Найти ограничение скобку Пуассона на лист $\begin{cases} h^2+u^2-v^2=0 \\ v \geq 0 \end{cases}$

(*) 1) Найти удобную параметризацию пов-ти и вычислить ск. П. в терм. этой парам.

---

$$\cancel{\begin{cases} h = r\,\mathrm{ch}\,s\cos\varphi \\ u = r\,\mathrm{ch}\,s\sin\varphi \\ v = r\,\mathrm{sh}\,s \end{cases}}
\qquad
\begin{cases} h = r\cos\varphi \\ u = r\sin\varphi \\ v = r \end{cases}$$

$\cancel{r = R > 0 = const}$

$\cancel{r^2 = h^2+u^2-v^2}$

$r, \varphi: \quad r > 0, \quad \varphi \in [0, 2\pi)$

$r = R = const > 0$

$\{h, u\} = \{h, x+y\} = 2v$

$\{h, v\} = 2u$

$\{u, v\} = -2h$

$\mathrm{tg}\,\varphi = \frac{u}{h}$

$\{r, \varphi\} = \{v, \varphi\}$

$\{v, \mathrm{tg}\,\varphi\} = \{v, \frac{u}{h}\} = \left(\frac{1}{h}\{v,u\} - \frac{u}{h^2}\{v,h\}\right) =$

$= \frac{1}{h}\cdot 2h - \frac{u}{h^2}\cdot(-2u) = \underline{2 + \frac{2u^2}{h^2}},$

$\{r, \mathrm{tg}\,\varphi\} = \{r, \varphi\}\frac{1}{\cos^2\varphi}$

$$\{r, \varphi\} = \cos^2\varphi \left(2 + \frac{2u^2}{h^2}\right) = \cos^2\varphi \left(\frac{2(h^2+u^2)}{h^2}\right) =$$

$$= \frac{\cos^2\varphi \cdot 2r^2}{r^2\cos^2\varphi} = 2.$$

$y \in C^\infty[0,1], \quad y(0) = A$

$$S[y] = \int_0^1 dx \left( \underbrace{\frac{1}{4}(y')^2 \log(y^2) + xy' + e^{\cos y}}_{L} \right)$$

a) $$\delta S[y] = \int_0^1 dx \left( \frac{\partial L}{\partial y}\delta y + \frac{\partial L}{\partial y'}\delta y' \right)$$

$$\frac{\partial L}{\partial y} = \frac{1}{4}(y')^2 \cdot \frac{1}{y^2} \cdot 2y + e^{\cos y}(-\sin y) =$$

$$= \frac{1}{2} \cdot \frac{1}{y}(y')^2 - \sin y \, e^{\cos y}$$

$$\frac{\partial L}{\partial y'} = \frac{1}{4} \cdot 2y' \ln y^2 + x = \frac{1}{2}y' \ln y^2 + x$$

$$\delta S[y] = \int_0^1 dx \left[ \left( \frac{(y')^2}{2y} - \sin y \, e^{\cos y} \right)\delta y + \left( \frac{1}{2}y' \ln y^2 + x \right)\delta y' \right] =$$

$$= \int_0^1 \left( \frac{(y')^2}{2y} - \sin y \, e^{\cos y} \right)\delta y \, dx + \left( \frac{1}{2}y' \ln y^2 + x \right)\delta y \Big|_0^1 -$$

$$- \int_0^1 \left[ \frac{1}{2}\left( y'' \ln y^2 + y' \cdot \frac{2}{y} \right) + 1 \right] \delta y \, dx =$$

$$= \int_0^1 \left( \frac{(y')^2}{2y} - \sin y \, e^{\cos y} - \frac{1}{2}y'' \ln y^2 - \frac{y'}{y} - 1 \right)\delta y \, dx +$$

$$+ \left( \frac{1}{2}y' \ln y^2 + x \right)\delta y \Big|_0^1$$

b) $y(0) = A \Rightarrow \delta y(0) = 0$

$\delta y(1) \; \forall \Rightarrow \boxed{\frac{1}{2}y' \ln y^2 + x \Big|_{x=1} = 0}$

$F[y]$ на $C^2[0,1]$ : $y(1)=0$

$$F[y] = \int_0^1 dx\left((y')^2 - 2xy\right)$$

---

$$\Delta F[y] = \Delta \int_0^1 dx\left((y')^2 - 2xy\right) =$$

$$= \int_0^1 dx\left((y' + (\delta y)')^2 - 2x(y+\delta y)\right) - \int_0^1 dx\left((y')^2 - 2xy\right) =$$

$$= \int_0^1 dx \left\{ \cancel{(y')^2} + 2y'\delta y' + (\delta y')^2 \cancel{- 2xy} - 2x\delta y \cancel{- (y')^2} \cancel{+ 2xy} \right\} =$$

$$= \int_0^1 dx\left(2y'(\delta y)' + (\delta y')^2 - 2x\delta y\right)$$

$$\delta F[y] = \int_0^1 dx\left(2y'(\delta y)' - 2x\delta y\right) =$$

$$= \int_0^1 2y'(\delta y)'\,dx - \int_0^1 2x\delta y\,dx =$$

$$2y'(\delta y)' + 2y''\delta y$$

$$= \int_0^1 (2y'\delta y)'\,dx - \int_0^1 2y''\delta y\,dx - \int_0^1 2x\delta y\,dx =$$

