

а) $H(q, p) = \frac{p^2}{2m} + \frac{m\omega^2 q^2}{2}$ $\{q_0, p_0\} = 1$

$$\begin{cases} \dot{p} = -\frac{\partial H}{\partial q} = -m\omega^2 q \\ \dot{q} = \frac{\partial H}{\partial p} = \frac{p}{m} \end{cases}$$

б) $a = a(0) \cdot e^{-i\omega t}$ где $a = \sqrt{\frac{m\omega}{2}} (q + \frac{ip}{m\omega})$

$$a = a(0) \cdot e^{-i\omega t} = a(0) \cdot (\cos(-\omega t) + i\sin(-\omega t)) = a(0) \cdot (\cos(\omega t) - i\sin(\omega t)) =$$

$$= \sqrt{\frac{m\omega}{2}} (q_0 + \frac{ip_0}{m\omega}) (\cos(\omega t) - i\sin(\omega t))$$

Преположим беря к мнимого части:

$$\sqrt{\frac{m\omega}{2}} q = \sqrt{\frac{m\omega}{2}} (q_0 \cos(\omega t) + \frac{p_0}{m\omega} \sin(\omega t))$$

$$\frac{p}{m\omega} = \frac{p_0}{m\omega} \cos(\omega t) - q_0 \sin(\omega t)$$

$$q = q_0 \cos(\omega t) + \frac{p_0}{m\omega} \sin(\omega t)$$

$$p = p_0 \cos(\omega t) - q_0 m\omega \sin(\omega t)$$

$$\{q, p\} = \{q_0 \cos(\omega t) + \frac{p_0}{m\omega} \sin(\omega t), p_0 \cos(\omega t) - q_0 m\omega \sin(\omega t)\} =$$

$$= \{q_0, p_0\} \cos^2(\omega t) - \{p_0, q_0\} \frac{\sin^2(\omega t)}{m\omega} \cdot m\omega = \cos^2(\omega t) + \sin^2(\omega t) = 1$$

в) $F_1(q_0, q, t)$

$$p_0 = \frac{\partial F_1}{\partial q_0} = \frac{(q - q_0 \cos(\omega t)) \cdot m\omega}{\sin(\omega t)}$$

$$F_1 = \frac{(q \cdot q_0 - \frac{q^2}{2} \cos(\omega t)) \cdot m\omega}{\sin(\omega t)} + f(q, t)$$

$$\beta = -\frac{\partial F_1}{\partial q} = -\frac{(q - q_0 \cos(\omega t)) \cdot m\omega}{\sin(\omega t)} \cdot \cos(\omega t) + q_0 m\omega \sin(\omega t)$$

$$F_1 = \frac{(\frac{q^2}{2} - q q_0 \cos(\omega t)) m\omega \cos(\omega t)}{\sin(\omega t)} + q_0 q m\omega \sin(\omega t) + f(q, t) =$$

$$= -\frac{q^2 m\omega \cos(\omega t)}{2 \sin(\omega t)} + \frac{m\omega q_0 q (\cos^2(\omega t) + \sin^2(\omega t))}{\sin(\omega t)} + f(q, t) =$$

$$= -\frac{q^2 m\omega \cos(\omega t)}{2 \sin(\omega t)} + \frac{m\omega q_0 q}{\sin(\omega t)} + f(q, t)$$

$\tilde{H} = H + \frac{\partial F_1}{\partial t}$ p_0, q_0 - константы $\Rightarrow H = 0 \Rightarrow \tilde{H} = \frac{\partial F_1}{\partial t}$

$$\frac{p^2}{2m} + \frac{m\omega^2 q^2}{2} = \tilde{H} = \frac{\partial F_1}{\partial t}$$

$$\frac{p^2 + m^2 \omega^2 q^2}{2m} = \frac{p_0^2 + m^2 \omega^2 q_0^2}{2m}$$

$$\tilde{H} = \frac{p_0^2}{2m} + \frac{m\omega^2 q_0^2}{2} = \frac{p_0^2}{2m} + \frac{m\omega^2 q_0^2 (1 - \cos^2(\omega t))}{2 \sin^2(\omega t)} =$$

$$= \frac{p_0^2 \sin^2(\omega t) + m \omega q_0^2 - m \omega q_0^2 \cos^2(\omega t)}{2 m \sin^2(\omega t)} = \frac{m \omega (m q_0^2 + \frac{2 p_0^2}{m \omega^2} \sin^2(\omega t) - p_0^2 \cos^2(\omega t))}{2 p_0^2 \sin^2(\omega t)} =$$

$$= \frac{m \omega (m q_0^2 - \frac{2 p_0^2}{m \omega^2} \sin(\omega t) \cos(\omega t) - 2 q_0^2 \cos^2(\omega t) + q_0^2)}{2 p_0^2 \sin^2(\omega t)} =$$

