

$$\dot{\vec{x}} = \frac{c^2}{\varepsilon}\vec{p} = \frac{c^2}{\varepsilon}\vec{p}_0$$

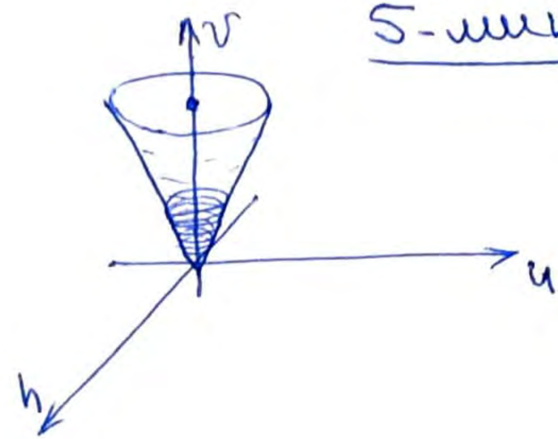
$$\vec{x}(t) = \frac{c^2}{\varepsilon}\vec{p}_0 t + \vec{x}(0), \quad \vec{x}(0) = 0$$

$\boxed{\vec{p}(t) = \vec{p}_0;\ \vec{x}(t) = \frac{c^2}{\varepsilon}\vec{p}_0 t}$, где $\varepsilon = c\sqrt{\vec{p}_0^{\,2}+m^2c^2}$

5-минутка

$\boxed{18.20.21}$

в $\mathbb{R}^3$: $\begin{cases} h^2+u^2-v^2=0 \\ v>0 \end{cases}$



$h = v\cos\varphi$, $u = v\sin\varphi$, $v = v$

$\begin{cases} v \in (0, +\infty) \\ \varphi \in [0, +2\pi) \end{cases}$

$f(h, u, v) = 0$ в $\mathbb{R}^3$

$h = h(\xi, \eta)$, $u = u(\xi, \eta)$, $v = v(\xi, \eta)$

$f(h(\xi,\eta), u(\xi,\eta), v(\xi,\eta)) \equiv 0$

$\{h, u\} = 2v$, $\{h, v\} = 2u$, $\{u, v\} = -2h$

$\cos\varphi = \dfrac{h}{v} \Rightarrow \{\cos\varphi, v\} = -\sin\varphi\,\{\varphi, v\}$

$\left\{\dfrac{h}{v}, v\right\} = \dfrac{1}{v}\underbrace{\{h, v\}}_{2u} = \dfrac{2u}{v}$

$\Rightarrow \{\varphi, v\} = -2$.

$$L(\vec{x}, \dot{\vec{x}}) = -mc^2 \sqrt{1 - \frac{\dot{\vec{x}}^2}{c^2}}$$

константа

a)

$$p_x = \frac{\partial L}{\partial \dot{x}} = -mc^2 \frac{1/2}{\sqrt{1 - \frac{\dot{x}^2}{c^2}}} \left(- \frac{2\dot{\vec{x}}}{c^2} \right) =$$

$$= + \frac{m\cancel{c^2}\dot{\vec{x}}}{\cancel{c^2}\sqrt{1 - \frac{\dot{x}^2}{c^2}}}$$

$$= \frac{m\dot{\vec{x}}}{\sqrt{1 - \frac{\dot{x}^2}{c^2}}} \Rightarrow \sqrt{1 - \frac{\dot{x}}{c^2}} = \frac{m\dot{x}}{p_x}$$

$$H = p_x \dot{\vec{x}} + L = \frac{m\dot{\vec{x}}}{\sqrt{1 - \frac{\dot{x}^2}{c^2}}} \dot{\vec{x}} + mc^2 \sqrt{1 - \frac{\dot{x}^2}{c^2}} \Bigg|_{\dot{\vec{x}} = \frac{p_x^2 c^2}{\sqrt{m^2c^2 + p_x^2}}}$$

$$p_x \sqrt{1 - \frac{\dot{x}^2}{c^2}} = m\dot{x}$$

$$\dot{x} = \sqrt{\frac{p_x^2 c^2}{m^2c^2 + p_x^2}}$$

$$H = \frac{m}{\sqrt{1 - \frac{p_x^2}{m^2c^2 + p_x^2}}} \cdot \frac{p_x^2 c^2}{m^2c^2 + p_x^2} + mc^2 \sqrt{1 - \frac{p_x^2}{m^2c^2 + p_x^2}} =$$

$$=$$

$$\boxed{H = \frac{p_x^2 c}{\sqrt{m^2c^2 + p_x^2}} + \frac{m^2c^2}{\cancel{p_x}} \cdot \frac{\cancel{p_x} c}{\sqrt{m^2c^2 + p_x^2}} = \frac{p_x^2 c + m^2 c^3}{\sqrt{m^2c^2 + p_x^2}}}$$

б)