$$-2\int_0^1 \delta y\,dx\,(y'' + x)$$

$$= -2\int_0^1 dx\,(y'' + x)\,\delta y + 2y'\delta y\Big|_0^1 = 0$$

$$y(1) = 0 \Rightarrow \delta y\big|_{x=1} = 0$$

$$y'' + x = 0$$

$$y'' = -x$$

$$y' = -\frac{x^2}{2} + C_1$$

$$y = -\int \frac{x^2}{2}\,dx + C_1 x = -\frac{1}{2}\cdot\frac{x^3}{3} + C_1 x + C_0$$

$$y(x) = -\frac{x^3}{6} + C_1 x + C_0 \quad (*)$$

Граничное условие в т. $x = 0$

$$2y'\big|_{x=0} = 0 \qquad (\delta y \text{ - произв. в т. } x = 0)$$

$$y(1) = -\frac{1}{6} + C_1 + C_0 = 0 \qquad \Rightarrow C_0 = \frac{1}{6}.$$

$$y'(0) = -\frac{x^2}{2} + C_1\Big|_{x=0} = +C_1 = 0 \Rightarrow C_1 = 0$$

$$\Rightarrow \boxed{y(x) = -\frac{x^3}{6} + \frac{1}{6}}$$

17:40.

мат. т. в $\mathbb{R}^2$ движ под действ $F$: $\vec{F} = (F_x, F_y)$

$\forall x, y \in \mathbb{R}$: $\begin{cases} F_x = -2xy - \dfrac{(1+x)^2}{1+x^2} \\ F_y = -x^2 + \dfrac{2y}{1+y^2} \end{cases}$

а) Показать: $\vec{F}$ – потенциальна $\Rightarrow$ найти $U(x,y) = ?$

б) Работа $\vec{F}$ при движ. по дуге окр

$x^2+y^2=1$ от $P(1,0)$ до $Q(0,1)$ $= ?$

а) $\dfrac{\partial F_y}{\partial x} = \dfrac{\partial F_x}{\partial y}$ $(*)$

$\dfrac{\partial F_y}{\partial x} = -2x$

$\dfrac{\partial F_x}{\partial y} = -2x$

$\Rightarrow (*)$ выполнено

пр-во $\mathbb{R}^2$ односвязно $\Big\} \Rightarrow$

$\Rightarrow$ по лемме Пуанкаре $(*)$ явл-ся достаточным условием $\Rightarrow$

$\Rightarrow$ сила $\vec{F}$ потенциальна.

$\exists U(x,y)$: $\dfrac{\partial U}{\partial x} = -F_x = 2xy + \dfrac{(1+x)^2}{1+x^2}$

$\dfrac{\partial U}{\partial y} = -F_y = x^2 - \dfrac{2y}{1+y^2}$

$\int\left(1+\dfrac{2x}{1+x^2}\right)dx = x + \int\dfrac{dx^2}{1+x^2}$

$U(x,y) = +\int dx \left(2xy + \dfrac{(1+x)^2}{1+x^2}\right) = \cancel{2}y\dfrac{x^2}{\cancel{2}} + \int\dfrac{(1+x)^2}{1+x^2}dx =$

$= x^2y + \ln(x^2+1) + x + c(y)$

$$\cancel{x^2} + C'(y) = \cancel{x^2} - \frac{2y}{1+y^2}$$

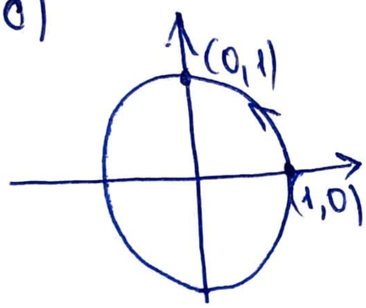
$$C'(y) = -\frac{2y}{1+y^2}$$

$$-\int \frac{dy^2}{1+y^2}$$

$$C(y) = -2\int \frac{y}{1+y^2}\,dy = -\ln(y^2+1) + C$$

$$\boxed{U(x,y) = x^2y + \ln(x^2+1) + x - \ln(y^2+1) + C}$$

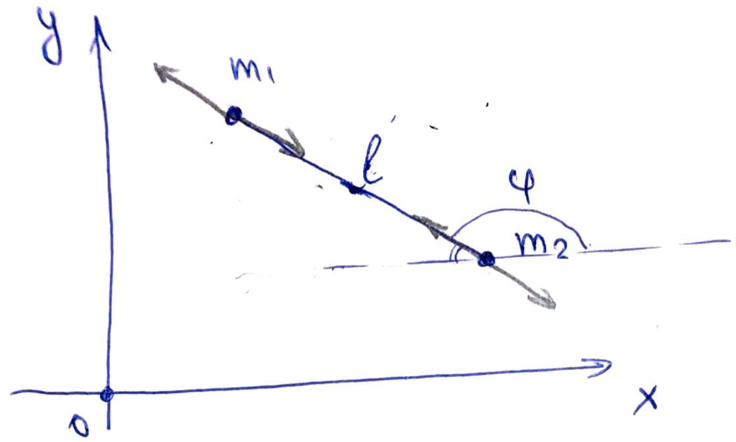
б)




