Мельникова Дана

№1.

а) $H(q,p) = \frac{p^2}{2m} + \frac{m\omega^2 q^2}{2}$ $\qquad \{q_0, p_0\} = 1$

$$\begin{cases} \dot{p} = -\frac{\partial H}{\partial q} = -m\omega^2 q \\ \dot{q} = \frac{\partial H}{\partial p} = \frac{p}{m} \end{cases}$$

В задаче №2 из ДЗ 6 получили, что $a = a(0)\cdot e^{-i\omega t}$, где $a = \sqrt{\frac{m\omega}{2}}\left(q + \frac{ip}{m\omega}\right)$

$\sqrt{\frac{m\omega}{2}}\left(q + \frac{ip}{m\omega}\right) = a = a(0)\cdot e^{-i\omega t} = a(0)\cdot(\cos(-\omega t) + i\sin(-\omega t)) = a(0)\cdot(\cos(\omega t) - i\sin(\omega t)) =$

$= \sqrt{\frac{m\omega}{2}}\left(q_0 + \frac{ip_0}{m\omega}\right)(\cos(\omega t) - i\sin(\omega t))$

Приравниваем вещ. и мнимую части: $\sqrt{\frac{m\omega}{2}}\, q = \sqrt{\frac{m\omega}{2}}\left(q_0\cdot\cos(\omega t) + \frac{p_0}{m\omega}\cdot\sin(\omega t)\right)$

$\frac{p}{m\omega} = \frac{p_0}{m\omega}\cdot\cos(\omega t) - q_0\cdot\sin(\omega t)$

$q = q_0\cdot\cos(\omega t) + \frac{p_0}{m\omega}\sin(\omega t)$ $\qquad p_0 = \frac{(q - q_0\cdot\cos(\omega t))}{\sin(\omega t)}\cdot m\omega$

$p = p_0\cdot\cos(\omega t) - q_0 m\omega\sin(\omega t)$

$\{q,p\} = \{q_0\cdot\cos(\omega t) + \frac{p_0}{m\omega}\sin(\omega t),\ p_0\cdot\cos(\omega t) - q_0\cdot m\omega\sin(\omega t)\} =$

$= \{q_0, p_0\}\cdot\cos^2(\omega t) - \{p_0, q_0\}\cdot\frac{\sin^2(\omega t)}{m\omega}\cdot m\omega = \cos^2(\omega t) + \sin^2(\omega t) = 1$

$\{p,p\} = \{p_0\cos(\omega t) - q_0 m\omega\sin(\omega t),\ p_0\cos(\omega t) - q_0 m\omega\sin(\omega t)\} = -\{p_0, q_0\}\cos(\omega t)\cdot m\omega\cdot\sin(\omega t) - \{q_0, p_0\} m\omega\sin(\omega t)\cos(\omega t) = -(-1) - 1 = 0$; $\{q,q\} = 0$

б) $F_1(q_0, q, t)$

$p_0 = \frac{\partial F_1}{\partial q_0} = \frac{(q - q_0\cdot\cos(\omega t))}{\sin(\omega t)}\cdot m\omega$

$$\boxed{F_1 = \frac{\left(q\cdot q_0 - \frac{q_0^2}{2}\cos(\omega t)\right)\cdot m\omega}{\sin(\omega t)} + \tilde{f}(q,t)}$$

$p = -\frac{\partial F_1}{\partial q} = -\frac{(q - q_0\cdot\cos(\omega t))\cdot m\omega}{\sin(\omega t)}\cdot\cos(\omega t) + q_0 m\omega\sin(\omega t)$

$F_1 = -\frac{\left(\frac{q^2}{2} - q q_0\cdot\cos(\omega t)\right) m\omega\cdot\cos(\omega t)}{\sin(\omega t)} + q_0 q m\omega\sin(\omega t) + f(q_0, t) =$