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$$\boxed{\dot{p}_x = -\frac{\partial H}{\partial x} = 0}$$

$$\dot{x} = \frac{\partial H}{\partial p_x} = \frac{(2p_x c)\sqrt{m^2c^2+p_x^2} - \frac{1}{2}\frac{2p_x}{\sqrt{m^2c^2+p_x^2}}(p_x^2c+m^2c^3)}{m^2c^2+p_x^2}$$

$$= \frac{2p_x c(m^2c^2+p_x^2) - p_x(p_x^2c+m^2c^3)}{(m^2c^2+p_x^2)^{3/2}}$$

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мат. т. в $\mathbb{R}^2$ движ под действ F: $\vec{F} = (F_x, F_y)$

$$\forall x,y \in \mathbb{R}: \begin{cases} F_x = -2xy - \dfrac{(1+x)^2}{1+x^2} \\ F_y = -x^2 + \dfrac{2y}{1+y^2} \end{cases}$$

а) Показать: $\vec{F}$ - потенциальна $\Rightarrow$ найти $U(x,y) = ?$

б) Работа $\vec{F}$ при движ. по дуге окр

$x^2+y^2=1$ от $P(1,0)$ до $Q(0,1)$ $= ?$

а) $$\frac{\partial F_y}{\partial x} = \frac{\partial F_x}{\partial y} \quad (*)$$

$$\frac{\partial F_y}{\partial x} = -2x$$

$\Rightarrow$ (*) выполнено

$$\frac{\partial F_x}{\partial y} = -2x$$

пр-во $\mathbb{R}^2$ односвязно $\Big\} \Rightarrow$

$\Rightarrow$ по лемме Пуанкаре (*) явл-ся достаточным условием $\Rightarrow$

$\Rightarrow$ сила $\vec{F}$ потенциальна.

$$\exists U(x,y): \quad \frac{\partial U}{\partial x} = -F_x = 2xy + \frac{(1+x)^2}{1+x^2}$$

$$\frac{\partial U}{\partial y} = -F_y = x^2 - \frac{2y}{1+y^2}$$

$$\int\left(1+\frac{2x}{1+x^2}\right)dx = x + \int\frac{dx^2}{1+x^2}$$

$$U(x,y) = +\int dx\left(2xy + \frac{(1+x)^2}{1+x^2}\right) = \cancel{2}y\frac{x^2}{\cancel{2}} + \int\frac{(1+x)^2}{1+x^2}dx =$$

$$= x^2 y + \ln(x^2+1) + x + C(y)$$

$$\cancel{x^2} + C'(y) = \cancel{x^2} - \frac{2y}{1+y^2}$$

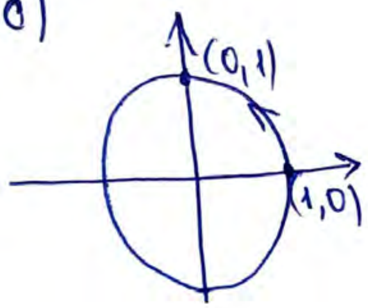
$$C'(y) = -\frac{2y}{1+y^2}$$

$$-\int \frac{dy^2}{1+y^2}$$

$$C(y) = -2\int \frac{y}{1+y^2}\,dy = -\ln(y^2+1) + C$$

$$\boxed{U(x,y) = x^2y + \ln(x^2+1) + x - \ln(y^2+1) + C}$$

б)



Т.к. сила потенциальна

$$A = U_н - U_к =$$

$$= U(1,0) - U(0,1) = \ln 2 + 1 - \ln 2 = 1.$$

$F[y]$ на $C^2[0,1]$: $y(1)=0$

$$F[y] = \int_0^1 dx\left((y')^2 - 2xy\right)$$

$$\Delta F[y] = \Delta \int_0^1 dx\left((y')^2 - 2xy\right) =$$

$$= \int_0^1 dx\left((y'+(\delta y)')^2 - 2x(y+\delta y)\right) - \int_0^1 dx\left((y')^2 - 2xy\right) =$$

$$= \int_0^1 dx\left\{\cancel{(y')^2} + 2y'\delta y' + (\delta y')^2 \cancel{-2xy} - 2x\delta y \cancel{-(y')^2} \cancel{+2xy}\right\} =$$

$$= \int_0^1 dx\left(2y'(\delta y)' + (\delta y')^2 - 2x\,\delta y\right)$$

$$\delta F[y] = \int_0^1 dx\left(2y'(\delta y)' - 2x\delta y\right) =$$

$$= \int_0^1 2y'(\delta y)'\,dx - \int_0^1 2x\,\delta y\,dx =$$

$$= \int_0^1 \overbrace{(2y'\delta y)'}^{2y'(\delta y)' + 2y''\delta y}\,dx - \int_0^1 2y''\,\delta y\,dx - \int_0^1 2x\,\delta y\,dx =$$

$$-2\int_0^1 \delta y\,dx\,(y''+x)$$

$$= -2\int_0^1 dx\,(y'' + x)\,\delta y + 2y'\,\delta y\Big|_0^1 = 0$$

$$y(1) = 0 \Rightarrow \delta y\big|_{x=1} = 0$$

$$y'' + x = 0$$

$$y'' = -x$$

$$y' = -\frac{x^2}{2} + C_1$$

$$y = -\int \frac{x^2}{2}\,dx + C_1 x = -\frac{1}{2}\cdot\frac{x^3}{3} + C_1 x + C_0$$

