Error correcting codes: combinatorics

1. Show that a code with minimal distance d can fix up to $d-1$ *erasures* — unknown bits at fixed positions.

2. Are there exists $(n,k,d)_q$–codes as follows (recall that q is alphabet size, n is block size, k is number of information symbols, d is the minimal distance):

a) $[n,n,1]_q$; b) $[n,n-1,2]_q$; c) $[11,5,5]_2$; d) $[20,7,5]_2$; e)* $[22,15,3]_2$?

3. Find $(n,k,d)_q$ satisfying Hamming condition but with $d > n-k+1$

4. Propose a $(n,n-1,2)_q$–code that can detect transposition of any pair of symbols (when symbols exchange their places).

5. Suppose $C \subset V = \mathbb{F}_q^n$ be a linear code. By *dual code* denote $C^* = \mathrm{Ann}(C) \subset V^*$. So if C is $(n,k,d)_q$-code, then C^* is $(n,n-k,d')_q$-code for some d'.

a) Show that for natural dual bases we have generator and parity check matrices for C^* coincide with parity check and generator matrices for C respectively.

b) Show that group of repeating symbol corresponds to check sum of this group in the dual code.

c) Find a minimal distance for the code, dual to Hamming code.

d) Suppose that C is a cyclic polynomial $(n,k,d)_q$-code generated by a polynomial $g(x)$. Show that C^* is a cyclic code generated by a polynomial $x^k h(x^{-1})$, where $h(t) = (t^n - 1)/g(x)$.

e) Is dual to polynomial code always polynomial?

6. For a given graph with m vertices and n edges we associate $(n,n-m,d)_q$ code as follows. Bits are enumerated by edges, a vector is a code word iff sum of bits for all edges with a common vertex is zero.

a) Find a graph associated with repeating code.

b) Find minimal distance of the code associated with n-gone.

c) Find minimal distance of the code associated with a simplex on m vertices.

d) Find minimal distance of the code associated with *Petersen graph*