Т.к. сила потенциальна

$$A = U_н - U_к =$$

$$= U(1,0) - U(0,1) = \ln 2 + 1 - \ln 2 = 1.$$




Обобщенные координаты:

$$\underbrace{x,\ y}_{\text{коорд. } m_2},\ \varphi$$

а) $x_{m_1} = l\cos\varphi + x$

$y_{m_1} = l\sin\varphi + y$

$$T_{кин} = \frac{m_1\left(\left((l\cos\varphi + x)^{\cdot}\right)^2 + \left((l\sin\varphi + y)^{\cdot}\right)^2\right)}{2} + \frac{m_2(\dot{x}^2 + \dot{y}^2)}{2} =$$

$$= \frac{m_2}{2}(\dot{x}^2 + \dot{y}^2) + \frac{m_1}{2}(\dot{x}^2 + \dot{y}^2 + l^2\dot{\varphi}^2 + 2l\cos\varphi\,\dot{x}\dot{\varphi} + 2l\sin\varphi\,\dot{y}\dot{\varphi})$$

$$U = -G\frac{m_1 m_2}{l}$$

$$\boxed{L = T_{кин} - U = \frac{m_1}{2}(\dot{x}^2 + \dot{y}^2 + l^2\dot{\varphi}^2 + 2l\cos\varphi\,\dot{x}\dot{\varphi} + 2l\sin\varphi\,\dot{y}\dot{\varphi}) + \frac{m_2}{2}(\dot{x}^2 + \dot{y}^2) + G\frac{m_1 m_2}{l}}$$

$$L_x := \frac{d}{dt}(m_2\dot{x} + m_1\dot{x} + m_1 l\dot{\varphi}\cos\varphi) = 0$$

$$L_y := \frac{d}{dt}((m_1 + m_2)\dot{y} + m_2 l\dot{\varphi}\sin\varphi) = 0$$

$$L_\varphi := \frac{d}{dt}(m_1 l^2\dot{\varphi} + m_1 l\cos\varphi\,\dot{x} + m_1 l\sin\varphi\,\dot{y}) - (m_1 l\cos\varphi\,\dot{y}\dot{\varphi} - m_1 l\sin\varphi\,\dot{x}\dot{\varphi}) = 0$$

$\frac{\partial L}{\partial t} = 0 \Rightarrow$ Выполняется закон сохр. энергии

$F[y]$, $y \in C^2[0,1]$, $y(1)=0$ 23.03.21.

$$F[y] = \int_0^1 dx\left((y')^2 - 2xy\right) \Rightarrow y_{экстр}(x) = ?$$

$$\Delta F[y] = F[y+\delta y] - F[y] = \underbrace{\int_0^1 dx\left(2y'\delta y' - 2x\delta y\right)}_{\delta F} + o(\|\delta y\|)$$

$$\delta F[y] = 2\int_0^1 dx\left(y'\delta y' - x\delta y\right) =$$

$$= 2y'\delta y\Big|_0^1 - 2\int_0^1 dx\left(y'' + x\right) = 0$$

$y(1)=0 \Rightarrow \delta y(1) = 0 \Rightarrow y'$ в т. $x=1$ может принимать $\forall$ знач.

$y(0)$ не зафиксирован $\Rightarrow \delta y$ в т. $x=0$ м.б. $\forall \Rightarrow y'(0)=0$

$$\begin{cases} y'' + x = 0 \\ y(1) = 0,\ y'(0) = 0 \end{cases}$$

$$y(x) = -\frac{1}{6}x^3 + C_1 x + C_2$$

$C_1 = 0$, $C_2 = \frac{1}{6}$

$$y_{экстр}(x) = -\frac{1}{6}(x^3 - 1)$$

20.04.21

$$S[x,y] = \int dt\,(\dot{x}^2 y^{-4} + \dot{y}^2 t^2 - x^2 y^2 t)$$

$$\tilde{x} = e^{\varepsilon} x, \quad \tilde{y} = e^{a\varepsilon} y, \quad \tilde{t} = e^{b\varepsilon} t$$

$$S[\tilde{x},\tilde{y}] = \int d\tilde{t}\, e^{-b\varepsilon} \Big[ e^{2(b-1)} (\tilde{x}')^2 \, e^{4a\varepsilon} \tilde{y}^{-4} +$$

$$+ e^{2(b-a)\varepsilon} (\tilde{y}')^2 e^{-2b\varepsilon} \tilde{t}^2 - e^{-2a\varepsilon - 2\varepsilon} \tilde{x}^2 \tilde{y}^2 \cdot e^{-b\varepsilon} \tilde{t} \Big] =$$

$$= \int d\tilde{t} \,( e^{(2b-2-b+4a)\varepsilon} (\tilde{x}')^2 \tilde{t}^2 - e^{(2b-2a-b-2b)\varepsilon} (\tilde{y}')^2 \tilde{t}^2 -$$

$$- e^{-(2a+2b+2)\varepsilon} \tilde{x}^2 \tilde{y}^2 \tilde{t})$$

$$\begin{cases} b + 4a - 2a = 0 \\ 2a + b = 0 \\ 2a + 2b + 2 = 0 \end{cases} \qquad a = 1, \quad b = -2$$

При $\varepsilon = 0$ преобразование тождественны

$$\xi_{\varphi_0} = \frac{\partial \tilde{t}}{\partial \varepsilon}\Big|_{\varepsilon=0} = -2t$$

$$\xi_{\varphi x} = \frac{\partial \tilde{x}}{\partial \varepsilon}\Big|_{\varepsilon=0} = x$$

$$\xi_{\varphi y} = \frac{\partial \tilde{y}}{\partial \varepsilon}\Big|_{\varepsilon=0} = y$$

$$I = \frac{\partial L}{\partial \dot{x}} \xi_{\varphi x} + \frac{\partial L}{\partial \dot{y}} \xi_{\varphi y} + \left( L - \dot{x} \frac{\partial L}{\partial \dot{x}} - \dot{y} \frac{\partial L}{\partial \dot{y}} \right) \xi_{\varphi_0}$$

$$I = 2x\dot{x} y^{-4} + 2y\dot{y} t^2 + 2t \dot{x}^2 y^{-4} + 2t^3 \dot{y}^2 + 2x^2 y^2 t^3$$

5-минутка.

