$$t^{(m)} = f(x^{(m)}) \cong y = Xw + b$$

$$x^{(m)} \longrightarrow \phi(x^{(m)}) \Rightarrow y = \phi(X)w + b$$

$$\Rightarrow \mathcal{L}(y^{(m)}, t^{(m)}) = (y^{(m)} - t^{(m)})^2$$

$$\mathcal{E}(w, b) = \frac{1}{N}\sum_{i=1}^{N}(y^{(i)} - t^{(i)})^2 = \frac{1}{N}\sum_{i=1}^{N}(w \cdot \phi(x^{(i)}) + b - t^{(i)})^2$$

$$\boxed{\phi(x^{(i)}) \overset{not}{=} z^{(i)}}$$

$$\Rightarrow \mathcal{E}(w, b) = \frac{1}{N}\sum_{i=1}^{N}(w \cdot z^{(i)} + b - t^{(i)})^2 = \frac{1}{N}(w^T z + b - t)^T \cdot (w^T z + b - t)$$

$$w = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_D \end{pmatrix}, \quad b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_D \end{pmatrix}, \quad z = \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_D \end{pmatrix},$$

$\Rightarrow$ in scriere vectorială / matriceală $\Rightarrow$ $wz \Rightarrow w^T z$ sau $w \cdot z^T$.

$$\Rightarrow \mathcal{E}(w, b) = \frac{1}{N}(w^T z + b - t)^T (w^T z + b - t)$$

$$= \frac{1}{N}\left[((wz)^T + b^T - t^T)(wz + b - t)\right]$$

$$= \frac{1}{N}\left[(wz)^T wz + (wz)^T b - (wz)^T t + wz\, b^T + b^T b - b^T t - t^T wz - t^T b + t^T t\right]$$

$$= \frac{1}{N}\left[w^T z^T z w - 2 w^T z^T (b - t) + \underbrace{(b-t)^T(b-t)}_{a}\right]$$

$$\Rightarrow \boxed{\mathcal{E}(w) = \frac{1}{N}\left(w^T z^T z w - 2 w^T z^T a + a^T a\right)}$$

$\boxed{!}$ Se dorește minimizarea funcției de cost. $\Rightarrow$ derivata.

$\boxed{1}$ $P(w) = 2 w^T z^T a = 2 z^T w^T a$

$$P(w) = 2\left[\begin{bmatrix} z_{11} & z_{12} & \dots & z_{1m} \\ z_{21} & z_{22} & \dots & z_{2m} \\ \vdots & \vdots & & \vdots \\ z_{m1} & z_{m2} & & z_{mm} \end{bmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_m \end{pmatrix}\right]^T \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{pmatrix}$$

$$P(w) = 2\begin{bmatrix} z_{11} w_1 + \dots + z_{1m} w_m \\ z_{21} w_1 + \dots + z_{2m} w_m \\ \vdots \\ z_{m1} w_1 + \dots + z_{mm} w_m \end{bmatrix}^T \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{pmatrix}$$

$$P(w) = 2(z_{11}w_1 + \dots + z_{1m}w_m)a_1 + \dots + 2(z_{m1}w_1 + \dots + z_{mm}w_m)a_m$$

$$\Rightarrow \cancel{P(w) = 2\Sigma} \quad \frac{\partial P}{\partial w_1} = 2(z_{11}a_1 + \dots + z_{m1}a_m)$$

$$\frac{\partial P}{\partial w_2} = 2(z_{12}a_1 + \dots + z_{m2}a_m)$$

$$\frac{\partial P}{\partial w_m} = 2(z_{1m}a_1 + \dots + z_{mm}a_m)$$

$$(+)$$

$$\Rightarrow \boxed{\frac{\partial P}{\partial w} = 2z^T a}$$

$\boxed{2}$ $$Q(w) = w^T z^T z w = (w_1\ w_2 \dots w_m)\begin{pmatrix} z_{11} & z_{21} & \dots & z_{m1} \\ z_{12} & z_{22} & \dots & z_{m2} \\ \vdots & & & \\ z_{1m} & z_{2m} & \dots & z_{mm}\end{pmatrix}\begin{pmatrix} z_{11} & z_{12} & \dots & z_{1m} \\ z_{21} & z_{22} & \dots & z_{2m} \\ \vdots & & & \\ z_{m1} & z_{m2} & \dots & z_{mm}\end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_m \end{pmatrix}$$

$$Q(w) = (w_1 \dots w_m)\begin{pmatrix} z_{11}^2 w_1 + \dots + z_{1m}^2 w_m \\ z_{21}^2 w_1 + \dots + z_{2m}^2 w_m \\ \vdots \\ z_{m1}^2 w_1 + \dots + z_{mm}^2 w_m \end{pmatrix}$$

$$\Rightarrow Q(w) = w_1(z_{11}^2 w_1 + \dots + z_{1m}^2 w_m) + \dots + w_m(z_{m1}^2 w_1 + \dots + z_{mm}^2 w_m)$$

$$\Rightarrow \frac{\partial Q}{\partial w_1} = 2z_{11}^2 w_1 + 2z_{21}^2 w_1 + \dots + 2w_1 z_{m1}^2$$

$$\frac{\partial Q}{\partial w_2} = 2z_{12}^2 w_2 + 2w_2 z_{22}^2 + \dots + 2w_2 z_{m2}^2$$

$\boxed{3}$ $$\frac{\partial}{\partial w} a^T a = 0$$

$$(+)$$

$$\boxed{\frac{\partial Q}{\partial w} = 2z^T z w} \Rightarrow \boxed{\frac{\partial E}{\partial w} = \frac{\partial Q}{\partial w} - \frac{\partial P}{\partial w} = 2z^T z w - 2z^T a = 0}$$

$$\Rightarrow 2z^T z w - 2z^T a = 0$$

$$\Rightarrow 2z^T z w = 2z^T a \Rightarrow z^T z w = z^T a$$

$$\Rightarrow \boxed{w = (z^T z)^{-1} z^T \cdot a} = \boxed{(z^T z)^{-1} z^T (b - t) = w}$$

$$\Rightarrow \boxed{w = (\phi(x)^T \phi(x))^{-1} \cdot \phi(x)^T \cdot (b - t)}$$