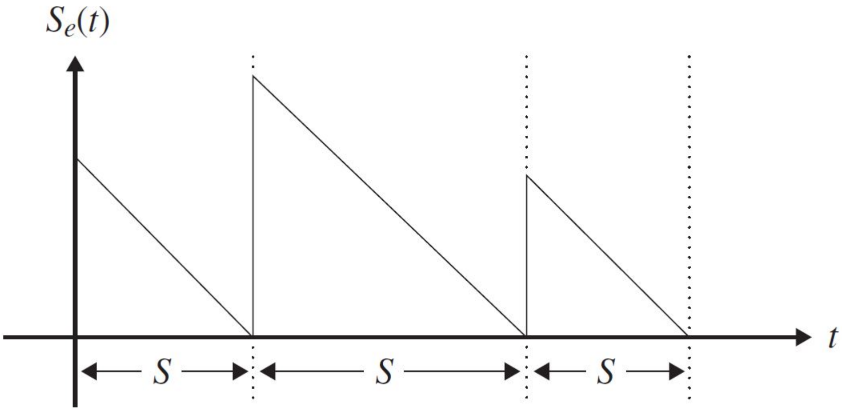
Scheduling: Packet Excess

Packet Excess

Definition of random variable S_e based on $S_e(t)$, where t is total time in busy state.



Concept of packet price R, may depend on S. E.g. $R = 1, S$. $R(t)$ as a total price (for unfinished the price could be any between 0 and its total price).

Theorem (Renewal-Reward) (wo/proof). Let $0 \leqslant ER < \infty$, $0 < ES < \infty$. Then with probability 1 we have $\frac{R(t)}{t} \to \frac{ER}{ES}$ for $t \to \infty$.

Using Renewal-Reward theorem to estimate ES_e in terms of ES.

1. Note that $ES_e = \{Time - averageExcess\} = \lim\limits_{s\to\infty} \frac{\int_0^s S_e(t)dt}{s}$.
2. $R(s) := \int\limits_0^s S_e(t)dt$
3. Finally $ES_e = \lim\limits_{s\to\infty} \frac{\int_0^s S_e(t)dt}{s} = \lim\limits_{s\to\infty} \frac{R(s)}{s} = \{Renewal - Reward\} = \frac{ER}{ES} = \frac{E\int_0^S (S-t)dt}{ES} = \frac{ES^2}{2ES}$.

Using Renewal-Reward theorem to estimate distribution function $F_e(k) = Pr\{S_e < k\}$ and corresponding probability density function $f_e(k)$.

Result: $f_e(k) = \frac{\overline{F_S(k)}}{ES}$.

Theorem 6 (wo/proof). $\int\limits_0^\infty e^{-sx} \int\limits_0^x b(t)dtdx = \frac{\widetilde{b}(s)}{s}$.

Laplace transform of Excess

Remind, that $\widetilde{B}(s) = \widetilde{S}(s + \lambda - \lambda\widetilde{B}(s))$.

Definition of B_W, calculus of $\widetilde{B_W}(s) = \widetilde{W}(s + \lambda - \lambda\widetilde{B}(s))$.

Calculus of $\widetilde{S_e}(s) = \frac{1-\widetilde{S_e}(s)}{sES}$.

Lemma. $T_{Q|busy}^{LCFS} = B_{S_e}$.

Calculus of $\widetilde{T_Q^{LCFS}}(s) = (1 - \rho) + \frac{\lambda(1-\widetilde{B}(s))}{s+\lambda-\lambda\widetilde{B}(s)}$.

Problems

1. Let packet length be distributed by the rule:

$$S = \begin{cases} \frac{1}{q} & \text{with probability } q \\ \frac{1}{1-q} & \text{with probability } 1-q \end{cases}$$

   Find ES_e.

2. (06/02:3) Express $EB = -\widetilde{B}'(0)$ in terms of S, ρ.

3. Express $EB^2 = \widetilde{B}''(0)$ in terms of S, ρ.

4. Express $-EB^3 = \widetilde{B}'''(0)$ in terms of S, ρ.