$= -\frac{q^2 m\omega\cdot\cos(\omega t)}{2\sin(\omega t)} + \frac{m\omega q_0 q(\cos^2(\omega t) + \sin^2(\omega t))}{\sin(\omega t)} + f(q_0, t) =$

$$= \boxed{-\frac{q^2 m\omega\cdot\cos(\omega t)}{2\sin(\omega t)} + \frac{m\omega q_0 q}{\sin(\omega t)} + f(q_0, t)}$$

$\tilde{H} = H + \frac{\partial F_1}{\partial t}$ $\qquad p_0, q_0$ – интегралы движения $\Rightarrow H = 0 \Rightarrow \tilde{H} = \frac{\partial F_1}{\partial t}$

$\frac{p^2}{2m} + \frac{m\omega^2 q^2}{2} = \tilde{H} = \frac{\partial F_1}{\partial t}$ $\qquad \tilde{H} = \frac{p_0^2}{2m} + \frac{m\omega^2 q_0^2}{2} = \frac{p_0^2}{2m} + \frac{m\omega^2 q_0^2(1 - \cos^2(\omega t))}{2\sin^2(\omega t)} =$

$\frac{p^2 + m^2\omega^2 q^2}{2m} = \frac{p_0^2 + m^2\omega^2 q_0^2}{2m}$

$$= \frac{p_0^2 \cdot \sin^2(\omega t) + m^2\omega^2 q_0^2 - m^2\omega^2 q_0^2 \cdot \cos^2(\omega t)}{2m\sin^2(\omega t)} = \frac{m\omega^2 \cancel{m} q_0^2 + \frac{p_0^2}{m\omega^2}\cdot \sin^2(\omega t) - \cancel{m} q_0^2 \cdot \cos^2(\omega t))}{2\cancel{m}\sin^2(\omega t)} =$$

$$= \frac{m\omega^2 (q^2 - \frac{2q_0 p_0}{m\omega}\cdot \sin(\omega t)\cdot\cos(\omega t) - 2q_0^2 \cos^2(\omega t) + q_0^2)}{2\sin^2(\omega t)} =$$

$$m\omega^2 (q_0^2 + \frac{p_0^2}{m^2\omega^2}\cdot\sin^2\omega t - q_0^2 \cos^2\omega t$$

$$2q_0\cos\omega t(q_0\cos\omega t + \frac{p_0}{m\omega}\sin\omega t)$$

$$(q = q_0 \cdot \cos(\omega t) + \frac{p_0}{m\omega}\sin(\omega t))$$

$$q^2 = q_0^2 \cdot \cos^2(\omega t) + \frac{2q_0 p_0}{m\omega}\sin(\omega t)\cdot\cos(\omega t) + \frac{p_0^2}{m^2\omega^2}\sin^2(\omega t)$$

$$\cancel{= \frac{m\omega^2 [q^2 - \frac{q_0}{m\omega}\sin(\omega t)\cdot\cos(\omega t)\frac{(q - q_0\cdot\cos(\omega t))}{\sin\omega t}\cdot\omega m + q_0^2 (1 - 2\cos^2(\omega t))]}{2\sin^2(\omega t)}} =$$

$$\cancel{= \frac{\omega^2 [q^2 - q_0\cdot\cos(\omega t)\cdot(q - q_0\cdot\cos(\omega t)) + q_0^2(1 - 2\cos^2(\omega t))]}{2\sin^2(\omega t)}} =$$

$$\cancel{= \frac{\omega^2 [q^2 - q\cdot q_0\cdot\cos(\omega t) + q_0^2\cos^2(\omega t) + q_0^2 - 2q_0^2\cdot\cos^2(\omega t)]}{2\sin^2(\omega t)}}$$

$$= \frac{m\omega^2 [q^2 - 2q_0\cos\omega t\cdot q + q_0^2]}{2\sin^2(\omega t)}$$

$$F_1 = -\frac{m\omega^2 q^2}{2}\cdot\frac{\operatorname{ctg}(\omega t)}{\omega} + q_0 q \frac{\omega^2 m}{\omega \sin\omega t} - \frac{m\omega^2 q_0^2}{2}\cdot\frac{\operatorname{ctg}(\omega t)}{\omega} + f(q, q_0)$$

