## Entropy and information

**1.** Find $H(X)$, where the distribution of $X$ is:

a) uniform on $n$ points;

b) uniform on a segment $[a, b]$;

c) exponential on a segment $(0, +\infty)$ with density $\lambda e^{-\lambda x}$;

d)* Weibull distribution on a segment $(0, +\infty)$ with density $\frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k}$

e)* Cauchi distribution on a line with density $\frac{\pi^{-1}}{1+x^2}$

**2.** Show that the following distributions $X$ maximize $H(X)$:

a) uniform among all discrete distributions on $n$ points;

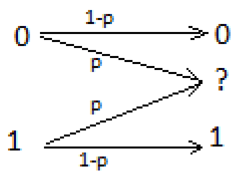
b) uniform among all continuous distributions on a given segment

c) exponential among all continuous distributions with a given expectation on a segment $(0, +\infty)$

**3.** a) Propose discrete distributions $X$, $Y$ and an element $y \in Y$ such that $H(X|y) > H(X)$

b) Propose discrete distributions $X$, $Y$ and $Z$ such that $I(X; Y|Z) < I(X; Y)$

**4.** Find capacity of a *binary erasure channel*: it sends a bit (0 or 1), and receives the same bit with probability $1 - p$ and a new symbol ? with probability $p$




**5.** Let $c_1 : \{0, 1\}^n \to \mathbb{R}$ и $c_2 : \{0, 1\}^m \to \mathbb{R}$ be constellations on a plane. Introduce their *Cartesian product* $c_1 \times c_2 : \{0, 1\}^{n+m} \to \mathbb{R}^2$ as a constellation on a plane sending a bit sequence $b_1 \times b_2$ to a point $c_1(b_1) \times c_2(b_2)$.

Find a) SER, b) BER, c) BICM mutual information for $c_1 \times c_2$, knowing them for $c_1$ и $c_2$

**6*.** Show that the minimal number of comparison used to order $n$ objects is $O(n \log n)$

**Hint.** Express in bits amount of necessary for ordering information and use Stirling's formula.