$$L = -mc^2\sqrt{1-\frac{\dot{\vec{x}}^2}{c^2}} \qquad L_u \simeq \frac{m\dot{\vec{x}}^2}{2} + O\left(\frac{1}{c}\right)$$

а) $$p_i = \frac{\partial L}{\partial \dot{x}_i} = \frac{m\dot{x}_i}{\sqrt{1-\frac{\dot{\vec{x}}^2}{c^2}}} \Rightarrow \vec{p} = \frac{m\dot{\vec{x}}}{\sqrt{1-\frac{\dot{\vec{x}}^2}{c^2}}}$$

$$\vec{p}^2 = \frac{m^2\dot{\vec{x}}^2}{1-\frac{\dot{\vec{x}}^2}{c^2}} \Rightarrow \dot{\vec{x}}^2 = \frac{\vec{p}^2c^2}{\vec{p}^2+m^2c^2}$$

$$1-\frac{\dot{\vec{x}}^2}{c^2} = \frac{m^2c^2}{\vec{p}^2+m^2c^2} \Rightarrow \vec{p} = \frac{\dot{\vec{x}}}{c}\sqrt{\vec{p}^2+m^2c^2} \Rightarrow$$

$$\Rightarrow \dot{\vec{x}} = \frac{\vec{p}c}{\sqrt{\vec{p}^2+m^2c^2}}$$

$$E = \dot{\vec{x}}\frac{\partial L}{\partial \dot{\vec{x}}} - L = \frac{mc^2}{\sqrt{1-\frac{\dot{\vec{x}}^2}{c^2}}}$$

$$H = E\Big|_{\dot{\vec{x}}=\dot{\vec{x}}(\vec{p})} \Rightarrow \boxed{H = c\sqrt{\vec{p}^2+m^2c^2}}$$

б) $$\begin{cases} \dot{x}_i = \dfrac{\partial H}{\partial p_i} = \dfrac{cp_i}{\sqrt{\vec{p}^2+m^2c^2}} \\ \dot{p}_i = -\dfrac{\partial H}{\partial x_i} = 0 \Rightarrow p_i(t) = p_i = \mathrm{const} \end{cases}$$

в) $$H = c\sqrt{\vec{p}^2+m^2c^2} = \mathrm{const} = \varepsilon$$

$$\dot{\vec{x}} = \frac{c^2}{\varepsilon}\vec{p} = \frac{c^2}{\varepsilon}\vec{p}_0$$

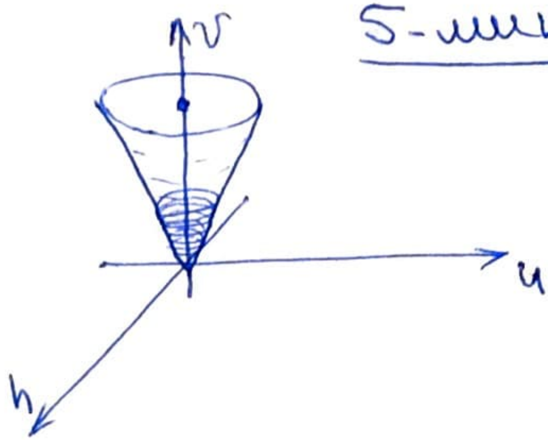
$$\vec{x}(t) = \frac{c^2}{\varepsilon}\vec{p}_0 t + \vec{x}(0), \quad \vec{x}(0) = 0$$

$$\boxed{\vec{p}(t) = \vec{p}_0; \ \vec{x}(t) = \frac{c^2}{\varepsilon}\vec{p}_0 t}, \text{ где } \quad \varepsilon = c\sqrt{\vec{p}_0^{\,2}+m^2c^2}$$

5-минутка.

$\boxed{18.20.21}$

в $\mathbb{R}^3$: $\begin{cases} h^2+u^2-v^2=0 \\ v>0 \end{cases}$




$h = v\cos\varphi,\ u = v\sin\varphi,\ v = v$

$\begin{cases} v\in(0,+\infty) \\ \varphi\in[0,+2\pi) \end{cases}$

$f(h,u,v)=0$ в $\mathbb{R}^3$

$h=h(\xi,\eta),\quad u=u(\xi,\eta),\ v=v(\xi,\eta)$

$f(h(\xi,\eta),u(\xi,\eta),v(\xi,\eta))\equiv 0$

$\{h,u\}=2v,\ \{h,v\}=2u,\ \{u,v\}=-2h$

$\cos\varphi=\dfrac{h}{v} \Rightarrow \{\cos\varphi, v\} = -\sin\varphi\{\varphi,v\}$

$\{\cos\varphi, v\} = \left\{\dfrac{h}{v}, v\right\} = \dfrac{1}{v}\underbrace{\{h,v\}}_{2u} = \dfrac{2u}{v}$

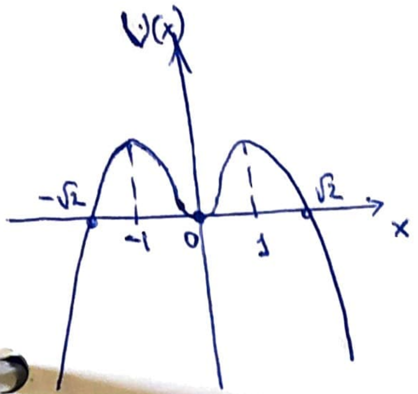
$\Rightarrow \{\varphi, v\} = -2.$

# Разбор пятиминутки    Держись !!    02.02.21

$U(x) = 2x^2 - x^4$ , $m = 2$

Фазовый портрет - зависимость $\dot{x}(x)$

(1) $\ddot{x} + 2x - 2x^3 = 0 \leftarrow m\ddot{x} = F = -U'(x)$




$\ddot{x} + 2x - 2x^3 = 0 \Leftrightarrow \begin{cases} \dot{x} = y \\ \dot{y} = -2x + 2x^3 \end{cases}$ (1')

