Вариант 9

Пусть

$$\omega = xy^3z\,dx + 5x^2y^2z\,dy + xy^2\,dz \quad \in \Omega^1(\mathbb{R}^3),$$

а

$$F : \mathbb{R}^2 \to \mathbb{R}^3,\ (u, v) \mapsto (u, v, u^2 + v^2)$$

– вложение плоскости в пространство в виде параболоида.

1) Вычислите значение ω на векторном поле $v = (6y + z)\frac{\partial}{\partial x} + (x + 3y)\frac{\partial}{\partial y}$.

2) Вычислите $d\omega$.

3) Вычислите $\omega \wedge d\omega$.

4) Вычислите $F^*\omega$.

5*) Напишите форму η, для которой $d\eta = \omega \wedge d\omega$.

Решения

Задача 1

$$dx(v) = dx((6y+z)\frac{d}{dx} + (x+3y)\frac{d}{dy}) = 6y+z$$
$$dy(v) = dy((6y+z)\frac{d}{dx} + (x+3y)\frac{d}{dy}) = x+3y$$
$$dz(v) = dz((6y+z)\frac{d}{dx} + (x+3y)\frac{d}{dy}) = 0$$

$$\omega(v) = xy^3z \cdot dx(v) + 5x^2y^2z \cdot dy(v) + xy^2 \cdot dz(v) =$$
$$xy^3z(6y+z) + 5x^2y^2 + z(x+3y) + xy^2 \cdot 0 = xy^2z(6y^2 + yz + 5x^2 + 15xy)$$

Задача 2

$$d\omega =$$
$$\frac{d}{dx}xy^3zdx \wedge dx + \frac{d}{dy}xy^3zdy \wedge dx + \frac{d}{dz}xy^3zdz \wedge dx + \frac{d}{dx}5x^2y^2zdx \wedge dy +$$
$$\frac{d}{dz}5x^2y^2zdz \wedge dy + \frac{d}{dx}xy^2dx \wedge dz + \frac{d}{dy}xy^2dy \wedge dz =$$

$$3xy^2zdy \wedge dx + xy^3dz \wedge dx + 10xy^2zdx \wedge dy + 5x^2y^2dz \wedge dy + y^2dx \wedge dz + 2xydy \wedge dz =$$
$$7xy^2zdx \wedge dy + (xy^3 - y^2)dz \wedge dx + (2xy - 5x^2y^2)dy \wedge dz$$

Задача 3

$$(xy^3zdx + 5x^2y^2zdy + xy^2dz) \wedge (7xy^2zdx \wedge dy + (xy^3 - y^2)dz \wedge dx + (2xy - 5x^2y^2)dy \wedge dz) =$$
$$xy^3z(2xy - 5x^2y^2)dx \wedge dy \wedge dz + 5x^2y^2z(xy^3 - y^2)dy \wedge dz \wedge dx + xy^27xy^2zdz \wedge dx \wedge dy =$$
$$(2x^2y^4z - 5x^3y^5z + 5x^3y^5z - 5x^2y^4z + 7x^2y^4z)dx \wedge dy \wedge dz =$$
$$4x^2y^4zdx \wedge dy \wedge dz$$

Задача 4

$$z = x^2 + y^2 \qquad dz = 2xdx + 2ydy$$
$$F^*\omega = F^*(xy^3zdx + 5x^2y^2zdy + xy^2dz) =$$
$$F^*(xy^3zdx) + F^*(5x^2y^2zdy) + F^*(xy^2dz) =$$
$$(xy^3z \circ F)(dx \circ F) + (5x^2y^2z \circ F)d(y \circ F) + (xy^2 \circ F)(dz \circ F) =$$
$$xy^3(x^2+y^2)dx + 5x^2y^2(x^2+y^2)dy + 2x^2y^2dx + 2xy^3dy =$$
$$(x^3y^3 + xy^5 + 2x^2y^2)dx + (5x^4y^2 + 5x^2y^4 + 2xy^3)dy$$

Задача 5

$$\nu = \frac{4}{3}x^3y^4z dy \wedge dz$$
$$d\nu = \frac{d}{dx}\frac{4}{3}x^3y^4z dx \wedge dy \wedge dz$$