Разбор к/р.

① а) $\{(\vec{r}_1,\vec{p}_2),(\vec{r}_2,\vec{p}_1)\} = \{r_{1i}\,p_{2i},\, r_{2j}\,p_{1j}\} =$

$= \delta_{ij}\,(p_{2i}, r_{2j}) - \delta_{ij}\,(r_{1i}, p_{1i}) = (\vec{r}_2\cdot\vec{p}_2) - (\vec{r}_1\cdot\vec{p}_1).$

б) $\{(\vec{r}_1,\vec{p}_2),\, \underbrace{[\vec{r}_2\times\vec{p}_1]_i}_{\varepsilon_{ijl}\, r_{2j}\, p_{1l}}\} = \{r_{1k}\,p_{2k},\, \varepsilon_{ijl}\, r_{2j}\, p_{1l}\} =$

$= \left([\vec{r}_2\times\vec{p}_2] - [\vec{r}_1,\vec{p}_1]\right)_i$

в) $\{(\underbrace{[\vec{r}_1\times\vec{p}_2]}_{\varepsilon_{ikl}})_i,\, (\underbrace{[\vec{r}_2\times\vec{p}_1]}_{\varepsilon_{jst}})_j\} = \delta_{ij}\left((\vec{r}_1\vec{p}_1) - (\vec{r}_2\vec{p}_2)\right) +$

$+ (r_{2i}\,p_{2j}) - (r_{1j}\,p_{1i})$

ε_{ijk}: $\varepsilon_{ija}\,\varepsilon_{kla} = \delta_{ik}\,\delta_{jl} - \delta_{il}\,\delta_{jk}$

$[\vec{a}\times[\vec{b}\times\vec{c}]] = \vec{b}\,(\vec{a}\,\vec{c}) - \vec{c}\,(\vec{a}\cdot\vec{b})$

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② $L = \frac{mv^2}{2} + \frac{e}{2c}\,(\vec{B},[\vec{r}\times\dot{\vec{r}}])$ $\qquad \vec{v} = \frac{d\vec{r}}{dt}$

а) $p_i = \frac{\partial L}{\partial \dot{r}_i} = m\dot{r}_i + \frac{e}{2c}\,[\vec{B}\times\vec{r}]_i \Rightarrow \dot{r}_i = \frac{p_i}{m} - \frac{e}{2mc}\,[\vec{B}\times\vec{r}]_i$

$\frac{\partial}{\partial \dot{r}_i}\left(\frac{e}{2c}\,B_i\,\varepsilon_{ijk}\,r_j\,\dot{r}_k\right) = \frac{e}{2c}\,B_j\,\varepsilon_{jki}\,r_k = \frac{e}{2c}\,\varepsilon_{ijk}\,B_j\,r_k = \frac{e}{2c}\,[\vec{B}\times\vec{r}]_i$

$H = \vec{p}\cdot\dot{\vec{r}} - L\big|_{\dot{\vec{r}}(\vec{r},\vec{p})} = \frac{\vec{p}^2}{m} - \frac{e}{2m}\,(\vec{B}\cdot[\vec{r}\times\vec{p}]) -$

$- \frac{m}{2}\left(\frac{\vec{p}^2}{m^2} - \frac{e}{m^2c}\,(\vec{B},[\vec{r}\times\vec{p}]) + \frac{e}{4m^2c^2}\,[\vec{B}\times\vec{r}]^2\right)$

$- \frac{e}{2mc}\left(\vec{B},\left[\vec{r}\times\left(\vec{p} - \frac{e}{2c}\,[\vec{B}\times\vec{r}]\right)\right]\right)\Big) =$

$= \frac{\vec{p}^2}{2m} - \frac{e}{2m}\,(\vec{B},[\vec{r}\times\vec{p}]) - \frac{e^2}{8mc^2}\,[\vec{B}\times\vec{r}]^2 + \frac{e}{4mc^2}\,(\vec{B},[\vec{r}\times[\vec{B}\times\vec{r}]])$

1) $[\vec{B}\times\vec{r}]^2 = \varepsilon_{iab} B_a r_b \varepsilon_{iks} B_k r_s = (\delta_{ak}\delta_{bs} - \delta_{as}\delta_{bk}) B_a B_k r_b r_s$

$= \vec{B}^2\vec{r}^2 - (\vec{B},\vec{r})^2$

2) $(\vec{B}, [\vec{r}\times[\vec{B}\times\vec{r}]]) = B_i \varepsilon_{ijk} r_j \varepsilon_{kab} B_a r_b =$

$= (\delta_{ia}\delta_{jb} - \delta_{ib}\delta_{ja}) B_i B_a r_j r_b = \vec{B}^2 \cdot \vec{r}^2 - (\vec{B}\cdot\vec{r})^2$

$$H = \frac{\vec{p}^2}{2m} + \frac{e}{2mc}(\vec{B}, [\vec{r}\times\vec{p}]) + \frac{e^2}{8mc^2}(\vec{B}^2\vec{r}^2 - (\vec{B},\vec{r})^2)$$