$$\boxed{= -\frac{m\omega q^2 \operatorname{ctg}(\omega t)}{2} + \frac{m\omega q_0 q}{\sin(\omega t)} - \frac{m\omega q_0^2 \cdot \operatorname{ctg}(\omega t)}{2} + f(q, q_0)}$$

$$\Rightarrow F_1(q_0, q, t) = -\frac{q^2\cdot m\omega\cdot\operatorname{ctg}(\omega t)}{2} + \frac{m\omega q_0 q}{\sin(\omega t)} - \frac{q_0^2\cdot m\omega\cdot\operatorname{ctg}(\omega t)}{2} + \text{const}$$

б) $F_2(q_0, p(t), t)$

$$p_0 = \frac{\partial F_2}{\partial q_0} = \frac{p + q_0 m\omega \sin(\omega t)}{\cos(\omega t)}$$

$$F_2 = \frac{p q_0 + \frac{q_0^2}{2} m\omega \sin(\omega t)}{\cos(\omega t)} + f_2(p, t)$$

$$q = \frac{\partial F_2}{\partial p} = q_0 \cdot \cos(\omega t) + \frac{p_0}{m\omega} \sin(\omega t) =$$

$$= q_0 \cdot \cos(\omega t) + \frac{p + q_0 m\omega \sin(\omega t)}{m\omega \cdot \cos(\omega t)} \cdot \sin(\omega t)$$

$$F_2 = q_0 p \cos(\omega t) + \frac{\frac{p^2}{2} + q_0 p m\omega \sin(\omega t)}{m\omega \cos(\omega t)} \cdot \sin(\omega t) + f_2(q_0, t) =$$

$$= \frac{m\omega \cos^2(\omega t) q_0 p + \left(\frac{p^2}{2} \sin(\omega t) + q_0 p m\omega \sin(\omega t) \cdot \sin(\omega t)\right)}{m\omega \cos(\omega t)} + f_2(q_0, t) =$$

$$= \frac{m\omega q_0 p + \frac{p^2}{2} \sin(\omega t)}{m\omega \cos(\omega t)} + f_2(q_0, t) = \frac{p q_0 + \frac{p^2}{2m\omega} \sin(\omega t)}{\cos(\omega t)} + f_2(q_0, t)$$

$$\tilde{H} = H + \frac{\partial F_2}{\partial t}$$

p_0, q_0 - инт-лы движения $\Rightarrow$ $H = 0 \Leftrightarrow \tilde{H} = \frac{\partial F_2}{\partial t}$, $\tilde{H}(p_0, q_0) = \frac{p_0^2}{2m} + \frac{m\omega^2 q_0^2}{2}$ —"—

$$F_2(q_0, p(t), t) = \frac{p q_0}{\cos(\omega t)} + \frac{\frac{q_0^2}{2} m\omega \sin(\omega t)}{\cos(\omega t)} + \frac{\frac{p^2}{2m\omega} \sin(\omega t)}{\cos(\omega t)} + \text{const} =$$

$$= \frac{p q_0}{\cos(\omega t)} + \frac{q_0^2}{2} m\omega \cdot \operatorname{tg}(\omega t) + \frac{p^2}{2m\omega} \cdot \operatorname{tg}(\omega t) + \text{const}$$

N2.

$$Q = -p, \quad P = q + Ap^2$$

а) $\{Q, P\} = \{-p, q + Ap^2\} = -\{p, q\} - 2pA\cancel{\{p, p\}} = 1$

$\Rightarrow$ Преобразование каноническое

б) $p = -Q = \dfrac{\partial F_1}{\partial q}$ $\qquad F_1 = -Q \cdot q + q_0$

$P = q + AQ^2 = -\dfrac{\partial F_1}{\partial Q}$ $\qquad F_1 = -q \cdot Q - \dfrac{AQ^3}{3} + Q_0$