$$(q = q_0 \cos(\omega t) + \frac{p_0}{m \omega} \sin(\omega t))$$

$$q^2 = q_0^2 \cos^2(\omega t) + \frac{2 p_0 q_0}{m \omega} \sin(\omega t) \cos(\omega t) + \frac{p_0^2}{m^2 \omega^2} \sin^2(\omega t)$$

$$\equiv \frac{m \omega (q^2 - \frac{p_0^2}{m^2 \omega^2} \sin^2(\omega t) \cos^2(\omega t) - \frac{2 p_0 q_0 \sin(\omega t) \cos(\omega t)}{\sin^2(\omega t)} + q_0^2 \cos^2(\omega t))}{2 p_0^2 \sin^2(\omega t)} =$$

$$= \frac{\omega^2 [q^2 - q_0^2 \cos^2(\omega t) - \frac{2 p_0 q_0 \sin(\omega t) \cos(\omega t)}{2 \sin^2(\omega t)} + q_0^2 (1 - \cos^2(\omega t))]}{2 \sin^2(\omega t)} =$$

$$= \frac{\omega^2 [q^2 - q_0^2 \cos^2(\omega t) + q_0^2 \sin^2(\omega t) + q_0^2 - q_0^2 \cos^2(\omega t)]}{2 \sin^2(\omega t)}$$

$$\Downarrow$$

$$\equiv \frac{m \omega^2 [q^2 - 2 q_0 \cos(\omega t) \cdot q + q_0^2]}{2 \sin^2(\omega t)}$$

$$F_1 = - \frac{m \omega^2 q^2}{2} \frac{\cos(\omega t)}{\omega} + q_0 \cdot q \frac{\sin(\omega t)}{\sin(\omega t)} - \frac{m \omega^2 q_0^2}{2} \frac{\cos(\omega t)}{\omega} + f(q, q_0)$$

$$= \left[\frac{m \omega q^2 \cos(\omega t)}{2} + \frac{m \omega q_0 q}{\sin(\omega t)} - \frac{m \omega q_0^2 \cos(\omega t)}{2} + f(q, q_0) \right]$$

$$\Rightarrow F_1(q_0, q, t) = - \frac{q^2 \cdot m \omega \cdot \cos(\omega t)}{2} + \frac{m \omega q_0 q}{\sin(\omega t)} - \frac{q_0^2 \cdot m \omega \cdot \cos(\omega t)}{2} + \text{const}$$

$$b) F_2(q_0, p(t), t)$$

$$p_0 = \frac{\partial F_2}{\partial q_0} = \frac{p + q_0 m \omega \sin(\omega t)}{\cos(\omega t)}$$

$$F_2 = \frac{p q_0 + \frac{q_0^2}{2} m \omega \sin(\omega t)}{\cos(\omega t)} + f_2(p, t)$$

$$q = \frac{\partial F_2}{\partial p} = q_0 \cdot \cos(\omega t) + \frac{p_0}{m \omega} \cdot \sin(\omega t) =$$

$$= q_0 \cdot \cos(\omega t) + \frac{p + q_0 m \omega \sin(\omega t)}{m \omega \cdot \cos(\omega t)} \cdot \sin(\omega t)$$

$$F_2 = q_0 p \cos(\omega t) + \frac{\frac{p^2}{2} + q_0 p m \omega \sin(\omega t)}{m \omega \cos(\omega t)} \cdot \sin(\omega t) + f_2(q_0, t) =$$

$$= \frac{m \omega \cos^2(\omega t) q_0 p + \left(\frac{p^2}{2} \sin(\omega t) + q_0 p m \omega \sin(\omega t) \sin(\omega t) \right)}{m \omega \cos(\omega t)} + f_2(q_0, t) =$$

$$= \frac{m \omega q_0 p + \frac{p^2}{2} \sin(\omega t)}{m \omega \cos(\omega t)} + f_2(q_0, t) = \frac{p q_0 + \frac{p^2}{2 m \omega} \sin(\omega t)}{\cos(\omega t)} + f_2(q_0, t)$$

$$\tilde{H} = H + \frac{\partial F_2}{\partial t}$$

p_0, q_0 - nur zur Gl. nummer $\rightarrow H = 0 \Rightarrow \tilde{H} = \frac{\partial F_2}{\partial t}$, $\tilde{H}(p_0, q_0) = \frac{p_0^2}{2m} + \frac{m \omega^2 q_0^2}{2}$ —

$$F_2(q_0, p(t), t) = \frac{p q_0}{\cos(\omega t)} + \frac{\frac{q_0^2}{2} m \omega \sin(\omega t)}{\cos(\omega t)} + \frac{\frac{p^2}{2 m \omega} \sin(\omega t)}{\cos(\omega t)} + \text{const} =$$

$$= \frac{p q_0}{\cos(\omega t)} + \frac{q_0^2}{2} m \omega \cdot \tan(\omega t) + \frac{p^2}{2 m \omega} \cdot \tan(\omega t) + \text{const}$$

