Контрольная работа по механике № 2

Струихиной Ксении

№ 2 $S[y(x)] = \int_0^{\pi/2} dx\left((y'')^2 - 81y^2 + 18y'x\right)$

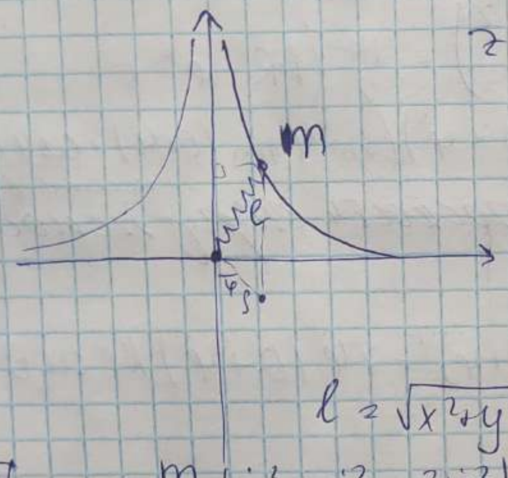
$\delta S[y+\delta y] - S[y] = \int_0^{\pi/2} dx\left(((y+\delta y)'')^2 - 81(y+\delta y)^2 + 18(y+\delta y)'x - \right.$

$\left. - (y'')^2 + 81y^2 - 18y'x\right) = \int_0^{\pi/2}\left((\delta y'')^2 - 81(\delta y)^2 + 18(\delta y)'x + 2y''\delta y'' - \right.$

$\left. - 162y\delta y\right)dx$

$\Rightarrow \delta S[y] = \int_0^{\pi/2}\left(2y''\delta y'' \ldots\right.$

№ 3



$z = \dfrac{1}{2(x^2+y^2)}$

Введем цилиндрические координаты

$\rho, z, \varphi, \quad \rho = \sqrt{x^2+y^2} \Rightarrow z = \dfrac{1}{2\rho^2}$

$l = \sqrt{x^2+y^2+z^2} = \sqrt{\dfrac{1}{2z} + z^2}$

$T_{кин} = \dfrac{m}{2}\left(\dot\rho^2 + \dot z^2 + \rho^2\dot\varphi^2\right) =$

$= \dfrac{m}{2}\left(\dfrac{\dot z^2}{8z^3} + \dot z^2 + \dfrac{\dot\varphi^2}{2z}\right)$

$z = \dfrac{1}{2\rho^2} \Rightarrow \rho^2 = \dfrac{1}{2z}$

$\dot z = -\dfrac{\dot\rho}{\rho^3} \Rightarrow \dot z^2 = \dfrac{\dot\rho^2}{\rho^6} \Rightarrow$

$\dot\rho^2 = \dot z^2 \cdot \rho^6 = \dfrac{\dot z^2}{8z^3} = (\rho^2)^3$

$U = \dfrac{k\left(\dfrac{1}{2z} + z^2\right)}{2}$

$L = T - U = \dfrac{m}{2}\left(\dfrac{\dot z^2}{8z^3} + \dot z^2 + \dfrac{\dot\varphi^2}{2z}\right) - \dfrac{k}{2}\left(\dfrac{1}{2z} + z^2\right)$

$\dfrac{\partial L}{\partial\varphi} = 0 \Rightarrow$ ЗСИ

$L_\varphi = \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial\dot\varphi}\right) - \dfrac{\partial L}{\partial\varphi} = 0 \quad \dfrac{d}{dt}\left(\dfrac{2\dot\varphi m}{4z}\right) = 0 \Rightarrow J = \dfrac{2\dot\varphi m}{2z} = const$ ЗСИ

$L_z = \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial\dot z}\right) - \dfrac{\partial L}{\partial z} = 0$

$\dfrac{d}{dt}\left(\dfrac{2m\dot z}{16z^3} + \dfrac{2m\dot z}{2}\right) - \left(-\dfrac{3m\dot z^2}{16z^4} - \dfrac{m\dot\varphi^2}{4z^2} + \dfrac{k}{4z^2} - \dfrac{2zk}{2}\right) = 0$

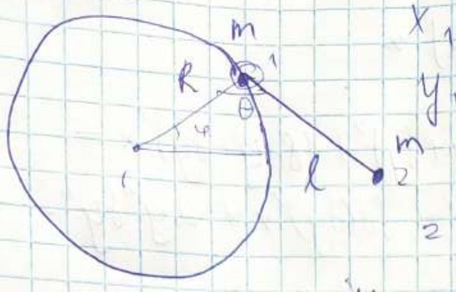
L явно не зависит от $t \Rightarrow$ выполняется

ЗСЭ: $E = T + U = const$

т.к. z есть в знаменателе, то $z \neq 0 \Rightarrow m\dot\varphi^2 \neq k \Rightarrow \boxed{\dot\varphi^2 \neq \dfrac{k}{m}}$

При $z = const$ $\dot z = 0$ $L_z\Big|_{z=const} = \dfrac{m\dot\varphi^2}{4z_0^2} - \dfrac{k}{4z_0^2} + z_0 k = 0 \quad z_0^3 = \dfrac{m\dot\varphi^2 - k}{4k}$

№ 4



$x_1 = R\cos\varphi \qquad \dot{x}_1 = -R\sin\varphi\cdot\dot\varphi$

$y_1 = R\sin\varphi \qquad \dot{y}_1 = R\cos\varphi\cdot\dot\varphi$

$x_2 = R\cos\varphi + l\,(\cos(\theta+\varphi-\pi)) =$

$= R\cos\varphi - l\cos(\theta+\varphi)$

$y_2 = R\sin\varphi - l\sin(\theta+\varphi)$

$T = \frac{m}{2}(R^2\dot\varphi^2) + \frac{m}{2}\Big((-R\sin\varphi\cdot\dot\varphi + l\sin(\theta+\varphi)(\dot\theta+\dot\varphi))^2 +$

