

23.03.21.

$$F[y], y \in C^2[0,1], y(1)=0$$

$$F[y] = \int_0^1 dx ((y')^2 - 2xy) \Rightarrow y_{\text{экстр}}(x) = ?$$

$$\Delta F[y] = F[y + \delta y] - F[y] = \underbrace{\int_0^1 dx (2y' \delta y' - 2x \delta y)}_{\delta F} + o(\|\delta y\|)$$

$$\begin{aligned} \delta F[y] &= 2 \int_0^1 dx (y' \delta y' - x \delta y) = \\ &= 2y' \delta y \Big|_0^1 - 2 \int_0^1 dx (y'' + x) = 0 \end{aligned}$$

$$y(1)=0 \Rightarrow \delta y(1)=0 \Rightarrow y' \text{ в т. } x=1 \text{ может принимать } \forall \text{ значения}$$

$$y(0) \text{ не зафиксирован} \Rightarrow \delta y \text{ в т. } x=0 \text{ м.б. } \forall \Rightarrow y'(0)=0$$

$$\begin{cases} y'' + x = 0 \\ y(1)=0, y'(0)=0 \end{cases}$$

$$y(x) = -\frac{1}{6}x^3 + C_1x + C_2$$

$$C_1=0, C_2=\frac{1}{6}$$

$$y_{\text{экстр}}(x) = -\frac{1}{6}(x^3-1)$$