(0,0), (1,0), (-1,0)

↑

точки покоя

⇓

стационарные реш-ия

$x = 0$, $x = 1$, $x = -1$

Линеаризуем в окр-ти особых точек

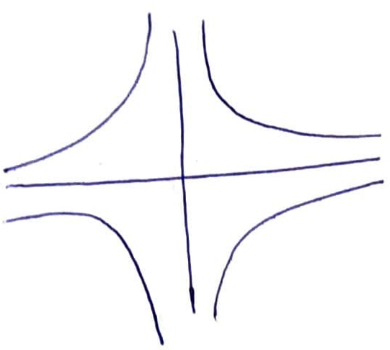
$(\pm 1, 0)$ $\dot{x} = y$

$\dot{y} = -2(x \mp 1) + 2(x \mp 1)^3 = 4x + \underbrace{2x^3 \mp 6x^2}$

$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = A_{\pm 1} \begin{pmatrix} x \\ y \end{pmatrix}$ , $A_{\pm 1} = \begin{pmatrix} 0 & 1 \\ 4 & 0 \end{pmatrix}$

$\chi_{\pm} = \lambda^2 - 4 \Rightarrow \lambda_{1,2} = \pm 2$ седло

неуст.



(0,0) $\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = A_0 \begin{pmatrix} x \\ y \end{pmatrix}$ , $A_0 = \begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix}$

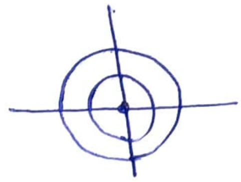
$\chi_0 = \lambda^2 + 2 \Rightarrow \lambda_{1,2} = \pm \sqrt{2} i$

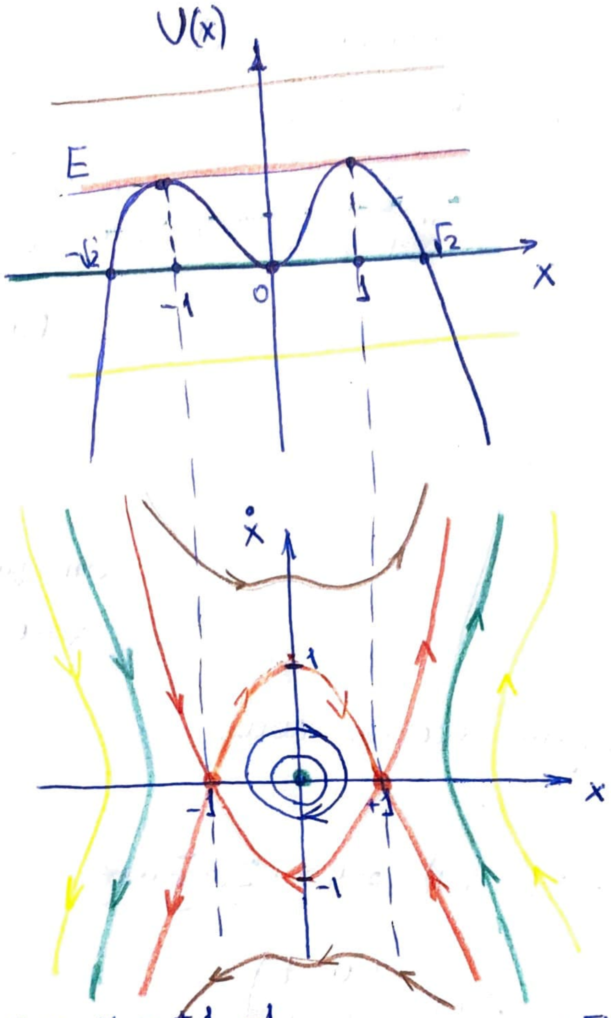
$\mathrm{Real}\ \lambda_{1,2} = 0 \Rightarrow$ лин. приближ. недостаточно, чтобы ответить на вопрос об устойчивости.

$\ddot{x} + 2x - 2x^3 = 0 \ |\cdot \dot{x}$

$\frac{d}{dt}\left(\frac{\dot{x}^2}{2} + x^2 - \frac{x^4}{2}\right) = 0 \Rightarrow E(x, \dot{x}) = \dot{x}^2 + 2x^2 - x^4$ - инвариант эволюции

$E|_{(0,0)} \simeq \dot{x}^2 + 2$






• $E = 1 \Longleftrightarrow x = +1, -1$

$U'(x) = 4(x - x^3) = 0$

$x = 0, \; x = \pm 1$

$U(x_0) = E = 1$

$\dot{x}^2 + 2x^2 - x^4 = 1$

$\dot{x}^2 = x^4 - 2x^2 + 1 = (x^2-1)^2$

$\dot{x} = x^2 - 1$

$\dot{x} = -x^2 + 1$

• $E = 0$

$\dot{x}^2 + 2x^2 - x^4 = 0$

$E = 0 \Rightarrow 3$ фазовые кр.

$E = -1 \Rightarrow 2$ фазовые кр

$E = 1/2 \Rightarrow 3$ фазовые кр.

$E = 1 \Rightarrow 8$ фазовых кр.

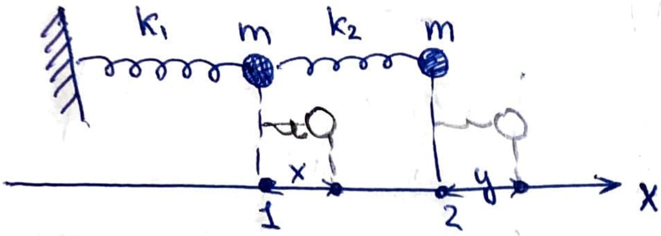
$E = 2 \Rightarrow 2$ фазовые кр.

