Task
Prove the equality:

$$\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}$$

Solution
We know that

$$\binom{n}{k} = \binom{n}{n-k}$$

Then

$$\sum_{k=0}^{n} \binom{n}{k}^2 = \sum_{k=0}^{n} \binom{n}{k}\binom{n}{n-k}$$

Then we can see that this is the same as choosing $n-k$ objectsfrom the set of power n and k from the other set with the same size. Then, considering the sum of $\binom{n}{k}\binom{n}{n-k}$ for all possible k, we get the number of ways to select n objects from a set of size $2n$.