$\Rightarrow F_1(q, Q) = -Q \cdot q - \dfrac{A}{3} \cdot Q^3 + \text{const}$

в) $p = \sqrt{\dfrac{P-q}{A}} = \dfrac{\partial F_2}{\partial q}$

$Q = -\sqrt{\dfrac{P-q}{A}} = +\dfrac{\partial F_2}{\partial P}$

$F_2 = -\dfrac{2A}{3}\left(\dfrac{P-q}{A}\right)^{3/2} + q_0$

$F_2 = -\dfrac{2A}{3}\left(\dfrac{P-q}{A}\right)^{3/2} + P_0$

$\Rightarrow F_2(q, P) = -\dfrac{2A}{3}\left(\dfrac{P-q}{A}\right)^{3/2} + \text{const} = -\dfrac{2}{3} \cdot \dfrac{(P-q)^{3/2}}{A^{1/2}} + \text{const}$

N3.

а) $F_{пост}$ $\Downarrow$ U лин. зависит от F

$U = \int F(q)dq$

$$L = T - U = \frac{m\dot{q}^2}{2} - qF + c$$

$$H = p\cdot\dot{q} - L = p\dot{q} - \frac{m\dot{q}^2}{2} + qF - c$$

$$p = \frac{\partial L}{\partial \dot{q}} = m\dot{q} \qquad \dot{q} = \frac{p}{m}$$

$$H = \frac{p^2}{m} - \frac{p^2}{2m} + qF = \frac{p^2}{2m} + qF - c$$

можем опустить

б) $Q = -p \qquad P = q + Ap^2$

$$p = -Q \qquad q = P - Ap^2 = P - AQ^2$$

$$\tilde{H}(Q,P) = \frac{Q^2}{2m} + (P - AQ^2)\cdot F$$

Можно привести к $\tilde{H}(P)$ выбором $A = \frac{1}{2mF}$: $\frac{Q^2}{2m} + PF - \frac{Q^2 F}{2mF} = PF$

в) $\tilde{H}(P) = PF$

$$\dot{Q} = \frac{\partial \tilde{H}}{\partial P} = F \qquad Q = Ft + Q^{(0)} \leftarrow const$$

const

$$\dot{P} = -\frac{\partial \tilde{H}}{\partial Q} = 0 \qquad P = P^{(0)} \leftarrow const$$

$$p = -Q = -Ft - Q^{(0)}$$

$$q = P - AQ^2 = P^{(0)} - \frac{1}{2mF}\cdot(Ft + Q^{(0)})^2 =$$

$$= P^{(0)} - \frac{1}{2mF}\left(F^2t^2 + 2Ft\cdot Q^{(0)} + (Q^{(0)})^2\right) = P^{(0)} - \frac{Ft^2}{2m} - \frac{tQ^{(0)}}{m} - \frac{(Q^{(0)})^2}{2mF}$$

N4.

$$F_2(q, P) = q^2 \cdot e^P$$

а) $p = \frac{\partial F_2}{\partial q}$ $\quad Q = \frac{\partial F_2}{\partial P}$

$p = 2q \cdot e^P$ $\quad Q = q^2 \cdot e^P$

$e^P = \frac{p}{2q}$

$P = \ln p - \ln 2q$ $\quad Q = q^2 \cdot \exp(\ln p - \ln 2q) = q^2 \cdot \frac{p}{2q} = \frac{pq}{2}$

б) $p = \frac{2Q}{q} = \frac{\partial F_1}{\partial q}$ $\quad \underline{F_1 = 2Q \cdot \ln q + q_0}$

$P = \ln(2Q) - \ln q - \ln 2q = \frac{\partial F_1}{\partial Q}$

$\frac{\partial F_1}{\partial Q} = \cancel{\left(2\ln q + q_0'(Q)\right)} = \ln \frac{2q^2}{2Q} = \ln \frac{q^2}{Q}$

$Q = e^y$

$\ln Q = y$

$\int \ln Q \, dQ = \int y \, de^y = y \cdot e^y - e^y = \ln Q \cdot Q - Q$

$\underline{F_1 = Q \cdot \ln q^2 - Q \cdot \ln Q + Q + const}$

$\Rightarrow F_1(q, Q) = 2Q \cdot \ln q - Q \cdot \ln Q + Q + const$