Семинар № 2.

$k_1 = 3k$
$k_2 = 2k$

19.01.21

⓪




$$\begin{cases} m\ddot{x} = -k_1 x - k_2(x-y) \\ m\ddot{y} = k_2(x-y) \end{cases}$$

III з-н Ньютона

$\vec{F_g} = -\vec{F_{пр}}$

$\ddot{X} = -AX$, $X(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$

$$A = \frac{1}{m}\begin{pmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{pmatrix} = \frac{k}{m}\begin{pmatrix} 5 & -2 \\ -2 & 2 \end{pmatrix}$$

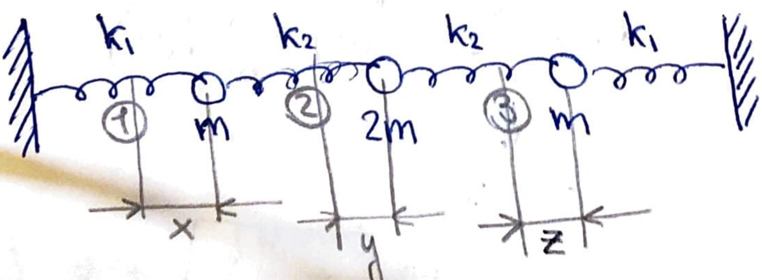
$\lambda_1 = \omega_1^2 = \frac{k}{m}$, $\lambda_2 = \omega_2^2 = \frac{6k}{m}$

$\Psi_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\Psi_2 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

Норм. моды $\Psi_i \cos\omega_i t$, $\Psi_i \sin\omega_i t$ $i = 1,2$

А как понять, что описывают моды?

①




$\begin{cases} k_1 = 3k \\ k_2 = 2k \end{cases}$

$$\begin{cases} m\ddot{x} = -k_1 x - k_2(x-y) \text{ почему нет } z\text{?} \\ 2m\ddot{y} = +k_2(x-y) - k_2(y-z) \\ m\ddot{z} = +k_2(y-z) - k_1 z \end{cases}$$

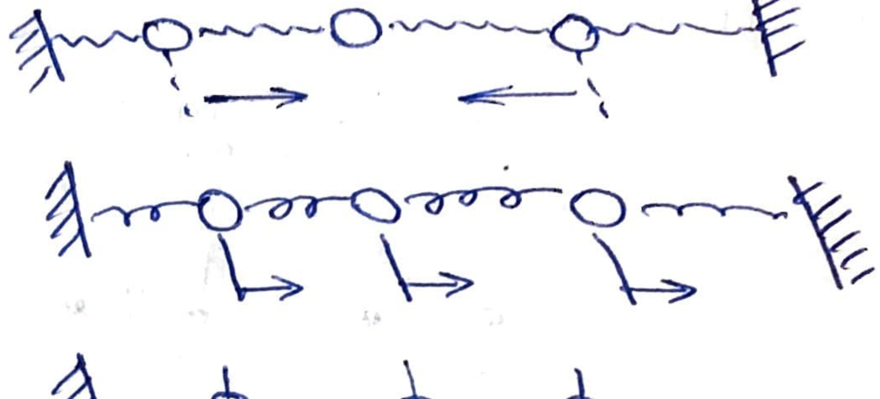
$$\ddot{X} = -AX$$

$$A = \frac{1}{m}\begin{pmatrix} k_1+k_2 & -k_2 & 0 \\ -\frac{k_2}{2} & k_2 & -\frac{k_2}{2} \\ 0 & -k_2 & k_1+k_2 \end{pmatrix} = \frac{k}{m}\begin{pmatrix} 5 & -2 & 0 \\ -1 & 2 & -1 \\ 0 & 2 & 5 \end{pmatrix}$$

$$\lambda_1 = \omega_1^2 = \frac{5k}{m} \longrightarrow \psi_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$\lambda_2 = \omega_2^2 = \frac{k}{m} \longrightarrow \psi_2 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

$$\lambda_3 = \omega_3^2 = \frac{6k}{m} \longrightarrow \psi_3 = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$$



2/3

$$L(\vec x, \dot{\vec x}) = -mc^2\sqrt{1-\frac{\dot{\vec x}^2}{c^2}}$$

константа

a)

$$p_x = \frac{\partial L}{\partial \dot x} = -mc^2 \frac{1/2}{\sqrt{1-\frac{\dot x^2}{c^2}}}\left(-\frac{2\dot{\vec x}}{c^2}\right) =$$

$$= +\frac{m\cancel{c^2}\dot{\vec x}}{\cancel{c^2}\sqrt{1-\frac{\dot x^2}{c^2}}}$$

$$= \frac{m\dot{\vec x}}{\sqrt{1-\frac{\dot x^2}{c^2}}} \Rightarrow \sqrt{1-\frac{\dot x}{c^2}} = \frac{m\dot x}{p_x}$$

$$H = p_x\dot{\vec x} + L = \frac{m\dot{\vec x}}{\sqrt{1-\frac{\dot x^2}{c^2}}}\dot{\vec x} + mc^2\sqrt{1-\frac{\dot x^2}{c^2}}\Bigg|_{\dot{\vec x} = \sqrt{\frac{p_x^2c^2}{m^2c^2+p_x^2}}}$$

$$p_x\sqrt{1-\frac{\dot x^2}{c^2}} = m\dot x$$

$$\dot x = \sqrt{\frac{p_x^2c^2}{m^2c^2+p_x^2}}$$

$$H = \frac{m}{\sqrt{1-\frac{p_x^2}{m^2c^2+p_x^2}}}\cdot\frac{p_x^2c^2}{m^2c^2+p_x^2} + mc^2\sqrt{1-\frac{p_x^2}{m^2c^2+p_x^2}} =$$

$$\boxed{H = \frac{p_x^2 c}{\sqrt{m^2c^2+p_x^2}} + \frac{m^2c^2}{\cancel{p_x}}\cdot\frac{\cancel{p_x}c}{\sqrt{m^2c^2+p_x^2}} = \frac{p_x^2c+m^2c^3}{\sqrt{m^2c^2+p_x^2}}}$$