$+ (R\cos\varphi\cdot\dot\varphi - l\cos(\theta+\varphi)(\dot\theta+\dot\varphi))^2\Big) = \frac{m}{2}\big(\cancel{2}R^2\cdot\dot\varphi^2 +$

$+ R^2\dot\varphi^2 + l^2(\dot\varphi+\dot\theta)^2 - 2\dot\varphi(\dot\theta+\dot\varphi)(\sin\varphi\sin(\theta+\varphi) + \cos\varphi\cos(\theta+\varphi)Rl)$

косинус разности

Т.к. точки лежат в плоскости окружности

то $U = 0$

$L = T = \frac{m}{2}\left(2R^2\dot\varphi^2 + \cancel{R^2\dot\varphi^2} + l^2(\dot\theta+\dot\varphi)^2 - 2\dot\varphi(\dot\theta+\dot\varphi)Rl\cos\theta\right)$

Т.к. $\frac{\partial L}{\partial\varphi} = 0$, то ЗСИ: $\frac{\partial L}{\partial\dot\varphi} = const$

$2mR^2\dot\varphi + 2l^2\dot\varphi + 2\dot\theta l^2 - 2(2\dot\varphi+\dot\theta) = const$

$\frac{\partial L}{\partial t} = 0 \Rightarrow$ ЗСЭ: $E = T + U = T - const$

№ 2 $S[y(x)] = \int_0^{\pi/2} dx\,((y'')^2 - 81y^2 + 18xy') =$

$\Rightarrow S(y+\delta y) - S(y) = \int_0^{\pi/2}$ — часть из 1 страницы (+)

$\delta S(y) = \int_0^{\pi/2}(2y''\delta y'' - 162y\delta y)\,dx =$

$= \int_0^{\pi/2} 2y''\,d(\delta y') - 162\int_0^{\pi/2} y\,\delta y\,dx =$

$= 2y''\delta y'\Big|_0^{\pi/2} - 2\int_0^{\pi/2}\delta y' y'''dx + \int_0^{\pi/2} 162y\,\delta y\,dx =$

$= 2y''\delta y'\Big|_0^{\pi/2} - 2y'''\delta y\Big|_0^{\pi/2} + 2\int_0^{\pi/2}\delta y\,y''''dy - 162\int_0^{\pi/2} y\,\delta y\,dx =$

$= \int_0^{\pi/2}(2y'''' - 162y) = 0 \quad 2y'''' - 162y = 0$

$y'''' - 81y = 0 \quad t^4 - 81 = 0 \quad t = \pm 3, \pm 3i$

$y = C_1e^{3x} + C_2e^{-3x} + C_3\cos(3x) + C_4\sin(3x)$

№1. $S[y] = 2y^2(\pi) + \int_0^\pi dx\,((y'(x))^2 - y^2(x) + 3y(x)\cos 2x)$

$y(0)\quad y(x) \in C^2[0,\pi]$

$S[y+\delta y] = 2y^2\pi + 4y(\pi)\delta y(\pi) + \underbrace{2\delta y^2(\pi)}_{O(\delta^2)} +$

$+ \int_0^\pi dx\,((y(x))^2 + 2y'(x)\delta y'(x) + \underbrace{(\delta y'(x))^2}_{O(\delta^2)} - y^2(x) - 2y(x)\delta y(x) -$

$- (\delta y(x))^2 + 3y(x)\cos 2x + 3\delta y'(x)\cos 2x)$ $\quad 2(y'\delta y)' - 2y''\delta y$

$S(y+\delta y) - S(y) = 4y(\pi)\delta y(\pi) + \int_0^\pi dx\,(\cancel{2y'(x)\delta y'(x)} -$

$- 2y(x)\delta y(x) + 3\delta y'(x)\cos 2x) = 4y(\pi)\delta y(\pi) +$

$+ 2y'\delta y\big|_0^\pi + \int_0^\pi dx\,(-2(y''+y)\delta y + (3\delta y\cos 2x)' + 6\delta y\sin 2x) =$

$= 4y(\pi)\delta y(\pi) + 2y'\delta y\big|_0^\pi + 3\delta y\big|_0^\pi - 2\int_0^\pi dx\,((y''+y)\delta y -$

$- 3\delta y\sin 2x)$

$\delta y(\pi)\cdot(4y(\pi) + 2y'(\pi) + 3) + \delta y(0)(-2y'(0) - 3) - 2\int_0^\pi dx\,(-$

$-(y'' + y - 3\sin 2x\,\delta y) = 0$

$4y(\pi) + 2y'(\pi) + 3 = 0$

$-2y'(0) - 3 = 0$

$y'' + y - 3\sin 2x = 0$

$y'' + y = 3\sin 2x \qquad y = -\sin 2x$

$y'' + y = 0 \qquad y'' = 4\sin 2x$

$y = \alpha\sin x + \beta\cos x \qquad y'' + y = 3\sin 2x$

$y(x) = -\sin 2x + \alpha\sin x + \beta\cos x$

$y'(x) = -2\cos 2x + \alpha\cos x - \beta\sin x$

$-2y'(0) - 3 = -2(-2+\alpha) - 3 = 0$

$4y(\pi) + 2y'(\pi) + 3 = 4(-\beta) + 2(-2-\alpha) + 3 = 0$

$4 - 2\alpha = 3 \qquad \alpha = 1/2$

$-4\beta - 4 - 2\alpha + 3 = 0 \Rightarrow \beta = -\frac{1}{2}$

$y(x) = -\sin 2x + \frac{1}{2}\sin 2x - \frac{1}{2}\cos 2x$