$$y(x) = -\frac{x^3}{6} + C_1 x + C_0 \quad (*)$$

Граничное условие в т. $x = 0$

$$2y'\big|_{x=0} = 0 \qquad (\delta y \text{ – произв. в т. } x = 0)$$

$$y(1) = -\frac{1}{6} + C_1 + C_0 = 0 \qquad \Rightarrow C_0 = \frac{1}{6}.$$

$$y'(0) = -\frac{x^2}{2} + C_1\Big|_{x=0} = +C_1 = 0 \Rightarrow C_1 = 0$$

$$\Rightarrow \boxed{y(x) = -\frac{x^3}{6} + \frac{1}{6}}$$

$y \in C^\infty[0,1]$, $\quad y(0) = A$

$$S[y] = \int_0^1 dx \left(\underbrace{\frac{1}{4}(y')^2 \log(y^2) + xy' + e^{\cos y}}_{L} \right)$$

a) $$\delta S[y] = \int_0^1 dx \left(\frac{\partial L}{\partial y}\delta y + \frac{\partial L}{\partial y'}\delta y' \right)$$

$$\frac{\partial L}{\partial y} = \frac{1}{4}(y')^2 \cdot \frac{1}{y^2}\cdot 2y + e^{\cos y}(-\sin y) =$$

$$= \frac{1}{2}\cdot\frac{1}{y}(y')^2 - \sin y\, e^{\cos y}$$

$$\frac{\partial L}{\partial y'} = \frac{1}{4}\cdot 2y' \ln y^2 + x = \frac{1}{2}y' \ln y^2 + x$$

$$\delta S[y] = \int_0^1 dx \left[\left(\frac{(y')^2}{2y} - \sin y\, e^{\cos y} \right)\delta y + \left(\frac{1}{2}y' \ln y^2 + x \right)\delta y' \right] =$$

$$= \int_0^1 \left(\frac{(y')^2}{2y} - \sin y\, e^{\cos y} \right)\delta y\, dx + \left(\frac{1}{2}y' \ln y^2 + x \right)\delta y \Big|_0^1 -$$

$$- \int_0^1 \left[\frac{1}{2}\left(y'' \ln y^2 + y'\cdot\frac{2}{y} \right) + 1 \right]\delta y\, dx =$$

$$= \int_0^1 \left(\frac{(y')^2}{2y} - \sin y\, e^{\cos y} - \frac{1}{2}y'' \ln y^2 - \frac{y'}{y} - 1 \right)\delta y\, dx +$$

$$+ \left(\frac{1}{2}y' \ln y^2 + x \right)\delta y \Big|_0^1$$

б) $y(0) = A \Rightarrow \delta y(0) = 0$

$$\delta y(1)\ \forall \Rightarrow \boxed{\frac{1}{2}y' \ln y^2 + x \Big|_{x=1} = 0}$$

$y \in C^\infty[0,1], \quad y(0) = A$

$$S[y] = \int_0^1 dx \Big(\underbrace{\tfrac{1}{4} (y')^2 \log(y^2) + xy' + e^{\cos y}}_{L} \Big)$$

a) $$\delta S[y] = \int_0^1 dx \left(\frac{\partial L}{\partial y} \delta y + \frac{\partial L}{\partial y'} \delta y' \right)$$

$$\frac{\partial L}{\partial y} = \frac{1}{4} (y')^2 \cdot \frac{1}{y^2} \cdot 2y + e^{\cos y} (-\sin y) =$$

$$= \frac{1}{2} \cdot \frac{1}{y} (y')^2 - \sin y \, e^{\cos y}$$

$$\frac{\partial L}{\partial y'} = \frac{1}{4} \cdot 2y' \ln y^2 + x = \frac{1}{2} y' \ln y^2 + x$$

$$\delta S[y] = \int_0^1 dx \left[\left(\frac{(y')^2}{2y} - \sin y \, e^{\cos y} \right) \delta y + \left(\tfrac{1}{2} y' \ln y^2 + x \right) \delta y' \right] =$$

$$= \int_0^1 \left(\frac{(y')^2}{2y} - \sin y \, e^{\cos y} \right) \delta y \, dx + \left(\tfrac{1}{2} y' \ln y^2 + x \right) \delta y \Big|_0^1 -$$

$$- \int_0^1 \left[\tfrac{1}{2} \left(y'' \ln y^2 + y' \cdot \frac{2}{y} \right) + 1 \right] \delta y \, dx =$$

$$= \int_0^1 \boxed{\left(\frac{(y')^2}{2y} - \sin y \, e^{\cos y} - \tfrac{1}{2} y'' \ln y^2 - \frac{y'}{y} - 1 \right)} \delta y \, dx +$$

$$+ \left(\tfrac{1}{2} y' \ln y^2 + x \right) \delta y \Big|_0^1$$

б) $y(0) = A \Rightarrow \delta y(0) = 0$

$$\delta y(1) \ \forall \Rightarrow \boxed{\tfrac{1}{2} y' \ln y^2 + x \Big|_{x=1} = 0}$$