б) 18:15

$$\boxed{\dot{p}_x = -\frac{\partial H}{\partial x} = 0}$$

$$\dot{x} = \frac{\partial H}{\partial p_x} = \frac{(2p_x c)\sqrt{m^2c^2+p_x^2} - \frac{1}{2}\frac{2p_x}{\sqrt{m^2c^2+p_x^2}}(p_x^2c+m^2c^3)}{m^2c^2+p_x^2}$$

$$= \frac{2p_x c(m^2c^2+p_x^2) - p_x(p_x^2c+m^2c^3)}{(m^2c^2+p_x^2)^{3/2}}$$

17:40.

мат. т. в $\mathbb{R}^2$ движ под действ $F$: $\vec{F} = (F_x, F_y)$

$\forall x, y \in \mathbb{R}$ : $\begin{cases} F_x = -2xy - \dfrac{(1+x)^2}{1+x^2} \\ F_y = -x^2 + \dfrac{2y}{1+y^2} \end{cases}$

а) Показать: $\vec{F}$ – потенциальна $\Rightarrow$ найти $U(x,y) = ?$

б) Работа $\vec{F}$ при движ. по дуге окр

$x^2+y^2=1$ от $P(1,0)$ до $Q(0,1)$ $= ?$

а) $\dfrac{\partial F_y}{\partial x} = \dfrac{\partial F_x}{\partial y}$ $(*)$

$\dfrac{\partial F_y}{\partial x} = -2x$

$\Rightarrow (*)$ выполнено

$\dfrac{\partial F_x}{\partial y} = -2x$

пр-во $\mathbb{R}^2$ односвязно $\Big\} \Rightarrow$

$\Rightarrow$ по лемме Пуанкаре $(*)$ явл-ся достаточным условием $\Rightarrow$

$\Rightarrow$ сила $\vec{F}$ потенциальна.

$\exists U(x,y)$ : $\dfrac{\partial U}{\partial x} = -F_x = 2xy + \dfrac{(1+x)^2}{1+x^2}$

$\dfrac{\partial U}{\partial y} = -F_y = x^2 - \dfrac{2y}{1+y^2}$

$\int\left(1+\dfrac{2x}{1+x^2}\right)dx =$

$= x + \int \dfrac{dx^2}{1+x^2}$

$U(x,y) = +\int dx \left(2xy + \dfrac{(1+x)^2}{1+x^2}\right) = \cancel{2}y\dfrac{x^2}{\cancel{2}} + \int \dfrac{(1+x)^2}{1+x^2}dx =$

$= x^2y + \ln(x^2+1) + x + c(y)$

$$\cancel{x^2} + C'(y) = \cancel{x^2} - \frac{2y}{1+y^2}$$

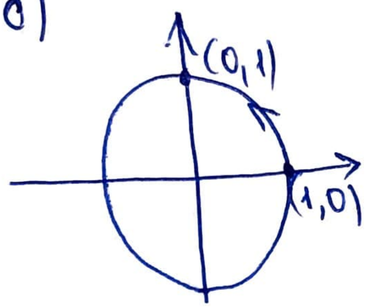
$$C'(y) = -\frac{2y}{1+y^2}$$

$$-\int \frac{dy^2}{1+y^2}$$

$$C(y) = -2\int \frac{y}{1+y^2}\,dy = -\ln(y^2+1) + C$$

$$\boxed{U(x,y) = x^2y + \ln(x^2+1) + x - \ln(y^2+1) + C}$$

б)




Т.к. сила потенциальна

$$A = U_н - U_к =$$

$$= U(1,0) - U(0,1) = \ln 2 + 1 - \ln 2 = 1.$$

$F[y]$ на $C^2[0,1]$ : $y(1)=0$

$$F[y] = \int_0^1 dx\left((y')^2 - 2xy\right)$$

---

$$\Delta F[y] = \Delta \int_0^1 dx\left((y')^2 - 2xy\right) =$$

$$= \int_0^1 dx\left((y'+(\delta y)')^2 - 2x(y+\delta y)\right) - \int_0^1 dx\left((y')^2 - 2xy\right) =$$

$$= \int_0^1 dx\left\{\cancel{(y')^2} + 2y'\,\delta y' + (\delta y')^2 \cancel{-2xy} - 2x\,\delta y \cancel{-(y')^2} \cancel{+2xy}\right\} =$$

$$= \int_0^1 dx\left(2y'(\delta y)' + (\delta y')^2 - 2x\,\delta y\right)$$

$$\delta F[y] = \int_0^1 dx\left(2y'(\delta y)' - 2x\,\delta y\right) =$$

$$= \int_0^1 2y'(\delta y)'\,dx - \int_0^1 2x\,\delta y\,dx =$$

$2y'(\delta y)' + 2y''\delta y$

$$= \int_0^1 (2y'\delta y)'\,dx - \int_0^1 2y''\,\delta y\,dx - \int_0^1 2x\,\delta y\,dx =$$

