1 Problem List 1

Задача 1.1. (3) The problem's input is numbers $n, k > 1$ and a list $a_1, \dots, a_n$ of positie integers. Construct an $O(nk)$ algorithm that computes $\max_{0<|i-j|\leqslant k} a_i \times a_j$, i. e. the maximal product of different elements with distance at most k. Try to construct an algorithm that uses $O(k)$ RAM (you can read the input sequence by elements).

Доказательство. $\square$

Задача 1.2. (3) There is an array of pairs $[(l_1, r_1), \dots, (l_n, r_n)]$. A pair (l_i, r_i) defines a segment $[l_i, r_i]$ on a line. Construct an $O(n \log n)$ algorithm that computes the Jordan measure of the union of the segments $\bigcup_{i=1}^{n} [l_i, r_i]$, i.e the union is a set of non-intersecting segments, the measure is the sum of their lengths.

Доказательство. $\square$

Задача 1.3. (5) The input is an array $a := [a_1, \dots, a_n]$ of different numbers. Construct an $O(n \log n)$ algorithm that cuts this array into the list of arrays such that the concatenation of sorted arrays from the list equals to the sorted array a. Moreover, the number of cuts should be maximal. More formally, cut is defined by a sequence of indices $i_1 < \cdots < i_k$ and consists of arrays

$$[a_1, \dots a_{i_1}], [a_{i_1+1}, \dots a_{i_2}], \dots, [a_{i_{k-1}+1}, \dots a_{i_k}].$$

In other words, we take the maximal number of continuous non-overlapping subarrays of a that covers a, sort them, and get the sorted array a as the result.

Доказательство. $\square$

Задача 1.4. (3) A peak of an array $[a_1, a_2, \dots, a_n]$ is an element a_i such that

$$a_{i-1} \leqslant a_i \geqslant a_{i+1}$$

for $1 < i < n$ or the only of the corresponding inequalities holds for $i \in \{1, n\}$ $(a_1 \geqslant a_2,\ a_n \geqslant a_{n-1})$. An array a of integers is stored in RAM. Construct an $O(\log n)$ algorithm that finds a peak of a.

Доказательство. $\square$

Задача 1.5. (6) An array $[a_1, a_2, \dots, a_n]$ of integers is stored in RAM. Construct an $O(n)$ algorithm that cuts the array into three parts $[a_1, \dots, a_i], [a_{i+1}, \dots, a_j]$, and $[a_{j+1}, \dots, a_n]$ such that at least two parts have positive sums of their elements. It is guaranteed that such a cut exists. The output is indices i and j.

Доказательство. $\square$