$\Updownarrow$

$H = \frac{1}{2m}(\vec{p} - \frac{e}{c}\vec{A})^2, \quad \vec{A} = \dots$

$\mathrm{rot}\,\vec{A} = \dots$

б) $\dot{\vec{r}} = \frac{\partial H}{\partial \vec{p}} = \frac{\vec{p}}{m} - \frac{e}{2mc}[\vec{B}\times\vec{r}]$

$\dot{\vec{p}} = -\frac{\partial H}{\partial \vec{r}} = \frac{e}{2mc}[\vec{p}\times\vec{B}] + \frac{e}{4mc^2}\underbrace{((\vec{B},\vec{r})\vec{B} - \vec{B}^2\vec{r})}_{\perp\vec{B}}$

в) $\{r_i, \dot{r}_j\} = \{r_i, \frac{p_j}{m}\} = \frac{1}{m}\delta_{ij}$

$\{p_i, \dot{p}_j\} = \frac{e^2}{4mc^2}(\vec{B}^2\delta_{ij} - B_i B_j)$

3) $\vec{F} = m\vec{g} = -\frac{\partial U}{\partial \vec{r}} \Rightarrow U = -m(\vec{g},\vec{r})$

$\vec{H} = \frac{p^2}{2m} - m(\vec{g},\vec{r})$

а) $\begin{cases} \dot{\vec{r}} = \frac{\partial H}{\partial \vec{p}} = \frac{\vec{p}}{m} & \vec{p}(0) = \vec{p} \\ \dot{\vec{p}} = -\frac{\partial H}{\partial \vec{r}} = mg & \vec{r}(0) = \vec{r} \end{cases} \Rightarrow \begin{cases} \vec{p}(t) = m\vec{g}t + \vec{p} \\ \vec{r}(t) = \frac{\vec{g}t^2}{2} + \frac{\vec{p}t}{m} + \vec{r} \end{cases}$

б) $\{p(t), r(t)\} = \{\vec{p}, \vec{r}\} = -1$

$F_1(\vec{r}, \vec{r}(t)) = ?$

$$\vec{p} = \frac{\partial F_1}{\partial \vec{r}}, \quad \vec{p}(t) = -\frac{\partial F_1}{\partial \vec{r}(t)}$$

$$\vec{p}(t) = \frac{m}{t}(\vec{r}(t) - \vec{r}) - \frac{m\vec{g}t}{2} = \frac{\partial F_1}{\partial \vec{r}(t)}$$

$$\Rightarrow F_1 = -\frac{m}{2t}(\vec{r}(t)-\vec{r})^2 - \frac{m(\vec{g},\vec{r})t}{2} + f(\vec{r}(t))$$

$\vec{p}(t)$ через $\vec{r}$ и $\vec{r}(t)$

$$\vec{p}(t) = m\vec{g}t + \vec{p} - m\vec{g}t + \frac{m}{t}(\vec{r}(t)-\vec{r}) - \frac{m\vec{g}t}{2} =$$

$$= \frac{m}{t}(\vec{r}(t)-\vec{r}) + \frac{m\vec{g}t}{2} = -\frac{\partial F_1}{\partial \vec{r}(t)}, \quad f_1 = \text{const}$$

$$F_1(\vec{r}, \vec{r}(t)) = -\frac{m}{2t}(\vec{r}(t)-\vec{r})^2$$

4. $Q = \ldots(\alpha,\beta)$; $P = \ldots(\alpha,\beta)$

a) $\{q,p\} = 1$

$\{Q,P\} = 1 \Rightarrow \alpha$ и β - restricted

$$\alpha p^{\alpha-1} e^{(\beta-1)q} + \beta p^{\alpha-1} e^{\beta q} - \alpha(2\alpha-\beta) p^{\alpha-1} q^{2\alpha-\beta-1} e^{\beta q} = 1$$

$\beta = 1$

$$\{Q,P\} = \underbrace{\alpha p^{\alpha-1}}_{\substack{\shortparallel \\ 1 \ \ \alpha=1}} + \underbrace{p^{\alpha-1}e^{q} - \alpha(2\alpha-1)p^{\alpha-1}q^{2\alpha-2}e^{q}}_{0}$$

$\Rightarrow \alpha = \beta = 1$

d) $F_3(p,Q)$

$$q = -\frac{\partial F_3}{\partial p}, \quad P = -\frac{\partial F_3}{\partial Q}$$

$q, P \leftarrow \cancel{q}\, p, Q$

$$q = \ln\left(\frac{Q}{p}\right), \quad P = \ln Q + \frac{P}{Q}$$

$$\ln\frac{Q}{p} = -\frac{\partial F_3}{\partial p} \Rightarrow \frac{\partial F_3}{\partial p} = \ln p - \ln Q$$

$$F_3(p,Q) = p(\ln p - 1) - p\ln Q + \xi(Q)$$

$$-\frac{\partial F_3}{\partial Q} = \frac{P}{Q} - \frac{\partial \xi}{\partial Q} = P + \ln Q + \frac{P}{Q} \Rightarrow \frac{\partial \xi}{\partial Q} = -\ln Q \Rightarrow \xi = -Q(\ln Q - 1) + \cancel{f(t)}$$

Ответ:

$$F_3(p,Q) = p(\ln p - 1) - Q(\ln Q - 1) - p\ln Q$$