$$-2\int_0^1 \delta y\,dx\,(y'' + x)$$

$$= -2\int_0^1 dx\,(y'' + x)\,\delta y + 2y'\delta y\Big|_0^1 = 0$$

$$y(1) = 0 \Rightarrow \delta y\big|_{x=1} = 0$$

$$y'' + x = 0$$

$$y'' = -x$$

$$y' = -\frac{x^2}{2} + C_1$$

$$y = -\int \frac{x^2}{2}\,dx + C_1 x = -\frac{1}{2}\cdot\frac{x^3}{3} + C_1 x + C_0$$

$$y(x) = -\frac{x^3}{6} + C_1 x + C_0 \quad (*)$$

Граничное условие в т. $x = 0$

$$2y'\big|_{x=0} = 0 \qquad (\delta y \text{ – произв. в т. } x = 0)$$

$$y(1) = -\frac{1}{6} + C_1 + C_0 = 0 \qquad \Rightarrow C_0 = \frac{1}{6}.$$

$$y'(0) = -\frac{x^2}{2} + C_1\Big|_{x=0} = +C_1 = 0 \Rightarrow C_1 = 0$$

$$\Rightarrow \boxed{y(x) = -\frac{x^3}{6} + \frac{1}{6}}$$

$y \in C^\infty[0,1], \quad y(0) = A$

$$S[y] = \int_0^1 dx \Big( \underbrace{\tfrac{1}{4}(y')^2 \log(y^2) + xy' + e^{\cos y}}_{L} \Big)$$

a) $$\delta S[y] = \int_0^1 dx \left( \frac{\partial L}{\partial y}\delta y + \frac{\partial L}{\partial y'}\delta y' \right)$$

$$\frac{\partial L}{\partial y} = \frac{1}{4}(y')^2 \cdot \frac{1}{y^2} \cdot 2y + e^{\cos y}(-\sin y) =$$

$$= \frac{1}{2} \cdot \frac{1}{y}(y')^2 - \sin y \, e^{\cos y}$$

$$\frac{\partial L}{\partial y'} = \frac{1}{4} \cdot 2y' \ln y^2 + x = \frac{1}{2} y' \ln y^2 + x$$

$$\delta S[y] = \int_0^1 dx \left[ \left( \frac{(y')^2}{2y} - \sin y \, e^{\cos y} \right)\delta y + \left( \frac{1}{2}y' \ln y^2 + x \right)\delta y' \right] =$$

$$= \int_0^1 \left( \frac{(y')^2}{2y} - \sin y \, e^{\cos y} \right)\delta y \, dx + \left( \frac{1}{2}y' \ln y^2 + x \right)\delta y \Big|_0^1 -$$

$$- \int_0^1 \left[ \frac{1}{2}\left( y'' \ln y^2 + y' \cdot \frac{2}{y} \right) + 1 \right] \delta y \, dx =$$

$$= \int_0^1 \left( \frac{(y')^2}{2y} - \sin y \, e^{\cos y} - \frac{1}{2} y'' \ln y^2 - \frac{y'}{y} - 1 \right)\delta y \, dx +$$

$$+ \left( \frac{1}{2}y' \ln y^2 + x \right)\delta y \Big|_0^1$$

б) $y(0) = A \Rightarrow \delta y(0) = 0$

$\delta y(1) \ \forall \Rightarrow \boxed{\frac{1}{2} y' \ln y^2 + x \Big|_{x=1} = 0}$

$y \in C^\infty[0,1], \quad y(0) = A$

$$S[y] = \int_0^1 dx \left( \underbrace{\frac{1}{4}(y')^2 \log(y^2) + xy' + e^{\cos y}}_{L} \right)$$

a) $$\delta S[y] = \int_0^1 dx \left( \frac{\partial L}{\partial y}\delta y + \frac{\partial L}{\partial y'}\delta y' \right)$$

$$\frac{\partial L}{\partial y} = \frac{1}{4}(y')^2 \cdot \frac{1}{y^2}\cdot 2\cancel{y} + e^{\cos y}(-\sin y) =$$

$$= \frac{1}{2}\cdot\frac{1}{y}(y')^2 - \sin y\, e^{\cos y}$$

$$\frac{\partial L}{\partial y'} = \frac{1}{4}\cdot 2y' \ln y^2 + x = \frac{1}{2}y' \ln y^2 + x$$

$$\delta S[y] = \int_0^1 dx \left[ \left( \frac{(y')^2}{2y} - \sin y\, e^{\cos y} \right)\delta y + \left( \frac{1}{2}y' \ln y^2 + x \right)\delta y' \right] =$$

$$= \int_0^1 \left( \frac{(y')^2}{2y} - \sin y\, e^{\cos y} \right)\delta y\, dx + \left( \frac{1}{2}y' \ln y^2 + x \right)\delta y \Big|_0^1 -$$

$$- \int_0^1 \left[ \frac{1}{2}\left( y'' \ln y^2 + y' \cdot \frac{2}{y} \right) + 1 \right] \delta y\, dx =$$

$$= \int_0^1 \boxed{\left( \frac{(y')^2}{2y} - \sin y\, e^{\cos y} - \frac{1}{2}y'' \ln y^2 - \frac{y'}{y} - 1 \right)} \delta y\, dx +$$

$$+ \left( \frac{1}{2}y' \ln y^2 + x \right)\delta y \Big|_0^1$$

б) $y(0) = A \Rightarrow \delta y(0) = 0$

$\delta y(1)\ \forall \Rightarrow \boxed{\frac{1}{2}y' \ln y^2 + x \Big|_{x=1} = 0}$