N2.

$$Q = -p, \quad P = q + Ap^2$$

$$a) \{Q, P\} = \{ -p, q + Ap^2 \} = -\{p, q\} - 2pA\{p, p\} = 1$$

\Rightarrow Преобразование каноническое

$$b) p = -Q = \frac{\partial F_1}{\partial q} \quad F_1 = -Q \cdot q + q_0$$

$$P = q + A Q^2 = -\frac{\partial F_1}{\partial Q} \quad F_1 = -q \cdot Q - \frac{A Q^3}{3} + Q \cdot q_0$$

$$\Rightarrow F_1(q, Q) = -Q \cdot q - \frac{A}{3} Q^3 + \text{const}$$

$$b) \quad p = \sqrt{\frac{P-q}{A}} = \frac{\partial F_2}{\partial q}$$

$$Q = -\sqrt{\frac{P-q}{A}} = + \frac{\partial F_2}{\partial P}$$

$$F_2 = -\frac{2A}{3} \left(\frac{P-q}{A} \right)^{3/2} + q_0$$

$$F_2 = -\frac{2A}{3} \left(\frac{P-q}{A} \right)^{3/2} + P \cdot q_0$$

$$\Rightarrow F_2(q, P) = -\frac{2A}{3} \left(\frac{P-q}{A} \right)^{3/2} + \text{const} = -\frac{2}{3} \cdot \frac{(P-q)^{3/2}}{A^{1/2}} + \text{const}$$

N3.

$$a) F_{\text{roer}} \quad L = T - U = \frac{m\dot{q}^2}{2} - qF + c$$

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Умножаем на F

$$H = p\dot{q} - L = p\dot{q} - \frac{m\dot{q}^2}{2} + qF - c$$

$$U = \int F q dq$$

$$p = \frac{\partial L}{\partial \dot{q}} = m\dot{q} \quad \dot{q} = \frac{p}{m}$$

$$H = \frac{p^2}{m} - \frac{p^2}{2m} + qF = \frac{p^2}{2m} + qF - c \quad \text{константа}$$

$$b) Q = -p \quad P = q + Ap^2$$

$$\tilde{p} = -Q \quad q = P - Ap^2 = P - AQ^2$$

$$\tilde{H}(Q, P) = \frac{Q^2}{2m} + (P - AQ^2) \cdot F$$

Можно переписать $\tilde{H}(P)$ в виде $A = \frac{1}{2mF} : \frac{Q^2}{2m} + PF - \frac{Q^2 F}{2mF} = PF$

$$b) \tilde{H}(P) = PF$$

$$\dot{Q} = \frac{\partial \tilde{H}}{\partial P} = F \quad Q = Ft + Q^{(0)} \leftarrow \text{const}$$

$$\dot{P} = -\frac{\partial \tilde{H}}{\partial Q} = 0 \quad P = P^{(0)} \leftarrow \text{const}$$

$$p = -Q = -Ft - Q^{(0)}$$

$$q = P - AQ^2 = P^{(0)} - \frac{1}{2mF} \cdot (Ft + Q^{(0)})^2 =$$

$$= P^{(0)} - \frac{1}{2mF} \cdot (F^2 t^2 + 2Ft \cdot Q^{(0)} + (Q^{(0)})^2) = P^{(0)} - \frac{Ft^2}{2m} - \frac{tQ^{(0)}}{m} - \frac{(Q^{(0)})^2}{2mF}$$

N4.

$$F_2(q, P) = q^2 \cdot e^P$$

$$a) \quad p = \frac{\partial F_2}{\partial q}$$

$$Q = \frac{\partial F_2}{\partial P}$$

$$p = 2q \cdot e^P$$

$$Q = q^2 \cdot e^P$$

$$e^P = \frac{p}{2q}$$

$$P = \ln p - \ln 2q$$

$$Q = q^2 \cdot \exp(\ln p - \ln 2q) = q^2 \cdot \frac{p}{2q} = \frac{pq}{2}$$

$$b) \quad p = \frac{2Q}{q} = \frac{\partial F_1}{\partial q}$$

$$\underline{F_1 = 2Q \cdot \ln q + q_0}$$

$$P = \ln(2Q) - \ln q - \ln 2q = \frac{\partial F_1}{\partial Q}$$

$$\frac{\partial F_1}{\partial Q} = (2 \ln q + q'_0(Q)) = \ln \frac{2q^2}{2Q} = \ln \frac{q^2}{Q}$$

$$\underline{F_1 = Q \cdot \ln q^2 - Q \cdot \ln Q + Q + \text{const}}$$

$$\begin{aligned} Q &= e^y \\ \ln Q &= y \\ \int \ln Q dQ &= \int y d e^y = y e^y - e^y \\ &= \ln Q \cdot Q - Q \end{aligned}$$

$$\Rightarrow F_1(q, Q) = 2Q \cdot \ln q - Q \cdot \ln Q + Q + \text{const}$$